Digital Voice and Picture Communication

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Lecture - 19

Theory of Wavelets

.... discuss about the topic which we had introduced in the last class so we will elaborate further in this and that is about the theory of wavelets.

Now in the last class we had pointed out some of the deficiencies of the discreet cosine transform and then we had said that as an alternative to the DCT the discreet wavelet transform as emerged as a very powerful technique where a significant amount of compression ratio can be achieved as compared to the DCT and it does not produce the artifacts like the blocking artifact what we had discussed in the last class. so as a theory of wavelet as you recall, that in the last class, we had developed a class of functions which we had defined like this that we had talked about phi r, s where r and s happen to be integer and this is a function of the variable x. So in this case you can take that x is a continuous variable. So x is a variable in the continuous space whereas r and s they happen to be integers and phi r, s of the continuous variable x is given by 2 to the power r by 2 phi 2 to the power (r x minus s) this is what we had already shown in the last class. So this class of functions phi r, s (x) what we can derive from different values of r and s this entire set of functions we will be calling as the scaling functions.

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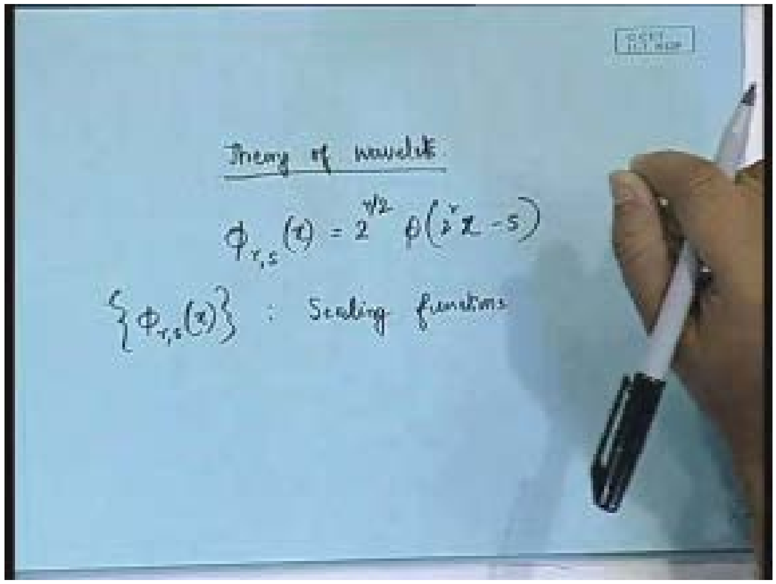

Now definitely that if we include all the possible values of r and s and we also include some function; if you choose a function such that with all r and s it should be possible for us to cover the entire square integrable real space. So, if we define the square intergrable real space as the L square R space then the entire L square R space should be covered by the set of functions phi r, s. But now let us say that we fix the R that is to say what we call as the scaling parameter, if we fix that to some value, let us say that we fix r equal to r 0 where r 0 is a specific value of r, so, by considering that r is equal to r 0 we will be able to generate a set of function that we will then write as phi r 0, s of the continuous variable x.

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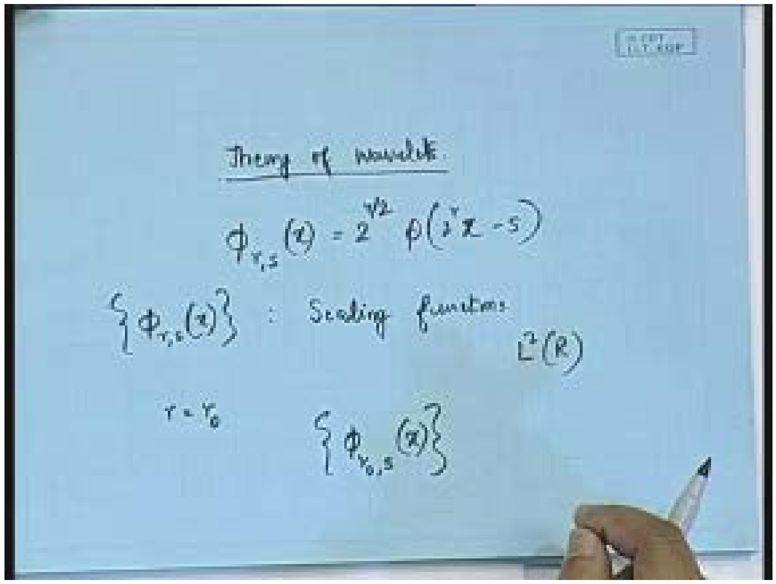

Now, when we fix up r 0 in that case what we are doing is that the only parameter that we will be able to vary to spend the square integrable space is this parameter s that is to say the shift parameter s. Now definitely it will not be possible for us to cover the entire square integrable real space if you only vary s keeping r equal to r 0 a constant quantity, is it okay?

See, in the last class we had seen an example where we had defined a scaling function which was basically defined from 0 to 1 it was having a value equal to unity and otherwise at all other points it was having a value of 0. now we can create a So, if we call that function as phi 0, 0 we had shown that when we consider phi 1, 0 then its width gets half. Now, if I ask that is it possible for me to cover the entire real space by using phi 0, 0 the answer is clearly known because I will not be able to analyze function which is having a width less than unity using that scaling function phi 0, 0.

Likewise if I take phi 1, 0 then I can analyze functions which have got a width of 0.5 or higher than 0.5. Supposing some width is 1.2 and I want to analyze a width of 0.5 that is to say using phi 1, 0 function what we had shown as an example in the last class, then we will not be able to go up to 1.2 because our resolution then is 0.5 so it will be 1 or the next approximation will be with 1.5. So definitely there is a limitation of the total subspace that it can cover. So if I fix up r to be equal to some quantity r 0 then this set of function phi r 0, s will cover only a limited subspace of the entire square integrable space L square R.

So let us denote that pictorially this subspace let us denote by this (Refer Slide Time: 07:06). So this subspace we will be getting by keeping r equal to r 0. So we call this subspace as VR 0. or let us say that if we keep r is equal to 0 because r is equal to 0 basically leads to the scaling function which we considered like the phi 0, 0 so phi 0, 0 is the form of the basic scaling function. So with r is equal to 0 we will be having the width which is equal to the basic function phi 0, 0.

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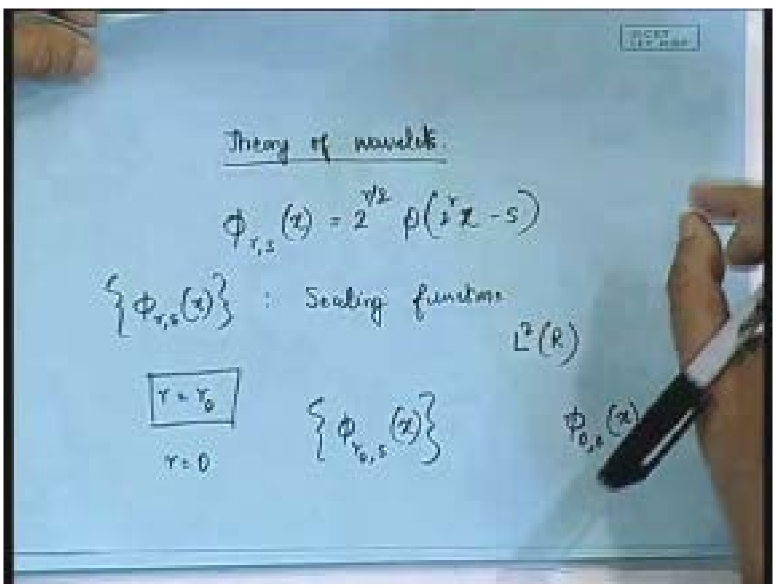

So, if I take r is equal to 0, I can define the corresponding subspace as V 0. Now supposing I increase r by a value of 1. So if it is r 0 I just make it r 0 plus. So by making it r 0 plus 1 I increase the amplitude by a factor of root 2 because it is 2 to the power r by 2 so if I increase the value of r by 1 then another multiplication factor of 2 to the power of half that is root 2 that comes in.

