Digital Voice and Picture Communication

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Lecture - 05

Mu Law and Optimum Quantizer

Today we are going to continue our discussions on the quantizer for speech signal and especially we will deal with the mu law and optimum quantizers.

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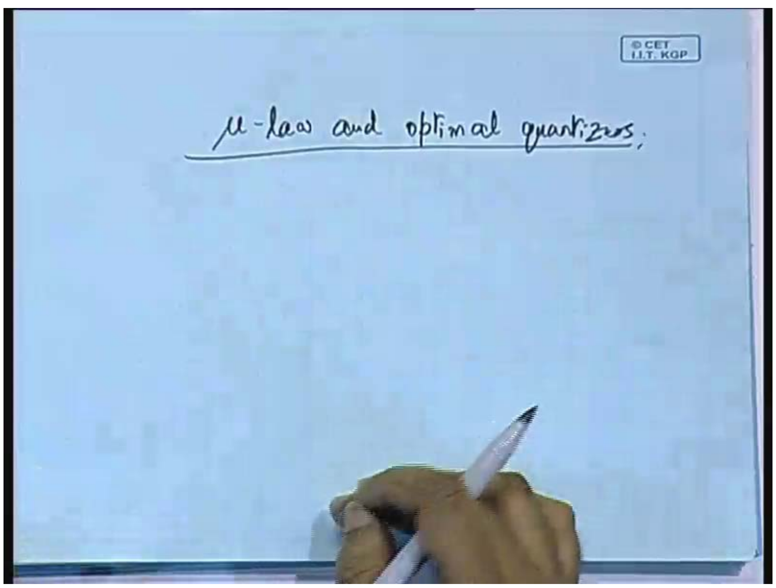

Now we have described in the last class the logarithmic encoder and the decoder and there you must have noted that in the case of logarithmic encoder and decoder the signal to noise ratio is not dependent upon the variance of the signal and it is only dependent upon one upon sigma epsilon square which means to say that only the steps size is the deciding factor. So this really makes it a very good candidate for designing the quantizers for speech signals and we are going to consider this. In fact one of the difficulties which one faces with the logarithmic encoder decoder combination is that the logarithmic encoder decoder essentially has got a very large dynamic range. And in fact the dynamic range here happens to be infinite and that is why we have to design with very large number of step sizes if we have to make it practical. So it is not actually practical. We have to really modify the scheme.

And also, another aspect which we have noted in the last class is that we are providing some approximation to the logarithmic function. In fact you have seen that the exponential approximation we are doing, only that is leading to the derivation that the signal to noise ratio was independent of sigma x square.

Now we are going to have another logarithmic encoder decoder where the transfer characteristics are slightly altered. in fact what we have is some kind of a log diagram like this where the input will be the x(n) and then we will be having a transformation function which we call as F which will be a logarithmic function basically and then this will be followed by the quantizer which we are calling as the Q function and then we are following it up with the encoder and the encoded output will give us c of n and there will be the step size so the step size which is the delta that will be controlling both the quantizer as well as the encoder.

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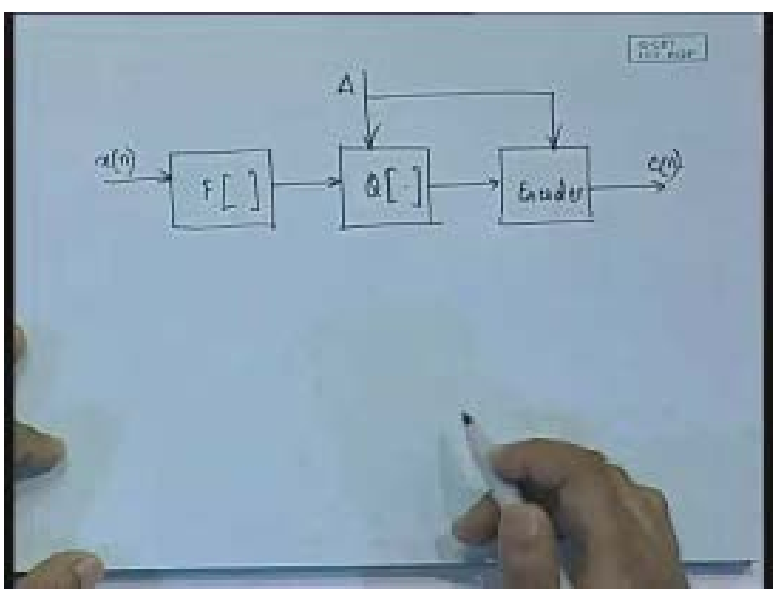

and in fact this The corresponding decoder will be that the input will be the c(n) and this will be followed by the decoder and then we are going to have the inverse transformation. So whatever F we are using, we have to use F inverse of this and that will generate the x cap of n which is going to be our reconstructed signal. So x of n is the original input and x cap of n is going to be the reconstructed output at the decoder end. So now we have to see what is the F function one can use instead of using the direct logarithm. Earlier we were saying that we will be using log of the Modulus of the x(n) and we will be also deriving the sign of the x(n); but instead, now we are going to consider a function which will take care of the deficiencies like utilizing the dynamic range as well as the approximation to the logarithmic characteristic.

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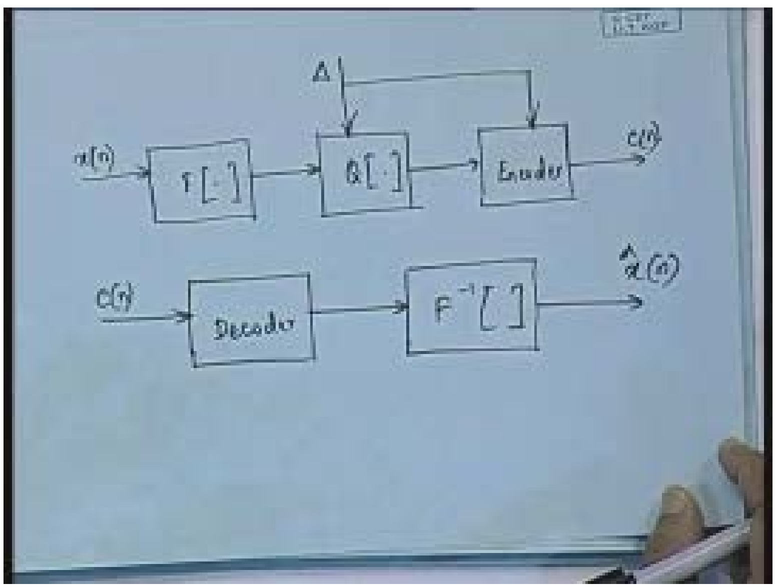

So the transfer function or rather this transformation if that is what we design is like this that we say y of n....... in fact y of n will be nothing but the output of this F (Refer Slide Time: 5:50) so here we have the y of n available as the output of this transformation log and we are going to write y of n as F of x(n) and this is given by the x max which is going to be the maximum amplitude of the signal, maximum possible amplitude and this multiplied by log of 1 plus mu into mod of x(n) divided by x max and this upon log of 1 plus mu and this to be multiplied by the sign of x(n) where sign of x(n) we already said that will be equal to plus 1 if x(n) happens to be positive and minus 1 if x(n) happens to be negative. So this is the expression and this is actually called as the mu law. So this is......... as per the mu law characteristic, the transformation F would be like this.

