

Digital Voice and Picture Communication
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Lecture - 04
Quantizers for Speech Signal

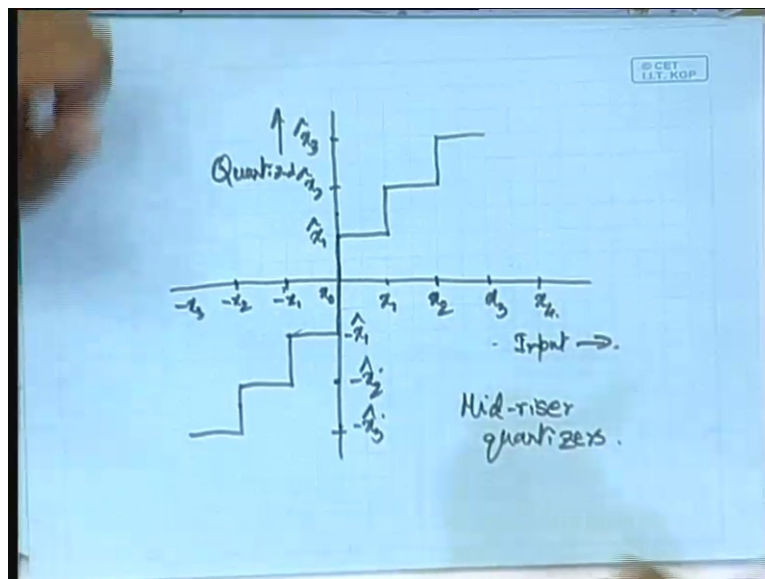
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So today, our topic is quantization of speech signals. Now, whenever we would like to have any waveform approximation coding or even for parametric coding also one of the essential steps that we require to store and process the digital samples is quantization. So, before quantization of course we require sampling of the speech signal. So, if we say that the speech signal is given by x as a function of time, in that case we are going to sample the speech where the samples will be denoted by $x(nT)$ where T is the sampling period and n is the index of the sample. So we have n that varies from....., if we start the sampling from n is equal to 0 then n is equal to 0 1 2 etc etc that means to say that at those time instances means x of capital T , x of $2 T$, x of $3 T$ etc there we are going to sample the speech signal and those samples which we can later denote by x of n

so on (Refer Slide Time: 4:58) so that when the input signal is between x_1 to x_2 then it will be denoted by the level x_1 cap. So this is the input and this is the quantized variable so x cap is actually the quantized value of x . So when it is between x_1 and x_2 then its quantized value is x_1 cap; when it is between x_2 and x_3 then its value will be x_2 cap; between x_3 and x_4 we would like to denote it by x_3 cap and so on and likewise on the negative side also we can have like here we can have minus x_1 cap (Refer Slide Time: 5:41) then minus x_2 to minus x_3 we can have minus x_2 cap, minus x_3 to minus x_4 we can have minus x_3 cap and so on. So this essentially realizes what is called as a mid-tread quantizer.

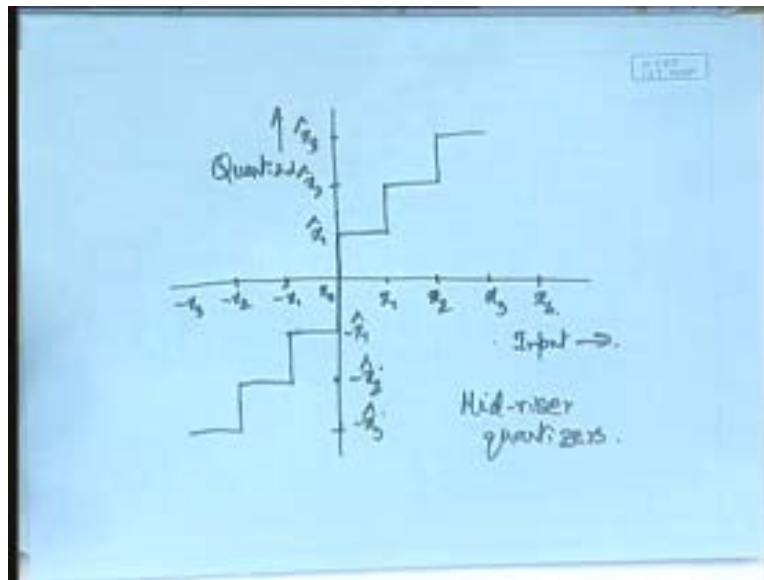
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And similarly there may be what is called as the mid-riser quantizer and the mid riser quantizer will have the steps like this that here there is no level that is at the 0, here you can see that it is a mid-tread (Refer Slide Time: 6:26) so corresponding to the value of input between minus x_1 to plus x_1 we have the quantized value as x_0 cap whereas in the case of mid riser no such x_0 s will be existing rather the quantization would go on like this. Like if we have this to be let's say x_0 then we have this as x_1 , then we have this as x_2 x_3 x_4 and so on; then corresponding to the signal between x_0 and x_1 we can have the quantized signal as x_1 cap; x_1 to x_2 we can have this as x_2 cap; then x_2 to x_3 we can have as x_3 cap and on the negative side also we can have

this as minus x_1 cap; then here we can have minus x_2 cap so that if the signal is between minus x_1 to minus x_2 then we can denote it by minus x_2 cap level and this we can denote by minus x_2 to minus x_3 could be minus x_3 cap and so on. So this type of quantizer we will be calling..... so this is the input variable and this is the quantized variable and this kind of quantizers they are referred to as mid riser quantizers.

