

**MEMS & Microsystems**  
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**Lecture No. #23**  
**Temperature Drift and Damping Analysis**

So this lecture is an extension of my earlier lecture where I was discussing on the piezoresistive accelerometer design and a case study and that is the piezoresistive accelerometer for avionics application. Two things are left and that is temperature drift and damping. So those two parameters are also important when you go for a piezoresistive accelerometer. Most important point is the temperature drift because the sensing element is made based on piezoresistive sensing and the piezoresistance is a temperature sensitive element that many times in many lectures I have emphasized that part.

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And now this lecture we will discuss little bit in detail with temperature what parameter is going to change and because of that your output is drifting from the desired value. So that is the major topic of discussion in this lecture. And the second aspect is the damping analyzation. How do you design the accelerometer particularly the upper glass upper cover plate and bottom cover plate as well as the gap between the cover plate. So that the proper damping is given to the vibrating structure vibrating sensing element which is the middle layer.

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**Temperature Drift in Piezoresistive Accelerometer**

- In response to a differential change of acceleration ( $\Delta g$ ), the differential output voltage ( $\Delta V$ ) of an ideally balanced bridge with assumed identical resistance change ( $\Delta R$ ) is given by

$$\Delta V = \frac{\Delta R}{R} V_s$$

R is zero stress resistance,  $V_s$  is the bridge supply voltage.

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So there some of the definition, first let us define. One is the sensitivity is one important aspect so that we will define. Before that the output voltage how much we get when the bridge is unbalanced so that I mention will be delta R by R into  $V_s$  where delta R is the change of resistance and R is the no stress, zero stress resistance and  $V_s$  is the supply voltage so according we will get the delta V.

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**Temperature Drift in Piezoresistive Accelerometer**

- The sensitivity (S) of the bridge is defined as relative change of output voltage per unit applied differential acceleration

$$S = \frac{\Delta V}{\Delta g} \cdot \frac{1}{V_s} = \frac{\Delta R}{R} \cdot \frac{1}{\Delta g}$$

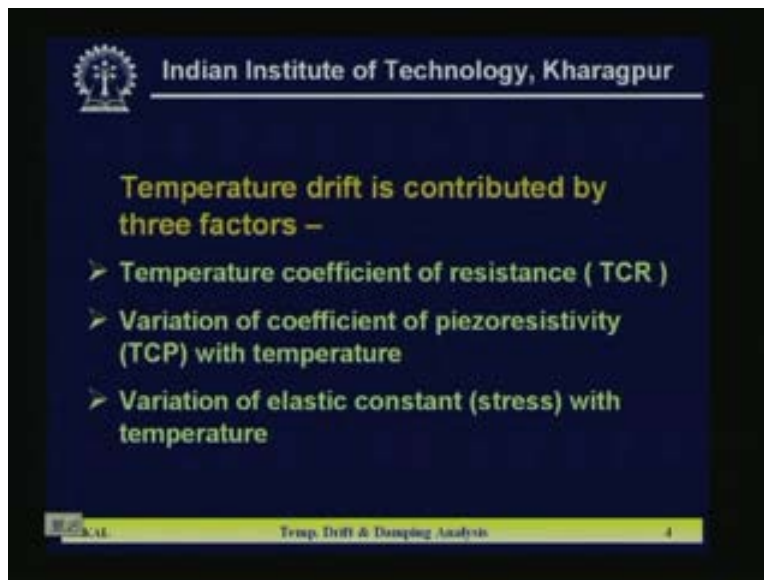
For a balanced current bridge, temp. variation changes the resistances of all piezoresistors equally so that output of the bridge remains zero

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Now the next point is sensitivity. Sensitivity of the bridge is defined as a relative change of the output voltage per unit applied differential acceleration. So that is the definition of the sensitivity and it is expressed mathematically by delta V by delta g dot one by  $I_b$  is

nothing but  $\Delta R / R \cdot \Delta T$ . For a balanced current bridge temperature variation changes the resistances of all piezoresistors equally, so that output of the bridge remains 0. So that means here with temperature the bridge resistance is going to change. But if all the resistances change equally, equally means either positive or negative direction whatever it is equally all positive direction then output voltage of the bridge should not change. Because it is basically the ratio of the resistances  $R_1 / R_2 = R_3 / R_4$ . So all the resistances are change equally so ratio remains same, in that case you may not get the change of the output. But the real life thing is not so. All the resistances will not change equally. So because of that reason you will get the output voltage which is which is temperature dependent; output voltage of the bridge which will be temperature dependent.

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Now we will see why it is not so. So before that we have to identify what are the parameters, what are the things, who can change the resistances values and the 3 factors which change the piezoresistances are: one is temperature coefficient of resistance TCR, that is because of the change of the mobility of the carrier by with temperature and with doping concentration. Because you know  $\mu$  mobility of the carrier is a function of doping concentration and temperature. Now doping concentration if we use a fixed value of doping concentration then all the resistance will have the same doping concentration then no change there. But temperature again with temperature mobility will change. Now there although the individual resistance will change because of the mobility change, but as a bridge concern all the bridges all the resistance will change equally.

So in that respect so first point temperature coefficient of resistance is not going to change of voltage at that much. So, now coming to the second point. Variation of coefficient of piezoresistivity with temperature which is TCP temperature coefficient of piezoresistivity, temperature coefficient of piezoresistor with temperature. Now here temperature coefficients of piezoresistances mean the resistances change because of the

stress. You are applying pressure or even if you apply acceleration, then stress is developed and because of that the resistance is going to change and who is the parameter, the piezoresistive coefficient which is pie11, pie44, and pie12 like that. So those pie values will change with temperature and that change is not same for all components with temperature.

If the pie11, pie44, pie14 change is same with temperature. Then it is not again going to create any problem either stress is compressive or stress is tensile; the temperature variation will be may be it is one negative, one positive; but with temperature the total change is same. So as a result the bridge output may not change, but it is not fact. So but real thing is that this change for difference stress component in different in something will increase and something will reduce in somewhere and this increase and reduction of those values will not in the same manner in the different temperature range. So that is the reason where the second point, second parameter, second factor is an important consideration when you go for temperature drift analysis of any of the piezoresistance.

There is a third point, third parameter is a variation of elastic constant with temperature elastic constants means the CR modulus or Young's modulus those values those constant. There are some mechanical, those are the mechanical the definite mechanical parameters and whether those elastic constant. That is basically coming from stress and strain AC of stress by strain is a modulus; young's modulus and CR modulus is there. We define all those parameter they are also temperature dependent. So their temperature dependence is how much we will see and because of that also the temperature, the resistance may change and which may change the output voltage. So these are the three parameters and one by one we will discuss who is going to change the resistance value how much.

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**Temperature Drift in Piezoresistive Accelerometer**

Electron and hole mobility in silicon:  
Empirical relations for the temperature as well as doping dependence of the carrier mobility in silicon are available and shown below

$$\mu_p(N, T) = 543 \left(\frac{T}{300}\right)^{0.57} + \frac{1.35 \times 10^8 T^{-2.25}}{1 + \frac{N}{2.35 \times 10^{17} \left(\frac{T}{300}\right)^{2.4}} 0.88 \left(\frac{T}{300}\right)^{0.146}} \text{ cm}^2/\text{V} \cdot \text{s}$$

$$\sigma = \frac{\Delta J}{\mathcal{E}} = q(n\mu_n + p\mu_p)$$

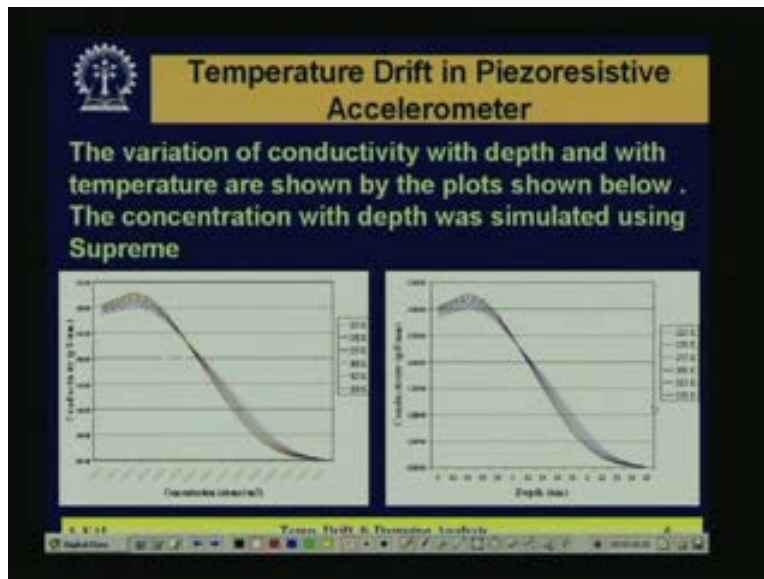
Temp. Drift & Damping Analysis

Now first parameter is the temperature TCR temperature coefficient of resistance. That is due to the electron and whole mobility in silicon. Then we know the empirical relations

