

Digital Voice and Picture Communication
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Lecture - 07
Differential Quantization

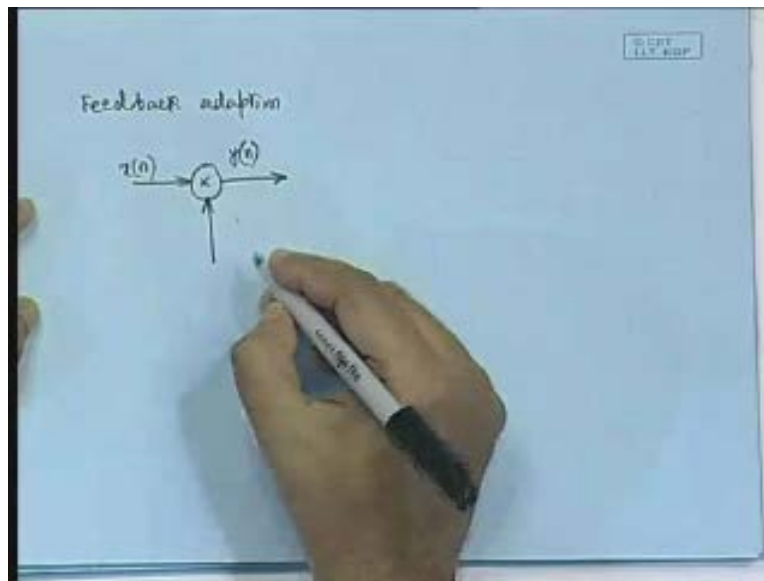
Last class we were discussing about the adaptive quantization and we have left certain discussions unfinished so we have to first just go through whatever unfinished aspects are there pertaining to the adaptive quantization. And today, after completing the adaptive quantization, we will go in for the differential quantization. Although I understand that those who have already gone through the courses in the Digital Communications already know this that is why I am going to go through much hurriedly as compared to what the digital communication course normally treats. And not only that the aspects of differential quantization I will be talking again in the perspective of speech communication. But before that we have to continue with our discussions related to the adaptive quantization so we will first be going to that.

Now, as you had seen that in the case of adaptive quantization we had discussed about one methodology which we had talked as the feedforward adaptation. In feedforward adaptation you had seen that what we are doing is that from the input samples itself we are estimating the power spectrum or the auto correlation we are estimating from the input signal and we are using that information in order to obtain the step size. So that was the basic philosophy behind the step size adaptation. And now, today, instead of going to the feedforward adaptation we will go through the feedback adaptation.

We had already talked about the feedback adaptation. Its basic philosophy we had talked about in the last class. And what we had said is that, in the case of a feedback adaptation, the step size has to depend based on the final codeword c of n what we are obtaining at the output. So from the c of n we have to derive the step size. The advantage is that, in this process we did not have to send this step size information or the gain information and instead of sending that into the

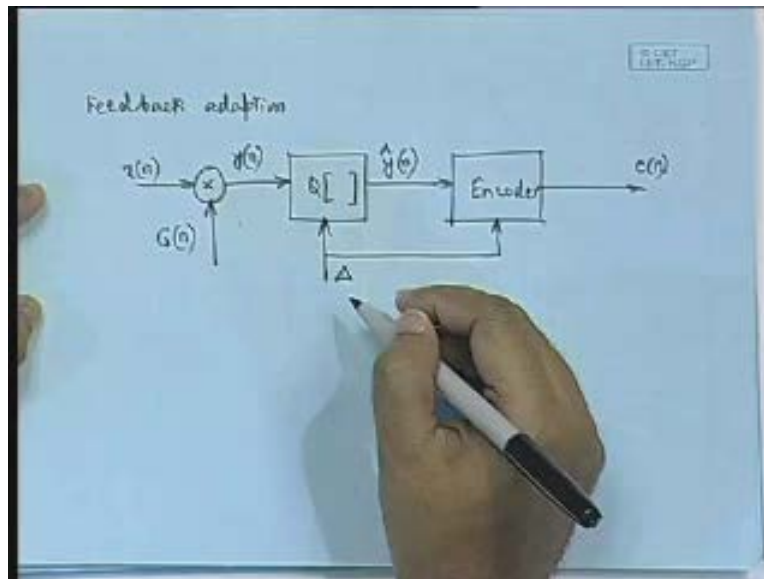
channel, since we are able to derive it from the output codeword likewise even at the decoder end also from the output codeword it should be possible for us to determine the step size or the gain. So the feedback adaptation block diagram would look like this that here we will be having the x of n the samples (Refer Slide Time: 4:06) and now we are going to have the multiplier over here.

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So if we are adopting the gain adaptation then the block would look like this that x of n and this we have to multiply by the gain which we are calling as G of n and this will be quantized. So it goes to the quantizer block Q and the output of the quantizer will be y cap of n and then it will be followed by the encoder and then we will be having the output codeword which is c of n and the output of this..... and this Q as well as the encoder will be controlled by the step size.

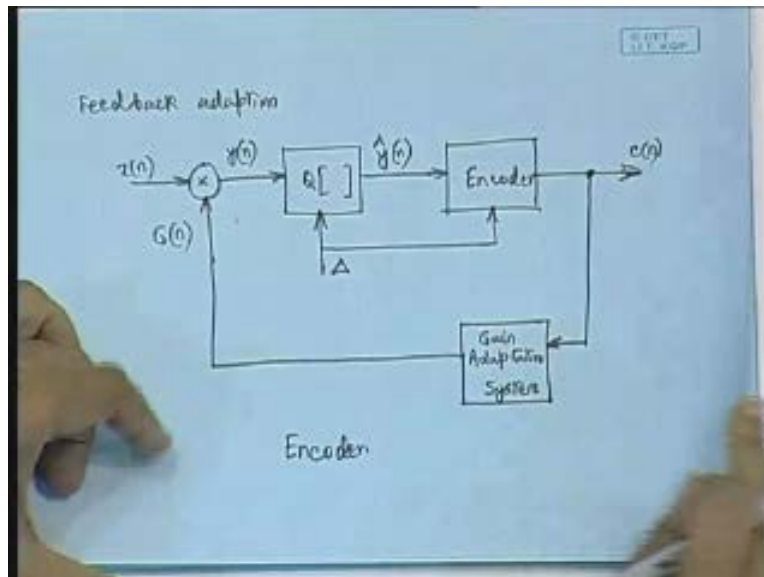
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Now because it is a gain adaptation the step size delta will be constant and we have to take this output from the $c(n)$. So based on the output codeword the gain adaptation system block will operate. This is the gain adaptation system (Refer Slide Time: 5:25) and the output of this gain adaptation would go to this multiplier block. So $x(n)$ multiplied by $G(n)$ will give us the y of n . this is the feedback adaptation.

