Digital Voice and Picture Communication

Prof. S. Sengupta

Department of Electronics and Communication Engineering

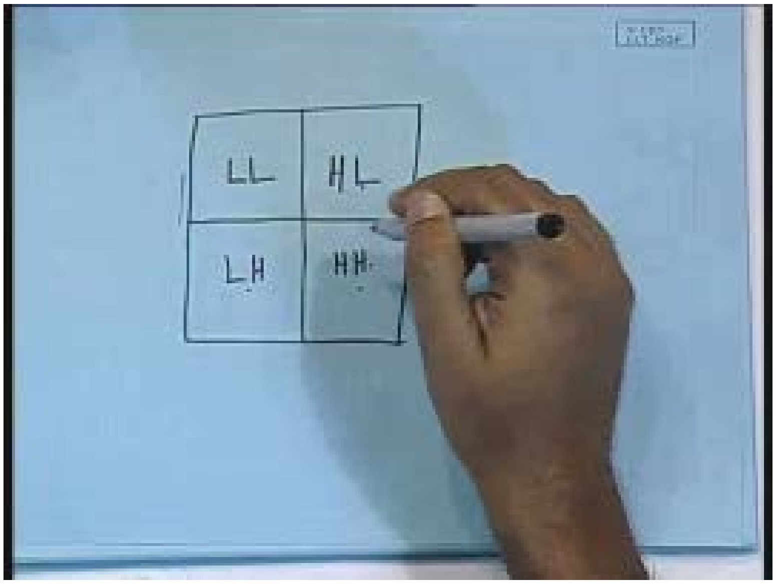
Indian Institute of Technology, Kharagpur

Lecture - 21

DWT on the Images and Its Encoding

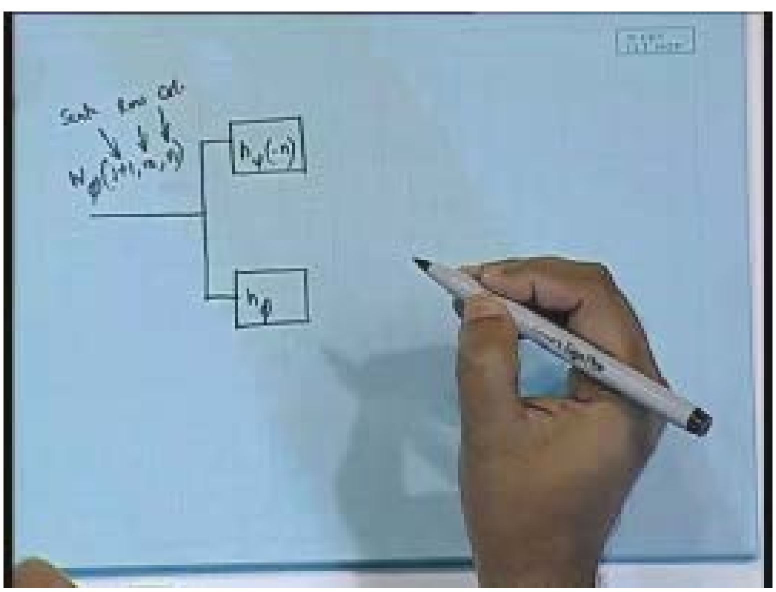
.... discussed about the discrete wavelet transforms as applied on the images, so DWT as applied on the images and also the way we are going to encode the DWT coefficients. So DWT on the images and its encoding; this will be the topic that we are going to cover today. Now, before now in the last class we were discussing about the way the discrete wavelet transform is applicable on the images. Now, as we had seen that we can apply the concept of the one dimensional discrete wavelet filters applicable into two dimensional way so that one can apply it on the images because images being a two dimensional signal so essentially what we have to do is in a block diagram form we can summarize it like this that if we have if we start with a scale let us say that we start with a scale j plus 1 in that case its filter image if we take as W phi (j plus 1 and with m and n) where m and n are the indices in the row and the column directions so m and n so m is the row index and n is the column index and j is the and j plus 1 will be the starting scale so this is the scale and what we can do is that from W phi (j plus 1, m, n) W the suffix phi indicates that it is the scaling function it is after application of the scaling function that means to say that it is the low pass filtered version of the signal meaning that when we apply it on the two dimensions then the subband that we are getting by filtering the image as low pass filtered along the rows and low pass filtered along the columns that means to say that the LL subband the picture which we had shown in the last class if you recall what we did in the last class was that after applying the discrete wavelet transform on the images we will be essentially getting four different subbands which we are designating as LL HL LH and HH these four subbands respectively where LL is the low pass filter. So essentially this is W phi so you can say that W phi to start with W phi could be the entire image itself but as we are going to show that such kind of application can be made to any scale that means to say that we once we obtain the subbands we can iteratively partition the subbands which we already obtained.

(Refer Slide Time: 3:49)



so we are going to show that Anyway first let me complete what I was trying to tell you. That means to say that if W phi (j plus 1, m, n) is the starting signal then this starting signal we are passing through two filters and one filter is what we are calling as h psi of minus n.

(Refer Slide Time: 4:52)



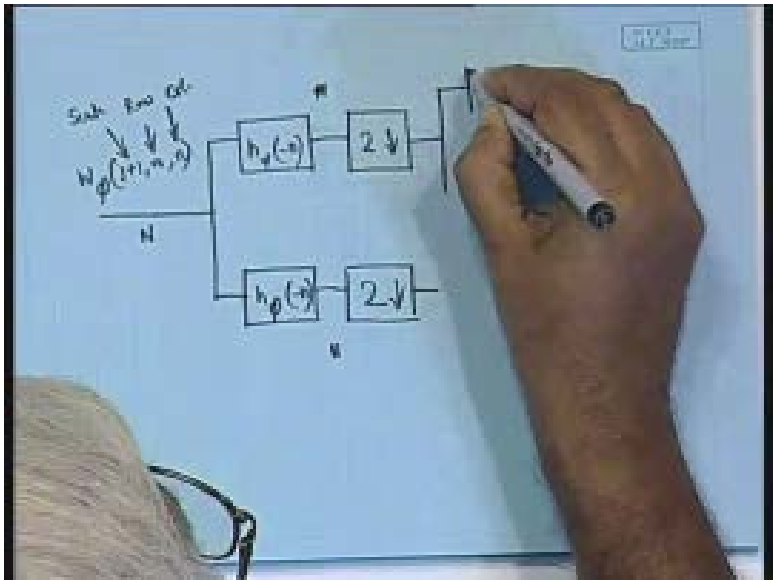
Now basically n is the index of the column so definitely this is the filter that we are applying along the columns. Now h psi means it is the wavelet filter applied on the columns. And likewise we also have to apply the scaling filter which we indicate as h of phi; h of phi minus n that will be the scaling function that will be the scaling filter or the low pass filter that we will be applying on the signal. now this is applied along the columns.

Now what we do is that because we are splitting up the signal into two essential two bands it is like one is the low pass filtered version and the other is the high pass filtered version; this is the high pass filtered version and h of phi filtered by h of phi gives us a low pass filter version of the signal.