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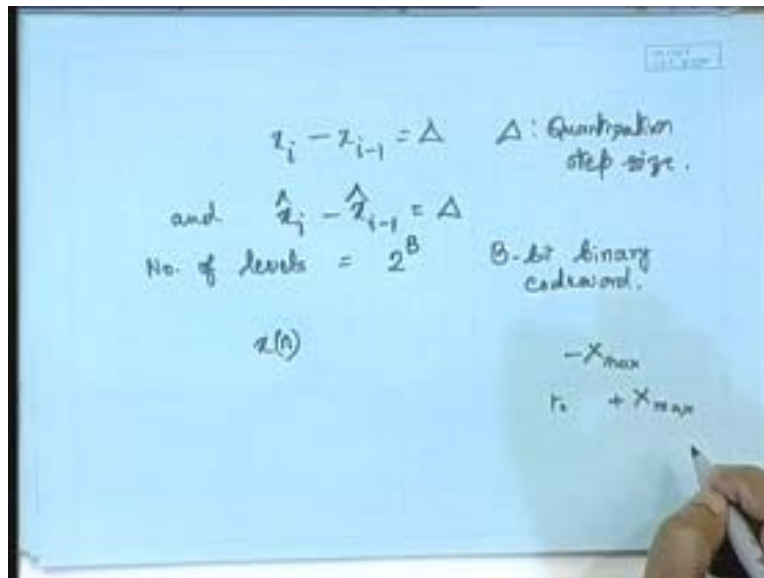


In fact the quantization step size, if we denote by a quantity delta then that is actually defined as x_i minus x_{i-1} that is equal to delta. So this is the quantization step size. So delta is nothing but the quantization step size and likewise for the quantized values also a similar step size will be there. We can also write x_i cap minus x_{i-1} cap that means to say that the difference between these two quantized levels, two successive quantized levels that also will be equal to delta. And now we assume that we have the number of levels that is to say the number of quantization levels we say as 2 to the power B **where B is the so** where each of the samples or each of the quantized values are represented by B-bit binary code word.

So we have a B-bit binary code word and the number of levels will be 2 to the power B. So what we can have is that the number of levels being 2 to the power B and if we assume that the samples,

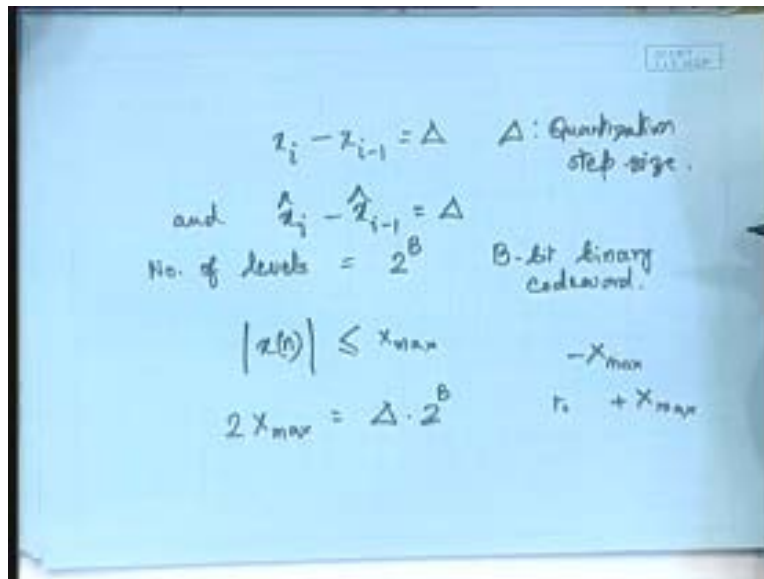
the samples now we are indicating by x of n where n is an integer so x of n can vary, let us say that it is going to have a dynamic range of minus x some value which we can call as minus x max to let us say that it can go up to plus x max.

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Therefore, it is minus x max to plus x max is the complete variation that is possible in the signal level and we can denote x of n as the mod value of x of n because x of n can be positive and also it can be negative. in the negative side its minimum value could be minus x max and on the positive side it could go up to plus x max **so that the overall** so that we can represent the same thing as mod of $x(n)$ is less than or equal to x max. And then if we have a total variation from minus x max to plus x max then this entire dynamic range; if you want to make use of this entire dynamic range for quantization then we can say that minus x max to plus x max which means to say that two times x max that is going to be delta that is to say the quantization step size into the total number of levels that is 2 to the power B.

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What is delta into 2 to the power B indicating?

because delta is Delta being the delta being one step size and 2 to the power B being the number of total steps then delta into 2 to the power B is nothing but indicating the overall range of the signal which we have assumed to be 2 to the power which we have assumed to be 2 x max if the variation is from minus x max to plus x max as we just now said.

Now the quantized samples as such we are going to represent by..... individual quantized samples then we will be calling as \hat{x} of n . So, corresponding to the sample x of n , its quantized value is going to be written as \hat{x} of n . Now, \hat{x} of n will be equal to x of n only when there is no quantization error. So \hat{x} of n will be equal to $x(n)$ provided there is no quantization error but think that when you have a quantization step size; take any of these cases mid-riser or mid-tread whatever you can see that when you take any such value that lies in the middle of this x_1 and x_2 let us say that we take some value over here (Refer Slide Time: 13:05) now its actual value is this much but because of the quantization we have already quantized this to the level x_2 cap. So this total step size being delta; so this being equal to delta that is the step size, you can see that..... whereas the actual value should have been somewhere over here we are having the quantized value to be x_2 which means to say that the maximum what you can

say the quantization error varies from minus delta by 2 to plus delta by 2. So, taking the quantization error also into consideration, we would like to write the $\hat{x}(n)$ as $x(n)$ that is the original sample plus a term we add $e(n)$ where $e(n)$ is the quantization noise.

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$$z_i - z_{i-1} = \Delta \quad \Delta: \text{Quantization step size.}$$