for the temperature as well as doping dependence of the carrier mobility in silicon is available in the literature, the empirical relation which is noted here  $\mu_p$  which is a function of  $N$  and  $T$ .  $N$  is a doping concentration and what is  $T$ ?  $T$  is the temperature this temperature  $T$  is absolute scale. So it is given by this empirical relation, so that why since it is empirical relation you will see the values is not a close from value which is 2.35 here 300, 0.88 here or 135. So those parameters you may ask how you got it. That is any of the empirical relation it not follow a regular pattern regular method by which some close form relation you will get mathematically.

Some of the parameters numerical values you have to adjust to get the agreement with the experimental results. So that is why those are known as empirical relations. So for  $\mu$  whole mobility or the electro mobility similar you can get it. The mobility with temperature and doping concentration is this expression where this  $N$  is coming here. For a fixed value of  $N$  then you can see for temperature change the  $\mu$  will change and if  $\mu$  changes the conductivity which is  $\Delta J$  by  $\epsilonpsilon$ .  $\epsilonpsilon$  is a field the  $q n \mu_n$  plus  $p \mu_p$  where the  $q$  is an electronic charge  $n$  and  $p$  are the doping concentration and  $\mu_n$  and  $\mu_p$  are electron whole mobility. So this  $\mu_n$  and  $\mu_p$  will change, accordingly  $\sigma$  will change and  $\sigma$  change means resistance will change.

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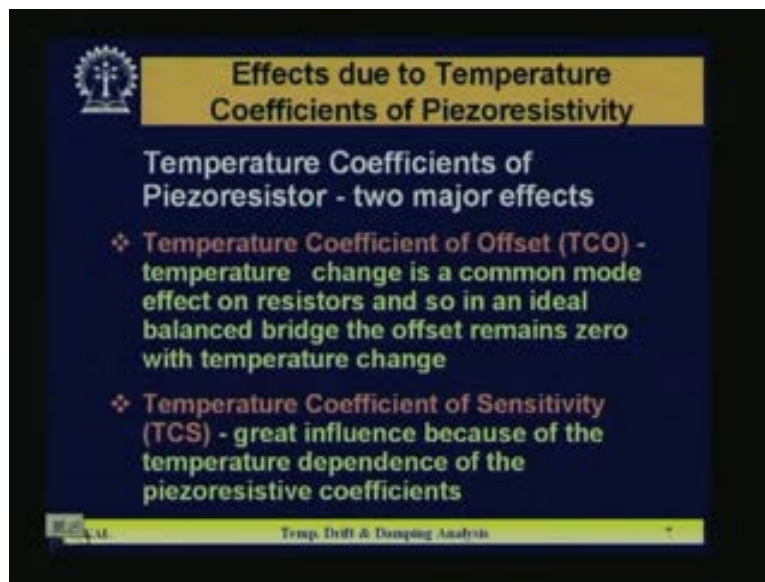


So that variation if you calculate and the plot looks like this. Here the concentration versus the conductivity, first plot and the second plot is the depth versus the conductivity. The junction depth as I mentioned in the last lecture that the total resistance value is also depending to some extent the junction depth. So here both plots are shown for different temperature. Temperature we have used 225 Kelvin to 350 Kelvin. So is a nearly where more than 100 degree see variation of the conductivity with concentration taking care of the change of mobility with temperature is shown in this curve. Now one unique feature of this both the curves is that you can see here that the temperature variation is almost

minimum for a typical doping concentration in this region. So now you can see here, so this point, anyway it is not coming.

So now you can see here this point which is here this where the temperature variation is minimum. So here again the width high this doping, higher doping concentration you can see the variation. That lower doping concentration you can see the variation with temperature the conductive variation. But in the middle region it is almost with temperature variation is not there. That means as well junction depth also is a sudden junction depth the conductivity variation with temperature is negligible. So those plots are for different temperature. Now that means to reduce the temperature variation, then the one parameter is the proper selection of the doping concentration in junction depth. So far the mobility variation is constant, the change of resistance with mobility variation. We have to select the doping concentration and junction depth properly so that temperature variation of the conductivity or resistance should be as minimum as possible so now this is the TCR.

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Now next parameter is the second parameter we can just we will see how that also going to change and that is a temperature coefficient of piezoresistors. That means piezoresistance coefficient effect so they are two major effects. One is the temperature coefficient of offset and the second is temperature coefficient of sensitivity. Temperature coefficient of offset is defined by the temperature change. This is a temperature change is a common mode effect on resistors and so in an ideal balanced bridge the offset remains 0 with temperature change. So now the offset is coming from for what reason, so that means the all the 4 bridges we found that there are total 8 resistances. And we assume the when under no stress condition all the resistance of the 4 bridges will be the same to show output voltage will be 0.

Bridges perfectly balanced, that is in ideal condition. But in practice all the resistances will not be the same. Even it is same you are making inter connection line. So those lines will induce some or will add some small value of resistances. There are contact points they will introduce some resistances as a result of which effective resistance of the arm all arms may not be exactly same and if it is not exactly same even in the zero stage condition you will have certain output voltage. So that is known as offset and that offset is not constant for different temperatures. Why? Normally if some offset is there they should not vary with temperature. But some variation you are getting because of the change of the contact resistance and the conducting line conductivity with temperature. So little bit variation will be there, because of that you will get the temperature coefficient of offset.

That mean with temperature the offset value will change and if the offset changes total of voltage also will change. So at the ultimately with temperature change the output what you are getting is basically the combination of all effects and they are very difficult to identify which factor will how much change is for what factor very difficult. But by calculation or by simulation you can know or I can just estimate which factor is much responsible for temperature dip? Who is less responsible for temperature dip? Second factor is temperature coefficient of sensitivity. This is very important great influence because of the temperature dependence of the piezoresistive coefficient. So that particular point we have to address much temperature dependence of the piezoresistive coefficient which I just mentioned pie 11, pie 44, and pie 14.

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**Bridge Excitation by Constant V vs Constant I**

Constant Bridge voltage,  $TCS_V$  is defined as

$$S = \frac{\pi_{44}(\sigma_l - \sigma_t)}{2\Delta g}$$

$$TCS_V = \frac{1}{S} \frac{\partial S}{\partial I} = \frac{1}{\pi_{44}} \frac{\partial \pi_{44}}{\partial I} + \frac{1}{(\sigma_l - \sigma_t)} \frac{\partial (\sigma_l - \sigma_t)}{\partial I}$$

Both the first and second term of expression for  $TCS_V$  are negative

Temp. Drift & Damping Analysis

So now lets see the constant again. Another thing is a bridge excitation by constant voltage and constant cut. So the bridge resistance bridges which were exciting, the supply voltage you can excite the bridge by constant current, you can excite the bridge by constant voltage; both are possible. Now here also it has been observed that the constant current and constant voltage excitation will not give the same performance. If we

compare this to we will find that some excitation is better than the other kind of excitation and that has been analyzed here you can see in this view graph that constant bridge voltage  $TCS_v$  temperature coefficient of sensitivity when V stands for constant voltage excitation and the S is defined by  $\pi_{44} \sigma_l - \sigma_t$  by  $2 \Delta g$ . These are longitudinal stress and transverse stress  $\sigma_l$  and  $\sigma_t$   $\pi_{44}$  is a piezoresistive coefficient and we have seen also the  $\pi_{44}$  if the most dominating factor among all other piezoresistive coefficient.