Now, whenever we are splitting up into two subbands like this in that case the band or rather to say the bandwidth of the signal the special bandwidth of the signal essentially gets halved in each of these subbands. So as a result what happens is that we will be getting redundant samples and one does not lose any information by having by throwing out the alternate samples that means to say that having a sub-sampling by a factor of 2. So what we have to do is to apply both on the h psi filtered image and also on the h phi filtered image we have to apply a sub-sampling by a factor of 2.

Sub-sampling means that we.............., I mean, out of two consecutive samples we pick up only one of them and discard the other which means to say that if we start with N number of samples over here in that case at the output of each of these filters we will be having N samples at the low pass filter as well as the high pass filter but individually their bandwidth is halved. So out of these N samples only N by 2 samples will be required for both low pass filter and for the high pass filter version. Now how do you get N by 2 samples you have to do a sub-sampling you have to throw out the alternate samples so this is a sub-sampling by a factor of 2 which you are doing in both these directions.

(Refer Slide Time: 8:10)



Then this sub-sampling you are doing along the columns, n being the column index. so when you were applying a column filter or rather the wavelet filter along the columns then sub-sampling also is along the columns so you do a sub-sampling by a factor of 2.

Now, after that what you do is that once you sub-sample this along the columns you can now apply the one dimensional filters along the rows. So what you do is that this signal you can split into two other bands: one band you call as h psi and this time we are going to call as h psi minus m. The index m is used for the row and here we are going to call it as h of phi that is the low pass filtered version: h phi of minus m. After the column filtering we are applying one low pass filter along the row and one high pass filter along the row. This is the low pass filter along the row and this is the high pass filter along the row.

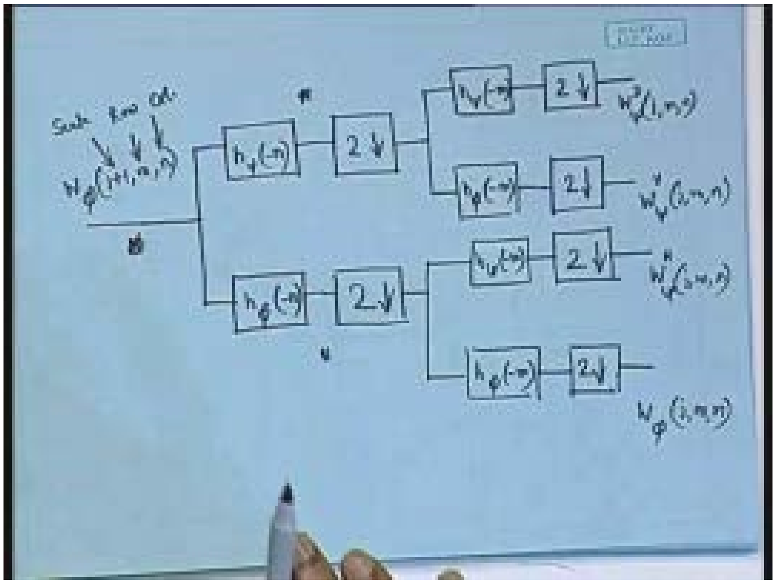
Now, individually again each of these subbands will contain half of the samples so what we have to do is to sub-sample by a factor of 2; just like the way we did sub-sample in the direction of the columns here we have to do a sub-sample in the direction of the rows. So what we do is that individually these two channels we have to do the sub-sampling and we will be getting these two outputs. This we are going to call as W psi (j, m, n) and this is............ and just look at the nature of this signal (Refer Slide Time: 10:00) and this signal we are deriving by high pass filtering along the column and high pass filtering along the rows which means to say that it is extracting the high pass along both rows and columns which means to say HH.

Basically HH subband is being referred to over here and high pass filtering extracts the edges so this is extracting the diagonal edges. So we are going to indicate this as W psi with a superscript d because it is extracting the edges along the horizontal as well as the vertical which means to say that it is extracting the diagonal edges. So it is W psi superscript the (j, m, n). So from the scale j plus 1 we have got into scale j.

Now what is this one (Refer Slide Time: 10:59)?
This one is going to be........... I mean, along the columns there is a high pass filtering but along the rows there is a low pass filtering. So because along the columns there is a high pass filtering we are extracting which edges; the vertical edges. We are going to call this as W psi superscript v (j, m, n) and for the low pass filtered signal along the column we can apply the two filters. Similarly, we can apply h psi minus m and we can apply h phi of minus m.

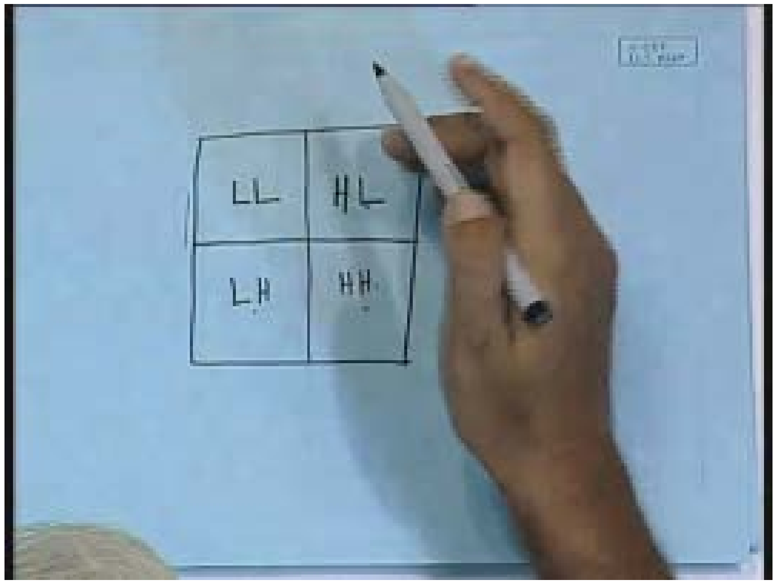
Hence, individually by very similar argument we have to do sub-sampling by a factor of 2 and what we have over here is just the opposite of what we had for W psi superscript v; why; because along the columns it is low pass filtered and along the rows this is high pass filtered that is why this channel we are going to call as W psi but with a superscript H (j, m, n) and this one is going to be what (Refer Slide Time: 12:37) low pass filtered along the rows low pass filtered low pass filtered along the columns, low pass filtered along the rows so this is the LL subband or rather we are not going to call it as W psi we are going to call it as W phi W phi (j, m, n).

(Refer Slide Time: 12:57)



Now look at something interesting. What was our starting point? Our starting point was W phi (j plus 1, m, n) and in this case what have we got; W phi (j, m, n) so what is the difference between this and this; only that j plus 1 is changed here to j. So we have come one scale down. We have come one scale down means that we have effectively lost the resolution by a factor of 2 along the horizontal and along the vertical directions. So you see the picture that we had shown in the last class.

(Refer Slide Time: 13:48)



So individually because we are doing sub-sampling by a factor of 2 along the rows and also along the columns we are effectively doing a sub-sample by a factor of 4. So one fourth of the sample, so LL subband contains one fourth of the total number of samples which are there in the image and likewise every subband is containing one fourth of the total number of samples.

