Digital Voice and Picture Communication

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Lecture - 18

DCT Quantization and Limitations

Discussing about the discrete cosine transforms and there we were also referring to the JPEG standard in which the DCT based image compression has been adopted as a compression technique in the JPEG standard. Now JPEG was also updated; there was a new standard called JPEG 2000 which came into being few years back and in that the discrete cosine transform was not considered and the wavelet transform instead, the discrete wavelet transform that was considered as a compression technique in the JPEG 2000. But DCT in fact is not only followed in the JPEG but even in the video sequences where we go in for the compression standards in the MPEG series the motion picture experts group MPEG-1 MPEG-2 they also adopted the DCT as a compression tool and the standards which have come up from the ITU the International Telecommunication Union regarding the video compression standards, they also have adopted either the DCT or some variants of the DCT like very recently in H.264 standard they have adopted transformation which is referred to as the integer transform which is a variant of the DCT but can be easily computed. Because one thing which you understand is that the DCT computation that involves a real multiplication because after all it is the cosine values which is a real number in the range of having a magnitude in the range of 0 to 1 so there it has to be represented by the real numbers and the multiplication also is informed.

Now, in the DCT discussion there were some points which I mean I could not get time to lay some emphases on certain points and one of those is the aspect of bit allocation. Because what we had essentially said is that the image is first divided into a set of non-overlapping blocks and on each block the DCT will be applied and then we will be finding that in each of these blocks one has to consider the coefficients and allocate certain bits to the coefficient because ultimately it is the bit stream that we are sending.

Now, on what philosophy is this bit allocation done?

Now, there are two basic policies: one is what is referred to as the zonal coding and the other is what is referred to as the threshold coding. Now, the essential philosophy of the bit allocation is that whenever you are considering a block to be encoded; a block of pixels means that it is a 8 by 8 that is there will be 64 pixels in the block so accordingly there will be 64 transform coefficients. Now what we will be doing is to measure the variances of each of these coefficients.

Now how do we measure the variance?

In order to measure the variance we need some statistical data. So what we have to do is to have an ensemble of such images and using those ensemble of images we should be able to find out the variances and wherever we will find that the variance is higher we will be allocating more bits and wherever the variances are lower the bit allocation will be less that is a very simple philosophy that we can follow.

So typically what we can consider is that if capital B is the total number of bits to be allocated and if we have totally M number of such coefficients M number of transform coefficients to which we are allocating the bits in that case what we have to do is that simply B by M becomes the average. But what we do is that for the ith coefficient we assign the bits which is equal to b i and that we can simply express like this that the number of bits allocated to the ith retained coefficient that will be given by B by M plus half log to the base 2 sigma square i; sigma square i is the variance that is associated with the ith coefficient and that minus 1 upon 2 M into summation i is equal to 1 to M log of sigma square i to the base 2. So there are M number of coefficients or rather to say you can say that M number of retained coefficients.

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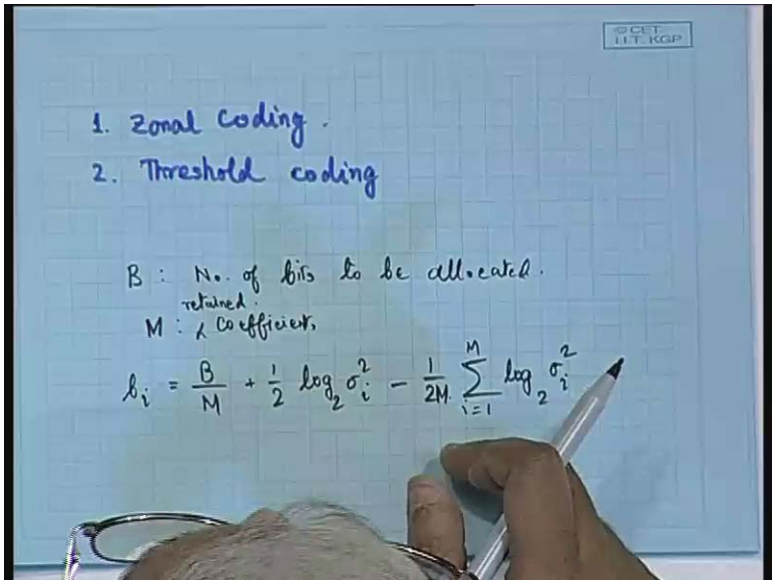

Now how do we retain the coefficient?