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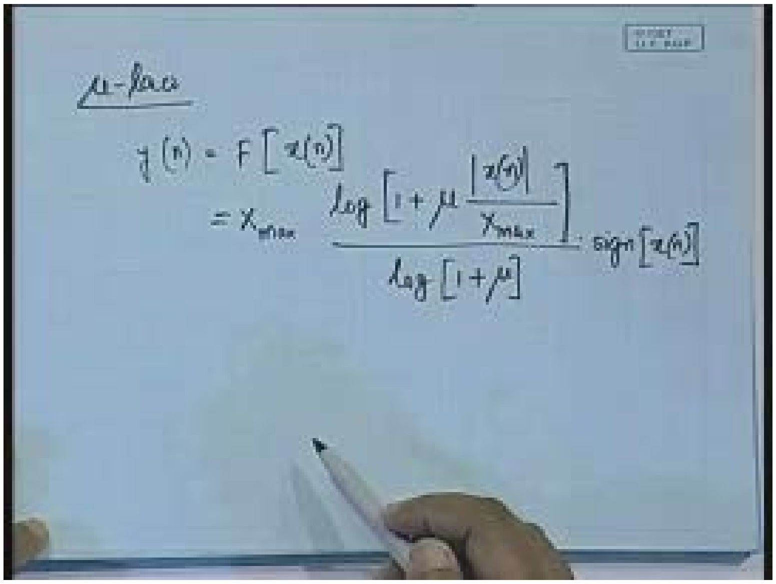

Now you can examine this equation y(n) is equal to x max multiplied by this expression, let us call that as equation 1 and you can see that if we substitute in this equation mu is equal to 0, but a direct substitution of mu is equal to 0 would mean that we get a 0 by 0 form so if we mathematically manipulate that by taking the derivative of the numerator and the denominator then what we derive is simply this that, you can derive that for mu is equal to 0 we are going to have y(n) to be equal to mod of x(n) into sign of x(n). This x max term gets cancelled ultimately. Here there is one x max term over here (Refer Slide Time: 8:16) and here there is another x max term. This x max term will ultimately come to the denominator if you see so that ultimately you will be left with mod of x(n) multiplied by sign of x(n) which is nothing but x(n). So y(n) is equal to x(n).

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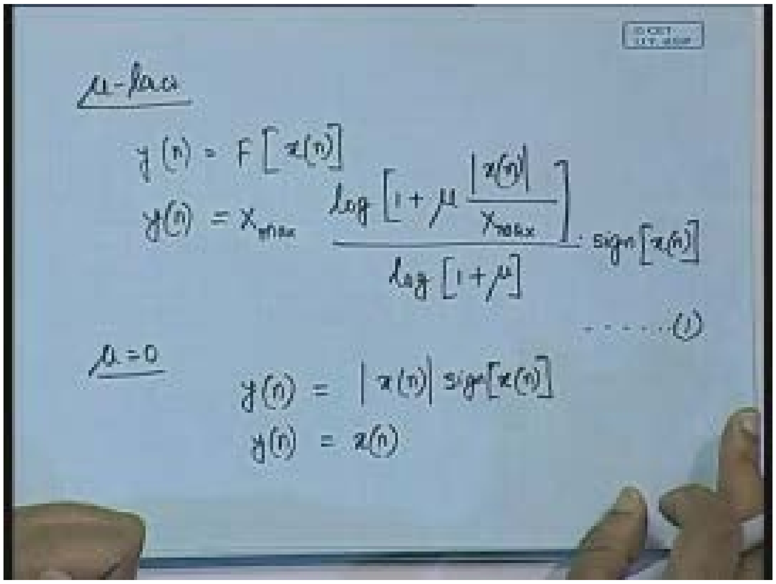

So, at mu is equal to 0 the characteristic that we will be obtaining if we happened to plot y(n) versus x(n) then the characteristic will be a completely linear characteristic that would pass through the origin. So it would be exactly like this (Refer Slide Time: 8:54) in fact with a 45 degree slope. So, if you are taking x(n); x(n) values if you are plotting in this direction and y(n) values you are plotting in this direction then x(n) is going to be equal then y(n) is going to be equal to x(n) so this is the curve that you will be having for mu is equal to 0. So somewhere at this point let us say that we have the maximum possible value of x(n) that is x max. So, when x(n) is equal to x max; likewise the y(n) also will be equal to x max according to this.

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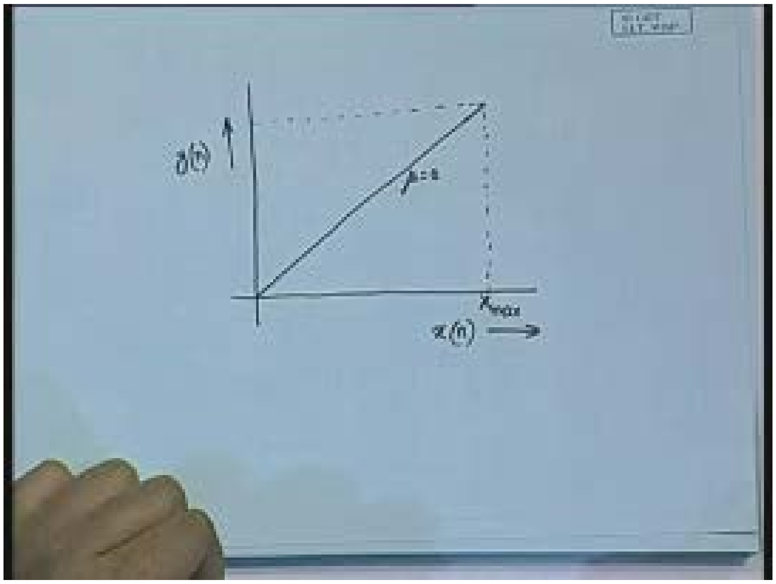

Now, if we go by this equation (Refer Slide Time: 9:36) and then try to plot the y(n) versus x(n) curves for different values of mu, we will be obtaining it like this. In fact this was for the case of mu is equal to 0 and then we can go to another extreme that is for very large mu. For large mu large values of mu if we plot this equation and also if we take the larger value of x(n) means x(n) which is more closer to x max as compared to x(n) closer to x(0); so when we take large mu and large x(n) then large mu and large x(n) would give rise to an approximation in the value of y(n) and we can write that mod of y(n) is equal to is approximately equal to x max into log mod x(n) upon x max.