So this sensitivity is defined by this relation. Now in the bottom relation if you see there  $TCS_v$  temperature coefficient of sensitivity when you use constant bridge voltage. That is why subscripted v is equal to  $1/S \cdot \frac{\partial S}{\partial T}$  that is there relation and that is equal to  $1/\pi_{44} \cdot \frac{\partial \pi_{44}}{\partial T} + \frac{1}{R} \cdot \frac{\partial R}{\partial T} + \frac{1}{(\sigma_l - \sigma_t)} \cdot \frac{\partial (\sigma_l - \sigma_t)}{\partial T}$ . Because there are two parameter  $\pi_{44}$  and  $\sigma_l$   $\sigma_t$ . So the  $\sigma_l$   $\sigma_t$  variation is expressed by  $1/\pi_{44} \cdot \frac{\partial \pi_{44}}{\partial T} + \frac{1}{R} \cdot \frac{\partial R}{\partial T} + \frac{1}{(\sigma_l - \sigma_t)} \cdot \frac{\partial (\sigma_l - \sigma_t)}{\partial T}$ . So that is temperature coefficient of sensitivity when you consider the constant bridge voltage. So in this particular expression both the first and second terms of expressions are negative. That means the first term and second term both are negative both are negative. Both are negative means the temperature coefficient sensitivity. If you apply the voltage excitation is in one direction change is adding together.

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**Bridge Excitation by Constant V vs Constant I**

Constant Bridge Current,  $TCS_i$  is defined as

$$TCS_i = \frac{1}{S} \frac{\partial S}{\partial T} = -\frac{1}{\pi_{44}} \frac{\partial \pi_{44}}{\partial T} + \frac{1}{R} \frac{\partial R}{\partial T} + \frac{1}{(\sigma_l - \sigma_t)} \frac{\partial (\sigma_l - \sigma_t)}{\partial T}$$

First and third term in expression for  $TCS_i$  are negative while the second term is positive for higher doping concentration thus the temperature sensitivity will be reduced

Bridge excitation by constant current source is preferred than constant voltage source

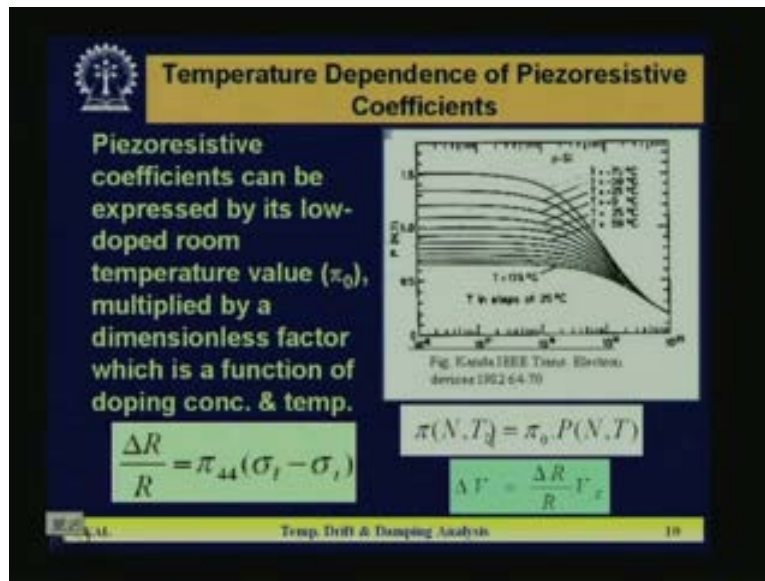
Temp. Drift & Damping Analysis

But if you go for the second thing which is constant bridge current excitation  $TCS_i$  then the temperature coefficient of sensitivity is given by this relation which is  $1/\pi_{44} \cdot \frac{\partial \pi_{44}}{\partial T} + \frac{1}{R} \cdot \frac{\partial R}{\partial T} + \frac{1}{(\sigma_l - \sigma_t)} \cdot \frac{\partial (\sigma_l - \sigma_t)}{\partial T}$ . Now here it has been observed that the first term and third term is negative. But the second term where  $1/R \cdot \frac{\partial R}{\partial T}$  is positive for higher doping concentration. All this positive negative I am talking that is for high doping concentration. Since for high doping concentration out of the 3 terms, middle is positive, first and the second term is negative.



So overall thing will go down. So that is why the bridge excitation by constant voltage and constant current will not be the same. So sensitivity where we will get more, so that we can chose either in which case we are getting much more sensitivity. Accordingly the excitation we can decide whether we go for constant current excitation and constant voltage excitation.

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Now I am coming to the temperature dependence of piezoresistive coefficient. So if we look into that, that means the piezoresistive coefficient are although there are various parameters  $\pi_{14}$ ,  $\pi_{44}$  and  $\pi_{11}$ . So I mentioned that  $\pi_{44}$  is the most dominating piezoresistive coefficient. So that is shown here the pie value which is function of doping concentration  $N$  and temperature  $T$  is basically product of two parameters; one is  $\pi_0$ .  $\pi_0$  is the low room temperature value is  $\pi_0$  and  $PNT$ .  $PNT$  is a function of  $N$  and  $T$  which is a dimension less quantity and that parameter variation is plotted in this curve. So  $\pi NT$  is equal to  $\pi_0$ .

$\pi_0$  into  $PNT$  and  $PNT$  is basically a curve is a dimension less quantity whose variation of the doping concentration and temperature was calculated by various people. One curve has been reproduced here from a paper, from IEEE transaction of electronic device 1982 by Kanda. So this plots shows how the  $PNT$  varies with varies with different temperature starting from 50 degrees to 75 degree centigrade and different concentration 10 to the power 15 to 10 to the power 20. So that variation that plots shows that the  $\pi NT$  at lower concentration varies much with temperature but at higher concentration the variation is less.

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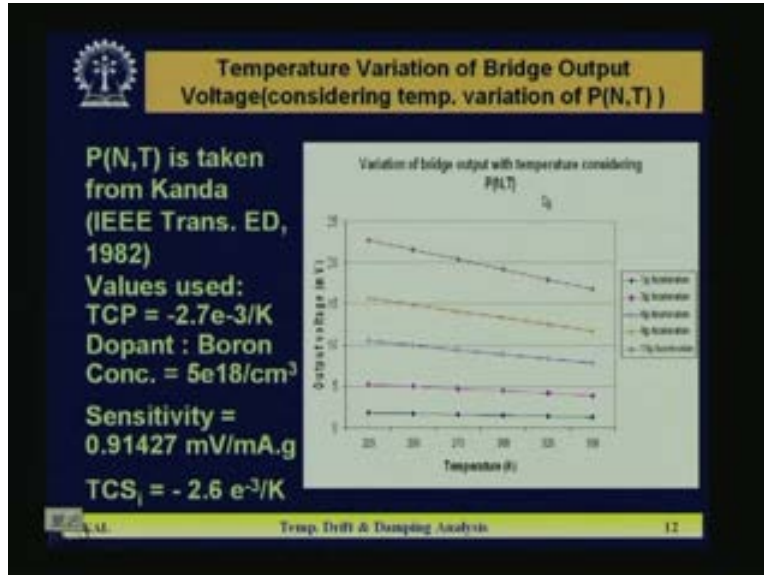
Temperature Dependence of Piezoresistive Coefficients

T K	P(N,T)	$\sigma$ (pS/ $\mu\text{m}$ )	$\pi_{11} \times e^{-5}$	$\pi_{12} \times e^{-5}$	$\pi_{44} \times e^{-5}$
225	1.2025	1.6378 e9	7.9365	-1.32275	166.065
250	1.135	1.6298 e9	7.491	-1.2485	156.743
275	1.0675	1.6221 e9	7.0455	-1.17425	147.42
300	1	1.6148 e9	6.6	-1.1	138.1
325	0.9325	1.6078 e9	6.1545	-1.0257	128.778
350	0.865	1.6013e9	5.709	-0.9515	119.456
375	0.7975	1.54e9	5.2635	-0.87725	110.135

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Now if we calculate those values, then we get all the 3 parameters  $\pi_{11}$ ,  $\pi_{12}$ ,  $\pi_{44}$  that plot is only for  $\pi_{44}$  PNT variation. PNT change is here with temperature 225 Kelvin to 375 Kelvin over a 100 degree Kelvin. PNTs are like that we assume at 300 the PNT is one and with different if you go the higher temperature the variation is like this. If we go lower temperature variation is like the PNT increases and in the higher temperature PNT reduces. Now with that you can calculate the sigma also. That is the conductivity and then  $\pi_{11}$ ,  $\pi_{12}$ ,  $\pi_{44}$  and if you look into that table, then the variation off the  $\pi_{44}$  what temperature is much more compare to the variation  $\pi_{11}$  and  $\pi_{12}$  with temperature is almost 7.4 here is 6.1 and 5.17 but here 166 to 210. So that variation the wide variation if you take the room temperature is 138.