$$\text{and } \hat{z}_i - \hat{z}_{i-1} = \Delta$$

$$\text{No. of levels} = 2^B \quad \text{B-bit binary code word.}$$

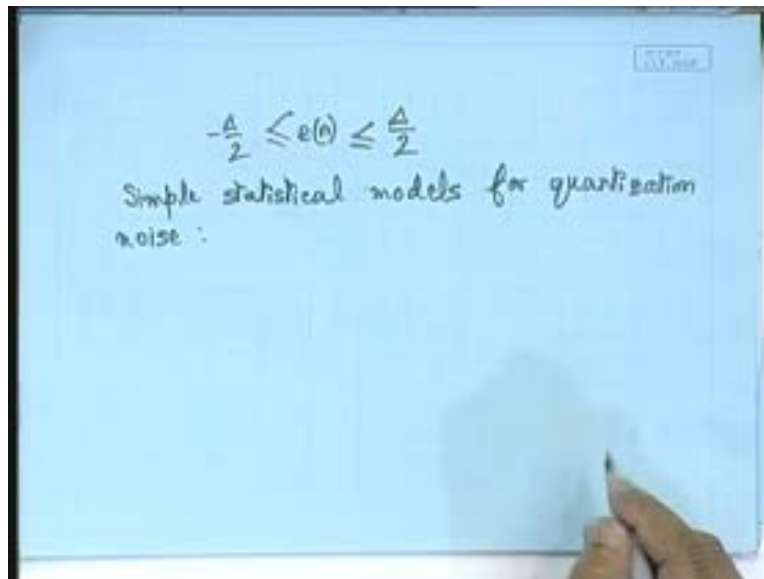
$$|x(n)| \leq X_{\max}$$

$$2X_{\max} = \Delta \cdot 2^B$$

$$\hat{x}(n) = x(n) + e(n) \quad e(n): \text{Quantization noise}$$

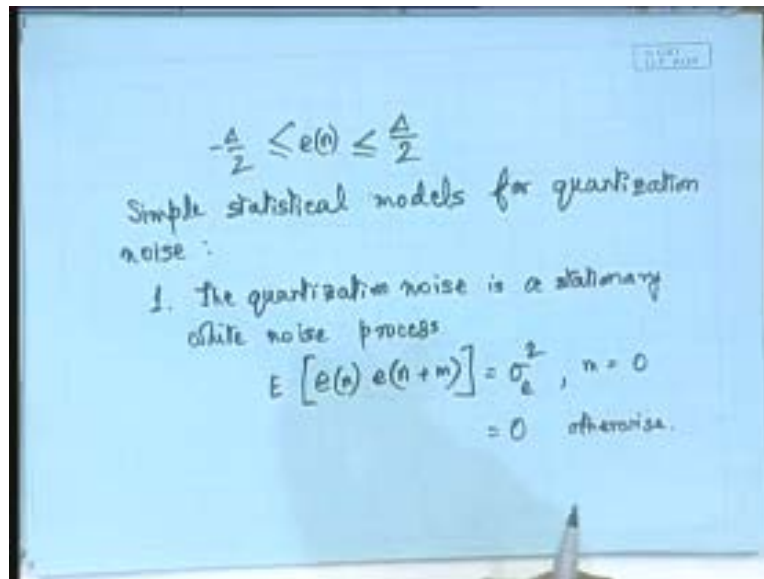
Obviously because delta happens to be the quantization step size we can definitely express this in terms of the inequality that $e(n)$ will then range between delta by 2 to minus delta by 2. Now, this being the quantization noise, we can assume some simple statistical models for quantization noise. And what are these simple statistical models?

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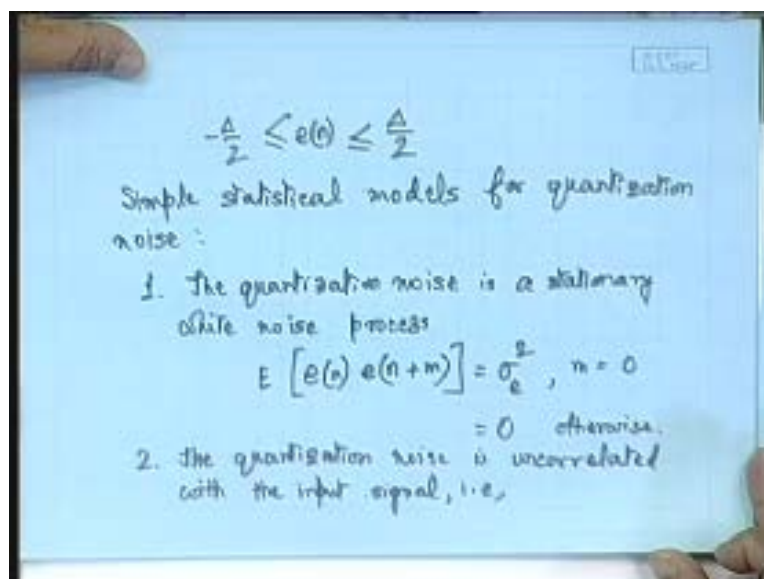
First of all we assume that the quantization noise is a stationary white noise process and under this assumption we can write that the expectation of $e(n)$ and its product with $e(n+m)$ where m is nothing but a displacement because $e(n)$ is the n th sample and $e(n+m)$ is a sample that is m samples ahead of this sample $e(n)$. Now $e(n)$ and $e(n+m)$ they should be ideally uncorrelated. That means to say that if it is a stationary white noise process then **no signal** no noise component is going to have correlation with the other noise components which means to say that $e(n)$ can have correlation with only $e(n)$ and it cannot have correlation with $e(n+1)$ or $e(n+2)$ or things like that $e(n+m)$ which means to say that we can write that this is equal to σ_e^2 where σ_e^2 we can assume to be the variance in the quantization noise so this is equal to σ_e^2 for n is equal to 0 and the expectation value is equal to 0 otherwise. So this **otherwise** condition obviously takes into account that the quantization noise is completely uncorrelated. So this is the first assumption that we are making about the statistical model of quantization noise.

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The second and a very valid assumption is that the quantization noise is uncorrelated with the input signal. Hence, this also we write that the quantization noise is uncorrelated with the input signal.

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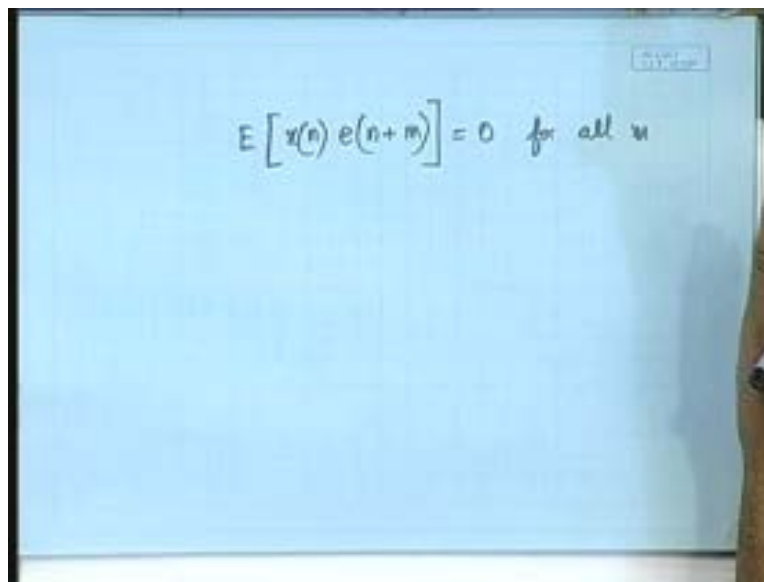


So how do we express that mathematically?