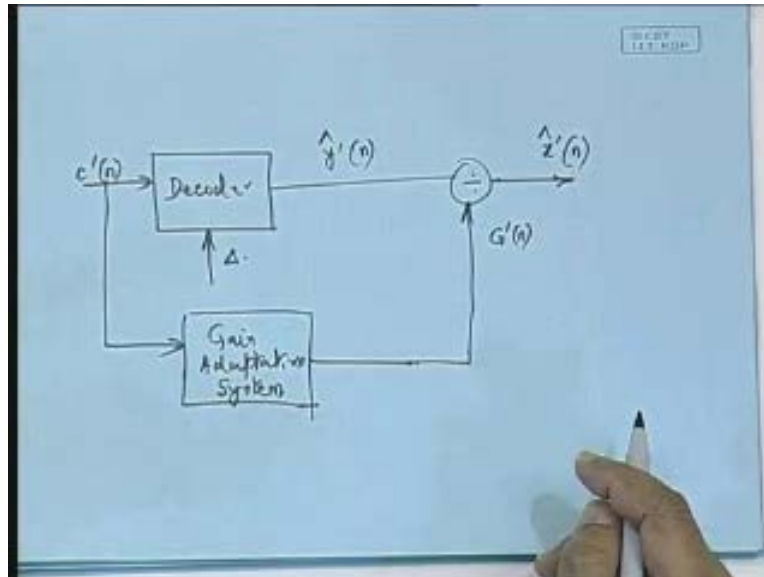
Now definitely you see in the channel that earlier we were sending the G of n ; in this case we need not have to send the G of n because c of n will be enough. Because what we are going to do at the decoder, so this is the encoder (Refer Slide Time: 5:57) so at the decoder what we are going to do is just the reverse of this.

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There we can just go through this, the c prime of n and that goes to the decoder so it is received as c prime of n decoder with the step size Δ and then we will be having the gain adaptation system. So gain adaptation will be in a similar way for both encoder and the decoder. Then we will be having here a division block and this will be G of n , not G of n exactly, it will be G prime of n . Because if we take this input to the c prime of n which is ideally equal to c of n in the case of noiseless channel but in the case that it is corrupted by the noise we call that as c prime of n and c prime of n will give rise to an adapted gain which we call as G prime of n and the decoder output we will call as y prime of n so y prime of n divided by G prime of n is going to give us the x prime of n which will be equal to x cap of n if we are going to have the noiseless channel.

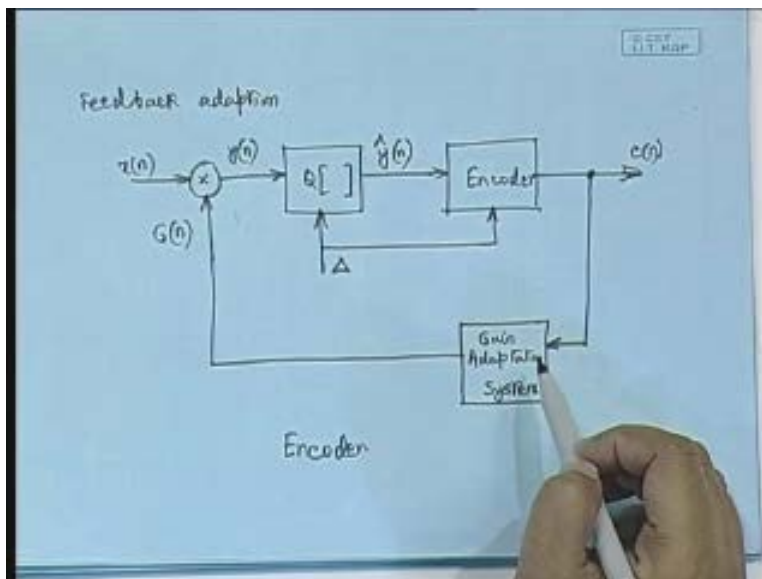
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So this will be the decoder block diagram. You can see that the basic philosophy remains the same, only thing is that here we can derive it from c prime of n , very clear, we did not have to send the gain information into the channel.

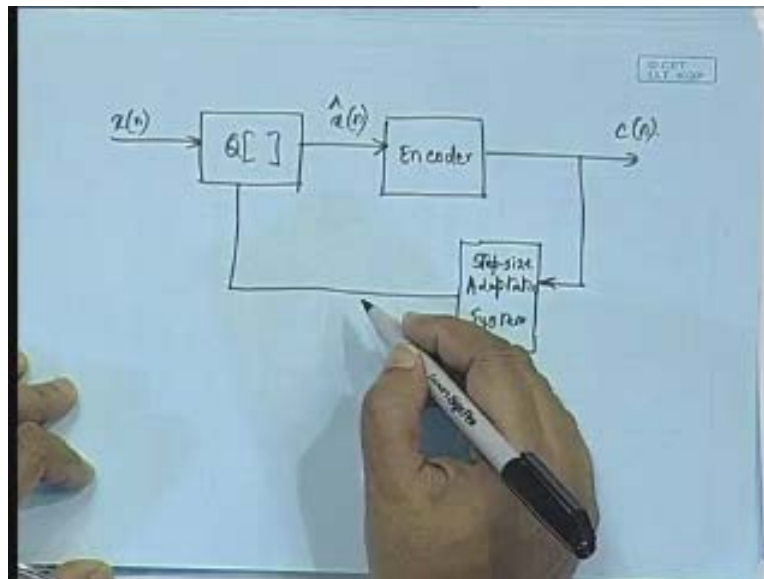
Now **what is** the advantage we talked of but every advantage is also associated with some kind of a disadvantage. Now if you look at the encoder block diagram you would notice one thing that this gain is dependent now upon the output codeword. Now on what factor is the output codeword dependent? Output codeword is dependent upon the quantization. So whatever output codeword we are having that is subject to this quantization noise which is there whereas earlier we were deriving our estimate directly from x of n ; here we will be deriving the estimate from the quantized value so there is a question of quantization noise and then whenever we are implementing this in circuit that time the step size Δ what we are having that step size Δ may not be the exact desired value of Δ so as a result of that there will be some errors in the codeword. So the codeword the output codeword will have some sensitivity to error which will influence the gain adaptation system. Next time we are going to talk about the step size adaptation system. So accordingly it is more sensitive to error as compared to the feedforward adaptation which we had seen earlier.

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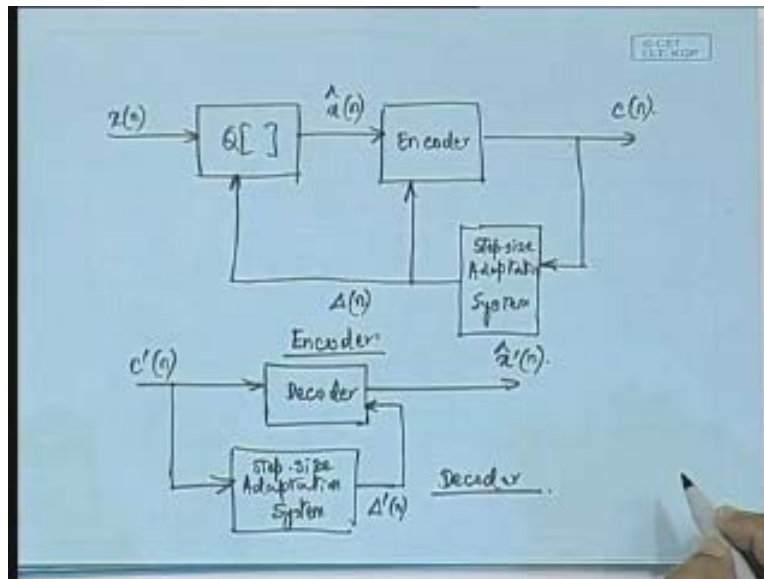
Feedforward adaptation let us see now. This is for the gain (Refer Slide Time: 9:18) and for the **step size adaptation** step size feedback adaptation that would go like this; that x of n will be the input and then we are having the quantizer Q and the output of the quantizer is \hat{x} of n and then we will be having the encoder and then the output codeword we will call as c of n and because it is feedback we are taking it from $c(n)$ and this block instead of the gain adaptation will be the step size adaptation. So this will be the step size adaptation system and the step size adaptation system will decide what? It will decide the delta.