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So now with that pie<sub>44</sub> variation if you calculate then we found that for boron doping concentration 5 into 10 to power 18 we found that TCP is minus 2.7 into 10 to power 3 power per Kelvin and sensitivity is 0.91427 millivolt per milliampere g, TCS is minus 2.6 10 to power minus 3 per Kelvin. Now for different g 1 to 13g you have calculated, the variation of the bridge at with temperature with temperature considering PNT. So as I told you the variation of three factors the middle one is by PNT pie change of pie with temperature. So that considering that effect only we just got the output voltage which is 20 millivolt for 13g at room temperature, room temperature is here. So you see 20 millivolt you are getting and this, the top curve for 13g and started from 1g, this is 3g; this is 6g, 9g and 13g.

At 13g room temperature is here at the middle and you can get is a 20 millivolt. What I showed you in earlier view graphs last lecture 20 millivolt. But out of the 20 millivolt, then variation below room temperature and variation above room temperature if you look then you find that below room temperature variation is much more compare to that of above room temperature. You see the curves this is the room temperature value for here to 350 degree centigrade the slope is less compare to the slope from 225 to 275 this side. So that means the low temperature variation is much compare to the high temperature means above room temperature. If you go for 60 degree or 50 degree temperature more, so that variation is not that much.

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**Temperature Dependence of Elastic Constants**

Elastic coefficients or Elastic Stiffness Constants are the constants of proportionality between components of stress and strain

They are designated by the term  $C_{hk}$  (h and k are integers between 1 and 6) and for cubic crystal the values are given below :

$$C_{11} \approx 16.38 - 1.28 \cdot 10^{-3} T$$

$$C_{12} \approx 5.92 - 0.48 \cdot 10^{-3} T$$

$$C_{44} \approx 8.17 - 0.59 \cdot 10^{-3} T$$

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Now temperature dependence of elastic constants. So temperature dependence of elastic constants if you concentrate there what is parameter elastic stiffness constants? These basically are proportionality between components of stress and strain ratio of stress and strain is this stiffness constant. They are designated by the term  $C_{hk}$  where h and k are integers between one and six like the pie 1 1, 1 2, 1 3, 1 4, 1 5, 1 6, similarly here also,  $C_{11, 12, 13, 14, 15, 16}$ . In that way 6 by 6 matrix is there. So that h and k value changes between 1 and 6 and for cubic crystal the values are given here  $C_{11}$ ,  $C_{12}$ , and  $C_{44}$ . These are the dominating term and with temperature the relation is given here, the variation of the elastic constants  $C_{11, 12, 44}$  with temperature.

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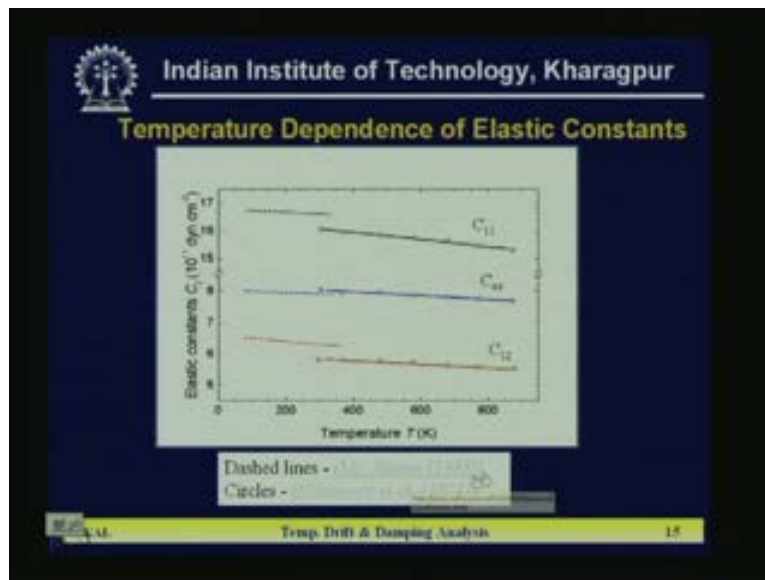
**Temperature Dependence of Elastic Constants (cubic crystal, Si)**

T (K)	$C_{11} \times 10^{11}$ (dyne/cm <sup>2</sup> )	$C_{12} \times 10^{11}$ (dyne/cm <sup>2</sup> )	$C_{44} \times 10^{11}$ (dyne/cm <sup>2</sup> )	E (Youngs Modulus MPa)	G (Shear Modulus) MPa
200	16.6	6.4	7.96	130.191E+3	7.962E4
225	16.6	6.4	7.96	130.191E+3	7.962E4
250	16.6	6.4	7.96	130.191E+3	7.962E4
275	16.6	6.4	7.96	130.191E+3	7.962E4
300	16.6	6.4	7.962	130.191E+3	7.962E4
325	16.568	6.38	7.94525	130.128E+3	7.9452E4
350	16.536	6.376	7.9305	129.873E+3	7.93E+4
375	16.504	6.364	7.9158	129.619E+3	7.91E+4
400	16.472	6.352	7.9010	129.36E+3	7.90E+4

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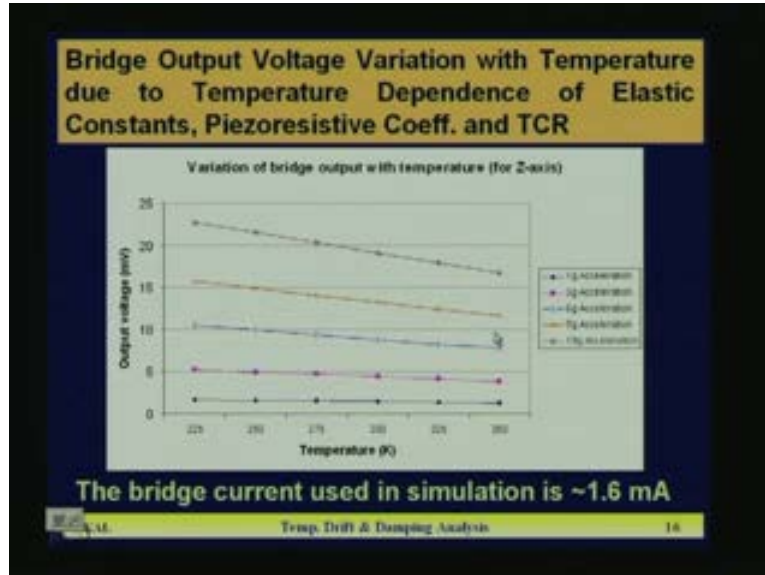
Now if we calculate temperature dependence of elastic constant, then we get this result.  $C_{11}$ ,  $C_{12}$ ,  $C_{44}$ , all are calculated for different temperature using there relations and there after that the Young's modulus and the CR modulus was also calculated and you can look here that from the values that you can see Young's modulus 130.19, so 191 here up to this because you see the below room temperature it is almost no change above room. Temperature the elastic constant change in the third may be second or third decimal place. So above room temperature here changes are there little bit, but below room temperature almost constant. This is 200 Kelvin here, 300 and this is 400 Kelvin. Similarly here also variation is not that much as in your the pie temperature coefficient of piezoresistance.