Again, to retain the coefficient we have to put some limitation on the variances. We have to put some threshold on the variances so that if it is higher than a certain threshold then we will be retaining that; if it is lower than certain threshold we will not retain it. That is how we will be forming a mask and that mask will contain only that mask will be only a binary mask containing 0s and 1s where the composition of the mask could be something like this.

Supposing here I draw a grid of 8 by 8; 3 4 5 6 7 8 oh I drew one extra this is an 8 by 8; now in this grid this one is the DC (Refer Slide Time: 11:15); now naturally DC represents the intensity so DC we have to retain. So DC we put 1 in that mask; if we have to retain we put 1, if we do not retain we put 0s. Then the next one also is a low frequency. Generally the low frequency components are quite rich and we preserve the low frequency components. So, in the zonal mask we will be keeping some of the low frequency components and depending upon the variances that means those types of images what are the variances if we consider an ensemble of image and compute the variances may be we will find that these coefficients are really essential for us and some of the very high frequency coefficients they may not be needed by us at all.

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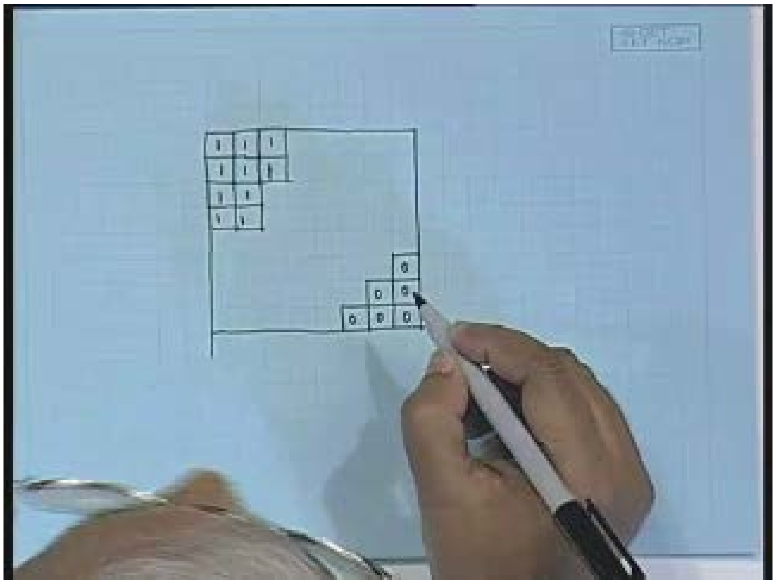

Like say for example, may be if we consider all these coefficients this could always be 0s because these high frequency components these will be having very low variance so we can discard them and in between whatever coefficients are there some of them can be 1s, some of them can be 0; so depending upon the nature of the image and its variance characteristics one can decide that which ones are we going to retain. So this type of a mask containing 1s and 0s where 1 indicates retained coefficients and 0 indicates discarded coefficients. This sort of a binary mask we refer to as a zonal mask.