So, increasing r 0 to increasing the value of r by unity results in the amplitude increased by a factor of root 2 and also its width is reduced by a factor of 2, because instead of r it is r plus 1. So now what happens is that, if we consider........... now the set of functions which will be having the value of phi r 0 plus 1, s or let us say that we consider r is equal to 0 in that case the subspace which will be covered by phi 1, 0 x this also will be a limited subspace. But tell me will it be having any relationship with V 0 subspace?

If we call this subset as......... if we call this subspace as V 1 then V 1 should form a super set as you people are saying. So V 0 will be contained within V 1. So we must have V 0 contained in V 1 so we should draw V 1 like this (Refer Slide Time: 9:47). this will be the total space that is the bend within this will be the subspace of V 1 which will include to the entire subspace of V 0. because as we said that whatever is possible to analyze with the phi 0, 0 it should be possible for us to analyze the same thing using phi 1, 0 also.

Now, if I consider now the subspace V 2 which includes the set of functions phi 2......... sorry this should be 1, s (Refer Slide Time: 10:29) so this also if I say phi 2, s of xp; if the V 2 subspace is defined like this then definitely V 1 will be contained within V 2. So now V 2 we should be drawing like this. So V 2 will be this total subspace, V 1 forms a part of it, V 0 forms even a sub part of V 1. So, that is how the subspaces are related to each other.

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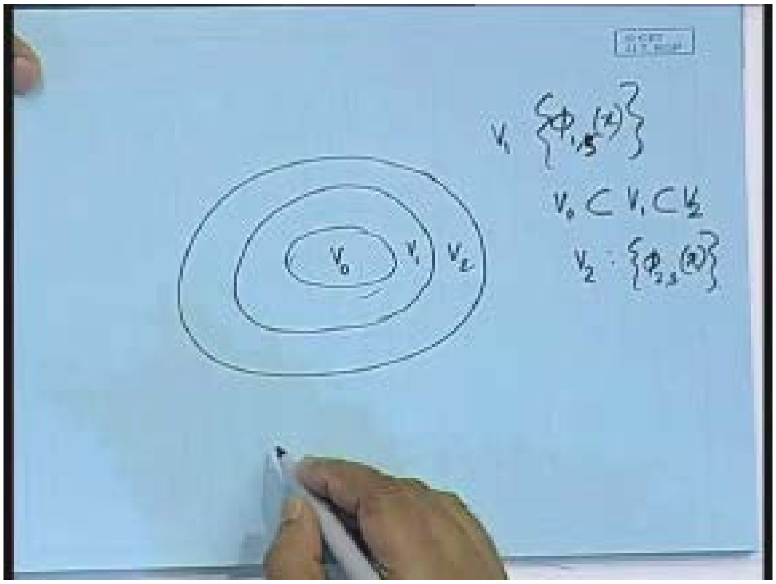

Therefore, it is possible for us to write in fact it is not mandatory that I have to use positive values for this r or positive values for s; in fact what I said is that r and s are integers so they could also have zeros or negative values. Thus, in general it is possible for me to then say that V minus infinity should be contained in all these subspaces; V minus 1 will be contained in V 0, will be contained in V 1, V 2 etc and everything will be contained within V infinity. Hence, this is going to be the subspace relationship.

Now if I have to consider any function any function which is lying within this space V 1 (Refer Slide Time: 12:12) can I approximate that function using this set of phi 1, s (x)? It should be possible for us to do because phi 1, s covers this entire subspace V 1. So if a function lies within the subspace V 1 it should be possible for us to analyze that using phi 1, s. Rather to say that Or we can also say that whatever function is there within the V 0 subspace even that also we should be able to analyze using the function or using using this this set of functions which cover the subspace V 1 that is to say the next higher subspace.

Therefore, going by that argument it is possible for us to write down the expression which we had written towards the end of the last class. now I had promised that I will be explaining you that so definitely it follows from this kind of an argument, the subspace coverage argument which we can definitely put forward to write down that phi of x will be equal to the summation of h of phi n where n is some....... okay I will be telling you what n is; so root 2 phi 2 x minus n. now let us see the significance of this equation and this is to be summed up over n.

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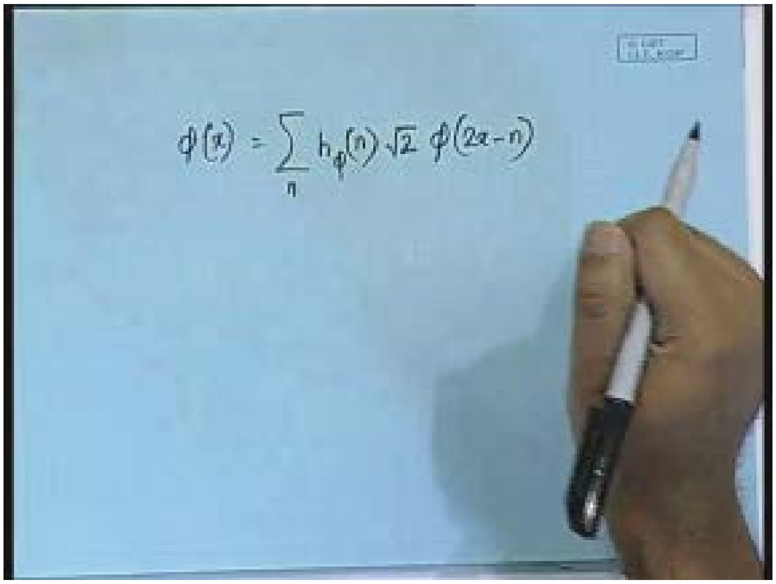

Now what is the function that I am approximating? phi of x that means to say that I want to approximate the scaling function that covers that lies within this V 0 space and I want to use V 1 space to do that (Refer Slide Time: 14:24). So in V 1 space whatever I am writing as phi x in the V 0 space in the V 1 space I have to write as phi 2 x but I have to obtain this function phi x using the shifted versions of this phi 2 x.

Remember, even in the last class the example which I was showing you that is to say the phi 0, 0 we can compose from the different shifted versions of phi 1, 0; did not we do that? We took the phi 1, 0 and shifted version then we also took phi 1, 1 so all these and individually this phi 1, 0 and phi 1, 1 had to be multiplied by some coefficient so this coefficient is this h of phi. Now n is the shifting parameter. So this n is also an integer. This is the same thing. i was calling earlier it Earlier I was calling it as s and now I am calling it as n. So if the shift parameter is let’s say n is equal to 0 that means to say no shift in that case I will be calling the corresponding coefficient as h phi 0. If I have shift parameter equal to 1 in that case I will be calling it as h phi 1. So what happens is that the shift parameters, every individual shift parameter will be associated with some coefficient.

Now, since it is possible for me to compose phi of x from different shifted versions weighted summation of shifted versions of the next higher space function that is to say phi of 2 x so we can write down phi of x using this expression.

Anybody having any doubts on my writing this equation?
h phi of n is definitely the weight. It is the weight or you can call it as the coefficient. So it is the coefficient which will be associated with phi 2 x minus n. So we are using this phi of x as a series form as a series summation of the shifted versions form phi 2 x. This is a very important relationship; and mind you we will be requiring this relationship later on also, so please remember this relation or we will be referring to this equation again in very near future.

Now let us also let us go back to the subspaces that we had created V 0 then we had V 1. Now V 1 completely includes V 0. V 2 completely includes V 1 also includes V 0. Now I can argue something like this that if I have supposing this space V 0 with me; supposing I have my phi 0, 0 function available and using that I can spend the entire V 0 space so I have got a scaling function scaling function phi 0, s so the set of function phi 0, s of x this spends entire V 0 space and we have this function available with us. Now I want to approximate a function that is lying in the V 1 space. Supposing it is somewhere between this V 0 space and V 1 space that means to say what I am drawing as the hatched space; so the hatching that I have created over here (Refer Slide Time: 18:39) this basically forms the difference in the subspaces V 1 and V 0 because V 1 covers the entire thing, V 1 includes V 0. So this hatched part basically includes the difference in the subspace.