We have to write again in terms of expectation only so we have to take the signal term. The signal term will be denoted by x of n and the noise term will be denoted by e of n . So we can write the correlation property as E of x of n into e of n plus m . So we take the signal and this is the error of the quantization noise component which is for the n plus m th sample.

But because we already said that the quantization noise is uncorrelated with respect to the input signal its value is going to be equal to 0 for all values of m for all values of n . So no matter even if m is equal to 0 we have $x(n) e(n)$; even $x(n)$ and $e(n)$ should be completely uncorrelated with each other. And take any other $e(n)$, take $x(n) e(n+1)$ or $x(n) e(n-1)$ or $x(n) e(n+2)$ anything these are going to be uncorrelated. So this is the second assumption that we are making about the statistical model of quantization noise.

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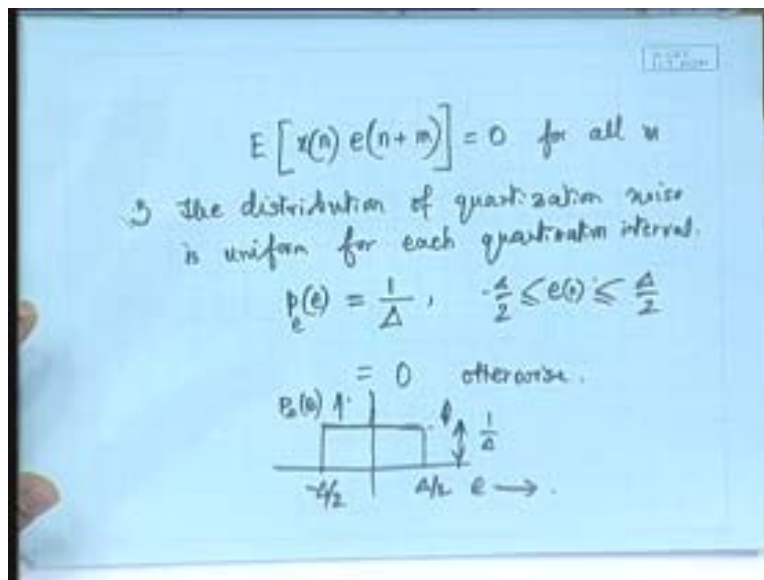

$$E[x(n)e(n+m)] = 0 \text{ for all } n$$

The third is that the distribution of quantization errors is uniform over each quantization interval so the distribution of quantization noise is uniform and it is uniform over all quantization interval, for each quantization intervals and we are going to express that as the probability

density function of e so $p(e)$ with respect e so it is the probability density function of the quantization error and because it is uniform how we are going to write we are going to write it as $1/\Delta$, Δ being the step size which I already mentioned.

Thus, this is equal to $1/\Delta$ where the value of e is going to be e or $e(n)$ whatever you write. Now we are not writing $e(n)$ every time; we can write it as $p(e)$ directly. The value of e rather lies between minus $\Delta/2$ to plus $\Delta/2$. So this is the range of the quantization errors and definitely the uniform probability density function is required that it is $p(e)$ of e is equal to $1/\Delta$ and this is equal to 0 otherwise. So it is of uniform PDF. So you can say that if you sketch it, it would be like this (Refer Slide Time: 21:21) that here it will be $\Delta/2$, here it will be minus $\Delta/2$ and this height is going to be **this height is going to be** $1/\Delta$. This is how you are going to plot if e is there and if $p(e)$ of e is plotted like this then you are going to have a uniform distribution as something like this. Hence, this is the third property.

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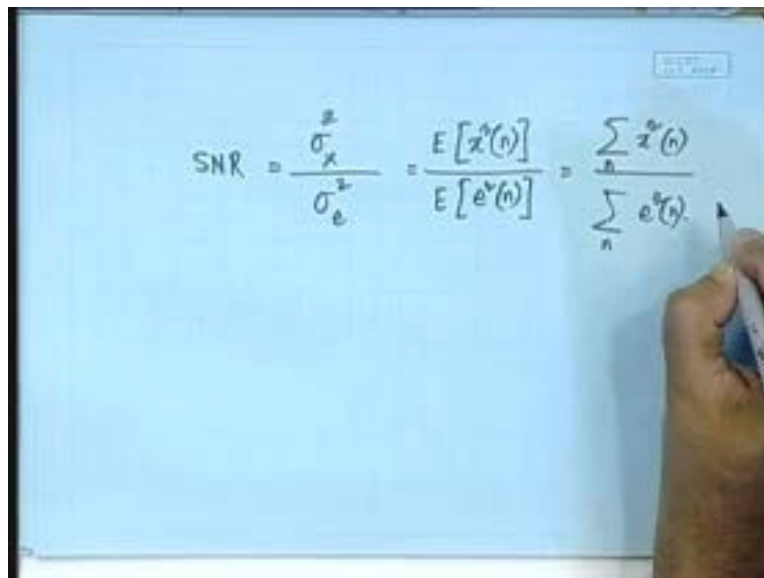


So three very very important not property rather I should say assumptions. So what are the assumptions is that it is a stationery white noise process which means to say that the error samples are completely uncorrelated with each other; second assumption is that the quantization

noise is uncorrelated with respect to the input signal; and the third property and the third assumption that we have made is that it is uniformly distributed. I mean, it may be something other than uniformly distributed also but of course there the expression would definitely change accordingly.

Now it has been well experimented for speech that these assumptions these three assumptions they hold good quite reasonably for speech signals. And taking this assumptions into consideration we can write now the signal to noise ratio of the speech signals like this that we can express the SNR, okay, the signal to noise ratio will be written as the signal variance which we write as sigma square x because x we are assuming to be the signal and the variance of the error signal is going to be written as sigma square e. So it is sigma square x by sigma square e and sigma square x is nothing but the expectation of $E[x^2(n)]$ and that of the error is $E[e^2(n)]$ so this is the energy of the signal and this is the energy of the error. And this can be written of course n summation form also. Summation over n x square n divided by summation over n e square of n.