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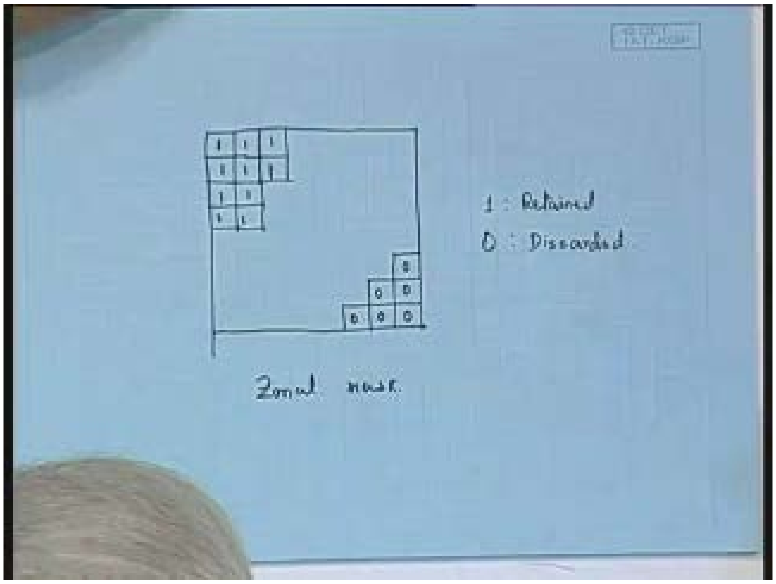

So, in the case of zonal coding what we do is that we apply a zonal mask like this and for each of the retained coefficients if you take that there are M number of retained coefficients (Refer Slide Time: 10:33) the bit allocation could be like this.

You just see that if you add up all these b i's where i is equal to 1 to m then what you are essentially getting is B because B by M is there for everything; there are M number of retained coefficients so B will be there and this is 1 by 2 M, a summation so if you are adding up for i is equal to 1, i is equal to 2 for the second term what you are getting is that it is leading to a term that is there in here so essentially you will be getting capital number of bits to be allocated. So the bit is allocated in proportion with the variance that is one philosophy of bit allocation.

The second philosophy that is followed is what is called as the threshold coding. Now, threshold coding is like this that in the block in the transformed block that you are having; let us say that it is an 8 by 8 block, in that case the transformed coefficients we represent as S (k 1, k 2) so k 1 for an 8 by 8 block k 1 would be 0, 1,........... up to 7 and k 2 also will be likewise 0, 1,................ up to 7 so these are the transformed coefficients and in the case of threshold coding what we do is that these transformed coefficients we divide by a matrix. So there is a matrix whose elements are given by T (k 1, k 2) where k 1 and k 2 also will be having a same index. That means to say that if it is a 8 by 8 block then we will have to use an 8 by 8 matrix for this. So T (k 1, k 2) is the corresponding element in the matrix and this matrix containing T (k 1, k 2) this is referred to as the quantization matrix.

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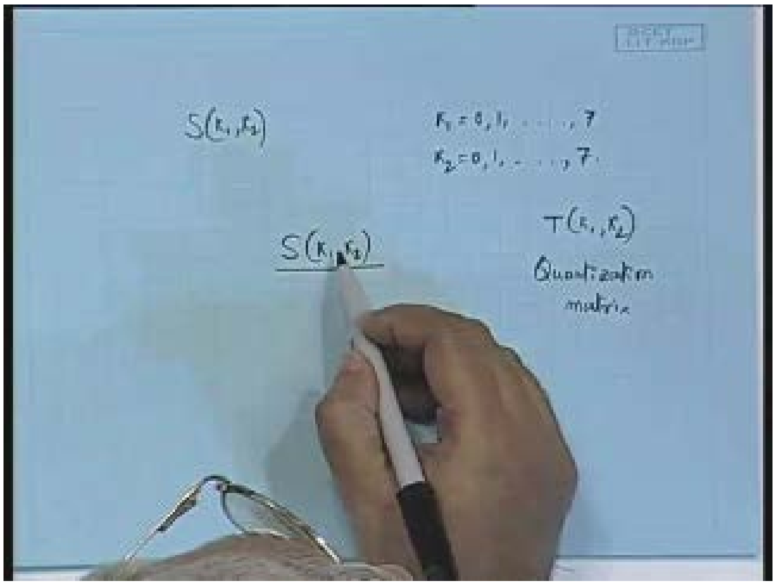