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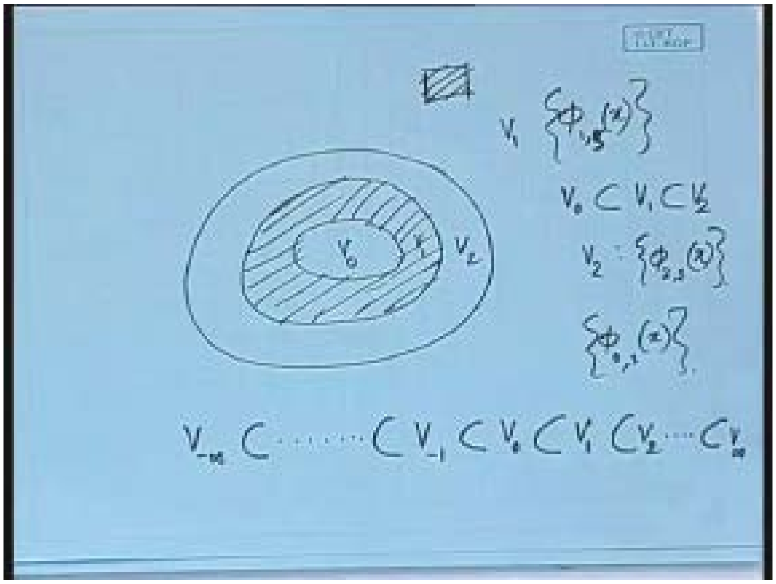

Now if I call the difference in these subspaces by another subspace which I call as W 0 so if I call that as W 0 in that case I can visualize it like this that I have got V 0 which is covered by the scaling function phi 0, s the set of scaling functions phi 0, s and then if I can create some function that spends only the difference space and I call it as W 0 in that case I can say that my V 1 is going to be equal to V 0 union with the W 0. I can write it by this symbol because this is a union of subspaces so V 0 union W 0 that forms the V 1.

Now, if I take the next difference of space that means to say that if I have W 1 in that case it is possible for me to realize V 2 as V 2 is equal to V 1 union W 1 and V 1 I can write as V 0 union W 0 so it is V 0 union W 0 union W 1. And I can go over to the next subspace; if I take the next difference subspace W 2 then I will be realizing V 3 as V 0 union W 0 union W 1 union W 2. So if I want to go over to V (n) in that case I can still start with V 0 and I can form unions with the W 0 W 1 W 2 etc etc subspaces up to W (n minus 1). So I have to take V 0 subspace which is formed by the scaling functions and then I have to form the unions with all these W 0 W 1 all these difference subspaces. So i must have I must develop a set of functions that essentially covers this difference subspace. And what kind of function can really realize this kind of a difference subspace.

Well, we had seen that........... if we look at the kind of the function that we had considered last time that means to say that I was defining a function which was like this (Refer Slide Time: 22:13) that x was here this is 0, this is 1 and I was having phi 0, 0 (x) which was having unit amplitude and we were having a function that was lying like this.

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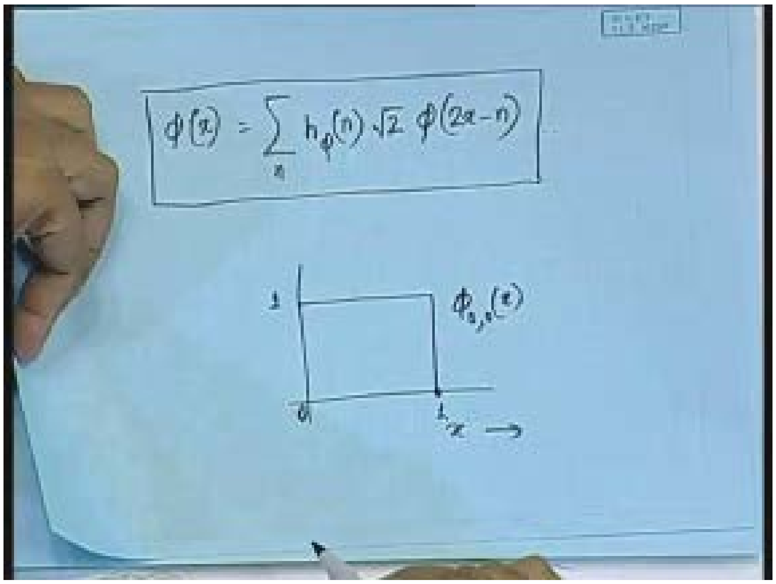

Therefore, if i take a function like this and apply it to over any signal then what I am doing I am doing a kind of an averaging that means to say a low pass filtering. But whenever I am considering a function that should cover the difference subspace then what we have to do is to take the difference in the spaces covered by the two low pass filters and what is that, that leads to a high pass filter.

So the class of filters that should cover this difference spaces has to be of some form of a high pass filter and the class of functions which are used to cover the difference spaces they are mathematically represented by a form like this and you will be very surprised to observe this form.

so this class of functions with which we will cover the difference subspace will be given by psi (x) equal to summation over n or rather to say first let me define the original form of the functions that was necessary. So I define a set of functions that is given by psi r, s of the continuous variable (x) which is written as 2 to the power r by 2 into psi of 2 to the power r into x minus s; surprised? Well, then what is the difference between this and this?

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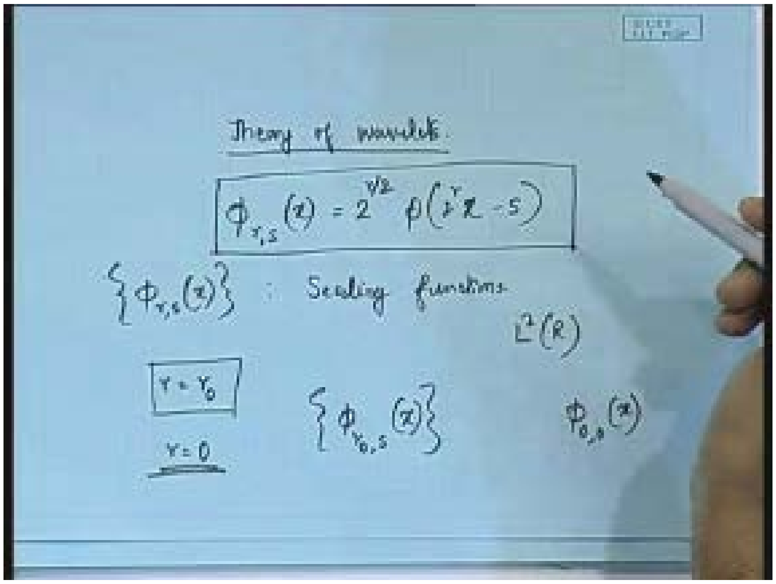

Yesterday we wrote this equation phi r, s in terms of phi (2 to the power r x minus s) and today I am writing psi r, s is equal to 2 to the power r by 2 psi (2 to the power r, s minus x); now, only the phi is replaced by psi.

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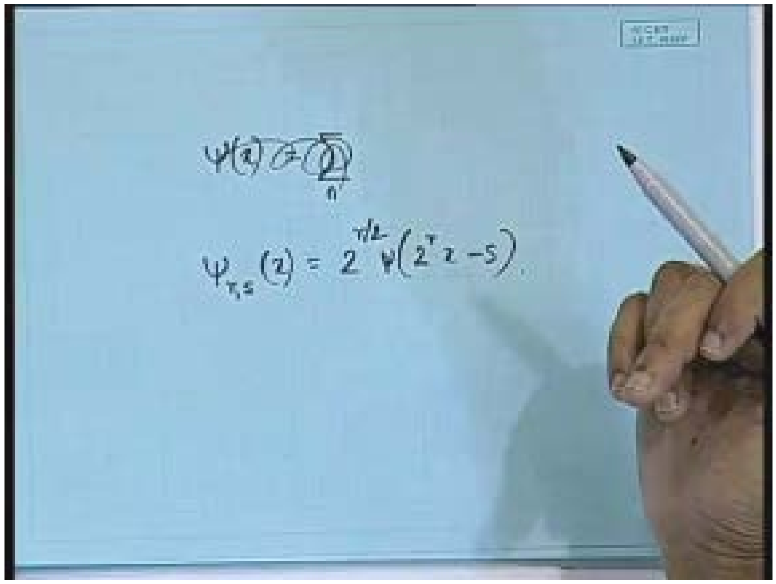

So the functional forms happen to be same but do not think that they are the same set of functions because we have to use these functions in order to cover the difference of space. So the kind of property which these classes of functions should have is that these functions, whenever I consider the shifted versions of these functions shifted versions of these functions they have to be orthogonal with respect to each other which means to say that if I had considered some function like this and then if I had considered its scaled version; supposing I consider its scaled version which becomes like this they are not really orthogonal to each other; I mean, phi 0, 0 and phi 1, 0 they are not orthogonal to each other. But in the difference of space the kind of function that we create have to be orthogonal and there we must form and we must have those functions which are oscillatory in nature, which should go to the positive, which should go to the negative and the area under that function area under those basis functions that should be equal to 0 because whatever positive area it is, that should get nullified with whatever area it covers in the negative.