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So now we are going to call that as $\delta(n)$ not as δ because this δ can change from sample to sample δ of n . So this will be the encoder and **on the same sheet I am also drawing the decoder**. So for decoder we will receive c prime of n and then we have the decoder block, the decoder output will be x cap prime n and then from c prime of n we will be deriving the step size adaptation. So here we have the step size adaptation system and the output of that we will now call as δ prime of n and δ prime of n will directly control the decoder. so this is what we are obtaining.

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We will now know the advantage and disadvantage. The advantage is that in this case also you see that there is no need to transmit $\Delta(n)$ into the channel and disadvantage-wise again the step size adaptation system will be more sensitive to error unlike what we would have got in case of the feedforward adaptation. So now you may notice one thing.

What are the different types of quantizers that we have studied so far?

We have studied the mu law quantize.... I mean uniform quantizer of course we all know but uniform quantizer as we already argued that we rule it out for the speech signals because it is not suited for the speech signal quantization. So instead we preferred rather than the uniform quantization we preferred the mu law quantizer. In fact the performance of the mu law seems to be much better as compared to the performance of the optimal quantizer which we talked of. But adaptive is indeed in a sense a better scheme because the adaptive quantizer essentially adapts its step size or the gain parameter dependent upon the variance of the signal. So it makes the quantizer well suited to the variance. This is what the advantage that one can claim.

Now in this process what we are doing is that we are able to achieve some compression in the quantized signal. Compression in what sense? **because you had seen that**..... I mean, in one of

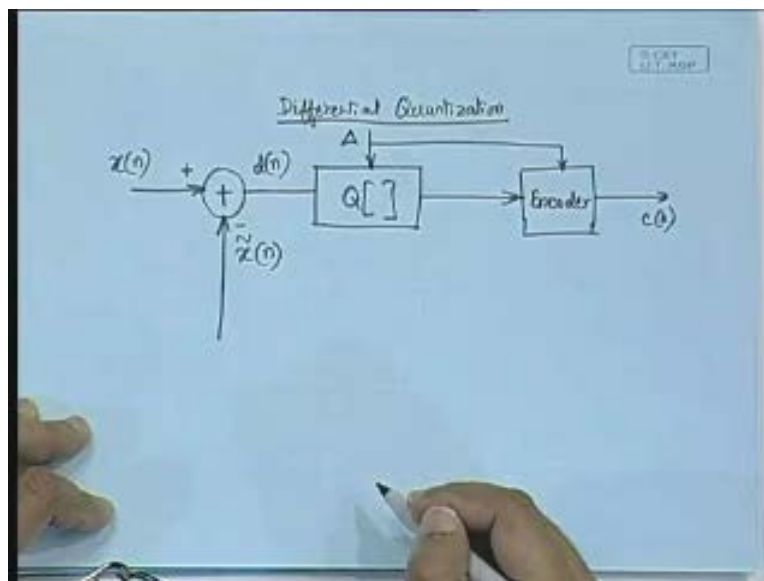
the classes I have given a comparison like this that whereas we have 11 bits, I mean we may require 11 bits for encoding a good high fidelity speech signal in the case of the uniform quantizer then for the same signal to noise ratio we will be requiring only 7 bits for mu law because it is making a better use of the signal variance. The step size being non-uniform is more suited to the distribution of the signal and that is why it is more adaptive to the step size variation and we had seen that instead of 11 bits we were requiring 7 bits.

In fact, in the adaptive quantization also we will be requiring somewhat lower bits may be sometimes 6 or sometimes 7. Now in this case the only fact that we are making use in order to compress the number of bits is that we are making the quantization more suited to the variance of the signal. But at the same time we are not exploiting one major feature of many signals including the speech signal and that is to say that from sample to sample there is a significant amount of correlation. All samples tend to be highly correlated especially in the immediate vicinity.

Now speech signal has got two important characteristics. Firstly, that speech is a relatively slow varying signal. So, slow varying signal obviously means that there will be correlation good correlation between the successive samples. Again we are saying the speech signal is having a quasi-periodicity. So, if you were taking the autocorrelation function which we have already discussed, in the case of autocorrelation function we find that the autocorrelation exhibits a good peak at the periodicity interval. Because of the quasi-periodicity, corresponding to the period we are going to receive some high peaks in the autocorrelation function. So now in this case we are not concerned about the periodicity aspect because periodicity is going to give us the good peaks as per as the autocorrelation function is concerned and from that we had worked in order to estimate the speech period. But in this case what we will be doing is that we will be exploiting the kind of redundancy that is present in between the samples. It is present in all the signals; no matter whether it is speech signal or any other forms of signal but for speech signal also it exists so that is why the inter-sample redundancy can be made use of and this is why we say that the speech signal will be very well suited to what is called as the differential quantization.

Now, in differential quantization the basic principle I believe that all of you will be knowing. But in the basic principle of differential quantization we have a block diagram which would go something like this: x of n and then we have got a summer block and here we are having the predicted signal. This is now \tilde{x} of n that is what we write and \tilde{x} of n is going to be a predicted version of this x of n , the incoming signal. And in this summer block what is being done is that, this signal this \tilde{x} of n is subtracted from x of n so that we get the difference of this and the difference is d of n . So d of n is nothing but x of n minus \tilde{x} of n and then we are going to have the quantizer Q and then we have the encoder and the encoded output. So this is the encoder (Refer Slide Time: 17:57) and the encoded output we are calling as c of n and these two things that is to say the quantizer as well as the encoder will be controlled by the step size which we are calling as the delta, the step size.