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A hand-drawn equation on a whiteboard showing the Signal-to-Noise Ratio (SNR) in three forms: as a ratio of variances, as a ratio of expectations, and as a ratio of summations. A hand is visible on the right side of the whiteboard, holding a marker.

$$\text{SNR} = \frac{\sigma_x^2}{\sigma_e^2} = \frac{E[x^2(n)]}{E[e^2(n)]} = \frac{\sum_n x^2(n)}{\sum_n e^2(n)}$$

And if we take the peak to peak quantizer range to be $2 \times \max$; this we have already shown. In fact we have already indicated this that is $2 \times \max$ is equal to Δ into 2 to the power B . So from this the quantization step size Δ can be expressed as two times $\times \max$ divided by 2 to the power B . And then one of the assumptions, the third assumption that we have made is that the quantization noise is uniformly distributed. And for uniformly distributed it is a very simple derivation that we would be having the error or the noise variance as σ^2_e which should be written as Δ^2 divided by 12 ; so that is how it comes for the uniform distribution case; uniform PDF for the quantization error and this is going to be expressed as..... if we just substitute this expression that is instead of Δ if we write $2 \times \max$ by 2 to the power B in that case we directly get it as $\times \max^2$ divided by 3 . It is coming out to be 3 because there is a two term over here so Δ^2 makes it 4 and there is a 12 in the denominator so in total we have a multiplication factor of 3 in the denominator. Therefore, it is 3 into 2 to the power $2B$ again because it is 2 to the power B for Δ ; for Δ^2 it is 2 to the power $2B$. So this is the σ^2_e term and now what we can do is that **we can make this** we can substitute this into our original SNR equation.

Hence, if we substitute this value of σ^2_e (Refer Slide Time: 25:57) into this this this expression σ^2_e into this σ^2_x by σ^2_e and in place of σ^2_e we write this, then we can write the corresponding signal to noise ratio as..... I mean, if we take this to be equation number 1 and if we take this to be the equation number 2 in that case simply substituting equation number 2 in equation number 1 we get the SNR as 3 into 2 to the power $2B$ because this expression is coming in the denominator of σ^2_e so 3 into 2 to the power $2B$ that goes to the top and in the denominator term this $\times \max^2$ is there and σ^2_x which is in the numerator term if it is brought on the denominator side then we have to express this as $\times \max$ by σ this quantity whole square. Therefore, this is going to be our signal to noise ratio.

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The image shows a whiteboard with the following handwritten equations:

$$\text{SNR} = \frac{\sigma_x^2}{\sigma_e^2} = \frac{E[x^2(t)]}{E[e^2(t)]} = \frac{\sum_n x^2(t)}{\sum_n e^2(t)} \quad \text{.....(1)}$$

$$\Delta = \frac{2x_{\max}}{2^b}$$

$$\sigma_e^2 = \frac{\Delta^2}{12} = \frac{x_{\max}^2}{3 \cdot 2^{2b}} \quad \text{.....(2)}$$

$$\text{SNR} = \frac{3 \cdot 2^{2b}}{\left[\frac{x_{\max}}{\sigma_s}\right]^2}$$

Now this is just a ratio and if we would like to express the signal to noise ratio in dB, we are going to write it like this.

So, expressing in dB we can write the SNR and let us write within bracket (dB) SNR dB would be $10 \log$ to the base 10 of this signal to noise ratio and this is already expressed in terms of the power so this is $10 \log$ $10 \sigma_x^2$ by σ_e^2 from its original definition of the signal to noise ratio, so we are just taking $10 \log$ 10. And if we are substituting all these things, means instead of σ_x^2 by σ_e^2 if we simply substitute this (Refer Slide Time: 28:02) and then take the logarithm then we can just very easily verify that it comes out as $6B$ plus 4.77 , the 4.77 is coming, you can well understand that there is a 3 over here, \log of 3 to the base 10 happens to be 0.477 and then everything gets multiplied by 10 so that is why you get 4.77 as the term arising out of that isolated 3 .

And this 2 to the power $2B$ of course is the one that is ultimately contributing to this $6B$ quantity and then this x_{\max} by σ_x the term is left over at the denominator of this signal to noise ratio expression. Since this is in the denominator and we are taking the signal to noise ratio as 10

log 10 of this entire quantity the denominator obviously gives rise to a minus sign so it is minus 20 log to the base 10 of course as X_{max} by σ_x .

So we can write the whole thing as $6B$ plus 4.77 minus $20 \log$ to the base 10 X_{max} by σ_x and just very simply you can see that if we said our value of σ_x if we said to be just four times σ_x . So if we said that then we get the SNR in (dB) as simply $6B$ minus 7.2 ; this we can say as equation number 3.

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Expressing in dB,

$$\text{SNR (dB)} = 10 \log_{10} \left[\frac{\sigma_s^2}{\sigma_e^2} \right]$$

$$= 6B + 4.77 - 20 \log_{10} \left[\frac{X_{max}}{\sigma_x} \right]$$

If we set $X_{max} = 4\sigma_x$

$$\text{SNR (dB)} = 6B - 7.2 \dots \dots \dots (3)$$

Now this equation that we have written so far this is valid subject to certain assumptions. What are these assumptions?

The first assumption is that we can list what kind of assumptions we make in this process. First is that the signal fluctuation is rapid, the signal fluctuation is rapid and that is why the statistical model of noise what we have already assumed those three assumptions which we stated and the statistical model of noise is valid.