As a result of that this set of functions although its functional form happens to be similar to that of the scaling functions this is a different class of functions essentially and the set of functions psi r, s of x this set of function we refer to as the wavelet functions.

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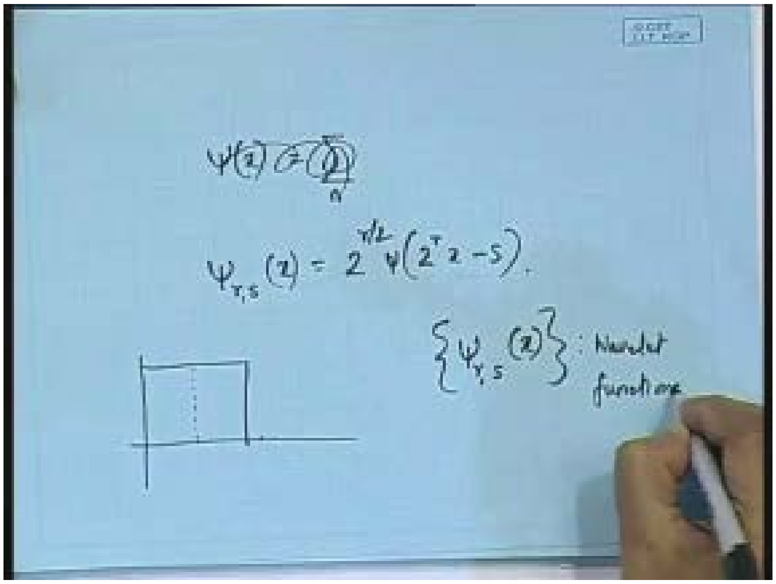

Thus, what we essentially require is that if I want to realize let us say for example the V 2 subspace; I need V 0, I need W 0, I need W 1 which means to say that I need a scaling function to realize the V 0, I need wavelet function to realize W 0, I need another wavelet function W 1 (Refer Slide Time: 27:28); and mind you, even the wavelet function also follows the same relationship; means psi r, s is related to psi (2 to the power of r, s x minus s) by a very similar way.

Therefore now, from the property that these set of functions have to fulfill it is possible for us to develop a relationship between the psi x and phi x and what results is like this. Let me put forward that argument in a better way.

Supposing I want to realize this V 2 subspace, V 2 means V 0 union W 0 union W 1. Now I can do it this way that it is possible for me that this V 2 can be realized or rather to say if I consider the difference space................ let me let me let me put the form this way that supposing I want to realize this difference space W 1, I want to cover I want to spend this difference space W 1 now how can I cover it? I cannot cover using V 0, I cannot cover using W 0 but I can cover using V 2.

If I now take .... with V 0 I cannot cover with V 1 I cannot cover but with V 2 but I can cover this subspace.

Since I can cover it with V 2; so what is this? This is a difference in the subspace between V 1 and V 2 so the difference in subspace between V 1 and V 2 that can be realized using the shifted forms of V 2 shifted forms of the functions that spend V 2. So it should be possible for me to write down psi of x as a series summation again summed up over n h psi of n into root 2 phi (2 x minus n) why 2 x is because I am considering psi of x and psi of x can be spent by a scaling function which has got a half width as that of this psi x so that is why it becomes phi of 2 x. This is because as I was telling you that W 1 can be realized by V 2. So, if you take W 1 to be this psi x then it can be realized by V 2 which has to be phi of 2 x and the shifted versions of this that is why this minus n quantity comes and the summing up is done over n and in order to realize this psi of x the coefficients h which will be associated with it will not be h phi of m as we had taken last time but instead a difference set of coefficients h psi of n.

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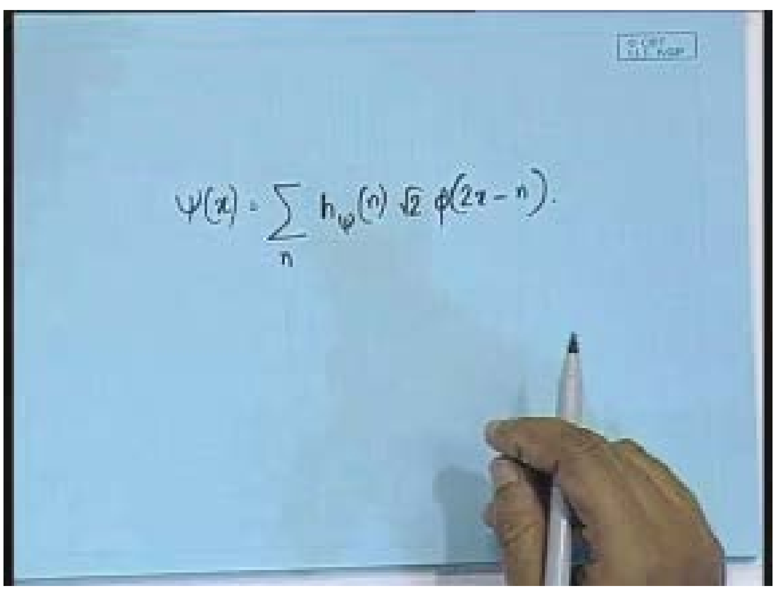

Remember, again I go back to this equation.

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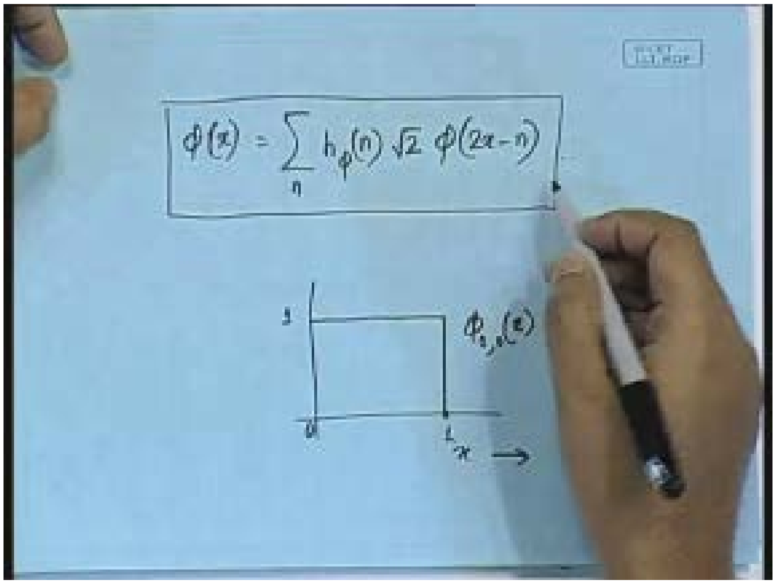

Functional form-wise they are the same. But here I have the coefficients as h phi of n when I am going to realize V 1; when I want to realize V 1 using V 2 this is the functional form that I am using. But when I am going to realize W 1 from V 2 this is the functional form that I am using.

Anybody having any doubts?

Hence, this is the equation that we will be using. This is the equivalent form.