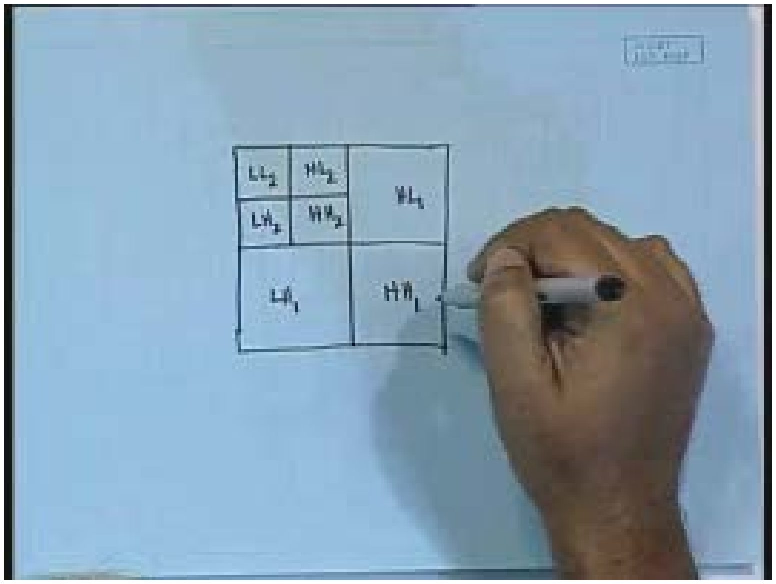
We are just indicating, the way of representation like this means that in this part or in this quadrant we are accommodating all the pixels which are basically the LL filtered version which means to say along the rows' low pass filter, along the columns' low pass filter these type of signals we are putting into the LL format.

Now essentially what we are getting is that individually these four bands......... now these four bands individually can be partitioned or analyzed further. We can take up the LL subband we can take up the HL LH or HH any of these four subbands we can pick up and we can do a similar partitioning or similar sub-partitioning like the way we did for the original signal. So original signal partitioning we got this 4. Now let us say that we take up this HL subband. So we will be partitioning this HL subband into 4, here there will be the LL subband within this HL, this will be the HL within this HL, this will be the LH within this HL, this will be HH within this HL and then any of these sub-partitions also can be analyzed. Likewise LH can be analyzed, HH can be analyzed, LL also can be analyzed.

Now which one is normally taken out of these four subbands?
Now, as I have told much time, that images are generally very rich in the low frequency content. So whenever we are doing signal analysis on the images essentially what we try to look for is that we do the analysis further on the low pass filter version of the images. So LL we have already got one low pass filter version so it is the and if we look at the image we will be seeing that as if to say that the original image has been squeezed to a one fourth size image and put into the LL subband and most of the and the richest information, I mean, out of all these four subbands you will find the maximum information is contained within this LL subband only. So logically what we should do is to partition this subband. And mathematically also it is very easy because now having obtained W phi (j, m, n) from W phi (j plus 1, m, n) I can use this W phi (j, m, n) use that as the starting point to the next one and obtain four subbands over here at the j minus 1 scale. So here i will be obtaining W psi (j minus 1) W psi v (j minus 1) W psi h (j minus 1) here I will be obtaining W phi (j minus 1).

So if I analyze LL subbands essentially what we are going to get is something like this that this was our original image size, we split up into four subbands and let us say that we keep the HL LH and HH intact as it is but we analyze the LL subband further. That means to say that we will be obtaining four sub sub-partitions of this LL subband which means to say that here I will be getting one LL subband but this LL subband we will not write as LL, we will use a subscript let us say a subscript 2 we use two indicating that this is the second level of the composition so this is LL 2, this will be HL 2, this will be LH 2 and this will be HH 2 and just to keep the consistency we mark HL as 1, LH 1 HH 1 where the number that I am writing in the subscript essentially indicates the level of partitioning.

(Refer Slide Time: 19:08)



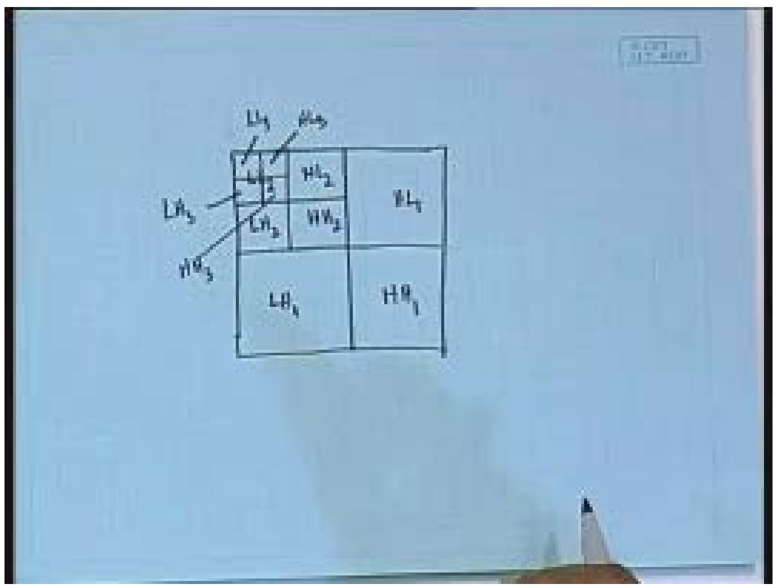
Therefore, totally we now have seven subbands as you look at it. There are seven subbands, four subbands at level 2 and three subbands at level 1.

Now, again what happens is that individually any of these four subbands can be analyzed and again we will be finding that it is the low pass filtered version which contains the maximum number of maximum amount of signals. So LL 2 will be richest in the information as compared to the HL 2 LH 2 and HH 2 so it will make more sense in partitioning this LL 2 subband further. So we now decide to partition LL 2 partition it into four subbands, here this we will be getting as LL 3 (Refer Slide Time: 20:09), this we will be getting as HL 3, this will be our LH 3, this will be the HH 3 so we have come one more step further. That means to say that this is a typical three level decomposition and nothing prevents us from making a fourth level of decomposition or fifth level of decomposition so I can go and go further and further down which means to say that actually what is the physical concept of the scale; what was our highest scale; highest scale was the original image.

Now original image contains the maximum resolution and after the first level of decomposition every subband is containing one fourth of the resolution. So there is a bit of coarseness which we have incorporated after the first level of decomposition. Then second level further coarseness, third level further coarseness like that we are going from the finer domain to coarser and coarser domain of analysis.

Essentially what is the purpose?
Remember, what we talked in the beginning that the whole objective of doing the wavelet analysis is a localization and in this case we are talking about the space frequency localization.