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Expressing in dB,

$$\text{SNR (dB)} = 10 \log_{10} \left[\frac{\sigma_x^2}{\sigma_e^2} \right]$$
$$= 6B + 4.77 - 20 \log_{10} \left[\frac{x_{\text{max}}}{\sigma_x} \right]$$

If we set $x_{\text{max}} = 4\sigma_x$

$$\text{SNR (dB)} = 6B - 7.2 \dots \dots \dots \textcircled{1}$$

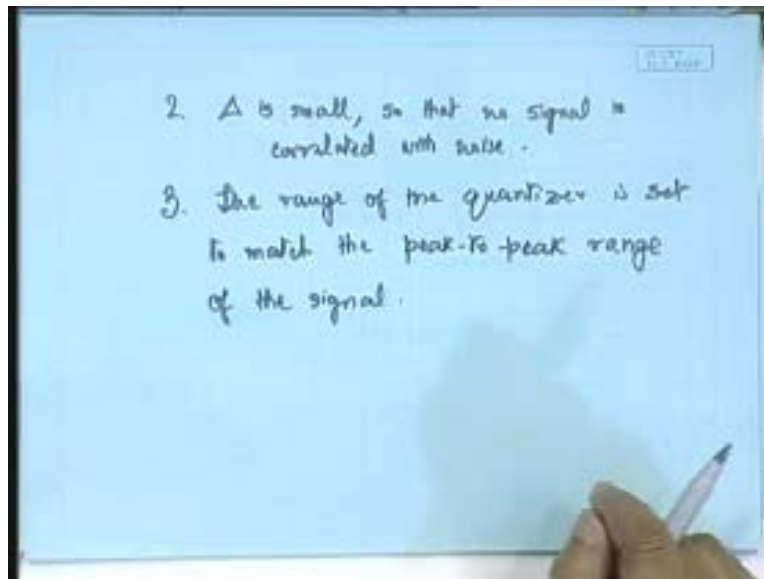
Assumptions : 1. The signal fluctuation is rapid and the statistical model of noise is valid.

Mind you, the statistical noise, the statistical model of noise is valid for very rapid signal fluctuations because then only you can take that the noise signals are uncorrelated with respect to each other and noise is also uncorrelated with respect to signal. So without this, without rapid signal fluctuation you cannot have that assumption anymore.

The second assumption is that you have delta as small and only if you have delta to be small then there is no signal that is correlated with the noise. Delta is small so that no signal is correlated with noise. In fact for large delta that is what you are going to have. If you have a very large step size then your quantization error simply follows your signal then the error then the noise and the signal they get correlated with each other. So, if delta is small no signal is correlated with noise.

The third assumption is that the range of the quantizer is set so as to match the peak to peak range of the signal, the peak to peak range of the signal.

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Now, this is a very **what you can say** it is a very important assumption. In fact you must have already seen that we have talked about this sometime back already because we have been taking..... that delta we were taking to be delta into 2 to the power B we were taking to be 2 times X max. So, because we have already taken that we took this point into consideration that this range is matched with the peak to peak range of the signal.

Now, for the first two assumptions that we have made that is reasonably valid. Especially for delta to be small what we require; we require that the number of step sizes should be large because that is only what can make the delta to be small. So it is seen that for speech signal the first two assumptions are quite valid for number of levels greater than 2 to the power of 6 and generally for speech signals it is so; for speech quantization the number of levels should be greater than that of 2 to the power 6 so definitely that means to say that we will be having more than 6 bits for our representation of the quantized samples.

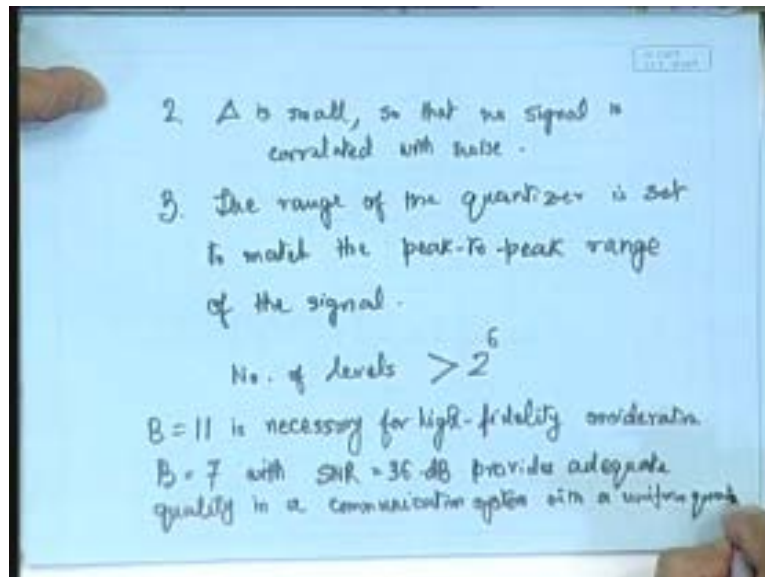
But one very important aspect of the speech signal is that between different speakers we have a very large variation in its signal strength. The signal levels in fact can vary by as much as 40 dB for different speakers and again for the same speaker the speech signal level varies drastically

between the voiced speech and the unvoiced speech. So, taking the speaker to speaker variation into consideration and voiced to unvoiced speech signal levels variation into consideration these two factors leads to the fact that we really have a very tough task in fulfilling the property number 3 because the property number 3 says that quantization is set to match the peak to peak range of the signal but for which signal; if we say that it matches with the voiced signal..... for the voiced signal if we can happen to match this range then whenever the speaker goes to the unvoiced part of the speech then definitely that range cannot be satisfied because the unvoiced speech is going to have a much lower dynamic range which means to say that for the unvoiced part of the speech we are going to have a very poor SNR, the SNR may be good for the voiced speech but for unvoiced speech it is going to be very bad. Then if it is matched to one speaker, for a speaker having a good speech signal level, we change the speaker, that is, instead of one person if somebody else talks over the same speech signal quantization system then there is going to be a very drastic variation because from speaker to speaker also we have a large variation.

Therefore, from this point of consideration **the simple sampling is not** the simplest scheme of quantization is not adequate. In fact we have to go in for something like adaptive quantization which we are going to talk about in the later lecture. But now at least we can say that what is actually used for the speech quantization process.

It is generally seen that for high fidelity consideration B is equal to 11 that means to say 11-bit representation is necessary. So B is equal to 11 is necessary **for high quality** for high fidelity considerations whereas it has been seen that B is equal to 7 with SNR equal to 36 dB provides adequate quality in a communication system. So this fact we remember; in a communication system and of course with a uniform quantizer.