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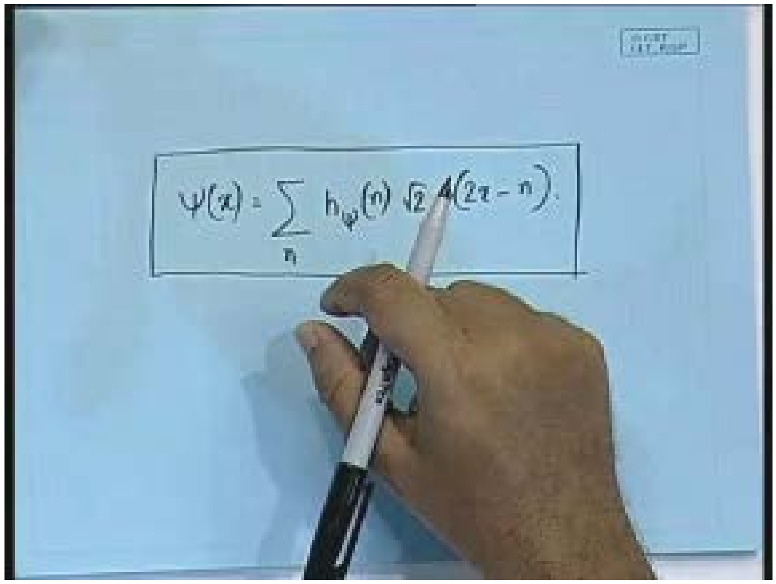

What does this basically tell us?

This gives us a relationship between the wavelet function and the scaling function having half width or scaling function in the next higher subspace. A wavelet function can be realized using a series summation of shifted versions of scaling functions of the next higher subspace. This also is a very interesting relationship and definitely it is possible for us to mathematically obtain; by putting forward the conditions of orthogonality it is possible for us to extract a relationship between this h phi of n and h psi of n.

Now, before going into this, let us see that if I define my phi of x to be like this that phi of x is equal to 1 when x lies between 0 and 1 and is equal to 0 otherwise. If I start with this scaling function then the corresponding wavelet function that gets realized is like this: it is given by psi of x and psi of x will be equal to 1 for x lying between 0 and 0.5 and this is equal to minus 1 when x lies between 0.5 so it is 0.5 less than or equal to x so the equality goes to the minus 1 and this is less than 1 and this is equal to 0 elsewhere. So the corresponding psi x functions psi x function is like this.

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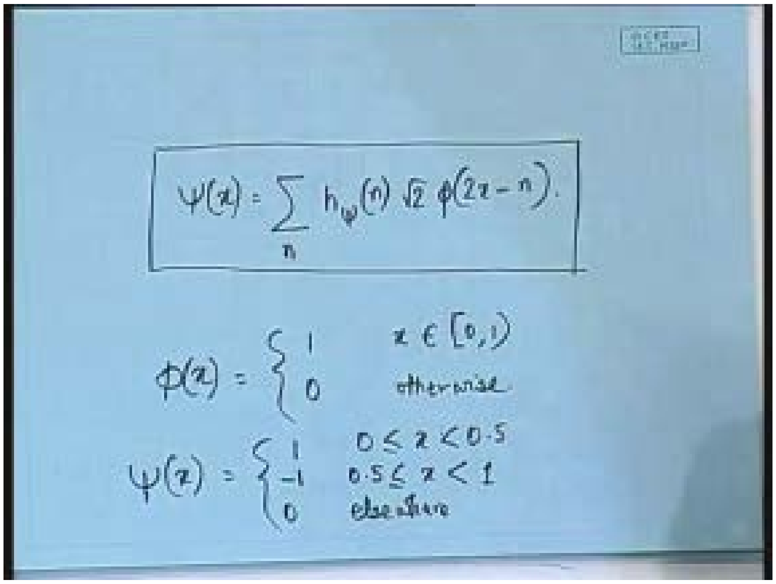

Now we can show the psi x functions plot as follows. If I take that this is a distance of 0.5, this is a distance equal to unity in that case and if this is a this is having a height equal to unity so again 1 2 3 4 four divisions over here so this much is our minus 1 and this is 1 and for all other values this is equal to 0.

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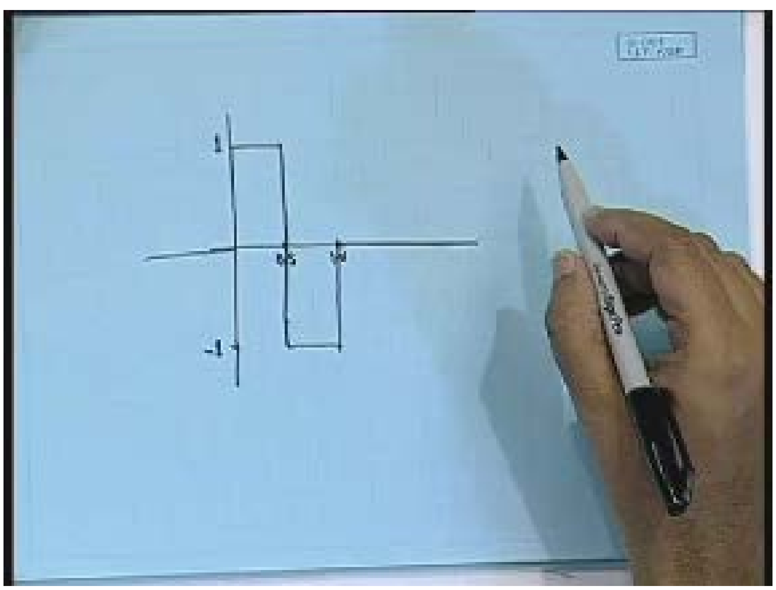

This is the form of corresponding form of psi x. In fact this kind of function this is a wavelet function (Refer Slide Time: 35:13) and this wavelet function is referred to as Haar wavelet. So definitely you can see that by our basic definition that means to say that when I say psi r, s (x) is equal to 2 to the power r by 2 psi of 2 to the power r, x minus s and if you put r equal to 0 s is equal to 0 that means to say that you are trying to realize psi 0, 0 of x then psi 0, 0 of x becomes equal to psi of x and psi of x is of this form so you can call this as psi x or you can call this as psi 0, 0 x. So this is Haar wavelet, and this one is the corresponding scaling function we call as Haar scaling function.

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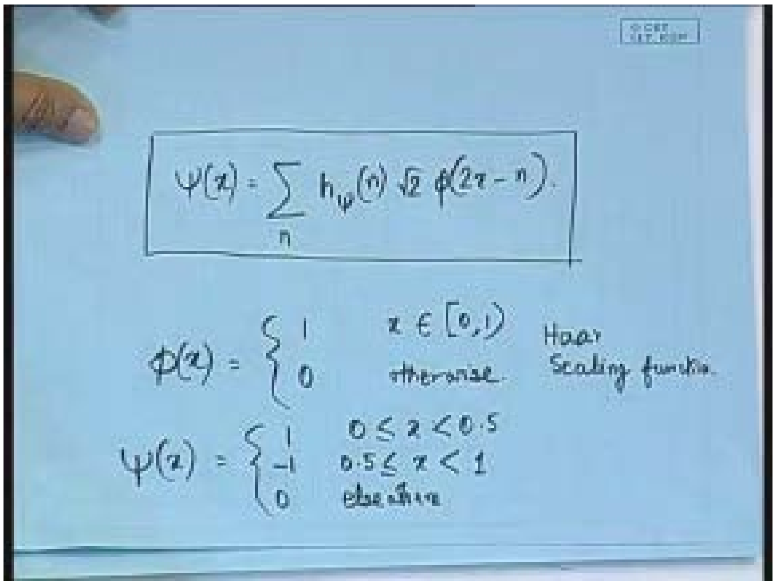

So all that it means to say is that, in order to analyze a function that lies in the real space in the square integrable real space what we require to do is that we first consider a scaling function subspace and then we add up the wavelet function subspaces or the difference subspaces to it in order to realize the in order to spend the function. So this is our form of the Haar wavelet that we were considering.

Now, again like the way I did last time if I ask you that how do I realize psi 1, 0 of x that is very simple; what you have to do is to apply this equation (Refer Slide Time: 37:20). So, if you apply this equation then what you get psi 1, 0 (x) becomes equal to 2 to the power r is equal to 1 so it is root 2 so it is root 2 times psi this is 2 x root 2 times psi 2 x so psi of x is the basic function. So what it means to say is that its amplitude will be root 2 times so its amplitude........... now I can write with the different color, let us say that I use this color so this amplitude is now root 2 and what will be its width; its width gets half now. So the function the corresponding function would be like this; this also would go to minus root 2 and then come here.