So what we do is that the DCT coefficient at (k 1, k 2) position will be divided by the quantization corresponding quantization matrix element T (k 1, k 2). Now this is definitely this S (k 1, k 2) is a real number. Now you are dividing it by some matrix and the division is definitely going to be a real number. But if we want to represent it, if you want to quantize it there you have to truncate. So you can truncate this quantity to the nearest integer (Refer Slide Time: 13:38) so you can write as NINT where the NINT function basically refers to the nearest integer and then this quantity you can represent as S dot (k 1, k 2). So, S dot comma S dot (k 1, k 2) will be represented as nearest integer of this quantity.

So what is essentially S dot (k 1, k 2)?
S dot (k 1, k 2) is nothing but the quantized DCT coefficient. So this is a quantized DCT coefficient.

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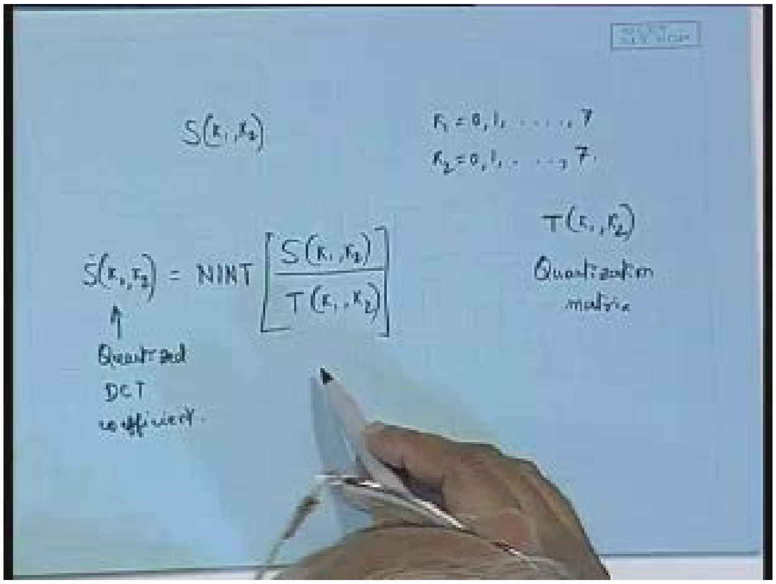

Now what is important note is that how does one find out that what this T (k 1, k 2) is going to be? Now what values do we put into T (k 1, k 2) or what is the consideration for designing this quantization matrix?

Well, to do that lot of psychovisual experiments are performed and it is seen from the luminance and the chrominance data; it has been analyzed very extensively that our human visual system is responsive to which luminance frequencies and to which chrominance frequencies; frequencies meaning spatial frequencies. So accordingly, after such extensive studies were made, purely from a psychovisual consideration purely from psychovisual consideration this kind of quantization matrix was designed. But mind you, do not think that a single quantization matrix can be universal. because after all the extent of this sensitivity could also depend as you know upon the distance from which you are viewing and the elimination conditions, so these things are also very important. So accordingly there will be some variations, that is why the people who framed the standard did not specify one single quantization matrix but rather they specified a set of quantization matrix which can be used under various conditions.

Therefore the essential philosophy is that the S (k 1, k 2) coefficients has to be divided by the quantization matrix whose elements are psychovisually motivated and then the quantized DCT coefficients S dot (k 1, k 2)s are obtained.

Now as you had seen, in the last class we had presented the block diagrams and all these things where the quantized coefficients will thereafter be I mean, we do a zigzag coding and the zigzag coding is followed by the entropic coding because from the zigzag what we do is that we form run level pairs. In run level pairs what we do is that............. when we have an array like this (Refer Slide Time: 17:20); supposing this is an 8 by 8 array 3 4 5 6 7 8 and again here 4 5 6 7 8; 2 3 4 5 6 7 8 so this is an 8 by 8 array and then in order to then after scanning these coefficients what we do is that we form a run level pair and this run level pair means that what will be the runs of zeros that we have which is terminated by a nonzero level.