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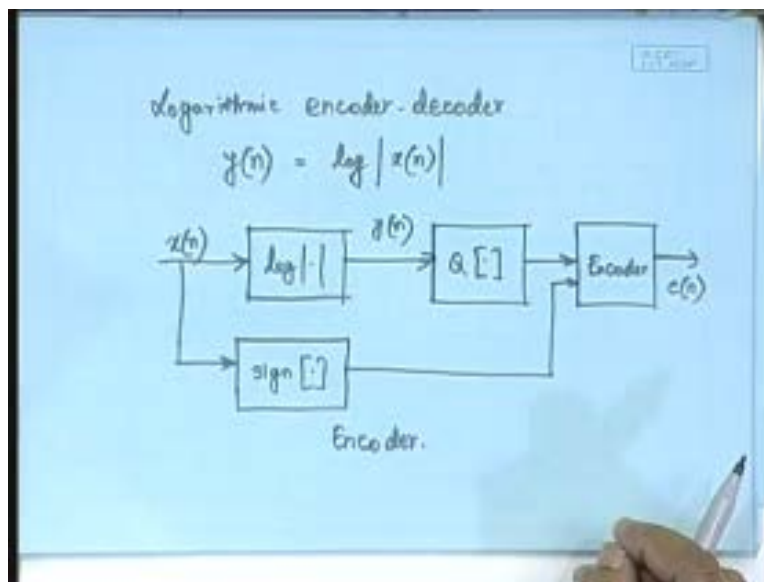
Again mind you that whatever expressions we wrote down those expressions are valid for uniform quantization because only for uniform quantization you can have that Δ^2 by 12 coming in. So now we can see another aspect. That means to say that to take the large variation in the dynamic range of speech signals into consideration. One thing which we employ is a logarithmic encoder and decoder because by taking the logarithm we can appropriately take care of the dynamic range because logarithmic transformation essentially reduces the dynamic range of the original signal and then it is easier for us to handle.

So how do we have the logarithmic encoder and the decoder?

We have to consider the blocks like this; so logarithmic encoder decoder and they will be using a transformation which is given by y of n equal to \log mod of x of n . So we have to realize this and how do we realize that; we take x of n that is the speech sample as an input and then we have a logarithmic transformation block. So it is \log mod so definitely this output will be nothing but y of n . But then after making the logarithmic transformation for the signal $y(n)$, now instead of taking the quantization for x of n we take the quantization for y of n .

After y of n , after obtaining y of n we put the quantization which we just write as Q and this is the quantized output and then we also take..... because we are taking the modulus what we have lost in this case is the sign of the original signal x of n , original sample x of n sign that must be preserved so we have a block that preserves the sign. So this is sign of x of n that is what we will be taking. So sign value which is a one bit representation that whether it is positive or whether it is negative, this 1 bit information has to be encoded and then also the quantized value of $y(n)$. So there will be an encoder that accepts the quantized signal and also the sign of $x(n)$ and this gets encoded into the bit stream. So $c(n)$ is the bit stream corresponding to the n th sample. So this is what we are having at the encoder end.

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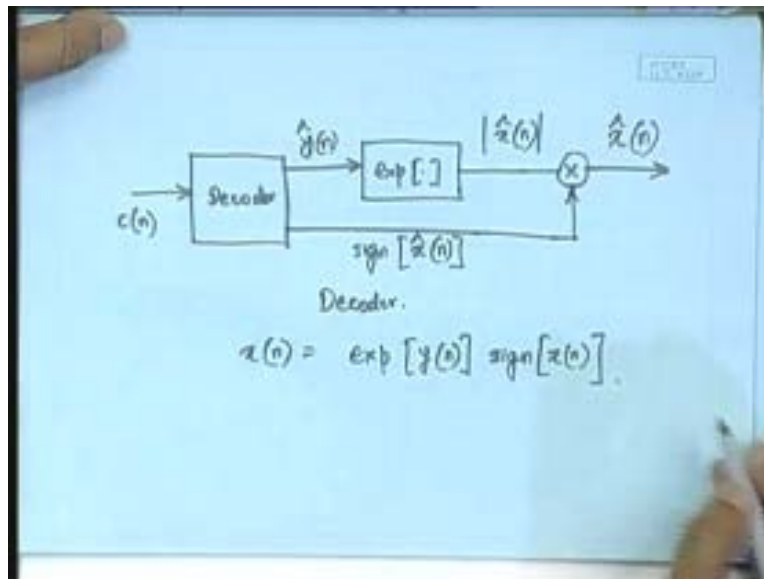
No, at the decoder end we must have a corresponding inverse transformation. Because here we have made logarithmic transformation, so definitely to get x of n back; obviously we will not be able to get x of n directly back because there is a quantizer in between and quantizer is definitely going to provide losses. So, instead of x of n we are going to have \hat{x} of n which will be ultimately obtained and that will be obtained at the decoder. So the decoder block diagram would go like this that it will accept this c of n that means to say these encoded bits will be received as

an input to the decoder and then ultimately it has to obtain $\hat{x}(n)$ as an approximated form of $x(n)$. The decoder block diagram can be shown here (Refer Slide Time: 42:57).

This is $c(n)$ coming as an input here. We have the decoder, then we have $y(n)$, we write it as $\hat{y}(n)$ and then we have here the exponential. Because you have taken the logarithm you have to take the exponential just to get back this signal and the exponential of this actually helps you to get back the mod value of $x(n)$. Because this $\hat{y}(n)$ what we have taken is definitely the mod value because we already saw it for the encoder part that here (Refer Slide Time: 43:42) it is the magnitude of $x(n)$ which is already taken and over that the logarithmic transformation has been applied so here also we get mod of $\hat{x}(n)$. But $c(n)$ contains the sign information also.