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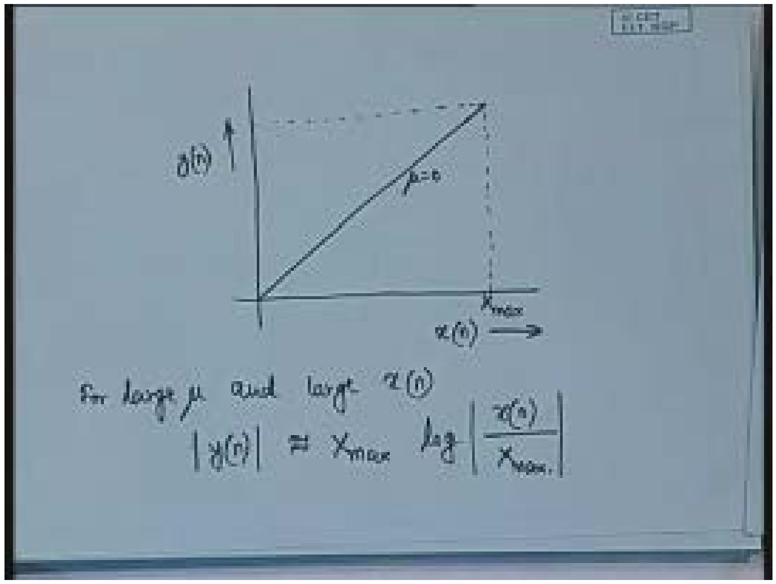

This is for large mu (Refer Slide Time: 11:02) and in fact if we directly go by the plot corresponding to this equation that is equation number 1, we will be obtaining the characteristic as something like this that for increased value of mu what do we expect? Should the curve be like this or should it be like this? What you feel? Should it go like this or should it go like this? Because this will approximate the logarithmic characteristics more so this will be upward so we will be having a characteristic like this for......... let us say if we take mu is equal to 5 we may find a characteristic which will lie above this mu is equal to 0 characteristic. So mu is equal to 0 is actually the case of no compression; you see that there is no signal compression that is happening because whatever is the value of x(n), y(n) is having the same value whereas here you can see that for these values of x(n) you will be finding that the incremental change in y(n) is much smaller and if we increase the value of mu further then we will be observing the characteristic like this. So we may observe........... let us say for mu is equal to 50 we may observe a curve like this; for mu is equal to 255 we may be having a curve like this (Refer Slide Time: 12:41). So this will be for the large values of mu we will be having like this.

So this is a typical characteristic that we will be obtaining. So this is x max, so we will be obtaining for y(n) versus x(n) for different values of mu.

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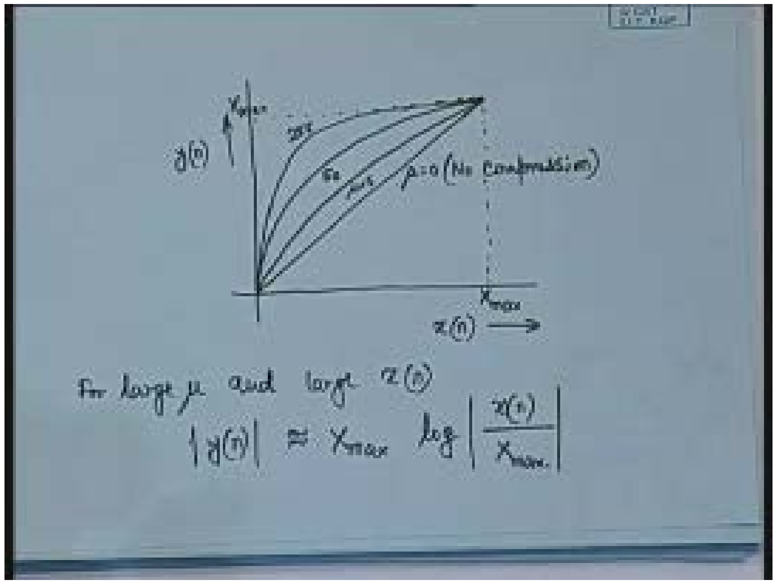

Now, in terms of the characteristic one can observe that if we are plotting the value of x cap n versus x(n); so x cap n means that at this point (Refer Slide Time: 13:25) when we quantize it so this was y(n) so y(n) is after the transformation and after the quantization this we are calling as the x cap of n so this x cap of n if we plot against x(n) then the characteristic will be something like this that we will be having (Refer Slide Time: 00:13:57 min); if you plot x(n) in this direction and plot x cap n in this direction then we will be observing a nature very similar to what we had just shown for y(n) versus x(n) because it is ultimately the y(n) which will be quantized not the x(n) directly and for different values of x(n) rather to say corresponding to the decision levels.

So, if we mark the decision levels corresponding to x(n) then we will be observing; suppose this is x 1, this is say x 2, this is say x 3 and this is say the value of the ultimate x max, then we will be observing that the value of x cap would be something like this that here we will be observing the x 1 cap so x 1 cap would go like this and then we will be observing this to be the x 2 cap, this will be the x 3 cap then finally there may be some more intermediate levels like this but if it is just having let us say 1 2 3 four levels on this side...................in fact this will not be four levels, this will be eight levels; why? Why eight levels is because the signal is a bipolar signal so x of n can have samples which are positive, can have samples which are negatives. So the actual range will be minus x max to plus x max.

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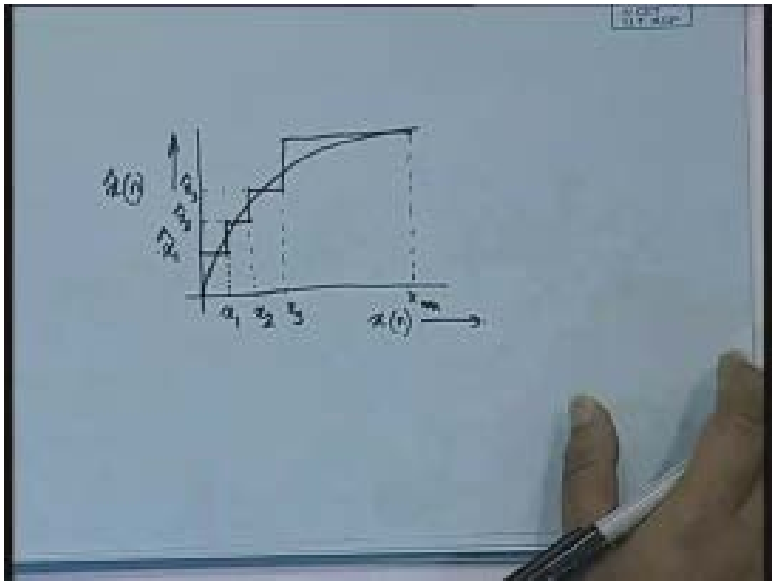

Since we are having four levels; in this example since we are having four levels corresponding to the positive signal we will be correspondingly having four levels for the negative signals as well and this characteristic will be anti-symmetric about the origin. So if we just make an anti-symmetric version of this curve and then we plot x(n) over its range from minus x max to x max and just extend the characteristic on the negative direction with an anti-symmetry then what results is eight different quantization levels that is what we will be obtaining.