(Refer Slide Time: 21:51)

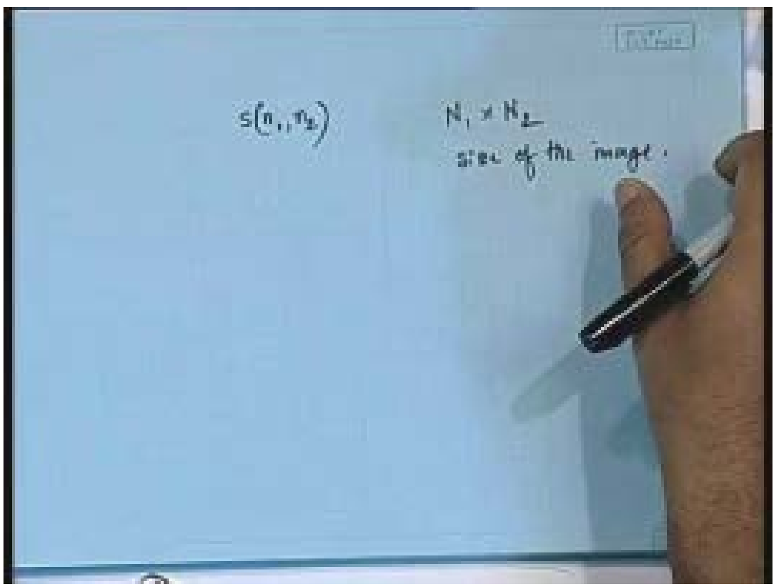


We want to know that at which spatial location or within which spatial location range we are going to have the higher frequency component and at which spatial location we are going to have lower frequency component and like that and so on. So, in order to have that kind of analysis or space frequency localization this sort of a partitioning is definitely going to help us.

Now this way of partitioning the LL subband and obtaining the further subbands out of it, this method of partitioning is called as dyadic partitioning. Now commonly dyadic partitioning is followed in the image because of the reason I told you that the images are rich in low frequency content and that is why this dyadic partitioning scheme where the LL subband will be decomposed into further and further coarser level that is adopted, so this is the total scheme of the dyadic partitioning and although it is not mandatory that you have do dyadic partitioning only because individually any of these subbands you can analyze. If it so happens that your images having rich high frequency contents it is having a reach HH band content, you can analyze the HH band and obtain and extract some further analyze further analysis you can do further analysis on that in order to derive your sequence.

This is very widely used and having known this........... so what we did essentially is that how to apply the wavelet on the images. Now the same thing one can mathematically write down like this that let us say that we take the image function to be s (n 1, n 2) and we take the image size to be capital N 1 by N 2 so this is the size of the image. N 1 by N 2 is the size of the image and the intensity function is indicated by s (n 1, n 2).

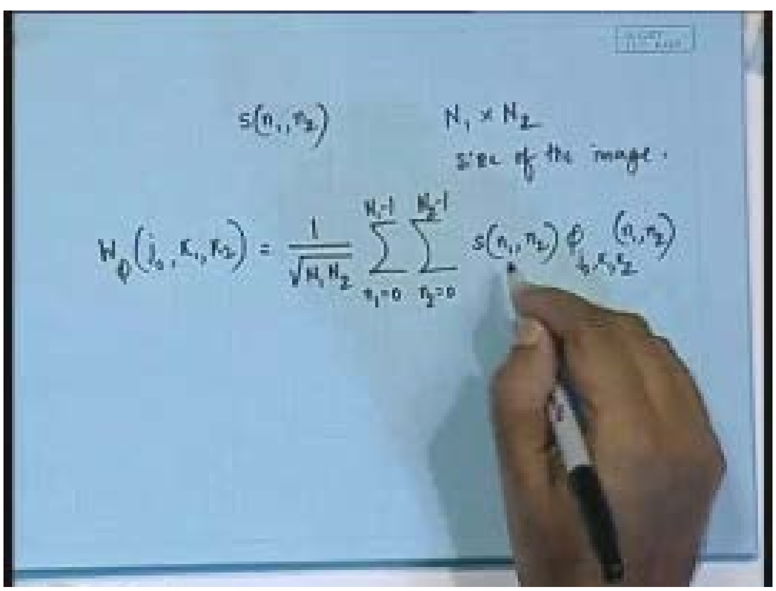
(Refer Slide Time: 24:41)



Now what we are doing in the wavelet transform basically? In this case (N 1, N 2) which is in the spatial domain will be transformed to the wavelet domain. So what we obtain so basically we will be obtaining two signals: one is the scaling filtered signal and the other will be the wavelet filtered signal and we can write down the mathematical expressions like this that: W phi if we apply the wavelet filter at scale j 0 let us say; so we are writing it as W phi j 0 and as the index we write as k 1 and k 2 the index in the n 1 along the n 1 direction we are writing as k 1 and the index along the n 2 direction we are writing as k 2. the reason why we are using the indices as k 1, k 2 instead of using n 1, n 2 is just to make a separation that n 1 and n 2 we are using for the image domain index and k 1, k 2 we are using for the wavelet filter domain index.

So W phi j W phi j 0 j 0 being the scale, k 1, k 2 will be the wavelet filtered image and this can be written as; a general form of two dimensional discrete wavelet transform would give us 1 upon root over N 1, N 2 and because it is two dimensional thing we will be having a double summation and the wavelet is being applied in this case scaling filter of course will be applied to the image signal which is s (n 1, n 2) and the index will be n 1 going from 0 to capital N 1 minus 1 and n 2 will go from 0 to capital N 2 minus 1. This is s (n 1, n 2) and then we have to multiply this s (n 1, n 2) by phi j 0 (k 1, k 2) this will be the filter and this filter will be applicable on N 1, N 2; at the position N 1, N 2 because n 1 and n 2s are varying over here this will be applied at a different position.

(Refer Slide Time: 27:24)

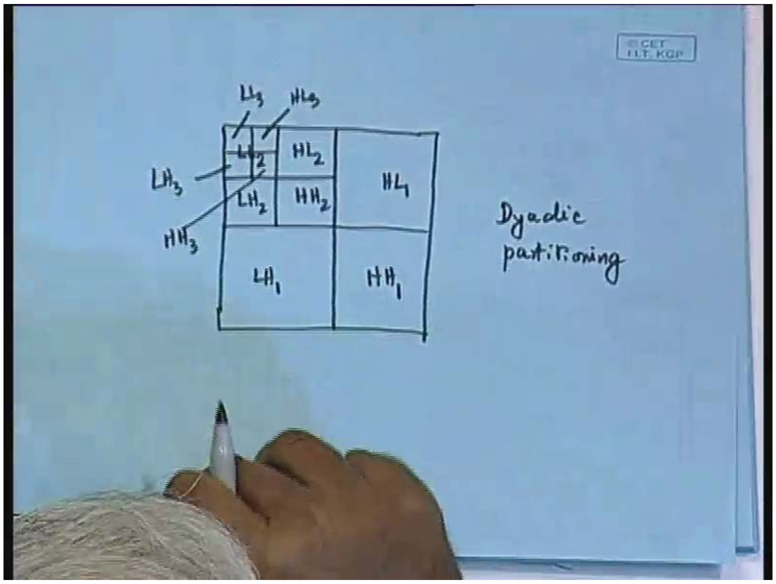


Because, at every point of the image we have to apply the corresponding filter and the product of this and then add it up that gives us the W phi. And similarly we will be obtaining the W psi (j 0, k 1, k 2) but mind you, we are applying the for the low pass filter we are having only one subband whereas for the high pass filter we are having three subbands. So essentially we are going to use a superscript i which i will be explaining very shortly. So W psi with a superscript i we are writing as 1 upon root over N 1 and N 2 and the summation process will be n 1 is equal to 0 to N 1 minus 1 n 2 is equal to 0 to N 2 minus 1 and this will be s (n 1, n 2) and this will be psi j 0 k 1, k 2 psi with a superscript i and parameters (n 1, n 2) and the significance of this i is that i can be either as the {H or the V or the D}. What are the significances of this {H, V and D}; horizontal, vertical and diagonal.