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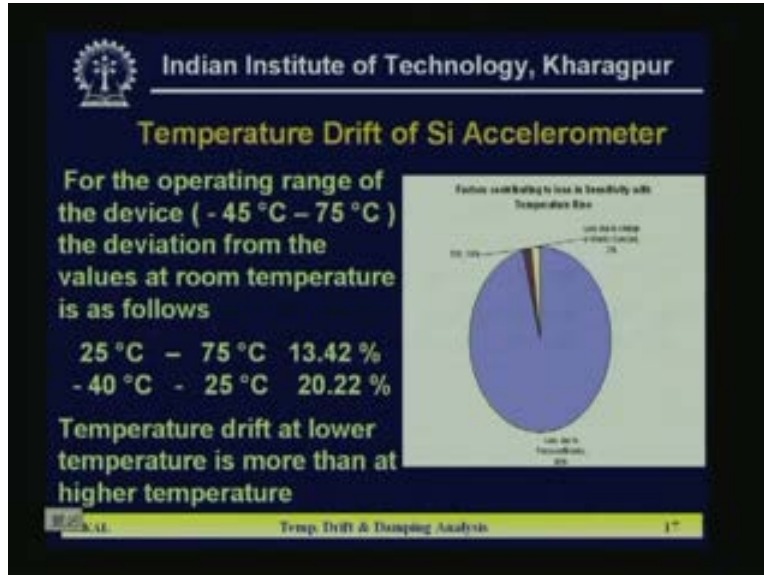
Now here the variation if you plot with temperature elastic constant variation is shown here the plots are taken from literature. One is by Skimin in 1953 and another is Nika Nikanorov et al by 1971. That is the circle line, these  $C_{11}$ ,  $C_{44}$  and  $C_{12}$  and dotted lines. Because two literature the curves are taken from there elastic constant variation with temperature.

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Now this curve shows the bridge output voltage variation with temperature. Due to temperature dependence of elastic constant, piezoresistive coefficient and TCR altogether. Because at the end it is very difficult to find out. I told you that who is going to play how much variation. So overall variation if you look, so that variation is plotted in this curve and you can see here that from 225 to 325 the total output voltage variation is not small for 1g. But you can see for 1g and 3g the total variation is almost varies very small negligible. But if you go for higher g for example here is say 6g, 9g and 13g, so at higher g the temperature dependent, temperature variation is much more from practical low g variation. Because their stress is less in low g and that may be the reason where a variation is much variation is not reflected there.

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So now it is a pie diagram. Here you can see the contribution due to different parameters. So now you can see this from the diagram to TCR is 1.9 percent, factors contributing to loss in sensitivity with temperature rise. So here the TCR variation is 1.9 percent loss due to change in elastic constant is 2 percent. So almost same in both the case and rest of the 96 percent is lost due to the piezocoefficient pie NT or PNT. So only 2 percent for TCR and 2 percent for elastic constant, rest of the 90 percent variation is for is responsible for change of piezoresistive coefficient. Second parameter which is much is dominant. Now here from minus 45 to 75 degree C we have calculated and then we found here from 25 degree C to 70 degree C variation is 13.4 percent and from minus 40 to 25 is nearly 20 percent. So temperature drift at lower temperature is more than at higher temperature.

So you have to take care of the situation when you are going to use this kind of sensors. The temp well below the room temperature and normally for space application the temperature is well below room temperature. So there the variation will be much if the sensor is exposed to the atmosphere. But if the sensor is inside the aircraft. So there aircraft temperature is maintain not very low is a near nearly 15 to 20 degree centigrade. So there it will not vary much but which is exposed to atmosphere, that temperature is very low. So there you have take care of then how to take care of it. So there are two thing something can be taken care of during your design values means your doping concentration I have shown you. That this different the parameters first or second or third parameter they are dependent on doping constant is an also if you choose doping concentration certain value, that temperature variation is minimum.

So there you can adjust to some, but that may not solve the full purpose because you can choose a doping concentration of the order of 10 to the by 19 or 3 in the 10 to the by 19 something like that. But for that value may be your temperature variation is less. But if that value will not satisfied your requirement of the ic resistances where we are going to make this signal condition circuits. So in that case you have to do certain compromise. So

because of that compromise temperature variation will be there and that can be taken care of by circuit technique. So you have to add certain the circuits and those circuit will take care of the variation. You have to have some look up table inside store in the memory of any of the processor. So then depending on the temperature the circuit parameter will change and at the output after that processor and if that signal will not vary with temperature. That is the normal way of reducing the temperature variation of any sensor.

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Temperature Drift Compensation Circuit

Temp. increases  $\rightarrow$  bridge resistance decreases  $\rightarrow$  bridge sensitivity reduces

If we made  $I_B$  to depend inversely on bridge resistance  $R_B$ , output voltage becomes temperature insensitive

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Some circuit techniques are shown from temperature drift compensation circuit. Here you can see, this is the bridge, left side the  $R_1$ ,  $R_2$ ,  $R_3$ , these total bridges and if you connect the bridge with a circuit like that. So this will give you to some extent temperature drift compensation circuit. If temperature increases so bridge resistance will decrease and sensitivity will reduce. But what they are doing the if you made  $I_B$ . This  $I_B$  which is the bias current  $I_B$  inversely, if you made  $I_B$  to depend inversely or bridge resistance  $R_B$  or full voltage becomes temperature insensitive and how to make that the  $I_B$  insensitive. So that is taken care by is circuit technique and that is shown at the top where this total resistance is given by  $R_t = R_1 + R_2$  and the  $R_t$  is a change which you can control externally depending on the available input data.



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### Temperature Drift Compensation Circuit

A negative impedance converter (NIC) implements a negative resistance  $(-R_N)$  in series with  $R_B$  such that  $I_B$  increase with temperature.  $R_N = (R_T R_1 / R_2)$ ,  $R_T$  can be adjusted externally

Temp. Drift & Damping Analysis 19

So these are some of another temperature drift compensation circuit is also shown here. That they use a negative impedance convertor NIC, the negative impedance convertor implements a negative resistance minus  $R_N$  in series with  $R_b$  such that  $I_B$  increase with temperature which is  $R_N$  equal to  $R_t R_1$  by  $R_2$  and that  $R_t$  can be adjusted externally. So that means resistance value is going to increase with temperature that can be reduced by connecting n negative impedance inverter. So that negative impedance inverter is design with the help of the circuit which is shown here. So there are different ways to reduce the temperature variation by circuit and similar circuit, similar chips has come out.

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### MAX1457 At-A-Glance

0.1% Accurate Signal Conditioner for Piezoresistive Sensor Compensation

Temp. Drift & Damping Analysis 20

And some of the chips showed here. This is the maximum 1457 chip, the internal diagram of the chip is shown here and basically signal conditioning piezoresistor sensor compensation circuit is available in the market which can accurate. This 1.1 percent the accurate signal condition. Then we here the temperature variation all other variation due to the either piezoresistor coefficient or less whatever it is, that variation is reduce to 0.1 percent by lot innovative circuit techniques and that chip is available and people are using a chip in a multimode packaging sensor and at the same time at the output they are connecting the chip. So that temperature compensation will be done.

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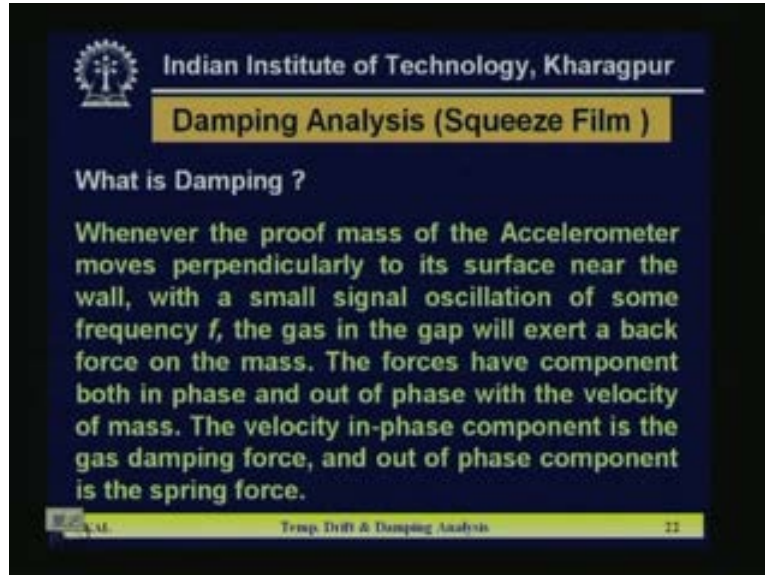
**MAX1457 At-A-Glance**  
**0.1% Accurate Signal Conditioner for Piezoresistive Sensor Compensation**

Part Number	PGA	Calibration Method	Accuracy (% FSO)	Output Type	Supply Voltage (V)	Typ. Max. Supply Current (mA)	Sensor Excitation Current Source (mA)	Package	Op. Range (°C)	Pin Count
MAX1457	344	Internal DACs updated from external EEPROM	0.1	Analog	4.5 to 5.5	22.8	0.1 to 2.8	28-Pin PDIP	-40 to +125	8

Temp. Drift & Damping Analysis

So this specification of maximum 1457 chip is shown here. So this can go temperature end up to plus 125 we can see minus 42 plus 125 and its variation in different temperature is a shown minus 42 plus 85 and 02 plus 75, that variation of the sensor excitation and current variation supply voltage and other full scale output variation, all the things are taken care of by this chip and this is a very popular chip as a temperature compensation or signal condition circuit piezoresistive MEMS.