Now with a characteristic like this, I mean, when we have x cap n versus x(n) being plotted this way now we can derive the expression for the signal to noise ratio in a manner very similar to what we did earlier for the uniform quantizers. For uniform quantizers, as you remember, we had obtained the signal to noise ratio which I am just reproducing for your reference that the signal to noise ratio was computed like this: (Refer Slide Time: 17:35); for uniform quantizers for uniform quantizer we had obtained the SNR in dB which was equal to 6B plus 4.77 minus 20 log to the base 10 of x max divided by sigma x and for mu law quantization what we are describing here. If we are following the similar derivation approach then one observes the SNR dB to be like this: SNR expressed as dB is equal to 6B plus 4.77 so after this the terms are equal but not beyond this. After this there will be a term that is 20 log to the base 10 1 plus mu minus 10 log to the base 10 and this will be 1 plus x max upon mu sigma x; this term square (Refer Slide Time: 19:08) plus root 2 into x max upon mu sigma mu sigma x.

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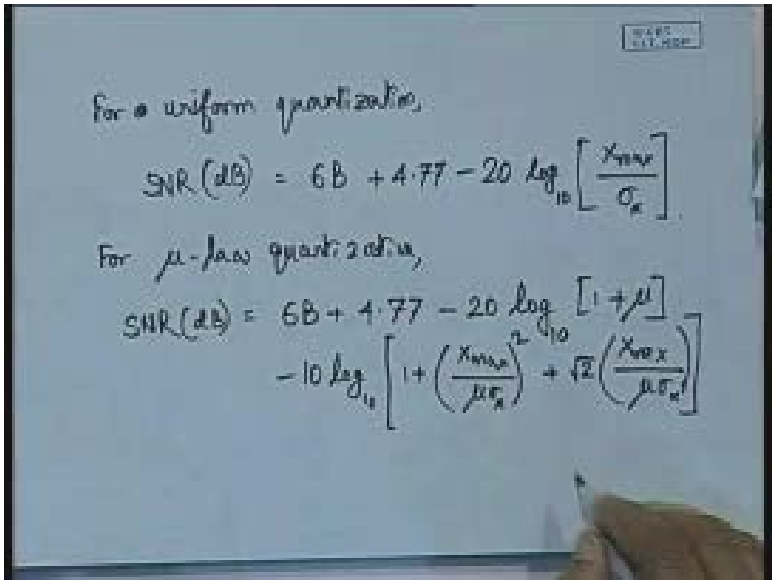

This is going to be the total expression you can see you can just compare these two equations and here you will be finding that here there is a direct term 20 log of x max by sigma x. In fact this is the term which we criticized (Refer Slide Time: 19:49) because we had said that if sigma x happens to be small; if the ratio of sigma x by x max that happens to be small in that case we are going to have a reduction in the signal to noise ratio.

Now, in this case also there is a reduction firstly that there are two terms which are contributing to the reduction. One is this minus 20 log of 1 plus mu but for a particular design of the quantizer mu is fixed so this is a constant quantity which will be subtracted. We are not bothered about that but we are more bothered about the term that is signal dependent. There is a sigma x term so you can see that x max by mu sigma x square but if this is a small quantity in this case there is a square term which is coming over here so this is further more reduced and you can see that this term is much less sensitive to this variation in x max by mu sigma x. In fact mu also happens to be a larger quantity. So if you are finding, so as a result of this the term that is going inside this logarithmic argument that term is not very sensitive. Therefore, as a result of that if we happen to plot the signal to noise ratio from this equation, if we plot signal to noise ratio against the quantity x max by sigma x in that case we will be finding more or less a constant, I mean, much less variation will be found whereas in this case we will be observing a fall although it is logarithmic fall.

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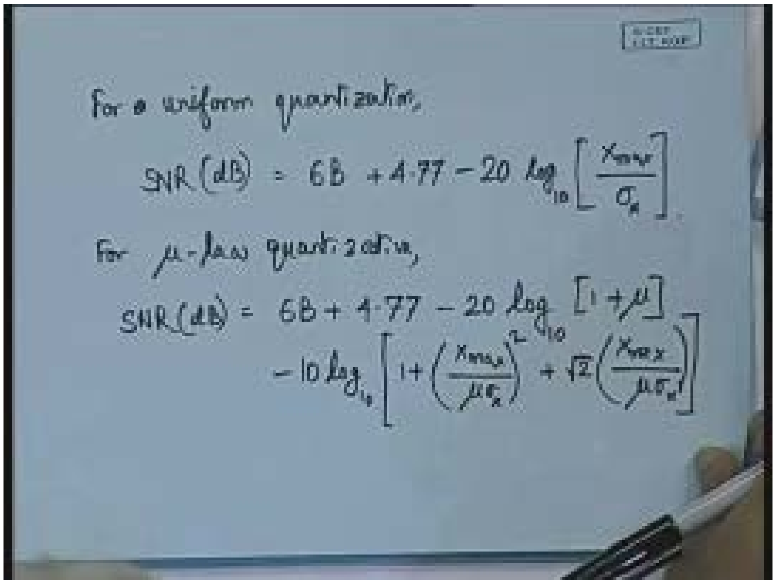

So now better to plot the two situations that is to say uniform quantizer and mu log quantization we can plot, on semi-log scale if we plot where this x max by sigma x will be plotted on the horizontal axis which will be on the logarithmic scale then the SNR in dB if we change it.........., so here if we happen to just take the........... if we happen to plot this then we will be observing a characteristics that is very similar to this. So we will be having 10 let us say that here we have 100 and here we have 1000 because we are going to plot it in the logarithmic scale so x max by sigma x we plot it like this and then the SNR is plotted this way (Refer Slide Time: 23:33). then we will be having, for B is equal to 5 let us say; see here you see (Refer Slide Time: 23:45) that it is dependent still upon the B that is the number of bits; so here it is 22 and we can see that it is like this. So this is for B is equal to 5 and then we will be observing that B is equal to 6 would be something like this; this is for B is equal to 6 and for B is equal to 7 this starts from 34 dB onwards and then we are observing it like this. So this is for mu is equal to 100 and we can see that here the variation is something of this nature that for B is equal to 5 then for B is equal to 6. Therefore, this is for the case of the uniform quantization. So this will be the characteristics for B is equal to 7. These solid lines are all for the mu law.