So, a simple example is that; supposing here we have a 0 say all these things are 0s and say here we have some nonzero value let us say minus 5 over here and here also some nonzero value was there let us say 10 was there so here what we have is that here the runs of 0s start so 1 2 3 4 5 6 7 8 there will be eight 0s and after eight 0s there is a level of minus 5. So we will call it as (8, minus 5). Supposing the next coefficient here is again nonzero one say 6 in that case how do we represent 6; the run is 0 but the level is plus 6 so the next one will be (0, 6). Supposing now we have 0 0 0 minus 1 so in this case what it will be 1 2 3 so three 0s so runs of three 0s followed by minus 1 so we will call it as (3, – 1). So this is how the run level pairs are formed.

Essentially please remember that what we have to do is to proceed in the zigzag scan fashion and pick up the runs of continuous zeros; count how many zeros you have included in that. If you have included eight 0s just say that your first quantity first parameter run is 8 and where the run ends it is invariably ending with a nonzero value so put that nonzero value as a level.

Now it will be possible that we will be having different combinations of run level pairs possible. Now it is not that all these run level pairs are equally probable to occur. Again one has to find out from an ensemble statistics that which run level pairs are highly probable and which run level pairs are not so probable. So based on that you follow it up with an entropy coding and entropy technique what it does is that it will find out that wherever the information content is less.................. in fact we will be allocating more bits to less probable symbols and we will be allocating less bits to more probable symbols.

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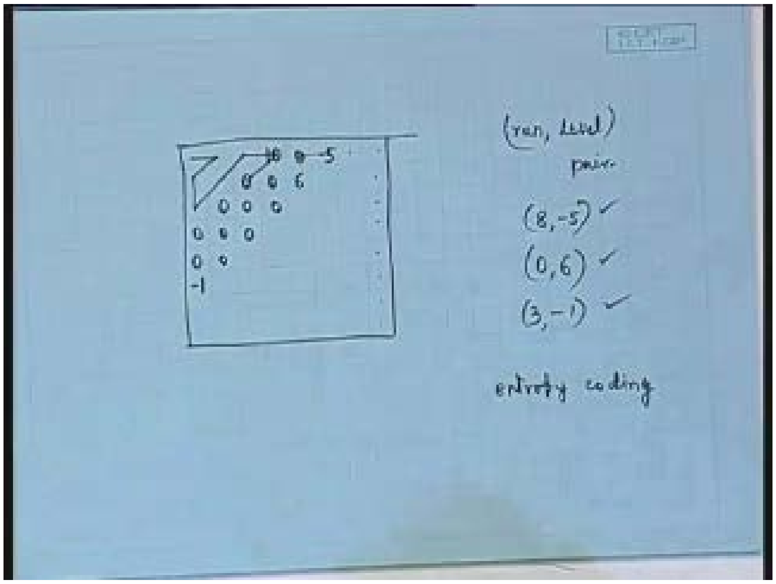

So what are the symbols?

The symbols will be this like say (8, minus 5) this is one symbol; (0, 6) it is one symbol; (3, minus 1) this is another symbol so like this there will be table which will be prepared and from that table one can find out that what should be the corresponding Huffman code and one can use the Huffman code for this. Or instead of Huffman code one can also go in for an arithmetic coding. Whether one goes in for an arithmetic coding or Huffman coding, some entropy coding technique is applied on the extracted run level sequences.

Hence, this is what we obtained and some of the salient points that one should mention is that is it so that the DCT is good all the time and in all respects. Well, every technique has got some short comings and DCT also has got certain basic short comings.