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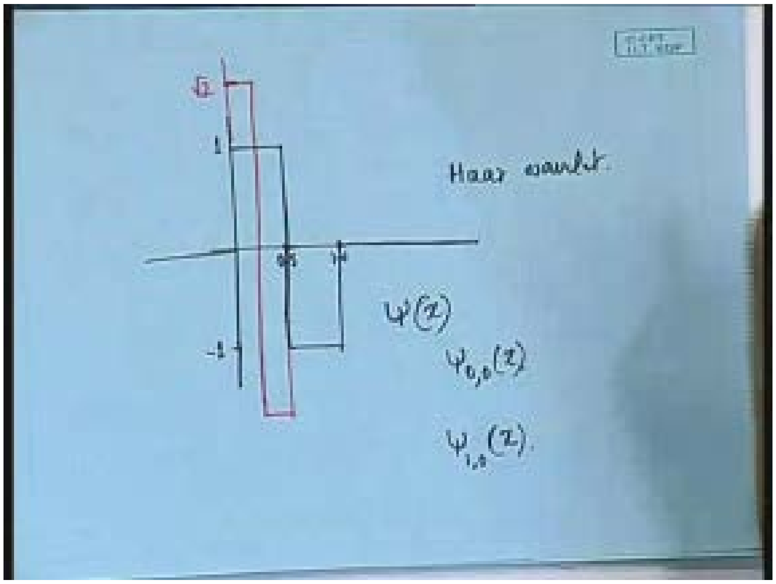

If I now ask you in that can you plot psi 1, minus 1 for me?

Well, pretty simple; so what you have to do is to give this a shift by one unit and whenever you are saying minus 1 then you have to give the shift to the left. You can give the shift to the left or you can give the shift to the right, so when s is equal to minus 1 you give a shift to the left. That means to say this whole thing you shift by one unit on the left. If you want to realize anything like psi 2, 3 well, you can work out; I mean, by using this basic expression you can work out any psi r, s in terms of the psi x function; only thing is that it will not be psi x it will be psi 2 to the power r into x. So according to the value of r that you are choosing your functional form will be realized that way. So just like the way we can create different scaled and shifted versions of scaling functions very likewise we can create scaled and shifted version of the Haar wavelet functions.

Why Haar wavelet?

In fact this is the way............ Haar wavelet is just an example but there are many other forms of wavelets. In fact, if you look at the literature that has been published over the last 15, 20 years you will be finding that a variety of wavelet functions have been proposed and some of these are used in the image compression domain so I will be mentioning about those things little later on.

Now what I mean to say is that it should be possible for us to now apply these scaling function and the wavelet function in order to do some filtering on the image. Because, as I was telling you that the basic objective why we want to use the wavelet transform is that we want to achieve space frequency localization with the image. We want to know that exactly at what position what frequency component exists; something very similar to a pinpointed short time Fourier analysis that within this time within this short span of time I detected this frequency again; I mean analogically in the image space, within this small region I have detected a high frequency. This is what the entire objective of the specific frequency localization is and how do I realize using the wavelet functions, that we will be seeing shortly.

Now look at some more aspects of it. Now we have got several very important relationships.

I have expressed psi of x in terms of phi of 2 x, I have also expressed phi of x in terms of phi of 2 x different shifted versions of this and what I said that x is a continuous variable. But in reality whenever we are working with the digital computers we do not work with the continuous sequence, we work with samples of the sequence. Samples means that it is only defined at some discrete points.

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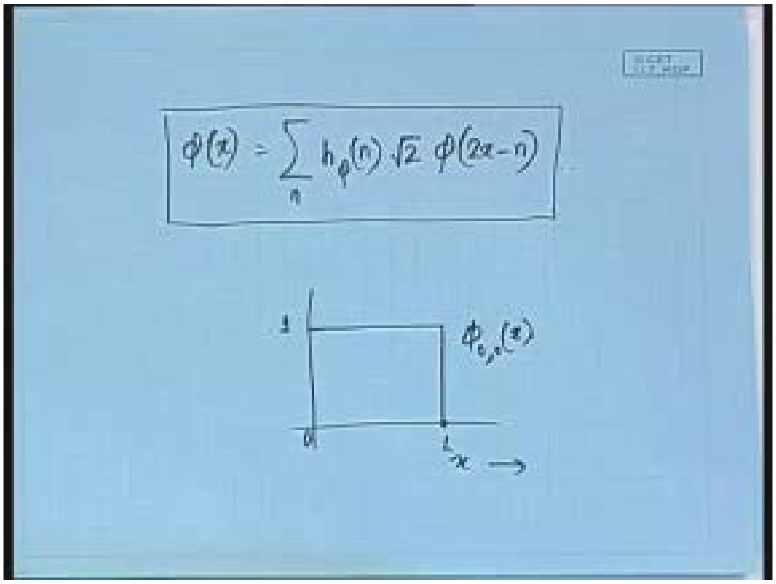

Therefore, in reality we should not be telling this as psi of x. generally the symbol x we reserve for the continuous variable. So whenever we come to the discrete variable we write the signal like the way we wrote............ even for the speech signals also we were calling it as s(n); the segment of the speech we were calling as s of n where n can assume values of 0, 1, 2, etc so these are the samples of s. And in this case we have to in fact take the samples, even for images we have to take the samples but the samples will be in two dimension. So I will be representing my signal as s (n 1, n 2) where n 1 can vary from 0, 1,.......... up to capital N minus 1 and n 2 also can vary from 0, 1,........... up to capital N minus 1 where capital N by capital N is the size of the image. If capital N by capital N is the size of the image, in that case my n 1 and 2 varies accordingly.

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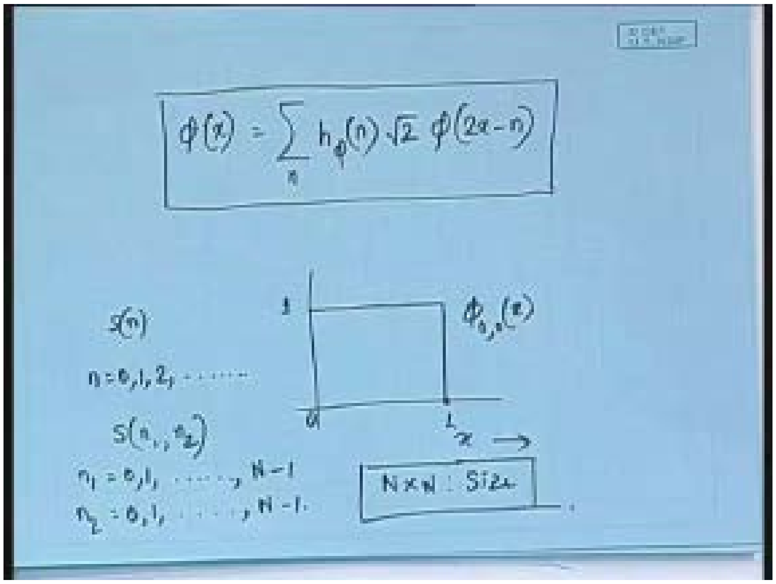

So we deal with discrete signals. Now, how to apply the discrete signals on that on these types of functions? I should not be talking in terms of phi of x but rather some samples of this phi of x. Now, before that let us also have a look at what all we can do using this set of functions phi of x and psi of x; how do we approximate any function.

Supposing I have got some continuous function only; supposing I have to realize any function f of x and what I have available with me is the set of function phi r 0, s that means to say that I choose a specific scale r 0 and can generate different shifted version of this and then I can use a set of psi r, s. So using this phi r 0, s; set of phi r 0, s and the set of psi r, s where r should be greater than or equal to r 0, then using these two sets of functions is it possible for us to approximate any function f of x; well, it is possible. In fact that is what is given by the wavelet series.