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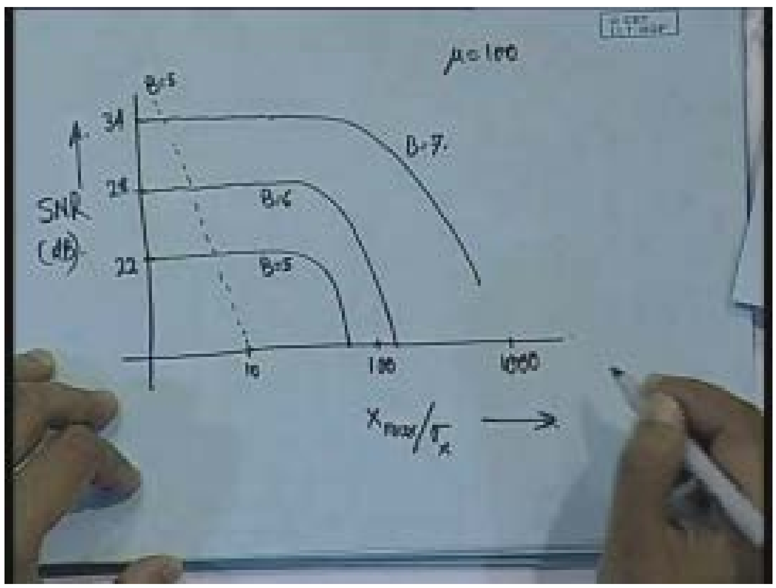

Now, for the uniform quantization, see, again because we are plotting in the logarithmic scale and this also is in the logarithmic scale (Refer Slide Time: 25:30) and the expression of SNR also is logarithmically changing, so we can expect a straight line for the uniform quantization. So you can see that B is equal to 5 characteristic would be like this and B is equal to 6 characteristic may be like this, B is equal to 7 characteristic could be like this and B is equal to 11 characteristic could be like this. So all these dotted lines are for uniform quantization.

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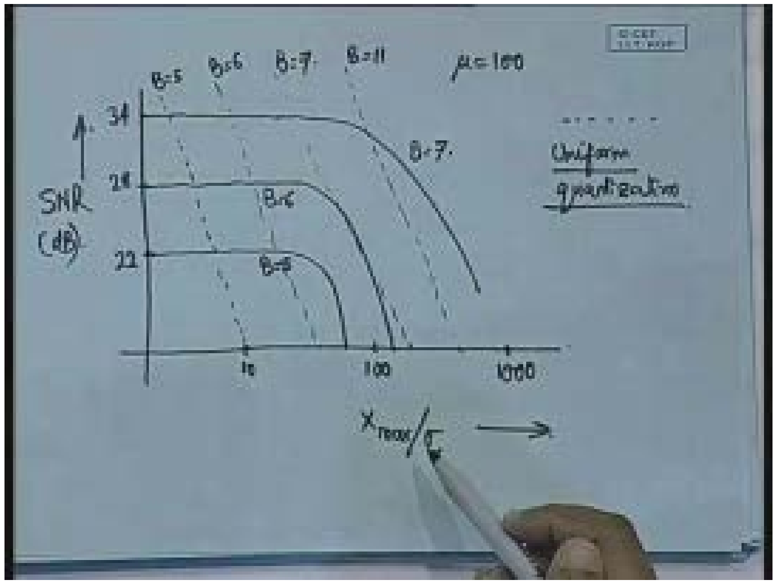

Now you can see what is the conclusion that we can draw out of it; that for lower value of x max by sigma x, lower value x max by sigma x means where the value of sigma x is closer to x max, means for very large signal variants we can see that the uniform quantizer is having a much more value of the signal to noise ratio; it is having a much better signal to noise ratio.

Just look at this point (Refer Slide Time: 27:12) here for B is equal to 5 and for a very low value of maybe this 2 or 3 will be the value of x max by sigma x over here, you can see that the B is equal to 5 curve happens to be at a much higher level as compared to the B is equal to 5 curve for the mu law. So definitely it leads to the conclusion that as if to say this is better. But again you see that for the variation of sigma, once it crosses this point may be that this is with x max by sigma x at a value of 7 and beyond that we are finding that this uniform quantization curve falls rapidly so in this case for B is equal to 5 the mu law characteristic is giving a much better SNR. It is more or less constant. It is only beyond certain stage that we are finding that for further larger value of x max by sigma x we are going to have a falloff. In fact this is quiet expected also from this equation (Refer Slide Time: 28:29).

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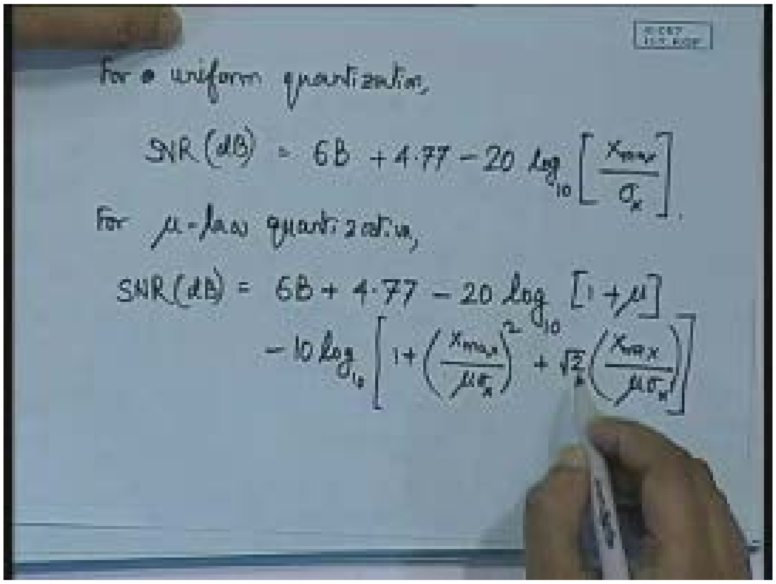

You can look at this equation that if this term......... if x max by mu sigma x that happens to be sufficiently large only then there will be some fall in the characteristics. This is what is happening for the mu law quantized version. But very interestingly you can see that for a reasonably higher value of x max by sigma x which is typically expected you can see that the value of the so you can see that the value is equal, the SNR is equal at this stage that is to say that near about 100 we are finding that the SNR is almost the same. So you can see that whatever value we are expecting with only 7 bits, I mean, for mu law with only 7 bits we are having the SNR to be equal to that of B is equal to 11. So this suggests that the characteristic will be much so i mean This really suggests that for the case of mu law we can go in for much lesser number of bits as compared to what we will be doing for the uniform quantization.