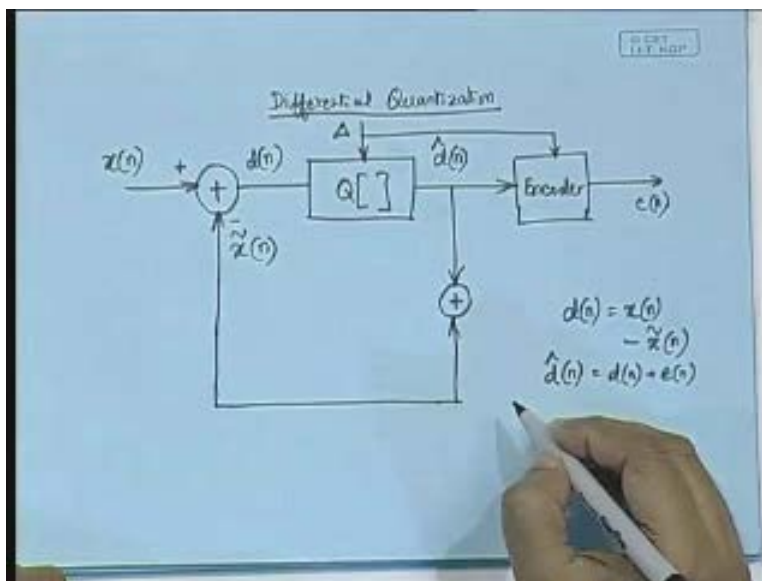
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Now this is d of n and at the output of the quantizer we will call this as \hat{d} of n . So this is the quantized version of the differential signal d of n . So this is \hat{d} of n and now what we are doing is that \hat{d} of n we are putting through a summer block. Now you see that if I add \hat{d} of n with \tilde{x} of n what would you expect to get? \hat{x} of n ; because you see, **now we can write down here very clearly** that d of n is going to be $x(n)$ minus \tilde{x} of n . So definitely d of n if I

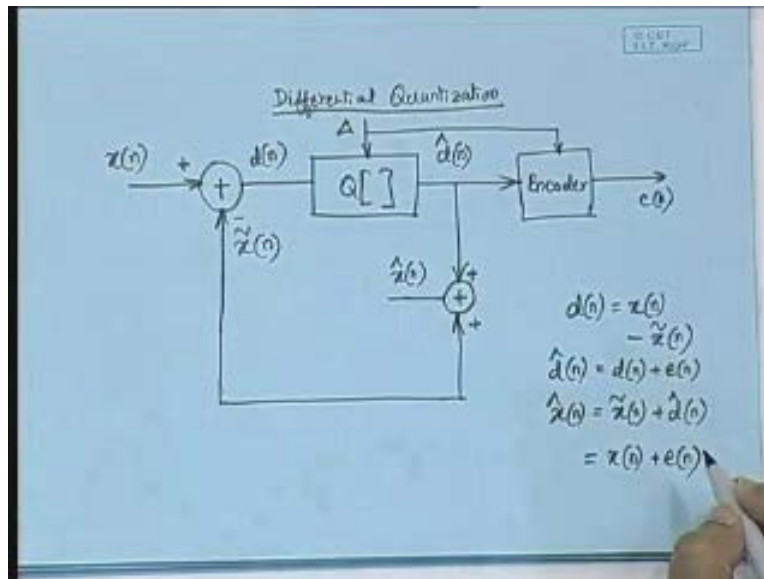
add with $\tilde{x}(n)$ I should get $x(n)$ but I am not being able to get $d(n)$ anymore because there is a quantization already being done. So instead of $d(n)$, because I am getting $\hat{d}(n)$ that is why we are going to get this as $\tilde{x}(n)$ and plus $\hat{d}(n)$ is going to give us $\hat{x}(n)$. **In fact we can write down all these things.**

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So $\hat{d}(n)$ we can write as $d(n) + e(n)$. What is $e(n)$? $e(n)$ is nothing but the quantization noise. Then we can write $\hat{x}(n)$; $\hat{x}(n)$ is nothing but $\tilde{x}(n) + \hat{d}(n)$. This you have already said; so $\tilde{x}(n) + \hat{d}(n)$ so both these will have a plus sign and here at this output we are going to get what we call as the $\hat{x}(n)$. So this is the $\hat{x}(n)$ or in other words the quantized form of this $x(n)$.

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Now this obviously implies, I mean, if x cap of n is x tilde n plus d cap of n just look at this that d cap of n is nothing but $d(n)$ plus $e(n)$ and $d(n)$ is $x(n)$ minus x tilde of n . So you just substitute this directly so what results is $x(n)$ plus $e(n)$. It is as if to say that we have quantized the original signals. Because if the original signal x of n is quantized we will be getting x cap of n and x cap of n we are going to write as x of n plus e of n . So the differential is not altering our basic relationship. Because the basic relationship still remains that the quantized form of signal that is x cap of n can be written always as x of n plus e of n . Then this x of n (Refer Slide Time: 21:23) this is fed to a block which we call as the P and this P is nothing but what is called as the predictor.

Therefore, P is a form of a predictor and based on this x cap of n we are having x tilde of n ; x tilde of n nothing but it is a prediction that is what we are doing; a predicted value. If the predicted value is good, in that case d of n will be a very small quantity so it is a good prediction. This is the overall scheme of the..... this is a generalized scheme of the differential quantizer. There are two very popular forms of differential quantizer which you all must be knowing: one is what is called as the delta modulation. Delta modulation is nothing but where this d of n is quantized to only two steps either to plus delta or to minus delta so it is a two level quantizer

whereas a more final level quantization is achieved by what is called as a the differential pulse code modulation or what is called as DPCM.

Now, in a generalized differential quantization scheme like this one can write down the signal to noise ratio expression in the following way. The signal to noise ratio is the expectation of x square n . This is nothing but the signal power. And the noise power is nothing but expectation of e square n . So this can be written as σ_x^2 upon σ_e^2 and this can be written as σ_x^2 by σ_d^2 into σ_d^2 by σ_e^2 . what is d ? d is nothing but the differential signal.

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$$SNR = \frac{E[x^2(n)]}{E[e^2(n)]} = \frac{\sigma_x^2}{\sigma_e^2} \cdot \frac{\sigma_d^2}{\sigma_e^2}$$

This we can write as a term which we call as G_P into SNR of Q . Because after all what is this σ_d^2 by σ_e^2 ?

You just see; look at this block diagram. To the quantizer d is the input signal and $\hat{d}(n)$ is the output; $\hat{d}(n)$ is nothing but $d(n)$ plus $e(n)$ so $e(n)$ as the noise or the error component, the quantization noise is e of n . So you see that σ_d^2 is the signal to this and σ_e^2 is the noise corresponding to this block diagram. So the σ_d^2 upon σ_e^2 is the quantized is the quantization SNR so this we are calling as SNR with a suffix Q indicating

that it is the signal to noise ratio due to quantization. And what is the other term; so this about the second term and what about the first term? Sigma square x by sigma square d; what is sigma square x? It is the variance of the original signal, and sigma square d happens to be the variance of the differential signal. Hence, there is a gain. If the variance of the original signal is higher as compared to the variance of the differential signal meaning that if sigma square x is larger than sigma square d that means to say that we have this gain that is G P should be greater than 1. So this implies that G P is greater than 1 and G P is greater than 1 means that the overall signal to noise ratio what we are obtaining from the differential quantization scheme will be higher than the normal signal to noise ratio of the quantizer.

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$$SNR = \frac{E[x^2(n)]}{E[e^2(n)]} = \frac{\sigma_x^2}{\sigma_e^2} \cdot \frac{\sigma_x^2}{\sigma_d^2} \cdot \frac{\sigma_d^2}{\sigma_e^2} = \underline{G_p} \cdot \underline{SNR_q}$$

$$\sigma_x^2 > \sigma_d^2 \Rightarrow G_p > 1.$$

So we have to really achieve that this G P should be on the higher side. Now, is it possible?

So, before going into any mathematical treatment let us try to intuitively approach the problem.