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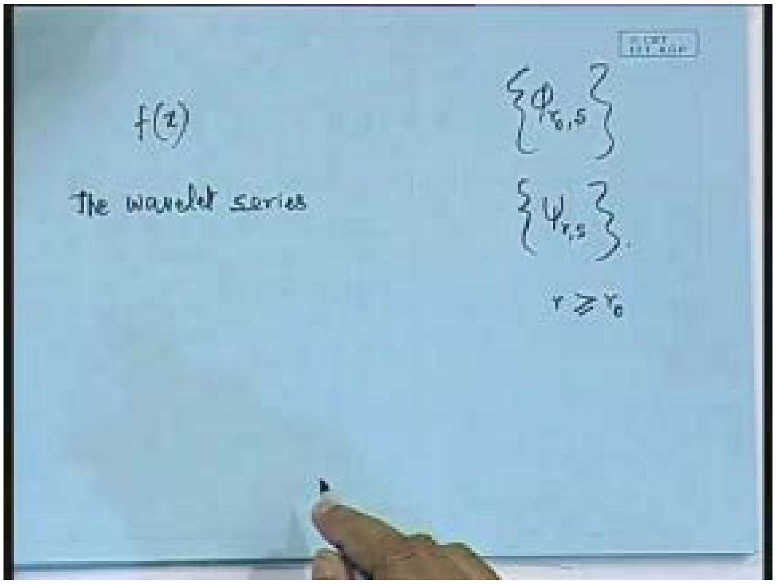

Hence, in wavelet series, the way we can express any function f(x) any function f(x) of course the condition is that f(x) must lie in the square integrable real space, L square R space. So f of x in terms of this wavelet and in terms of the scaling function and the wavelet function can be written as follows that I can write it as a series summation of a r 0, s into phi r 0, s of x and it is to be summed up over s because I have fixed r 0 I have a fixed scale, I have a fixed scale for the scaling function so I fix up r is equal to r 0 and I only generate the shifted versions of these scaling functions. So it is summed up over s this a 0 r, s into phi r 0, s (x).

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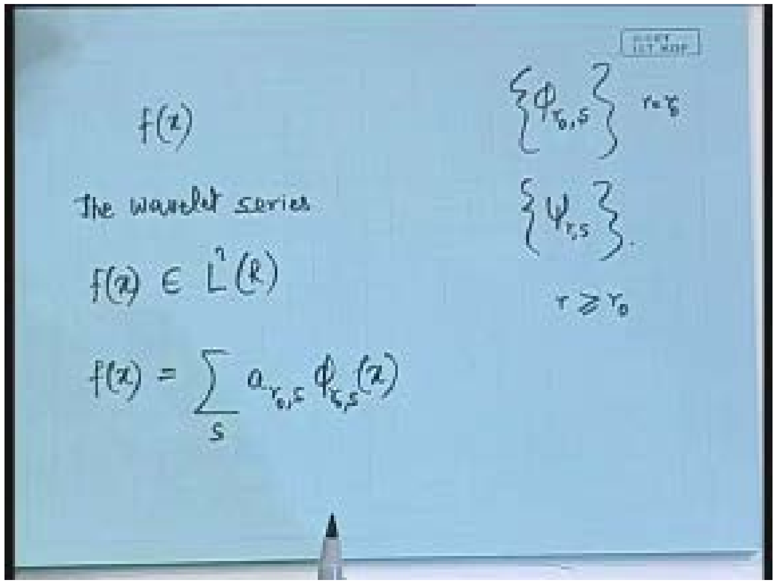

Now in this case what is this a r 0, s?

These are some of the coefficients. These are the corresponding coefficients which are associated with this set of functions phi r 0, s and I am using this phi r 0, s as well as these set of functions psi r, s as the basis functions. So using this I can cover v r 0 subspace but still I will not be able to approximate this function f(x) fully unless I take the difference subspaces in the next higher orders. So to do that what I have to do, I have to have a second term in my summation series and that will be given by summation............... in fact here it should be a double summation (Refer Slide Time: 48:14) summation r is equal to r 0, 2; in fact any general function f of x I do not know that up to what subspace it will go, it can go up to v infinity. So when I write the series summation in general it is better for me write the limits of the summation as r is equal to r 0, 2 infinity and then also the shifted version s and these set of coefficients which will be associated with this psi r, s I will be calling as b r, s so b r, s multiplied by psi r, s of x.

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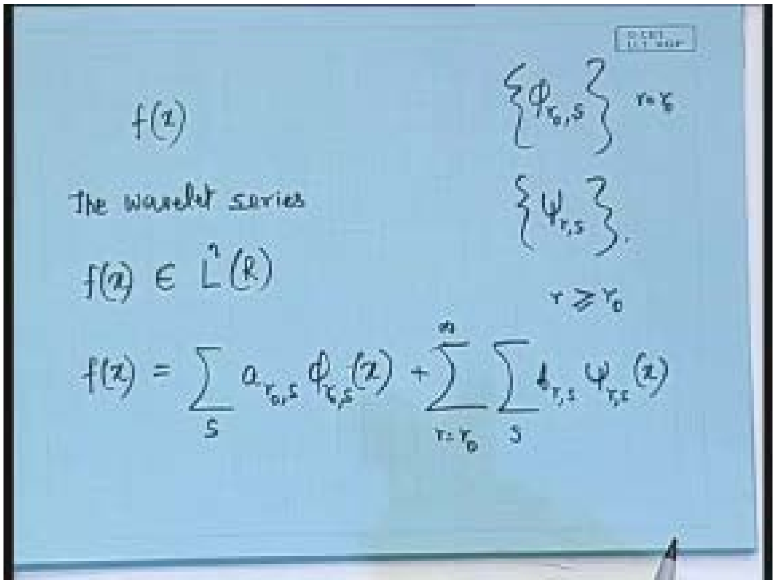

So what i have; I have shifted as well as the scaled version of the wavelet function. Just as an example, if my r 0 is equal to 0 let us say then as if to say that we are taking scaling function phi 0, s the set of scaling functions phi 0, s, but when it comes to the wavelet function there I am considering psi 0, s set, I am considering psi 1, s set, psi 2, s; psi 3, s up to psi infinity, s; all the different skill versions also I am considering, all the possible shifted versions also I am considering. So this will be the series summation form of f of x.

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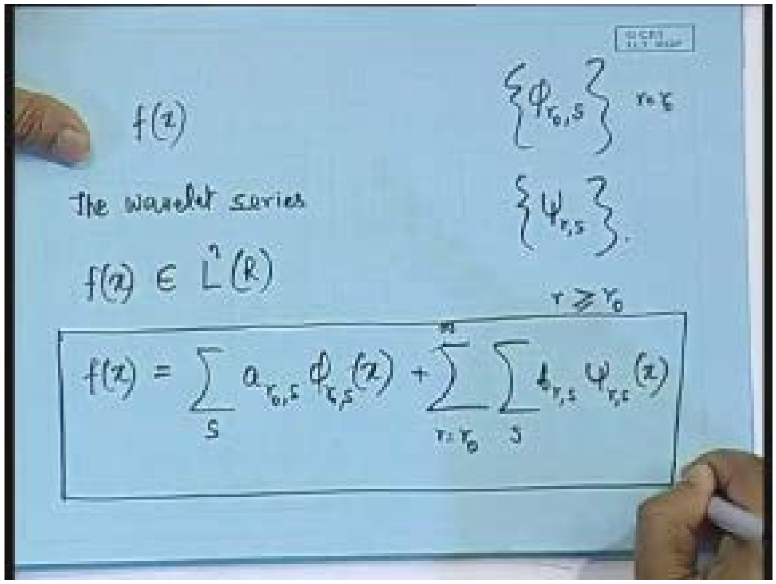

Now, as I said that the wavelet functions and the scaling function; in fact scaling function, whenever you are considering the scaling function at a particular scale that means to say phi r 0; I mean when you fix r is equal to r 0 then the different phi r 0, s that you are generating to different values of s those functions are; are they not orthogonal to each other because shifting you can do only in integer steps. So, if you are taking a scaling function of this nature; if you are taking a scaling function of this nature by what unit you can shift? You cannot shift by half unit, you cannot shift by one fourth unit; you have to shift by integer unit that means to say that when you consider phi 0, 1 (x) the whole thing as if to say shifts by 1 one unit. Or, whenever you take phi 0, 2 (x) the whole thing shifts by two units; when you take phi 0 minus 1 (x) the whole thing shifts to the left by one unit; then if I take the product of phi 0, 0 (x) with phi 0, 1 (x) or phi 0, 0 (x) I will take a product with any phi 0, s of x the product becomes equal to 0 unless s is equal to 0. So the set of phi r 0, s that is orthogonal for a fixed value of r 0 and psi r, s I said that they are orthogonal, this psi r, s the entire set is orthogonal with respect to each other; whether you take the higher scale or whether you take it in the same scale they are orthogonal to each other.