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Now then the second point is a damping analysis. So first we have to know what is the damping and what kind of damping we are going to use in this particular sensor and we are going to use a squeeze film damping. Squeeze film damping means a film means a gap between your sensing element and the fixed plate at the bottom will be maintain and because of the moment of proof mass the gap is going to change and the film is air or whatever thing you give you allow in between the sensing the element and the bottom or top plate that is going to squeeze and that will create pressure in opposite direction to the proof mass; proof mass of going to go downward.

So that means some spring force will be there and there it damping force. There are 2 forces working; you see as the accelerometer vibrates. So because of the flexure the spring action so there is spring force which will which will be generated into the structure and because of that some displacement will be there of the proof mass and at the same time the damping force. That means the film which is there or the air pressure which is there in the sensing middle plate to bottom plate, that will exert some damping force and they are trying to balance and they are to trying to come back the sensing element to its the rest the position so that is a basic damping thing.

So if you are going to define then whenever the proof mass for the accelerometer moves perpendicular to its surface near the wall with a small signal oscillation some frequency  $F$  the gas in the gap will exert a back force on the mass. That is a damping in opposite back force on the mass. The forces have component both in phase and outer phase with the velocity of mass. The velocity in phase component is the gas damping force and out of phase component is the spring force. Because there are two forces; one is the damping force, another is a spring force. So velocity in first component is known as the damping force and velocity {com} the outer phase component is the spring force. So those things you have to separate it out and you have to calculate.

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### Damping Analysis (Squeeze Film )

Damping coefficient (  $N/(m/s)$  ) and damping force (  $N/m$  )

The damping force is the component of the fluid force that is in-phase with the velocity oscillation. It is computed by integrating over the damping area of the pressure times the cosine of phase angle . The damping coefficient is ratio of this damping force to the amplitude of velocity of mass. The coefficient is a measure of dissipative forces of fluid.

Temp. Drift & Damping Analysis 13

Now the damping coefficient and damping force. The damping force is component on the fluid force that is in phase as I mention with the velocity oscillation. It is computed by integrating over the damping area of the pressure times, the cosine of the phase angle, the damping coefficient is ratio of this damping force to the amplitude of velocity of mass. The coefficient is measure of dissipative forces of fluid. Fluid means what either gas or if you use liquid, then liquid or gas normally gasses are used with certain pressure. So that is how much dissipative force you are going apply on the film, that is going to be decided by the damping factor. So these are the thing we have to select the damping factor and that in our design specification it is 0.7 plus minus 2. We have to achieve that goal and how do you get, how do you analysis this, that is shown here.

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### Damping Analysis (Squeeze Film )

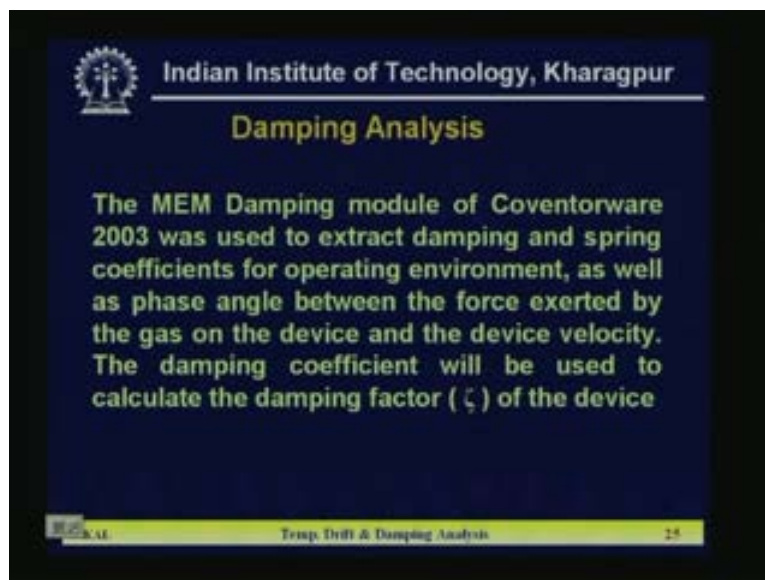
#### Spring coefficient and spring force

The spring force is the component of the fluid force that is out of-phase with the velocity of oscillation. The spring coefficient is the ratio of this spring force to the amplitude of displacement of mass. The coefficient provides a measure of restoring force resulting from external medium.

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Now the second point is the spring coefficient. Spring force is the out of phase that the velocity mass the force whose outer phase component is basically this spring force it is component of the fluid force. That is the outer phase with the velocity of oscillation. The spring coefficient again is the ratio of this spring force to the amplitude of displacement of mass. This spring force divide by the amplitude displacement of mass is known as the spring constant. The coefficient provides a measure of restoring force resulting from the external medium. So that is the spring coefficient and damping coefficient definition.

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### Damping Analysis

The MEM Damping module of Coventorware 2003 was used to extract damping and spring coefficients for operating environment, as well as phase angle between the force exerted by the gas on the device and the device velocity. The damping coefficient will be used to calculate the damping factor ( $\zeta$ ) of the device

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Now how do you make the analysis? Here we have used to again the Coventorware. The Coventorware 2003, one main damping module is there; microelectric mechanic system the damping module. That module we has been employed to find the value of the damping and spring coefficient for the operating environment as well as phase angle between the force exerted by the gas on the device and the device velocity. The damping coefficient will be used to calculate the damping factor of the device.

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### Damping Analysis

Let  $x$  be the displacement of the mass  $m$  relative to the frame. When the acceleration is  $a$ , the equation of motion for the mass is

$$F = m \ddot{x} + c \dot{x} + kx$$

where  $c$  and  $k$  are the damping coefficient and spring constant, respectively. Thus, the acceleration can be determined by measuring  $x$ .  
Rearranging and comparing with second order equation of motion

$$F(s) = s^2 mx + scx + kx$$

Temp. Drift & Damping Analysis 26

Now these are the analysis and it will give you much more idea of how the damping coefficient is calculated. So in this you can the force equation of motion is written is all of you known the equation is a  $F$  equal to  $m X$  double dot plus  $c X$  dot plus  $kx$  where  $c$  and  $k$  are the damping coefficient and spring constant respectively  $m$  is a mass and  $X$  is displacement. The acceleration can be determined by measuring the displacement  $X$ . Rearranging the equation here what is shown with the second order equation of motion the  $FS$  which is the force is given by a  $S$  square  $m x$ . So  $x$  is that  $X$  square is a double derivative, we know with respect to  $x$  plus  $s c x$  plus  $kx$ , the same equation is written in the fashion.

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### Damping Analysis

Rearranging and comparing with second order equation of motion

$$\frac{F(s)}{m} = s^2 x + s \frac{c}{m} x + \frac{k}{m} x$$

$$a = s^2 + 2\xi\omega_n s + \omega_n^2$$

We get

$$\omega_n = \sqrt{k/m} \quad \text{and} \quad \xi = \frac{c}{2m\omega_n} = \frac{c}{c_c}$$

where  $c$  is damping coefficient and  $c_c$  is critical damping

Temp. Drift & Damping Analysis 17

Now the rearranging again the  $F/s$  by  $m$  is given by  $s$  square  $x$  plus  $s$   $c$  by  $m$   $x$  plus  $k$  by  $m$   $x$ . Now this equation force by mass is acceleration  $F/s$  is a force and  $m$  is the mass. So force by mass is acceleration. It is given by  $s$  square plus twice  $\xi$   $\omega_n$  plus  $\omega_n$  square where the  $\omega_n$  is given by the equation under root  $k$  by  $m$  and this  $\xi$  or  $\xi$ . I think it is  $\zeta$  is given by  $c$  there twice  $m$  into  $\omega_n$ . This is equal to  $c$  by  $c_c$  where  $c$  is the damping coefficient and  $c_c$  is the critical damping and the critical damping is  $m$   $\omega_n$  this is  $c_c$ .