So remember that just little while back (Refer Slide Time: 00:29:13 min) we were using this notation: W psi {D, V, H} so D basically means high pass in both rows and columns; V means high pass along the column but low pass along the rows; H means just the opposite of that so that is why totally we have got three such versions so accordingly we have to apply the filter that is to say that when we are talking of psi H when we talk of psi H essentially psi H will mean that it will contain a low pass filter along the columns and it will contain a high pass filter along the rows. So accordingly the transformation kernel will be arranged. This is how you are obtaining; from s (n 1, n 2) you are obtaining the wavelet filter the scaling function filtered and wavelet filtered images; you are obtaining W phi and the three W psi filtered images.

Now, having known this transformation which we can call as the direct DWT as applied as applicable on the images. So this is the DWT in the direct form and because it is a reversible transformation we should be able to recover s (n 1, n 2) given this W phi and the three W psi’s.

(Refer Slide Time: 30:56)



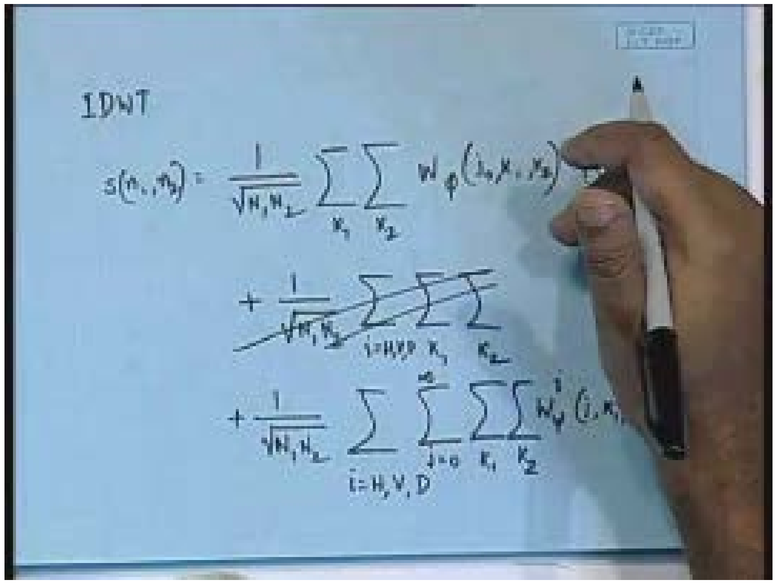
What I mean to say is that if somebody supplies to me this kind of a subband picture that means to say that somebody has provided the individual subband filtered images after application of the scaling and the wavelet filters seen along the rows and the columns I should be able to get back the image from there. Now how to get back that; that will be obtained by having an inverse discrete wavelet transformation.

What is the inverse discrete wavelet transform going to yield us?
It will give us s (n 1, n 2) out of this W phi and the three W psi's. so mathematically the inverse DWT or I can write in short form as the IDWT; inverse DWT expression would be s (n 1, n 2) this could be written as 1 by root over N 1, N 2; now in this case the convention that is followed is that both for the direct transform and the inverse transform this 1 by root over N 1, N 2 factor is used uniformly so that the direct filter structure and the inverse filter structure they remain the same because totally we require a normalization by factor of 1 by N 1, N 2. Hence application of the direct and the inverse; depends how you accommodate that 1 upon N 1, N 2 in the forward and the inverse; one way of doing it is to have 1 by square root of N 1, N 2 for both forward and the inverse.

So 1 by root over N 1, N 2 and then we have to write a double summation and that double summation would be over k 1 and k 2 applied on W phi (j 0, k 1, k 2) into phi phi with subscript j 0, k 1, k 2 with (n 1, n 2) where phi being the transformation kernel or the filter whatever way you talk of; so this will be the low pass filtered version and what we have to do is to add it up with 1 by N 1, N 2 1 by square root of N 1, N 2 and in this case in the summation process there will be not only summation over k 1 and k 2 but because we have to add up the three subbands we just put yet another index which we call as i is equal to 0 to i could be H or V or D i could be H, V or D and summation over k 1 and k 2 and again one more summation term we are missing over here that.................... see this is for one scale that we are obtaining (Refer Slide Time: 34:26).

Now typically we are going to have a multi-scale filtering situation; like in this case we have obtained a multi-scale filtering so essentially what we have to do is to add up over the scales also so what we require to do is that although the LL subband part we are doing from the highest subband and the highest subband if we consider to be the j 0 what we should do is that we should have yet another summation term; so better we rewrite it as plus 1 by root over N 1, N 2 summation over i and i could be H, V or D and then the scale factor we put as j so j is equal to 0 to................ we can typically write it as infinite because it depends that how many levels of partitioning we have done, so it is not actually infinite, it will be summed up to the levels of partition that you perform: summation over k 1, summation over k 2 and in this case it will be W psi of course with a superscript i indicating {H, V, D} over (j, k 1, k 2) where j being the running scale and this multiplied by psi again superscript i over j, k 1, k 2 (n 1, n 2).

(Refer Slide Time: 36:13)



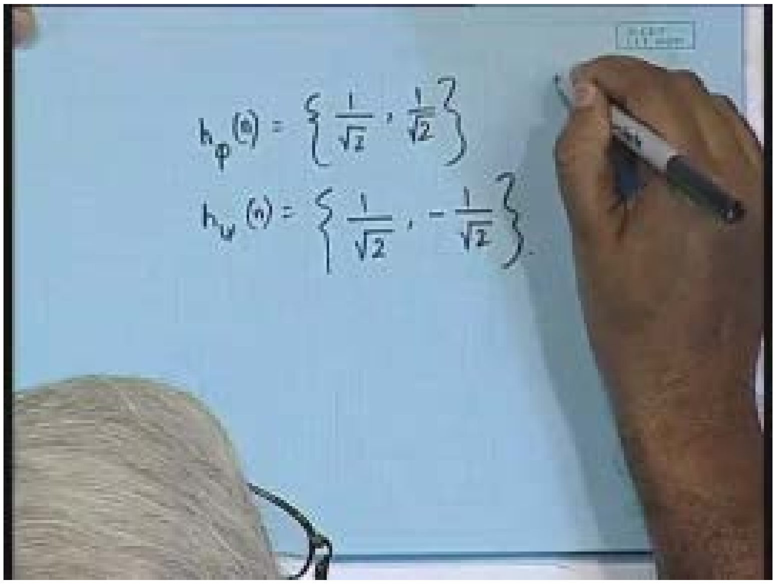
See, if we do that if we do all this entire summation process and add it up with the LL subband we can recover s (n 1, n 2) and s (n 1, n 2) it is not s cap (n 1, n 2) or s dot (n 1, n 2) because we have not applied any quantization so the signal is exactly recoverable.