Are you really going to expect this argument that sigma square x is going to be sigma square d?

Well, if you think more about it you will realize one thing that if the signal happens to have some correlation between the neighboring samples in that case obviously the sigma square d that is to say the differential signal what we are getting the differential signal is expected to be much smaller and it is expected to have a much smaller variance as compared to the original signal

variance. So as a result we can always expect intuitively that sigma square x should be greater than sigma square d. Better it is more increased value of G P we are going to have and increased value of G P is going to boost our overall signal to noise ratio significantly. So that is what we should expect. Hence, G P we are going to call as the gain due to differential quantization. So this is the gain due to differential quantization. So our objective should be to maximize G P by appropriate choice of the predictor system **maximize G P by appropriate choice of the predictor system**. Predictor system you know; predictor system is P. In our block diagram whatever P we had drawn over here so it is this P we are talking of (Refer Slide Time: 27:53). We have not defined this P yet. P can be defined in many different ways we like. Now, in order to predict this let us see that how the prediction can be made.

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The whiteboard contains the following handwritten text:

$$\text{SNR} = \frac{E[x^2(n)]}{E[e^2(n)]} = \frac{\sigma_x^2}{\sigma_e^2} = \frac{\sigma_x^2}{\sigma_d^2} \cdot \frac{\sigma_d^2}{\sigma_e^2} = G_P \cdot \text{SNR}_Q$$

$$\sigma_x^2 > \sigma_d^2 \Rightarrow G_P > 1$$

GP = gain due to differential quantization.

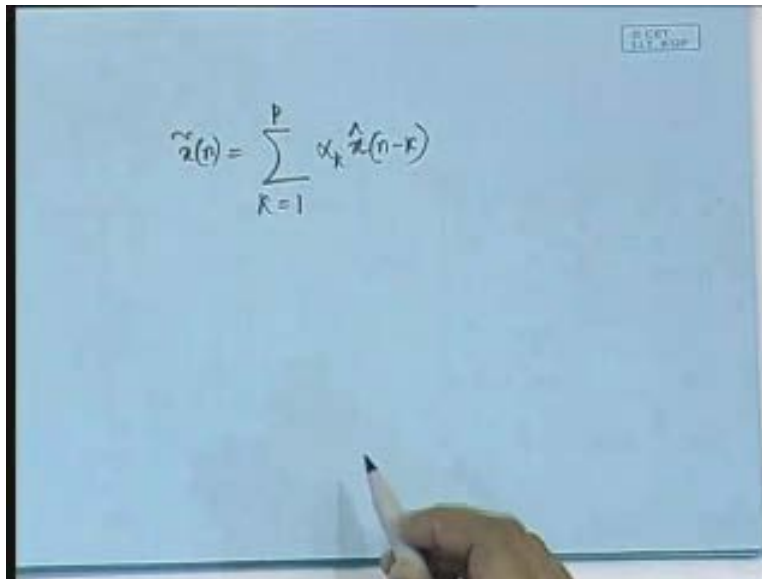
Objective: Maximize GP by appropriate choice of the predictor system P.

To predict \tilde{x} from x cap of n **we can use** the simplest predictor that we can use is what is called as a linear predictor.

Now, what is a linear predictor?

A linear predictor predicts a signal through a linear combination of the past signals. So in what form we can write? We can write it in this manner: $\hat{x}(n)$ this can be written as the summation of $\alpha_k x(n-k)$; k being the coefficient; α_k x cap of n minus k and k is equal to 1 to p .

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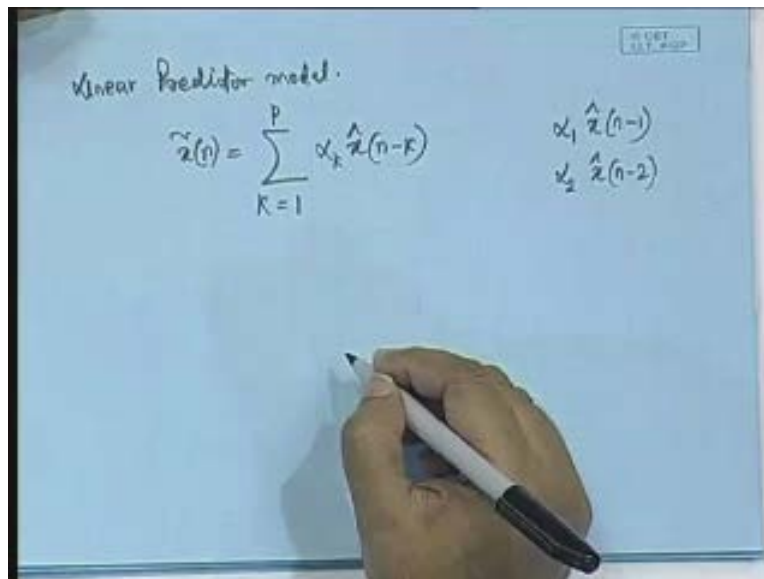

$$\hat{x}(n) = \sum_{k=1}^p \alpha_k \hat{x}(n-k)$$

Now let us have a look at the significance of this equation. When k is equal to 1 that is to say the first term in this summation we have α_1 times x cap n minus 1. What is n minus 1? Immediate previous sample and this is getting multiplied by α_1 . Now, if the signal x of n is going to have a good correlation with x cap of n minus 1 then the value of α_1 should be chosen..... high means how high? Can it exceed unity; it should not. So it should be close to unity. So you can have α_1 in the range of 0.9 or 0.95 0.98 0.99 so that is a good correlation value. Under very good correlation you can say that α_1 should be close to unity but not greater than unity definitely because greater than unity would mean that as if to say that my signal has increased from the previous sample. If it does not exactly become in that case we are going to have more values in the differential signal. So we do not have any difficulty because even if the differential signal is significant we have a means of quantizing that and then sending it into the channel.

Now, what is the second term?

In this the second term is going to be α_2 into $\hat{x}(n-2)$; $\hat{x}(n-2)$ is the sample which is just two samples before. So if the correlation between the present sample and two samples before is relatively less in that case we should have α_2 to be smaller; and why we are restricting this summation series to p **that means to say** is that we are taking the contributions only up to P past samples for all practical purposes. I mean, there are many systems you will see where the value of p is chosen to be quite small. In fact p is equal to 2 you will find systems design with p is equal to 2 for speech which means to say that okay take the contributions of only two channels. Anyway, in general the expression is that up to P samples we are inviting the contribution and the $\hat{x}(n)$ is expressed as a summation series of α_k into $\hat{x}(n-k)$. So this is what is called as the linear predictor model. So this is our linear predictor model.

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Now, if we take the z transform of this then what are we getting?

We are getting P of z to be equal to..... that is to say the z transform of the predicted signal we are writing it as P of z and this will be written as; k is equal to 1 to P α_k into z to the power minus k simply in this case by taking the z transform. So this means to say that **in the overall system** if the overall system has got a z transform H of z then H of z can be written as 1 minus

summation alpha k into z to the power minus k; k is equal to 1 to p so this is going be the z transform the equivalent z transform expression.