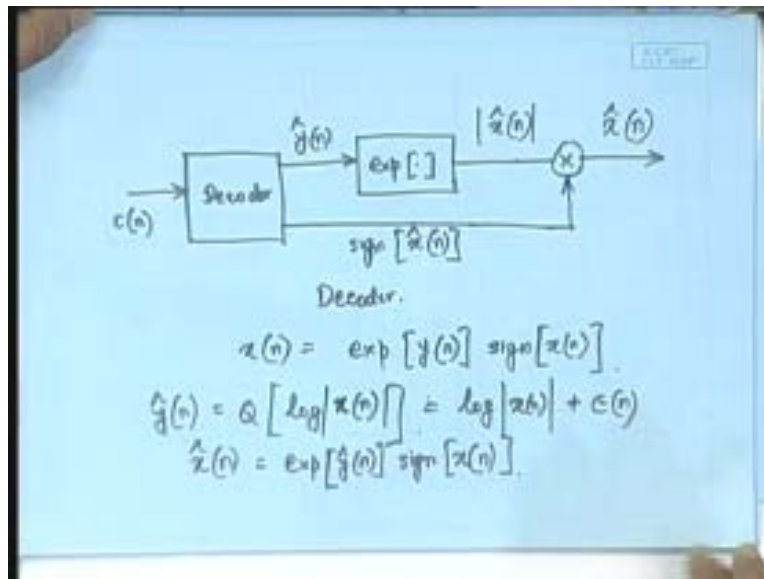
So, if we extract and separate out the sign information then what we can do is that we get this sign information. So this is sign of $\hat{x}(n)$ and if we multiply this $\hat{x}(n)$ mod of $\hat{x}(n)$ with the sign of $\hat{x}(n)$ where sign of $\hat{x}(n)$ is going to be either plus 1 for positive or minus 1 for negative then at the output of this multiplier we will be getting $\hat{x}(n)$. So this is our decoder and we can write the inverse transformation equation as $x(n)$ is equal to exponential of $\hat{y}(n)$ into sign of $\hat{x}(n)$. You see, this is exactly what we are going to do. The only thing is that here we are not saying $y(n)$ we are saying $\hat{y}(n)$. But exponential of $\hat{y}(n)$ is nothing but the mod of $x(n)$ and that is actually multiplied by the sign of $x(n)$. In this case we use the $\hat{x}(n)$.

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We can write from the encoder side, the encoder has got..... in fact here $y(n)$ and then after the quantization of $y(n)$ here we are going to have y cap of n . And in fact y cap of n we can therefore write as the quantized value of $\log \text{mod } x$ of n . You see, we directly get from here: $\log \text{mod } x(n)$ and then take the quantized value of that so you get y cap of n which is quantized value of $\log \text{mod } x(n)$ which of course we can write as $\log \text{mod } x(n)$ and then we can write this as $\log \text{mod of } x(n) \text{ plus epsilon } n$ because there will be quantization error. So the quantization error is denoted by $\text{epsilon } n$. And then **once obtained** once we obtain this y cap of n then x cap of n that is at the decoder end; x cap of n will be written by this equation directly: exponential of y cap n into sign of $x(n)$.

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Now here we simply have this; I mean, exponential y cap n can be written as mod of $x(n)$ mod of $x(n)$ into sign of $x(n)$ this directly follows from our encoder where y cap of n we are getting as or rather to say exponential of y cap n is mod of $x(n)$ into sign of $x(n)$ and then we also have the exponential of.....because you see we are taking just this y cap of n and y cap of n is \log of mod $x(n)$ plus $\epsilon(n)$ so this whole quantities $\epsilon(n)$ we have to take; this whole quantity's exponential we have to take so there will be a quantity exponential of this logarithm $x(n)$ which is simply the $x(n)$ and then there will be another term which is exponential of $\epsilon(n)$. So, exponential $\epsilon(n)$ is a term that we will be getting outside this. So this is exponential of $\epsilon(n)$ and this whole thing mod $x(n)$ sign $x(n)$ is nothing but $x(n)$ so this is $x(n)$ into exponential of $\epsilon(n)$.

If $\epsilon(n)$ is very small for very small $\epsilon(n)$ we can approximate this exponential $\epsilon(n)$ as a linear quantity so a very small $\epsilon(n)$ we can write as the x cap of n ; this whole expression we are making for x cap of n remember because we proceeded from this x cap of n (Refer Slide Time: 48:43) so x cap of n is this and for very small $\epsilon(n)$ I can write this as $x(n)$ rather I have to put an approximation sign approximately equal to $x(n)$ into..... instead of writing $\epsilon(n)$ we write as $1 + \epsilon(n)$ and this is equal to $x(n)$ plus $\epsilon(n) x(n)$ which

is equal to $x(n)$ plus $f(n)$ where $f(n)$ is equal to $x(n)$ epsilon n . And since $x(n)$ and epsilon n are assumed to be independent we have sigma square f equal to..... so $f(n)$ is nothing but the product of $x(n)$ and epsilon n and if the noise and the signal they are uncorrelated which we are assuming then we can have the variance of this f that is the product is equal to sigma square x into sigma square of epsilon so that the signal to noise ratio will be sigma square x divided by..... in this case we are going to have sigma square x into sigma square epsilon because it is sigma square x by sigma square f that is going to be the error term. So **x cap of n** because x cap of n is $x(n)$ plus $f(n)$ so that is why $f(n)$ is basically the noise term so that the noise power is sigma square f and we have sigma square x by sigma square f and sigma square f is written as sigma square x into sigma square epsilon.

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Handwritten mathematical derivation on a whiteboard:

$$= |x(n)| \operatorname{sign}[x(n)] \exp[\epsilon(n)]$$

$$\hat{x}(n) = x(n) \exp(\epsilon(n))$$

for very small $\epsilon(n)$,

$$\hat{x}(n) \approx x(n) [1 + \epsilon(n)] = x(n) + \epsilon(n)x(n)$$

$$= x(n) + \frac{f(n)}{x(n)}x(n)$$

where, $f(n) = x(n)\epsilon(n)$

$$\sigma_f^2 = \sigma_x^2 \sigma_\epsilon^2$$

$$\text{SNR} = \frac{\sigma_x^2}{\sigma_x^2 \sigma_\epsilon^2}$$

Now you see the beauty that from the numerator and the denominator this sigma square x that gets cancelled out and we simply get 1 upon sigma square epsilon to be the signal to noise ratio which means to say that the signal to noise ratio in this case for **the logarithmic transformation** by using the logarithmic transformation we see that the signal to noise ratio is independent of the signal variance. This we did not see for the regular quantizer, for the linear quantizer we did not see that but after the logarithmic transformation we see that the signal to noise ratio is becoming

1 by sigma square epsilon which means to say that it is not dependent on the signal and it is only dependent upon the quantization step size.

So this much for today and in the next class we are going to talk about adaptive quantizers as applicable to the speech signals. Thank you.