As long as the filter does not do any truncation, if we neglect any truncation error because of due to the filtering, assuming that the filter that we are applying is completely a reversible filter it should be possible for us to obtain s (n 1, n 2) directly without any signal degradation or whatever.

Now what we can do is that........... application of the filters. Now what type of filters are we using as the phi filter and as the psi filter?
It is very clear that we have to apply a two dimensional filtering over here and two dimensional filter can be realized by a cascaded form of low pass filter and high pass filter. So what we can do is that we use two filters; let us say that if we use h phi of n and h phi of n will be a low pass filter and if we take its coefficient as 1 by root 2 1 by root 2 which means to say that this is a two type filter; if we take a two tap filter for h phi of n having coefficient 1 by root 2 1 by root 2 and as h psi of n if we take another two type filter which will be having the coefficients 1 by root 2

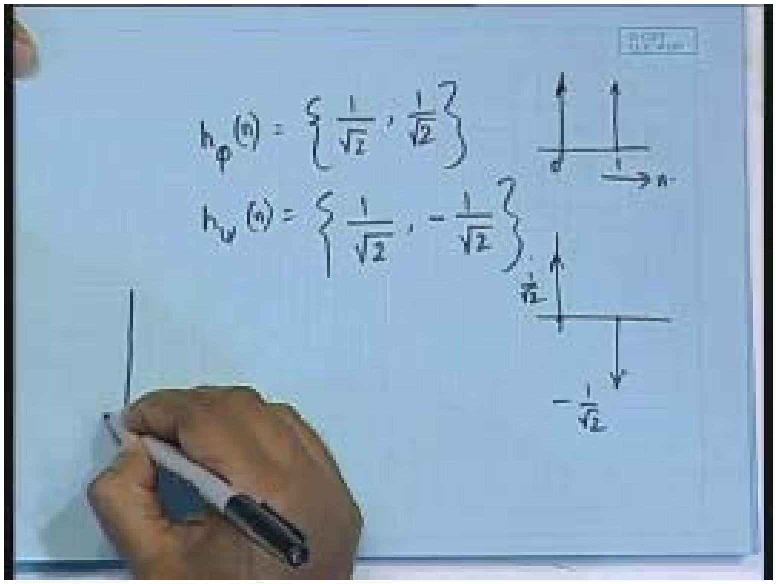
and minus 1 by root 2 then what type of filter is that? Anybody having any idea about what filters we are talking of? What is the sketch of the filter?

(Refer Slide Time: 38:45)



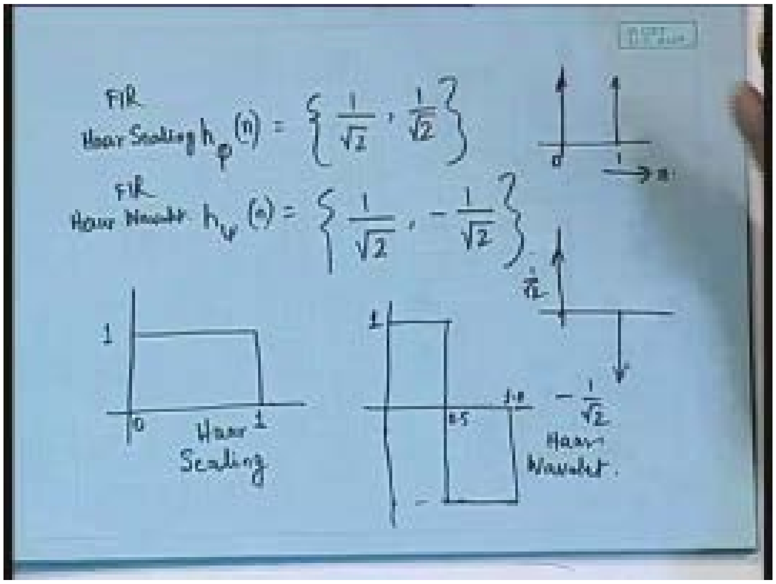
Here of course, this is an FIR filter so in this case we are having......... if we plot n over here so for n is equal to 0 and for n is equal to 1 we are having value equal to 1 by root 2 so this is for n equal to 0, this is for n equal to 1 whereas for h psi filter for n is equal to 0 we are having a value equal to 1 by root 2 and for n is equal to 1 we are having a value equal to minus 1 by root 2 so essentially for the low pass filter what we are doing is that we are adding up because it is a it is a process of averaging, so you apply the filter this filter so the two adjacent samples would get added up and here what happens is that when I apply h psi filter the two adjacent samples get subtracted. This is just a simple addition and this is a simple subtraction. And in fact, if you remember that this confirms with our definition of the Haar filter.

(Refer Slide Time: 40:09)



Haar filter we discussed that Haar low pass filter was............. of course that time we defined the continuous domain, so Haar low pass filter was this so here we were indicating the Haar wavelet or the Haar scaling function to be this and the Haar wavelet function was.................. here it will be up to 0.5 and then 0.5 to 1 it will be negative (Refer Slide Time: 40:37) So this is the and this amplitude was 1, this amplitude was 1 over here so this is so this is 1 so this this was in the continuous domain, this was the Haar scaling function continuous Haar scaling function and this was continuous domain Haar wavelet function, this is Haar wavelet, that one was Haar scaly. Now this is just the FIR filter version of Haar scaling. So this is FIR version of Haar scaling and h psi of n will be the FIR version of Haar wavelet.

(Refer Slide Time: 41:33)

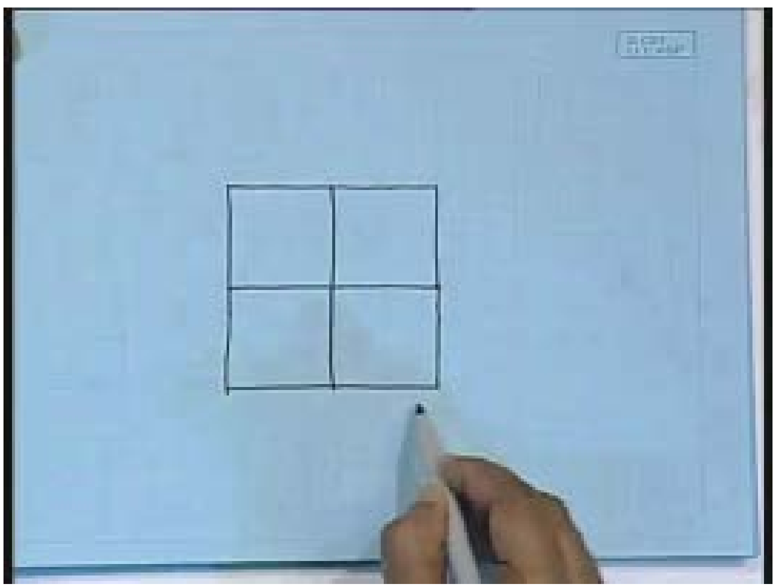


So, one can apply this very easily. In fact computationally it is a very simple thing. 1 by root 2 is a constant factor so you can multiply by 1 by root 2; with every level of scaling you have to do that 1 by root 2 factor of multiplication. But other than that it is only adding samples or subtracting samples. If it is low pass filter add the samples, if it is high pass filter subtract the samples. So conceptually it matches because any low pass filter would basically mean averaging and any high pass filter would mean a differentiation, the differentiation in the discrete domain means it is just subtraction or difference, difference in the discrete space. So this is what we will be obtaining.