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Linear Predictor model:

$$\hat{x}(n) = \sum_{k=1}^p \alpha_k \hat{x}(n-k)$$

$$P(z) = \sum_{k=1}^p \alpha_k z^{-k}$$

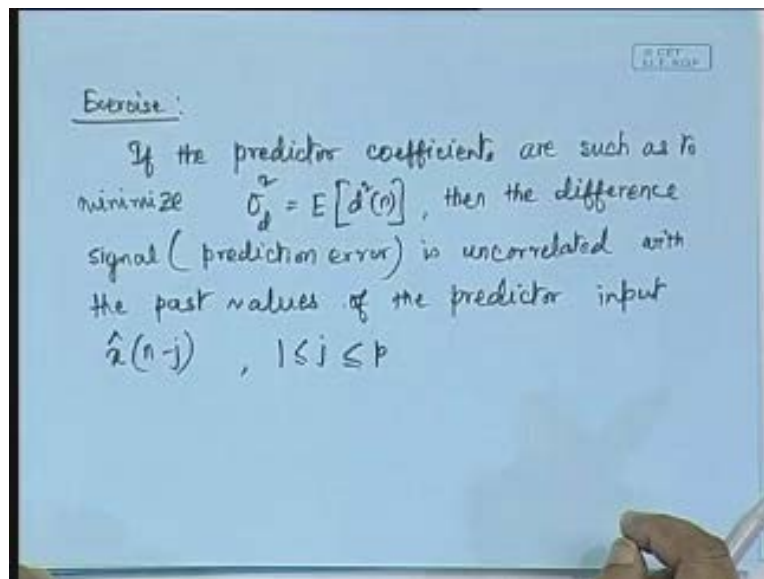
$$H(z) = \frac{1}{1 - \sum_{k=1}^p \alpha_k z^{-k}}$$

$\alpha_1 \hat{x}(n-1)$
 $\alpha_2 \hat{x}(n-2)$

Now I am assuming that many of these theories and expressions you have already done in the digital communication course so **that is why I am just** just to ensure that you already have the knowledge of this I am giving you a small assignment as an exercise which you can just try it out and the exercise is like this. What we want is that..... so as an exercise you please try to solve this. if you have any difficulty please refer to any standard book on Digital Communication and you will be able to solve it out. So this is.....

If the predictor coefficients **if the predictor coefficients**; predictor coefficients means in the expression that we have written down already there alpha k's are the predictor coefficients. So if the predictor coefficients are such as to minimize the signal variance that is sigma squared d the differential signal variance which is given as the expectation of d square n, **then the difference signal** then the difference signal that is to say **the prediction error** the prediction error is uncorrelated with the past values of the predictor input x cap n minus j for j lying between 1 and p.

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You may try to solve this. This will be fairly easy to solve. So, you take the linear predictor model and then you know that how to obtain the difference signal. The difference signal is nothing but $d(n)$ is $x(n)$ minus $\hat{x}(n)$ that is the difference signal. So you take the square of the differential signal and then the expression for σ_d^2 what you obtain that you have to differentiate with respect to the predictor coefficients; that is to say differentiate that with respect to α_1 α_2 up to α_p and then you will be able to prove this. So your exercise will be to prove this. So just try it out and that would really brush up your concepts pertaining to Digital Communication especially about dealing with the differential signals.

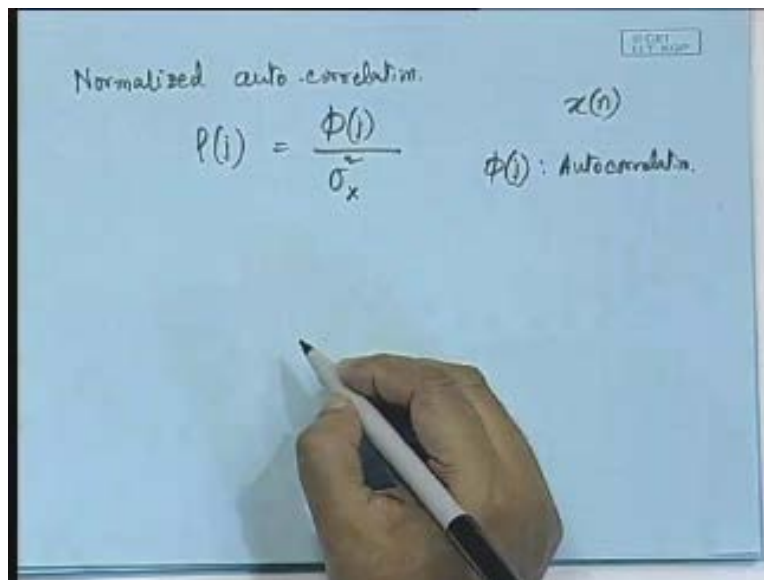
Now, without going into much of detailed mathematical analysis which you can always refer to the books, let me just give you the final results what one obtains pertaining to this.

see the autocorrelation if we call the autocorrelation as $\phi(j)$ j being the lag; so if the input sequence is $x(n)$ and we are obtaining the autocorrelation of this input sequence $x(n)$ we are denoting it by the function ϕ and $\phi(j)$ means that with a lag of j . So if we take j is equal to 1 that means to say with a lag of one sample; lag of one sample means that what is the autocorrelation with the immediate past sample. Because we are exploiting this fact that for the

case of differential quantization we can do the differential quantization very effectively when there is substantial correlation between the immediate past sample and the present one. So we can definitely expect that under such cases ϕ of 1 should be of a very high value; ϕ of 2 will be also somewhat high value but not as high as compared to ϕ of 1 and then the correlation value should decay with increasing value of j .

Again it will show up some increase when you have the quasi-periodicity; when you have the..... when you go to the periodicity of the signal there you will again observe higher values of ϕ of j but for this application j is equal to 1 2 that should be sufficient. And instead of defining the autocorrelation we divide the autocorrelation by the signal variance σ_x^2 and call this quantity as ρ of j . So ρ of j we are calling as the normalized autocorrelation. So this is called as the normalized autocorrelation. So ϕ of j is the autocorrelation and ρ of j is the normalized autocorrelation.

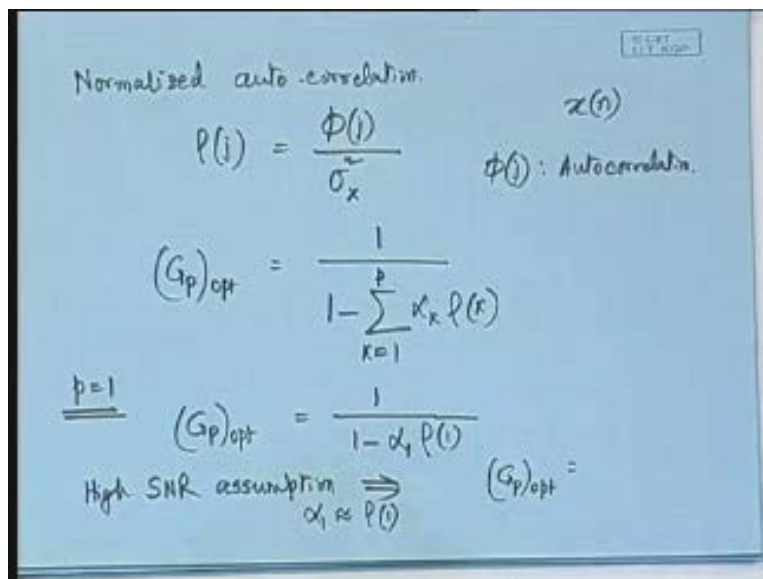
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Now it can be shown by analysis that the optimal value of the G P; G P remember, G P is the gain due to the differential coding, so (G P) optimal is given as $1 - \sum_{k=1}^p \rho(k)$; k is equal to 1 to p , this is the value of (G P) optimal. Therefore, now we take a