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### Damping Analysis Results

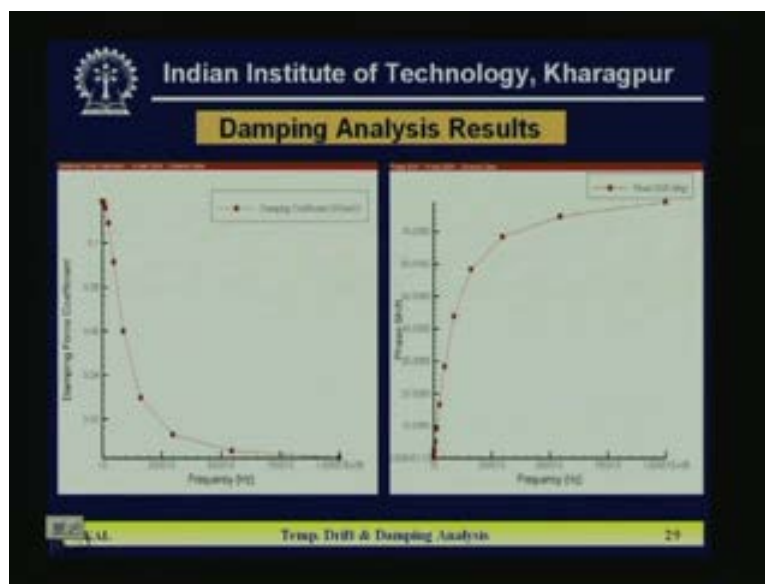
The spring force rises rapidly. The air captured in the cavity is squeezed since there is no real cavity at low frequency the air can escape with no problem and force is small at high freq air is held captive by its own inertia.

Damping level (Spring Force) vs Frequency (Hz)

Temp. Drift & Damping Analysis 18

Now the plots are shown here, one is the damping force Newton per meter and spring force Newton per meter. That green line is the with frequency the you can see the damping force is going have a peak value and then it is going to reduce and on the other hand the spring force with frequency is linearly increases and after some time it is going to be saturated. So these are the two different forces that are analysis has been done using the Coventorware directly. The spring force rises rapidly, you can see this spring force which is a green colored is rises ridely, rapidly it rises. The air captured in that cavity is quest since there is no real cavity at low frequency. The air can escape with no problem and force is small at high frequency air is held captive by its own inertia. So that is the inside the structure.

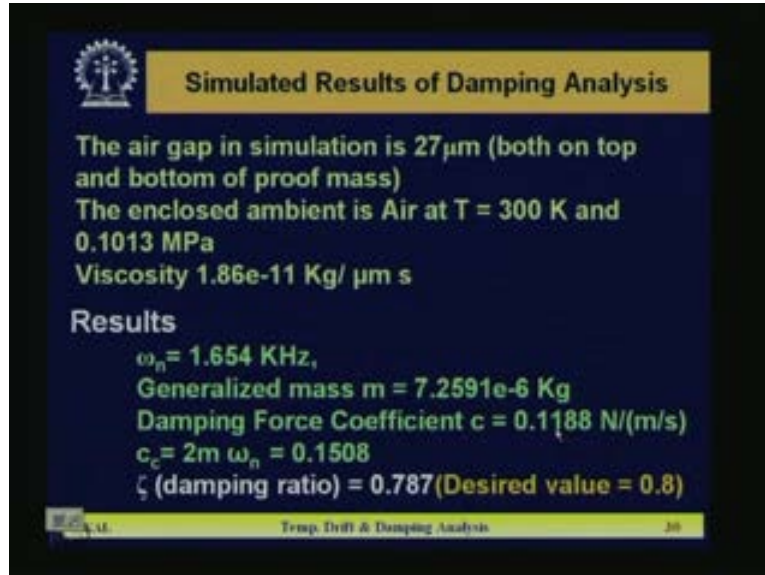
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Now here the damping force coefficient and the phase shift of the damping thing has been plotted. This is the force coefficient and the phase and here also we found that the variation at the low frequency is steep and the high frequency variation is not that much and in the earlier diagram you can see there is point where the both the spring and damping force are same is a cross over point and we have to take that particular point at which frequency it is same. And accordingly the  $\Omega_n$  value is decided and you can calculate the damping factors in both the cases.



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The air gap in this simulation is 27 micron. Using those analyses we got it both on top and bottom of the proof mass. The enclosed ambient is air at T equal to 300 Kelvin and 0.1013 mega Pascal. Viscosity of the fluid is assume to be  $1.86 \cdot 10^{-11}$  kg per micron second and with that simulation we got the result. The Omega n is 1.654 kilo hertz. Generalized mass is obtained as  $7.259 \cdot 10^{-6}$  kg. Damping force co-efficiency is obtained as 0.1188 Newton per meter second.  $c_c$  is calculated twice m into Omega n which is 0.1508 and damping ration which is important, we which we need it that value we got it with that simulation is 0.787 and desired values 0.8. So it is very close to that and actual value is 0.7 plus minus 0.2. So normally people state that value as 0.8, in this simulation you can get that value 0.787 which is very close to the desire value.

So in that way how we are going to get this value and that is basically obtained by changing the viscosity of the fluid, pressure of the fluid. Here the we are using the air damping, so that air means how much that the damping force will exert on the proof mark, that will be decided by the cavity or in the bottom and top plate and that is we kept is 27 micrometer and the viscosity of the medium and the pressure inside the chamber. Chamber means the three pieces are there top plate and bottom plate and middle plate there all these factors along with this structure and the structure the proof mass structure is a rectangular structure with flexure. Now sometimes the damping for small devices if you small devices if you total dimension is small. In that case the proof mass size will be again small. In that case in order to achieve the proper damping value which is close to 0.7 or 0.8, sometimes people go for the perforation of the proof mass itself.

If you perforate that, then there is an open clear path from bottom to top. So that if the middle sensing element vibrates up and down, so the fluid may flow through the proof mass as well as the open spaces surrounding the proof mass. The proof mass is at the middle, open space is the flexure and others are open; along with if you put some holes

through the proof mass, then also the damping factor will change to a great extent and this is important. So far as your speed of the devices is concerned and if you do not put the damping obviously it may go to resonance mode and in that case the total performance or the device may be damaged and not only that if you do not provide the proper damping settling time will be much more. So all these points will be there and that is why this particular parameter is also critical and you can design your structure.

This means, this basically is coming your structure as well as the medium which you are allowing into the cavity and the properties of that medium, fluid medium, those are the main parameters which is coming into the picture for calculation of the damping factor. So now with this I think we got all the specifications number one is 13g acceleration, number two is the model analysis the resonance frequencies will it should be above 100 hertz that we got it. Temperature analysis we did it and then found the variation due to different parameters and how to succumb in that problem. One is in the design means selecting the parameters. That is doping concentration junction etcetera. Other which you cannot do that can be compensated by using the circuit techniques. That we have seen and the temperature drift we have discussed and lastly which is another important parameter that is the offset.

How the offset can be reduced by some techniques of connecting the sensing elements over the Wheatstone bridge and as well as by changing the flexure position along the proof mass, that can be further improved. So all the aspects I discussed and with that you can get the complete analysis and now the next point for the realization is the making of the mask. After making mask layers, then next step is how to fabricate it. In my next lecture I am going to discuss on the fabrication process of realizing the complete structure and piezoresistances and the characterization of the deep sensing element. Means how much value you are getting each resistances and how much is the offset voltage you are getting, because of the miss match of the bridge resistances and how the offset is minimized with the certain circuit.

And what are they output signal you are taking, then that signal some noise will always incorporate how to reduce the noise at the output before going for signal conditioning. That means noise suppression or some circuit techniques you have to use, so that after the amplification you will get that much noise. So then actual static testing of the accelerometer and the dynamic testing means the vibration table if you put the accelerometer how the acceleration is outer volt is going to change with the acceleration, those things that means fabrication part and characterization of the accelerometer will be discussed in next lecture. Thank you very much.