In fact, still image compression system uses this wavelet in a big way and here essentially what we are doing is to analyze the signal into different subbands. Now so far we have not talked about the compression. If we are talking of still image compression then it must be seen that how this individual subband, this LL HL LH HH how individually these subbands can be compressed. Now, there must be some amount of redundant information which will be present. at least pima facie what we can say is that whenever we have got such different subbands, I have already said that the image will be rich in the low frequency content that means to say that LL will contain more information and the other subbands will be having a sparsity of information. Even if we truncate the higher frequency coefficients just like the way we did for the DCTs also; because DCTs although the motivation was a psychovisual motivation that was applied and that was the main reason for truncating the high frequency coefficients in the VCT. Even in wavelet also the high frequency coefficients would be quantized more severely or quantized more coarsely as compared to the low frequency sequence. So, in order to make effective use of the compression what we should do is that we should study that what is the inter-relationship between the different subbands; is there any inter-relationship? Is there any special inter-relationship; that is what let us study.

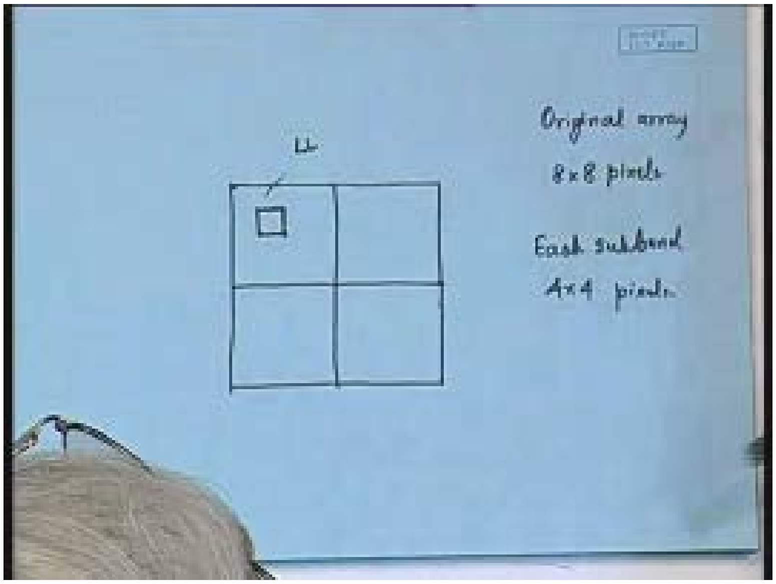
Let us say that we have only the first level of partitioning.

(Refer Slide Time: 45:29)



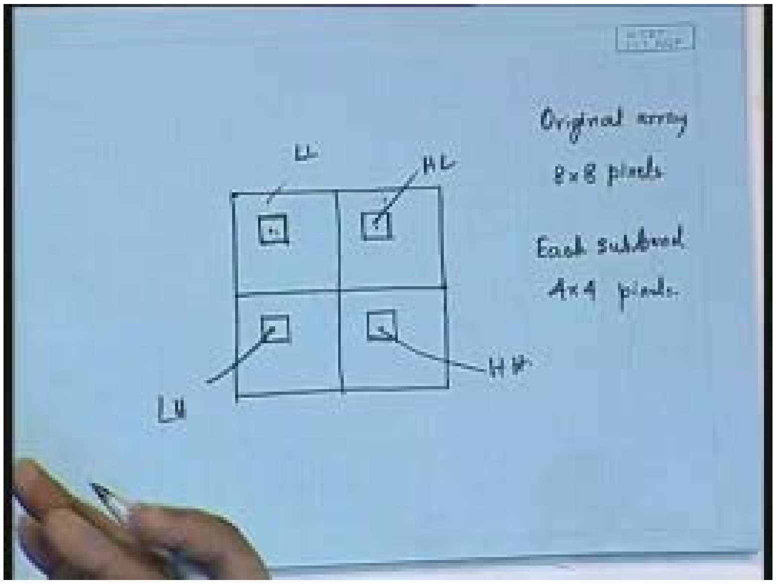
This was the original image and we have got four subbands LL LH .... now let us say I just wanted to draw it by using the grid of this paper I think you should be able to locate the individual grid positions in this picture. Now let us say that if I pick up any square within this, say I pick up this one and it is a pixel let us say (Refer Slide Time: 46:03) which means to say that the original size of the image was how many 8 by 8 pixels so 8 by 8 pixels was the original array so original array was 8 by 8 pixels and now I have got four subbands, so individually each subband each subband now contains an array of 4 by 4 pixels; now this is a 4 by 4 and let us pick up any particular pixel; let us say that this is the pixel that I have picked up in the LL subband; for simplicity I have assumed only the first level of decomposition.

(Refer Slide Time: 46:54)



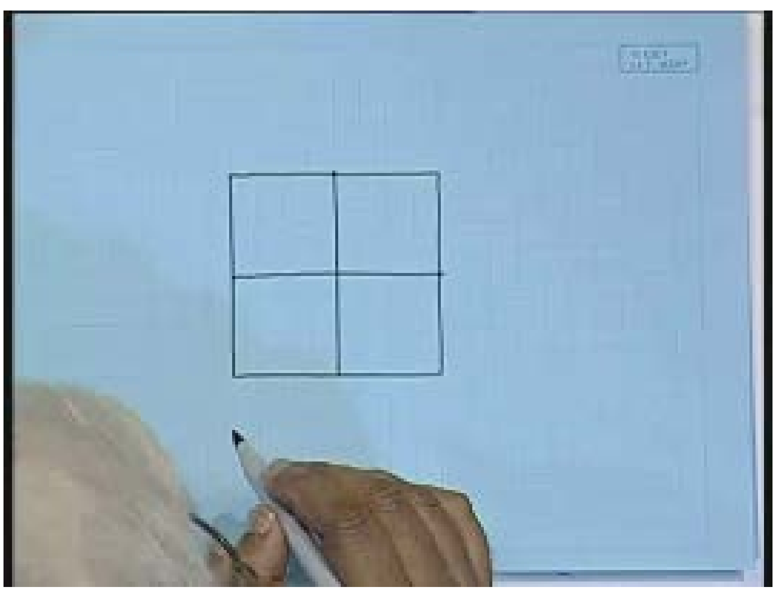
Now, spatially a position that is picked up over here which means to say that in terms of the image coordinate within this LL subband space if I take this to be reference then within the LL subband its coordinate is 1 1, so this is the 1 1 coordinate with respect to the LL subband. Now at the same position at the same pixel position 1 1 we are having the HL subband image, so HL subband image will be having this pixel position the corresponding pixel position in the LH will be this and the corresponding pixel position in this will be this so that I can say that this pixel is related to this pixel in a sense that these pixels the higher frequency versions or higher frequency coefficients corresponding to these pixels are available in this HL LH and HH subbands. So there is a relationship that this is as if to say related somehow this.