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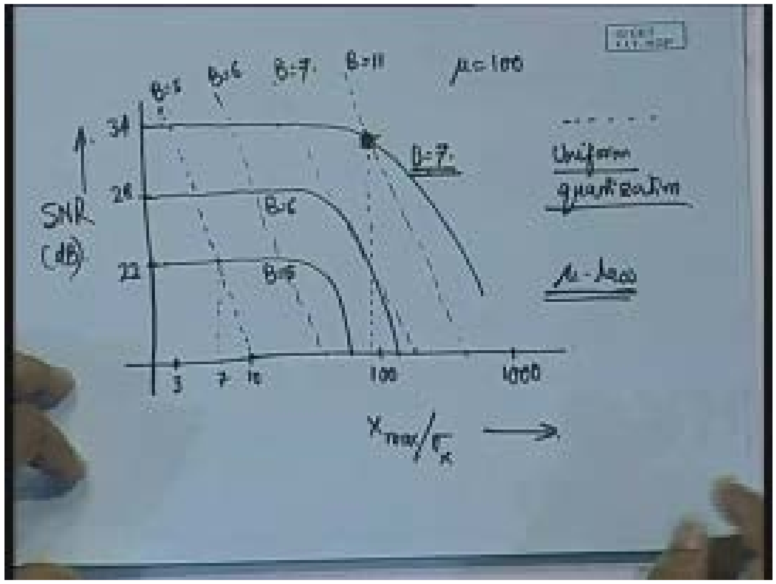

Then we can have a very similar plot for a higher value of mu. Let us say that we plot the same thing for mu is equal to 500 and in that case what are we going to obtain. This is for mu is equal to 500. Now tell me that, again look at this equation and tell me that if the value of mu is higher. Instead of 100 if we make it 500 what would you expect out of the SNR value? SNR will drop down. In fact it is seen that going by this equation the SNR will drop down. In this case I mean, from 100 to 500 if we use the actual values we will be finding that the SNR drops down by nearly 2.5 to 3 dB. So here we will be observing that it will be at 19.5 and then it drops down much later. So this is for B is equal to 5 and for B is equal to 6 it would be like this, for B is equal to 7 it would be like this. The values instead of 22 has come down to 19.5 here this has come down to 22.5 dB and here it has come down to 31.5 dB a uniform drop of 2.5 dB has happened and then again we will be observing, I mean I am not showing the other things but again for B is equal to 11 we can see that the characteristic makes an intersection at this particular point (Refer Slide Time: 32:36) which means to say that beyond this value of x max by sigma x we are going to find that this mu law is going to give us a better characteristic. In fact it will not intersect at two places; it will be something like this, not exactly what I have shown. So, beyond this value of x max by sigma x we will be finding that the mu law is going to perform better whereas for smaller values or x max by sigma x the uniform quantizer is going to perform better. But this means to say that although we may be having a better signal to noise ratio for the uniform quantizer case and the signal to noise ratio in this case suffers somewhat but this nature of the characteristic that it is totally uniform with respect to this, this helps a lot.

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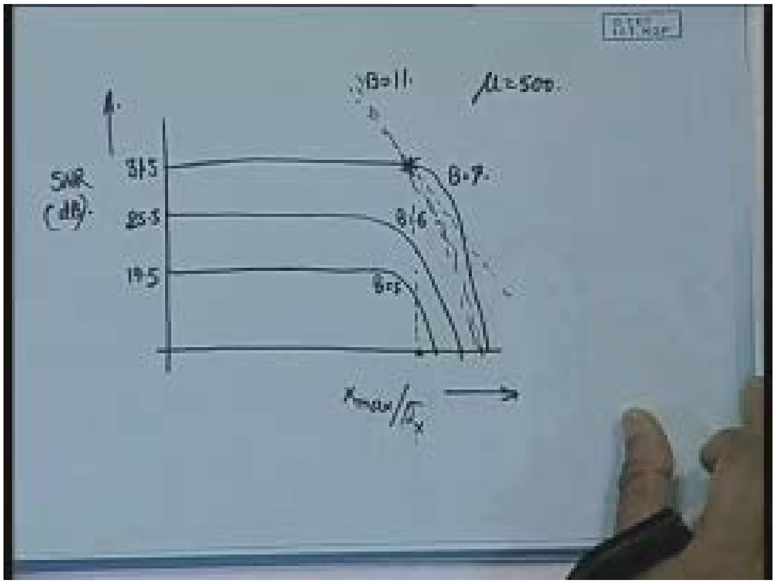

Hence, now we can go in for the case of the optimum quantizer. You see that this mu law quantizer this achieves a constant SNR over a wide range of signal variants. Now there is some sacrifice in the signal to noise ratio performance but instead if we consider the design of optimal quantizers.

In the case of optimal quantizers what we should do is that the quantization levels should match should match the signal variants and how do we do that. First thing is that the variants of the quantization noise we can write like this. it is given by sigma square E is equal to the expectation of e square n and this is equal to E of x cap n minus x(n) whole square and this let us call as the equation number 2.

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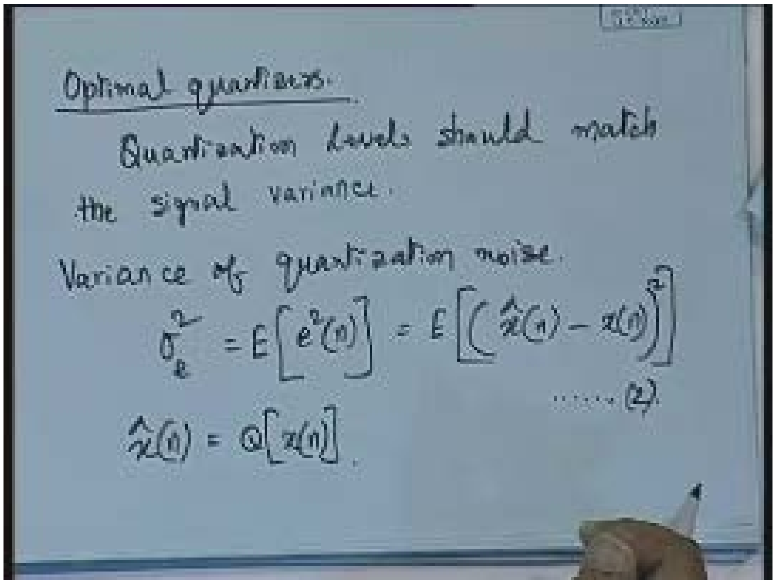

And in this case x cap n is equal to the quantized version that is Q of x(n) for an optimal quantizer. That means to say that x(n) will be directly followed by the quantizer. We are not using any other transformation. So it is just x(n) followed by the quantizer. So x cap n the quantized value is the Q of x(n) and in general we will be having M quantization levels. So if we use M quantization levels in that case we are going to have minus x cap minus M by 2 x cap of minus no not minus sorry x cap of minus M by 2 x cap minus M by 2 plus 1 so on x cap minus 1 then x cap 1 up to x cap M by 2. These are the set of M different quantization levels.