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Preview of Next Lecture

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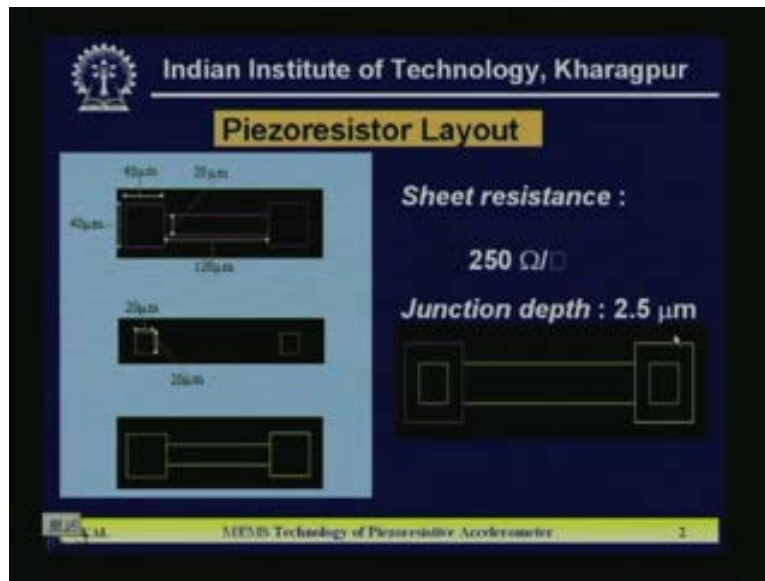
Lecture No. # 24

Piezoresistive Accelerometer Technology

We are discussing on a case study that was the development of MEMS accelerometer. That is piezoresistive accelerometer and in my last two lectures I have discussed in detail, on the design aspects of the accelerometer along with the external circuits necessary for

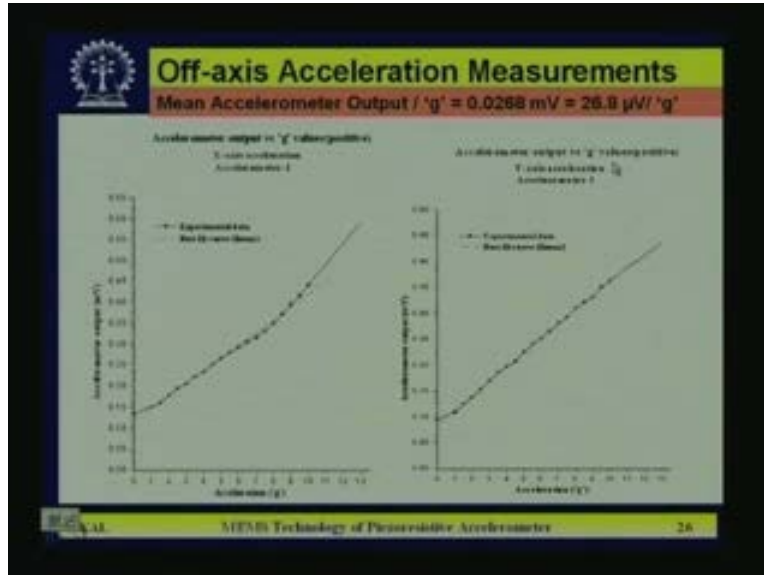
characterization the accelerometers. Today's lecture is on the development of the technical of the MEMS piezoresistive accelerometer. How the accelerometer is designed that has been told already. Now the technology development as well as packaging and characterization. How it can be done? That will be discussed in today's lecture. Then the case study will be completed.

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So for designing accelerometer the end of the design process is making the mask. Mask was not discussed in my last lecture. Today I will first initiate the discussion on the fabrication of mask and then we will switch over to the technology development. Now for the layout of the mask here first you have to design the resistance; piezoresistance and so far the design is concerned as same as the design resistor design used in integrated circuits. Already you know how in IC resistors are design. It is you have to know the sheet resistance of the layer which will give the desired resistance value. Now if you know the sheet resistance, then you have to select how many number of square are required to get that resistance. And in our design we found that our resistance value is nearly 1.5 kilo ohm in that range. But one thing I would like to mention that in any diffusion technique the resistance variation after fabrication is nearly 10 to 20 percent. In most of the ICs design in such fashion that even 20 percent variation of the resistance values will be accepted by the designer that mean design should be robust. So that with that resistance change the performance of your circuit will not hamper very much.

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Similarly here also we assume even 10 15 percent variation the complete circuit should work. But here you will get the value is a nearly here 26 microvolt. So that means there is highly 500 is z direction 500 microvolt and x and y direction only 26 microvolt per g. So of x axis sensitivity is less but we have to agree the fact that it is not as slow as design value. Because of various reasons of the fabrication, tolerance is not exactly showed which is required, which is estimated in our design. So according off axis sensitivity is not that high value but it is 500 is 26 at least one more than one order less.

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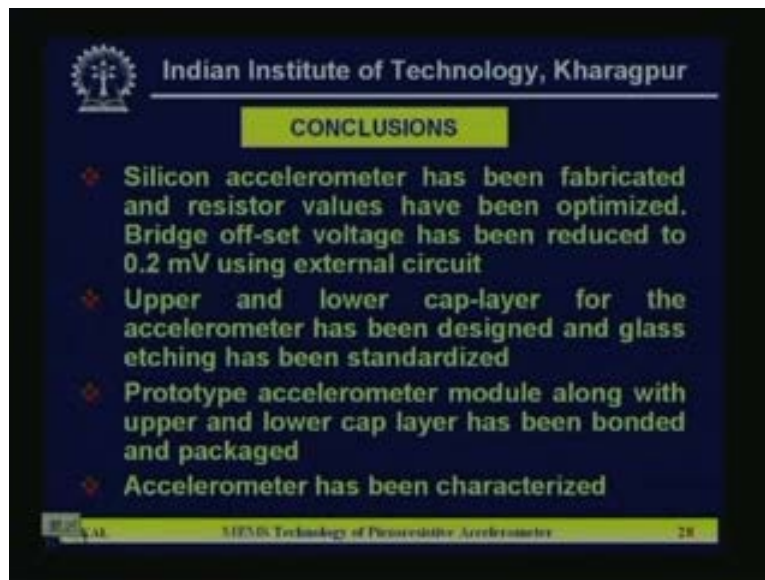
**Resolution studies of the accelerometers:**

Applied 'g'	Accelerometer Output Voltage Positive Z-axis	Accelerometer Output Voltage Negative Z-axis
2.00	1.074	-1.15
2.1	1.125	-1.204
2.2	1.172	-1.251
2.25	1.197	-1.277
2.3	1.225	-1.305
2.4	1.272	-1.36
2.5	1.319	-1.409

MEENS Technology of Piezoresistive Accelerometer 27

So now you see the resolution studies of the accelerometer then you can see here, you can measure the resolution in the z direction up to 0.25, this is a 2.25, and these are 2.3. Here you can see 0.05. 0.05g we could resolve because you can get the z axis accelerometer difference 1.197, 1.225 here. So difference is there, so that means up to 0.25g you can resolve. But again it could be resolved in the further low. But because of the measurement facilities not available to get milli g acceleration, that could not be measured. That is the limitation our test facility. But here still we can find that up to 0.25 g means 250 milli g it is resolved. So anyway so that is some within negative and positive at the z axis the resolution has been studied.

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So with that actually let us conclude that what we did today in today's lecture. So how the accelerometer piezoresistive accelerometer is fabricated, that is discussed in detail and the technology optimization has been done off set voltage circuit shows. That within 5 volt the offset can be reduce up to 0.2 milli volt using external circuit, that is also possible. How to make the upper and lower cap layer from pyrex glass that as been discussed in detail and accelerometer packaging substance bonding is shown and the external circuits are also connected with the main sensing element and at the end it has been characterized with the limitation of characterization facilities and all these thing shows that although it is not exactly the same as the design value very close to the design value we achieved the target specification. And with this I can conclude this lecture means case study on the fabrication of MEMS accelerometer using the piezoresistive pick up technique. Thank you very much.