Therefore, now I do not think that it should be difficult for you to tell me that what should be the solutions of this a r 0, s and b r, s any difficulty? What should be a r 0, s; how can I obtain from this series? [Conversation between Student and Professor: 52:32] multiply by.......... just like the way we obtained the coefficients in the Fourier series. Similarly for the wavelet series the coefficients will be given by a r 0, s, I can have as the integral f of x phi r 0 s (x) dx and how to obtain b r, s? b r, s will be the integral of f (x) psi r, s (x) dx. Using these equations it should be possible for you to compute the coefficients and then you can realize this function f of x.

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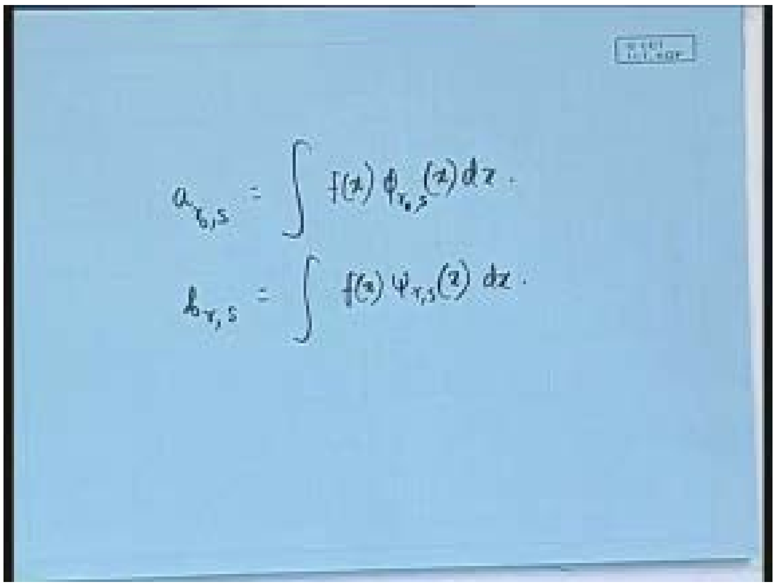

Now you can just do an example that let us say that you have a function f (x) which is given as e to the power x where x lies between 0 and 1 and it is equal to 0 otherwise. Supposing I define the function like this and I want you to realize this f(x) as a series summation of this form. When I say that I want you to express as a series summation in this form the entire objective is to expect that you should be able to compute a r 0, s and b r, s for different values.

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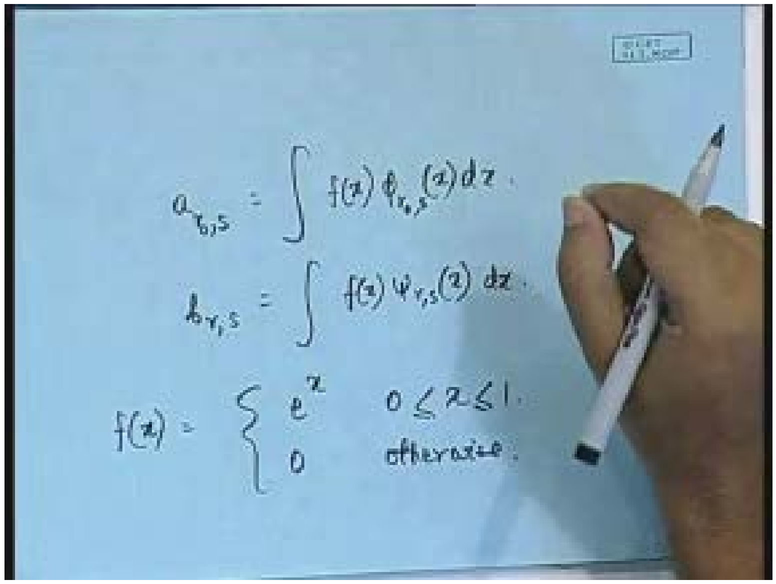

So what we can do is that we can straightaway use these two equations and I can say that for this a 0, 0 a 0, 0 is going to be the integral e to the power x phi 0, 0 (x) and then this is within the limit 0 to 1. And if I take the Haar scaling function and Haar wavelet as the basis function in that case what is this phi 0, 0 (x) within the limit 0 to 1; that is equal to unity. so essentially taking Haar scaling function And why I am saying that this is integral 0 to 1 is because the function is defined this way; f (x) is defined as e to the power x only within these limits 0 to 1 that is why......... I mean, outside 0 to 1 it is 0 that is why I have to consider the limit of integration only from 0 to 1 e to the power x phi 0, 0 (x) dx and that is equal to integral 0 to 1 e to the power x dx which is equal to nothing but e to the power x for value of so the limiting values of x are 0 and 1 which means to say that this becomes e minus e to the power 0, 1 that is to say e minus 1.

So e minus 1 becomes a 0, 0 coefficient and likewise b 0, 0 coefficients you should be able to determine; that has to be determined using this relationship phi x psi r, s (x) (Refer Slide Time: 56:14). So b 0, 0 (x) can be defined as the integral 0 to 1 again because of the function's definition itself that it is only within 0 to 1 e to power x psi in this case it becomes psi 0, 0 (x) dx and I am considering Haar wavelet and because of Haar wavelet what happens is that I have to write it like this: 0 to 0.5, I write this as e to the power x dx because between 0 to 0.5 the value of the function is equal to unity.

This is the function (Refer Slide Time: 57:05) this is the Haar wavelet function; so between 0 and 0.5 the value is unity and between 0.5 and 1 the value is minus 1. That is why I have to say that between the limit 0 to 0.5 it is e to the power of x dx and then between the limits 0.5 to 1 it will be minus e to the power x; so what I can do is minus e to the power x dx and this comes to this becomes equal to 2 into e to the power 0.5 minus e plus 1.

This is clear? Because it is e to the power 0.5 minus e to the power 0 that is 1 and there it is e to the power 1 minus e to power 0.5 so 2 times e to the power 0.5 it becomes. So this is one example that I can give you using which it should be possible for us to calculate this e 0, 0 and b 0, 0 b 0, 1 like that.

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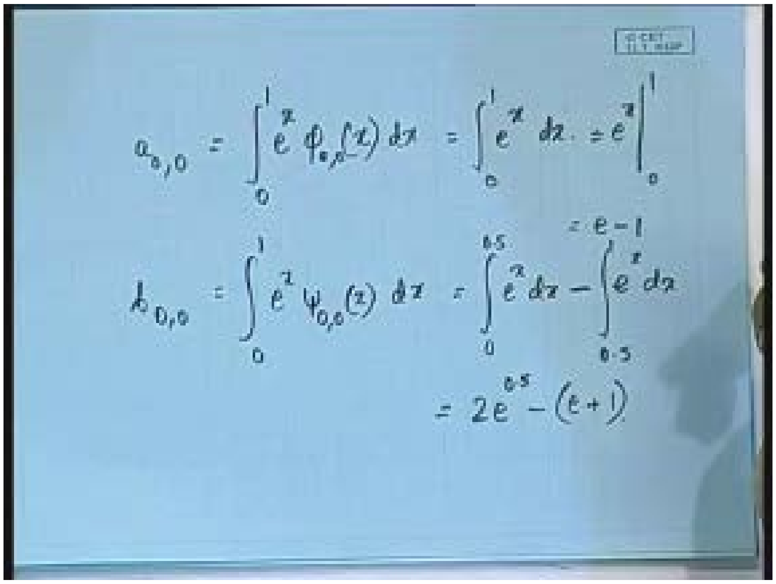

So it is just to show you that how to approximate continuous function f (x) using the continuous scaling function phi 0, 0 (x) and continuous scaling functions phi (x) and psi (x) and in the next class we are going to see that how to apply it on the discrete samples that is on the images; thank you.