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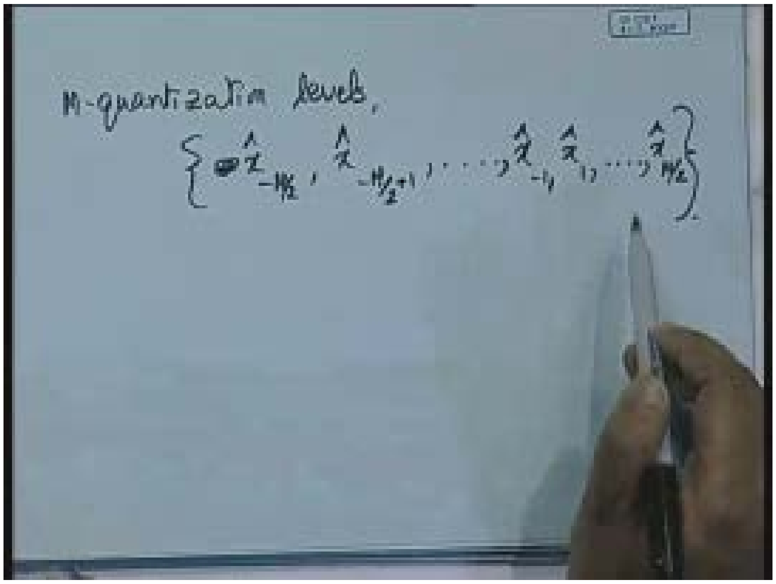

now what we do is that So in this case M will be considered to be an even number and the quantization level is associated so the quantization level is associated with associated with associated with the x j minus 1 to x j and this is denoted by x cap j. Now, when we have a zero mean amplitude distribution a zero mean amplitude distribution in that case we can define that x 0 will be equal to 0 and then furthermore we assume that the density function is non-zero for very large values of x.

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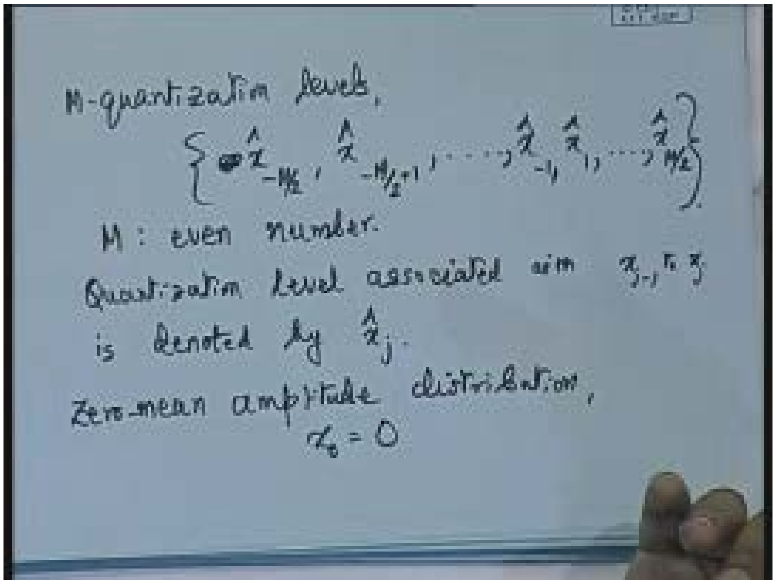

Now, where we can have such kind of situations; when we have the Laplacian PDF or a Gaussian PDF in that case it is going to have zero value only at infinity. So going by that so if the density function is non-zero for large amplitude then we can also assume that x plus or minus M by 2 that means to say that this level and this level we are setting as the plus minus infinity. Now with this assumption we can write down the variants. the variants is expectation of x cap n minus x(n) square and x(n) square is nothing but this x cap n minus x(n) whole square is nothing but e square of n and when we have these two that is to say the zero mean amplitude distribution and then for large signals we are having for large amplitude we are going to have the nonzero values in that case we can write down the sigma square e as this that means to say the expectation; the expectation can be written as sigma square e is equal to the integral of e square times the probability density function of the error p e (e) d(e) and this directly follows from here when we incorporate this assumption.

Now if we plot the error quantity that is to say e when we plot against the x, in that case we are going to have some kind of a situation like this that is supposing this is x 0 and if this is say x 1, if this is x 2 like that (Refer Slide Time: 40:41) and in this case we have x of minus 1 and here we have x of minus 2 all these different decision levels then corresponding to these decision levels we are going to have the value of the quantizer making a discontinuity. The error term will lead to a discontinuity corresponding to the case when x is equal to x 0 or x is equal to x 1 or x is equal to x 2 at those points and again mid-way between this x 0 and x 1 what is going to be the value of the error that is also going to be 0 at the mid points that means to say that if we say that x 0 plus x 1 by 2 or x 1 plus x 2 by 2 at these points the value of the error is going to be 0. But when x 0 is making a as but when the signal just reaches the x 0 in that case it is making a transition from the negative error to the positive error. Again at this point it is becoming 0 means that from the positive error it drops and beyond this if we increase the value of x further in that case we are going to have a negative error and then at the value of x 1 it is again going make a transition. And if it is a uniform quantizer then all these levels should have been same but in the case of non-uniform quantization we are going to have a value we are going to have a characteristic which may be like this.

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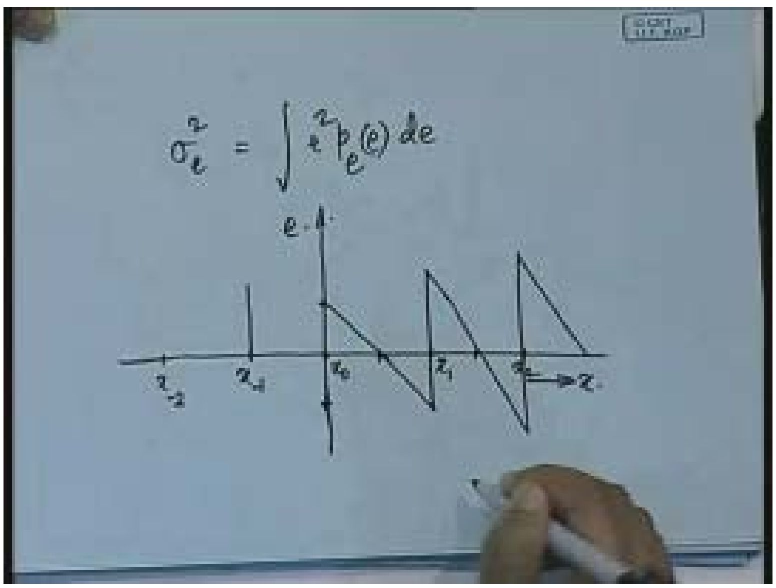

Again, just extending the same thing on the negative side we are going to see some kind of error characteristic which would be like this sorry it should be more like the mid-way and then here it will be x of minus 2. So the error versus x characteristic would be something like this. So this is the quantization error versus x.