very simple case. We take P is equal to 1, P is equal to 1 means where we are considering the contribution from the immediate past sample only; we are not considering the contribution from even the second past sample or beyond it. So, under the case of P is equal to 1 we can have a more simplified expression of the (G P) optimal and that is given as 1 minus..... in this case we are going to call it as alpha 1 so alpha 1 and rho of 1 **rho of 1**; so rho of 1 is what? What we were talking about the autocorrelation? phi of 1 is the autocorrelation with a lag of 1 immediate one sample and rho of 1 is the corresponding value of the normalized autocorrelation. So this is given by (G P) optimal is given by 1 upon 1 minus alpha 1 rho 1.

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Normalized auto-correlation.

$$p(l) = \frac{\phi(l)}{\sigma_x^2} \quad \phi(l): \text{Autocorrelation.} \quad x(n)$$

$$(G_p)_{\text{opt}} = \frac{1}{1 - \sum_{k=1}^p \alpha_k p(k)}$$

p=1

$$(G_p)_{\text{opt}} = \frac{1}{1 - \alpha_1 p(1)}$$

High SNR assumption $\Rightarrow \alpha_1 \approx p(1)$ $(G_p)_{\text{opt}} = \frac{1}{1 - p(1)^2}$

Now, **under now now** it can be shown that for high SNR assumptions so under high SNR assumptions one can mathematically obtain an approximated expression that G P of optimal will be given by..... I mean, under high SNR it can be shown that alpha 1 is approximately equal to rho of 1. This is intuitively okay because sometimes back only I was telling you that if a good correlation exists between the immediate past samples, we should have larger value of this alpha 1. So alpha 1 we make it as rho 1 which is basically indicative of the **correlation** normalized correlation in this case. Therefore, taking alpha 1 approximately equal to rho 1 we can approximately write down (G P) optimal as 1 upon 1 minus rho square 1 because we just

substitute **alpha is equal to** alpha 1 is equal to rho 1 into this expression (Refer Slide Time: 41:54) **can you read it** so this is 1 upon 1 minus rho square 1.

(Refer Slide Time: 42:00)

Normalized auto-correlation.

$$\rho(l) = \frac{\phi(l)}{\sigma_x^2}$$

$z(n)$
 $\phi(l)$: Autocorrelation.

$$(G_P)_{opt} = \frac{1}{1 - \sum_{k=1}^p \alpha_k \rho(k)}$$

$p=1$

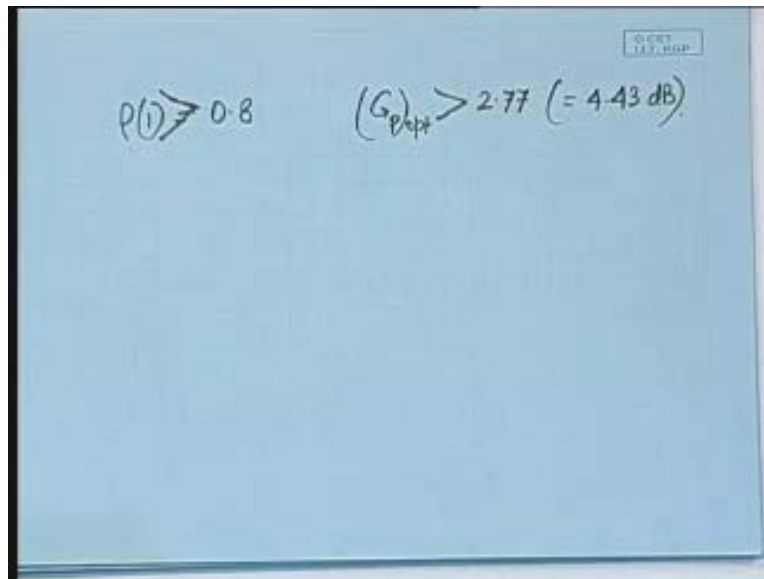
$$(G_P)_{opt} = \frac{1}{1 - \alpha_1 \rho(1)}$$

High SNR assumption $\Rightarrow \alpha_1 \approx \rho(1)$

$$(G_P)_{opt} \approx \frac{1}{1 - \rho^2(1)}$$

Now a typical value of rho 1 if we say that rho 1 is equal is greater than let us say 0.8; okay if we take a value of 0.8 then we will be seeing that the corresponding value of (G P) optimal (G P) optimal is to be computed as 1 upon 1 minus rho square 1. That means to say 1 by 1 minus 0.64 so 1 by 0.36 and if you are expressing the G P like that then we can show that correspondingly G P should have a value which is greater than 2.77 and 2.77 actually corresponds to; in dB scale it correspond to 4.43 dB. So you can see that this is what we are gaining by utilizing the differential quantization.

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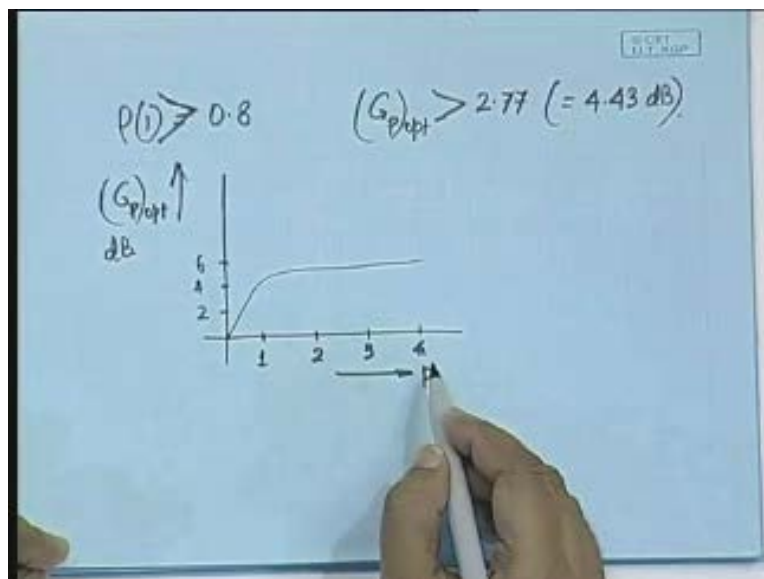
The image shows a whiteboard with two handwritten equations. The first equation is $\rho > 0.8$. The second equation is $(G_P)_{opt} > 2.77 (= 4.43 \text{ dB})$. There is a small logo in the top right corner of the whiteboard that reads "FOCUS 11.11.11".