You see what you are essentially doing. The first step is that you are dividing the image into a set of non-overlapping blocks and your block is of size let us say 8 by 8. Now each and every block you can quantize with different parameters like what to say is that do not think that we will be always doing S (K 1, K 2) divided by T (k 1, k 2). In fact it is possible that we can apply some scaling in the quantization process also. Like say for example, if we decide to multiply this T (k 1, k 2) by a number which is other than 1 that means to say that we are incorporating some scaling; say if we say M is equal to 2 and multiply this T (k 1, k 2) by a factor of 2 in that case what we are doing we are coarsely quantizing; we are coarsely quantizing it further. If we put N is equal to 3 we are furthermore coarsely quantizing.

Therefore, the extent of coarseness in quantization can vary from block to block that is one point that one should remember and as a result of that the statistics can vary from block to block. In fact the variance statistics also varies from block to block and as a result of that from block to block this variation becomes quite visible especially at very low bit rates. Whenever one is over quantizing the DCT coefficients then some artifacts are visible and those artifacts are referred to as blocking artifacts.

So blocking artifacts as the name implies; blocking artifacts means that from one block to the next block there will be some variations in the overall characteristics. Like say for example the simplest that you can think of is that supposing the overall intensity varies, the overall intensity is different; if this block is having some DC level and the next block is having a different DC level in that case what are you going to observe you are going to observe some kind of an artifact. After all, the DC value is also computed it is computed out of the local properties so the local properties also can change from this block and the next neighboring block may not have exactly identical local properties so as a result of that the overall intensity varies, the low frequency components would vary so this gives rise to some kind of gradations in the quantization of every individual block and this gives rise to some kind of a checkerboard effect; checkerboard effect means in the sense that, as if to say that, from the reconstructed image you will be able to find out that this is one block, this is another block so a blockiness an effect of blockiness would be there which is referred to as the blocking artifacts. This is a very serious problem and this kind of blocking artifacts is highly disturbing in DCTs especially at the very low bit rates.

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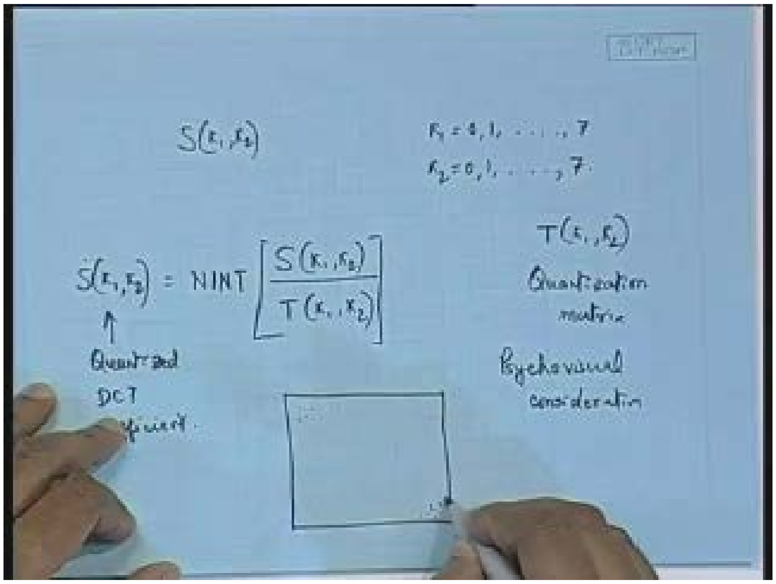

Again another phenomenon which is which happens is that after all what you are doing you are truncating the high frequency components. Because if you look at the composition of this T (k 1, k 2) matrix (Refer Slide Time: 27:01) there you will be finding that we give or rather from the psychovisual experiments whatever has been found it is found that our eyes are more sensitive to the low frequency low spatial frequencies as compared to the high spatial frequencies.

So it is possible that in the T (k 1, k 2) matrix you will be having quite lower values........ I mean, if this is your block then you will be having lower values of T (k 1, k 2) on this side and you will be having higher values of T (k 1, k 2) on this side. So, as a result of that what will happen is that many of the higher frequency components will be very severely truncated and because of that what happens is because of such severe truncations you will be losing some details in those parts of the image. Means wherever a block is reaching high frequency components you will be missing some details as a result of such kind of truncation. And if you are using it in a low bit rate application where your total number of bits in the bit pool itself is less in that case even to the low frequency components also you are assigning less bits and because of assigning less bits to it in very uniform region the regions which are otherwise quite uniform in nature, there you will be introducing some extra artifacts in the form of some graininess. So if you are over truncating the low frequency components it will result in what is called as graininess.