(Refer Slide Time: 48:18)



Now let see that what will be the state of this relationship when we have more levels of partitioning, let us say that when we have two levels of partitioning. In that case again I try to draw the same type of grid.

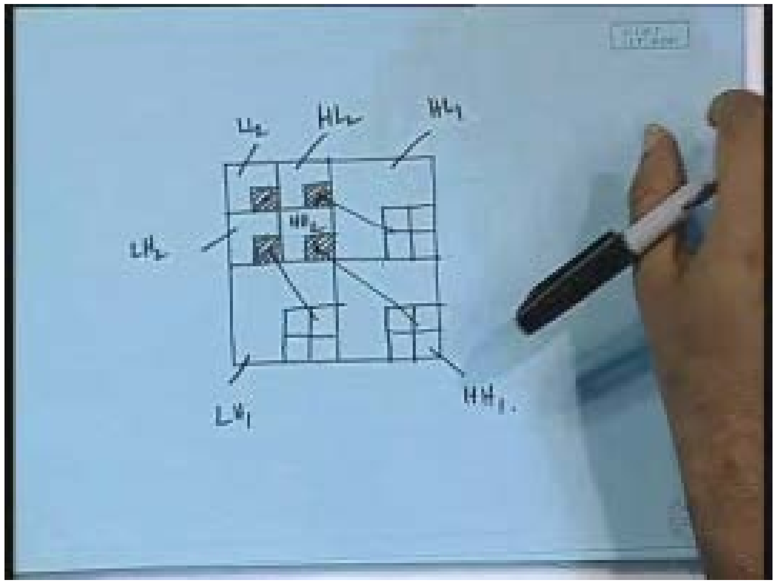
(Refer Slide Time: 48:56)



This is after the first level of decomposition and we make a second level of decomposition. So now individually these subbands are having 2 by 2 pixels. Let us say that I pick up this pixel from the top most LL subband. This will be having some correspondence with this; means its higher filtered version could be this in the corresponding........... I should call as HL to now this pixel could be what.................. we will be getting at HL............ at LH to this is what we will getting as HH 2.

Now this must have some kind of a relationship with these subbands also. But now see, if we call this as LL 2 and we call this as HL 2, this we are calling as LH 2 and this we are calling as HH 2. Now look at HL 2 and look at HL 1; (Refer Slide Time: 50:19) this is HL 1, this is LH 1, this is HH 1. Now if I ask you that find out a correspondence of this pixel with the corresponding pixels in the HL one subband you have a difficulty, why difficulty; because here the resolution is a 2 by 2 this is a coarse whereas here we are having a resolution of 4 by 4 this is a much final resolution. So one pixel in the HL 2 subband corresponds to how many pixels in the HL 1 subband? It is 4 because the resolution is higher in this. So what you are seeing one pixel in HL 2 will be 4 pixels in HL 1 so this corresponds to 4 pixels of............... ((00:51:18)). Similarly this HH 2 pixel would correspond to 4 pixels in HH 1, 1 pixel in LH 2 will correspond to 4 pixels in LH 1 and LL 2’s pixel is already related to these three.

(Refer Slide Time: 51:46)

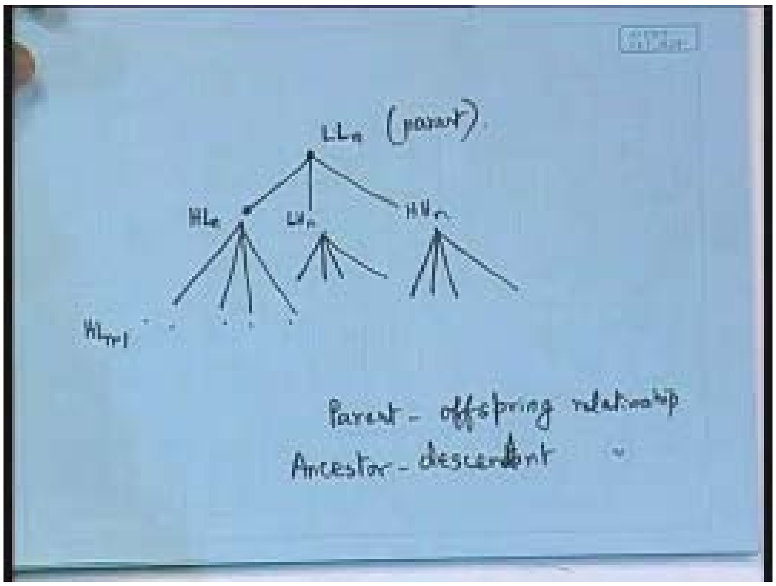


Now in this form what emerges is a kind of a data structure representation. What we can do is that, if we take this pixel, one pixel in LL 2 subband as a starting point then we can consider that its corresponding pixel in the HL 2 LH 2 and HH 2 these three other subbands its corresponding pixel positions will be considered as its descendants.

Individually each of these pixels will be having four descendants. So HL 2 this pixel will have four descendants over here, this will have four descendants; this is a picture that I have drawn with two levels of decomposition. If I draw it with three levels of decomposition in that case one pixel in HL 3 would correspond to four pixels in HL 2 and every one pixel in HL 2 will correspond to four pixels in HL 1 which means to say that one pixel in HL 3 would corresponds to sixteen pixels in HL 1. So there is a tree that evolves which means to say that pick up any pixels from the top most LL subbands.

Now consider any pixel from the top most LL subbands. Let us say that this is LL n; LL n will be the root of this tree, will be at the root of this tree and will be having its descendants in HL n LH n and HH n. Now every pixel in HL n will be having four descendants in what in HL n minus 1, HLn minus 1 it will be having four descendants; LH n will be having four descendants in LH n minus 1; HH n will be having four descendants in HHn minus 1. Individually each one of these will be having four descendants down in the HL n minus 2 or LH n minus 2 HH n minus 2 like this. So this is a kind of you can say that parent-offspring relationship; if you call this as the parent and this is called as parent-offspring relationship within the pixels or you can also call it as.......... some literatures describe this as ancestor-descendant relationship descendant sorry descendant relationship so. So, at three levels of decomposition you can that this is the grandfather, this is the father and this is the children like that.

(Refer Slide Time: 55:29)



This is this is how the tree a a tree structure evolves and why this tree structure is efficient for encoding the wavelet coefficients; I will discuss that in the next lecture and that will give you some insight as to how the wavelet coefficients are encoded, thank you.