In fact **rho value** for a perfect correlation ideal correlation $\rho = 1$ is going to be equal to 1 in that case you are going to have the (G_P) optimal as infinite. But that is of course a hypothetical case. But for all practical purposes we will be having $\rho = 1$ value in the range of 0.8 0.9 0.95; I mean, better the correlation is then intuitively one can see that with better and better correlation definitely we are going to gain further **by having** by utilizing the differential mode because in that case the differential signal variance would be much lower and that would in fact boost up the G_P that is to say the gain that we are obtaining. So gain of 2.77 or 4.43 dB this is also quite significant.

Now **you can see** you can just compare that just without increasing any bit we are able to obtain a gain of 4.43 dB whereas in the case of uniform quantizer, in order to improve the SNR by 6 dB we have to pay the price of adding 1 bit. So each bit of increase results in the overall bit rate increase obviously but it boosts the signal to noise ratio by 6 dB and in this case 4.43 dB is quite pessimistic. In fact if you are taking $\rho = 1$ to be in the range of 0.9 because that type of correlation quite often exists then without increasing the number of bits you can have 6 dB of improvement just by switching over from the normal quantization to the predictive quantization or what is called as the differential quantization.

Now a typical curve would look something like this (Refer Slide Time: 45:29) that if we draw a curve; say on this axis we have got P and then we take P as 1 here, 2 here, 3, 4 like this and here we have **(G P) optimal** (G P) optimal as 2 4 6 etc say 2 4 6 so this (G P) optimal is in dB; then one obtains a curve something like this for practical cases; for practical examples one obtains a curve like this. Therefore, you see that by taking P is equal to 0, now P is equal to 0 means what? P is equal to 0 means you are not taking any prediction; you are not using the differential quantization at all; P is equal to 1 means you have considered only up to one sample; P is equal to 2 may be marginally better but one thing is very clear that it is not significantly better.

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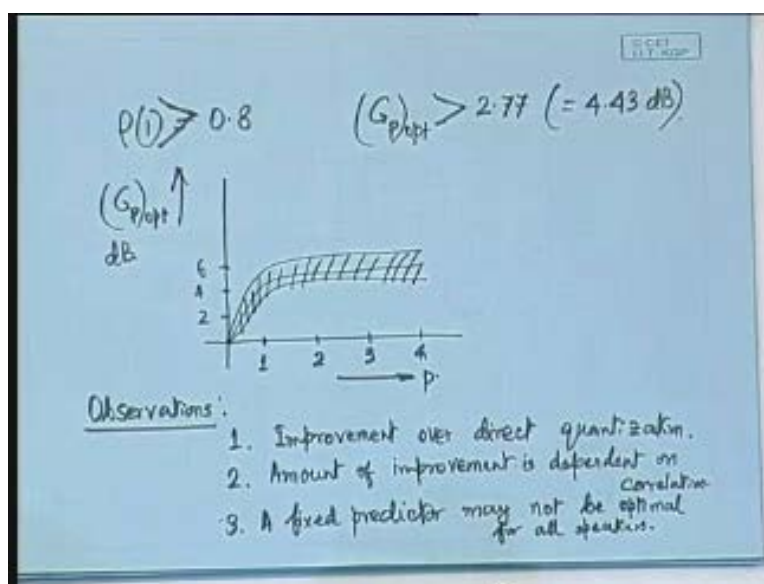


Therefore, if we go in for higher and higher values of P which means to say that going in for higher orders of predictions we are only marginally improving the gain and in fact there is no point in having higher order predictions because we will be unnecessarily complicating our design of the predictor because you will be needing more amount of memories and you will be needing increased complexity in the circuit so P is equal to 1 or 2 at the most is good enough. In fact most of the times you will be finding P is equal to 1 only and the range of values may lie, I mean, this curve **what I have drawn over here** (Refer Slide Time: 47:44) may lie somewhere in a band like this. So the best case improvement may be of the order of 6 dB; the worst case

improvement may be of the order of 3 dB and somewhere here **the thing will be there** the G P value would like something over there so what are our observations about the differential quantization?

Now our first observation is that the differential quantization yields improvement over direct quantization. So the first observation is definitely we conclude that it offers improvement over direct quantization and on what factor is the amount of improvement dependent upon; what is the major factor? Correlation; so, amount of improvement is dependent upon correlation and another aspect that should be shown is that a fixed predictor cannot be optimal for all speakers; this also is experimentally observed; a fixed predictor may not be optimal for all speakers.

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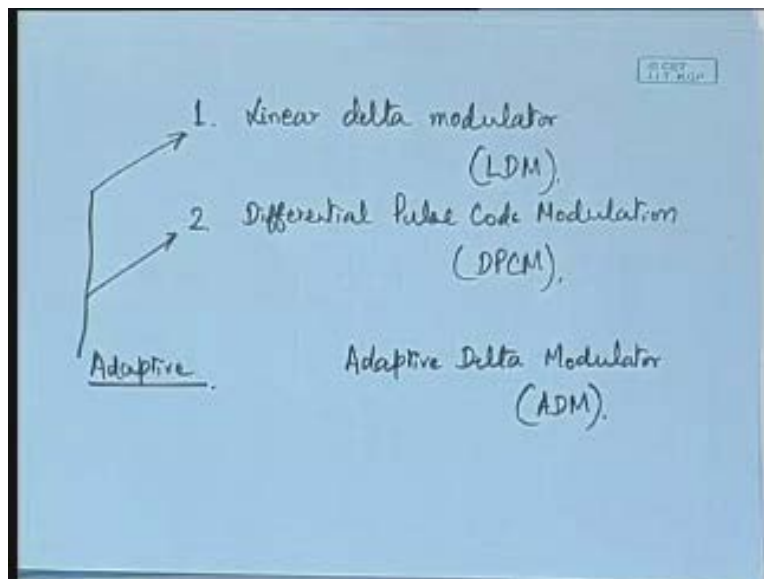
So what I mean to say is that you may be finding that for some speakers it may make some sense in taking two past samples also; means P is equal to 2 may be required for some speech applications; P is equal to 1 may be preferred for some speech applications and again the correlation; so the value of rho 1 rho 2 or alpha 1 alpha 2 that may vary quite a lot from speaker to speaker so that is why it is difficult, it is the characteristics of the speech that we may not able

to use a fixed predictor optimally for all different type of speakers. So that really tells us in a sense about the aspects of differential quantization.

We will be studying about two different differential quantizations: One what we already know is the delta modulator and we will be using the linear delta modulator and we are going to call that as the LDM; and the other is the **differential pulse code modulation** differential pulse code modulation which is a multi-level quantization for the differential signal. So differential pulse code modulation in short form it is called as DPCM.

And in fact just like the way we had seen that a fixed predictor is not going to be optimal so what is the solution? Even in the differential quantization also we have to try our adaptive concepts. So both LDM and DPCM should be made adaptive and that is what we are going to study that how to realize what is called as the adaptive delta modulator. Adaptive delta modulator in short form it is going to be called as the ADM.

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So in the next class we will be talking about LDM very briefly; more about the ADM and DPCM just to give you an idea that how this adaptive concept..... so likewise in the.....

like adaptive delta modulator we are also going to have adaptive DPCM or what is called as the ADPCM. So all these things we will be studying in the next class so till then thank you.