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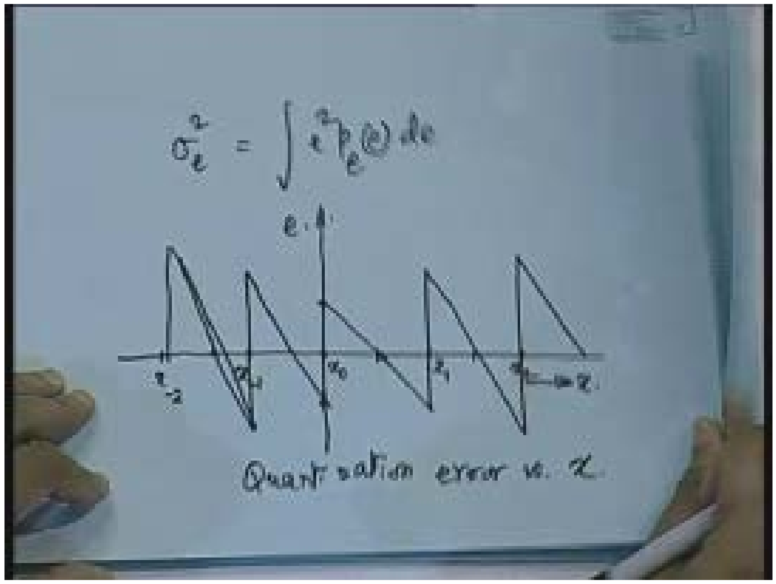

Now we can write down the error, the error is nothing but x cap minus x and then we can say that by making a so here with e is equal to x cap minus x we can write down that the probability of error the probability density function of error p e of e we can write down as p e of x cap minus x and this by change of variable we can write down as p of x by x cap into x by x cap and this could be defined as p x by p x of x. So, as a result of this we can write down the sigma square e as the summation i is equal to minus m by 2 plus 1 to M by 2 integral x of minus 1 to x i into x i cap minus x whole square into p(x) d(x) this is by direct substitution of this equation. the earlier integral equation what you have seen was in terms of the p e e so it was e square p e e the e and instead by change of variable we have made it as the e square remains as it is but it is changed to p(x) d(x).

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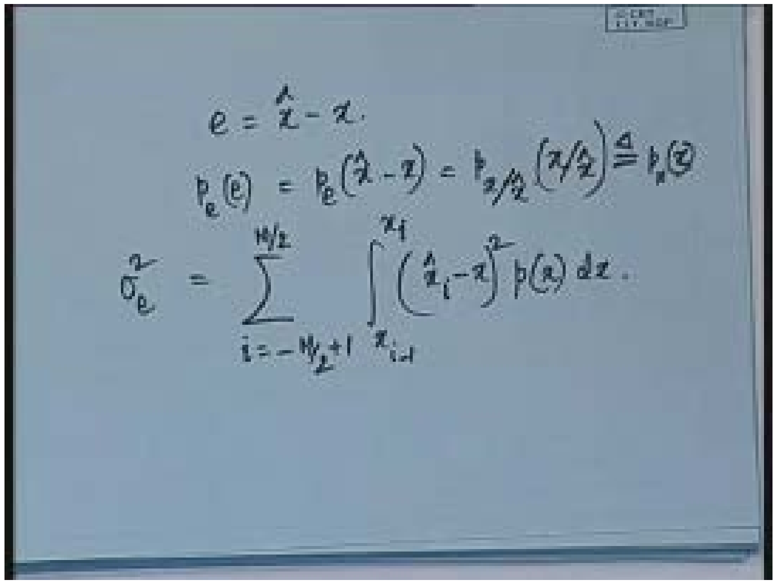

Now what we have to do is that this variance term this error variance term this noise variance term this has to be minimized with respect to what, now we have to design the quantizer. So designing the quantizer means what are the decisions that we have to make, we have to make the decision about x 1 to x n by 2 so all the decision levels have to be decided and then all the reconstruction levels also are to be decided. All these happen to be the variables so what we have to do is to differentiate this error term with respect to the quantities with respect to the individual quantities like this x i's and if we differentiate that and we equate them to zero in that case we are going to have a set of equations. Therefore, by differentiating by differentiating sigma square e sigma square e with respect to each parameter we can get this set of equations: integral x i minus 1 to x i of x cap minus x whole square p(x) d(x) that is equal to 0 for i is equal to 1 2 up to M by 2 and then we are going to have x i to be equal to half of x i cap plus x i cap i plus 1 for i is equal to 1 2 to M by 2 minus 1 and by our assumption we are having x 0 is equal to 0 and x plus or minus M by 2 is equal to plus minus infinity.

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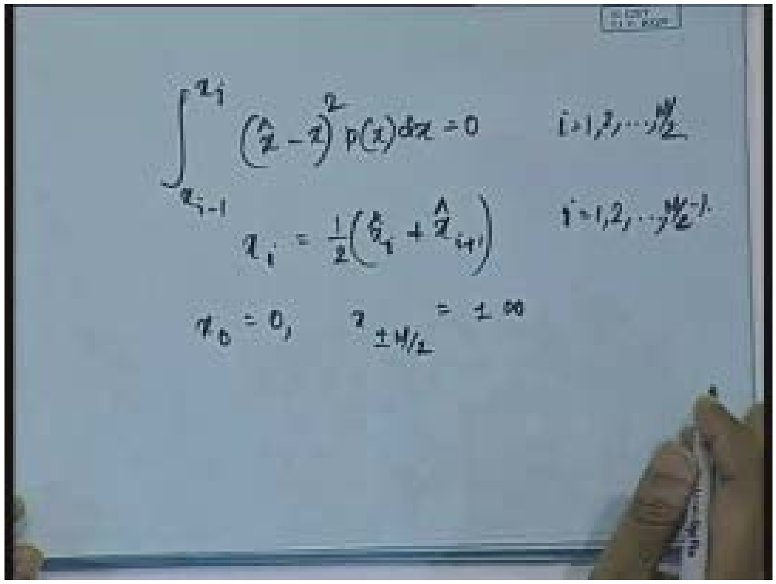

Now if we solve these equations in that case we are going to find out the value of these decision levels and the reconstruction levels and that give us an optimum quantizer. Now the point that we have discuss is that is optimum quantizer the best solution for speech coding? Unfortunately not; because although the optimal quantizer is going to yield a better signal to noise ratio because it is matched with the signal variance it is going to give a better signal to noise ratio but it performs poorly especially in the case when......... I mean, in between the speech waveforms we have got silence. So, for the speech signals the optimal quantizer may be okay but whenever we are having the silence in between; in the case of silence what is happening is that the signal amplitude is already too small and when the signal amplitude is too small in that case it makes a to and fro jump between the minimum quantization levels. So it will make a jump between x 1 cap and x 1 cap, I mean it will make a jump between x cap 1 and x cap minus 1 and if the step size of this happens to be large in that case it is going to produce a much larger error. Whereas in the case of the mu law compressor one can design the step size of the minimum signal to be much smaller as compared to the step size for the larger signal. Therefore, as a result of that for the silence duration the change in the quantized levels will be much less for the mu law as compared to what we will be getting for the optimum quantizer.

Therefore, although it is called optimum quantizer, for the speech signal just let us remember that mu law is going to serve as the best performance, is going to serve with a best performance because it is also taking care of the non-speech condition that is to say when the silent case is considered it is going to be much better. With this now we can go in for the discussions on the next version of the quantizer which will be actually the adaptive quantizer. Because you see that ideally what we should do is that we should design a quantizer that suits or that adaptively suits to the signal level where, as the signal level changes as the signal variance change we should have the change of the deltas adapted accordingly; the step sizes must be adapted accordingly and that is what we will be studying about, the adaptive quantization session case, in the next class. Thank you.