Thus, blocking artifact is one major artifact, graininess due to truncation of the low spectral coefficients or lack of details or rather to say blurring which is due to the truncation of high frequency components these are some of the major deficiencies that one faces in the discrete cosine transform.

Now what to what is being done is that because of these limitations it is found that beyond a certain bit rate it is not very advisable to use the DCTs because once the artifacts are visible to your eyes naturally you are not going to like those kinds of techniques for compression; you definitely look for better techniques.

In fact just to tell you what are the kinds of figures that people normally have as compression efficiency achievable in the DCTs you see that if you are observing a black and white image, a black and white image is represented by 8 bits per pixel so this is black and white or what you call as the monochrome image, so every monochrome image is having 8 bits per pixel. It is generally found that up to 0.25 bits per pixel we write it as bpp up to 0.25 bits per pixel you normally do not notice any serious degradation in the quality of the image. Now 0.25 bbp means what that already you are having, from 8 bits per pixel you are down to 0.25 bits per pixel which means to say a factor of 32 is to 1. So I am saying that the redundancies in most of the images are to the extent that 32 is to 1 of compression factor is not causing any visible degradation.

Now if instead of 32 is to 1 you go in for 64 is to 1; that means to say that from 0.25 bpp if you want to get to 0.125 bpp in that case the quality is going to suffer so you will be observing all these factors; all these artifacts that I was mentioning will be visible more and more you go in for higher compression factors. So there it is found that the wavelet transforms are very much useful.

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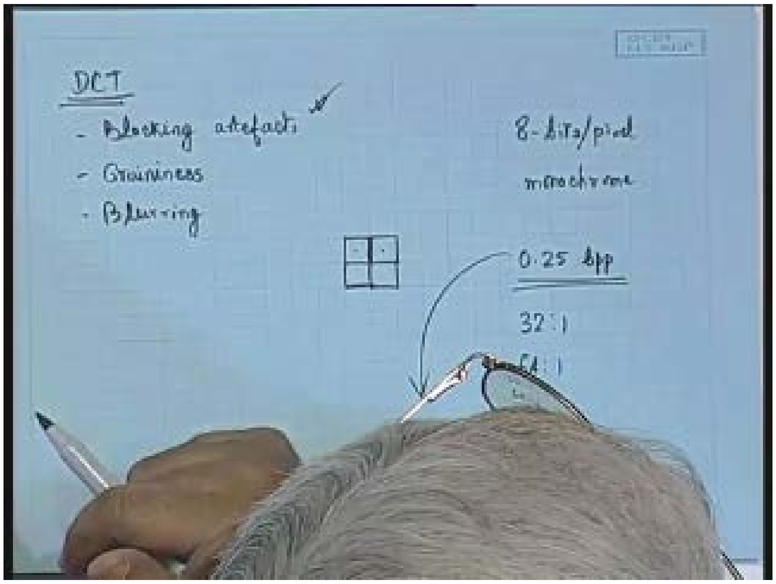

So, before introducing the wavelet transform I found that it will be I mean, it is a good idea to have some introduction to the theory of Wavelets. I do not know, many students some students in this group or those who are listening remotely, some of you may be familiar with the basic the theory behind the Wavelets but let us brush up the theory to some extent so that our understanding about the wavelet transform becomes little better.

In wavelet, basically the theory of the wavelet follows from this that, you see; what we are doing after all in the DCT, we are taking a block which is a local block of 8 by 8 and there we are having the there we are representing the block from the spatial domain to the frequency domain, now, when we represent it in the frequency domain what we are getting is a characteristic that is representative of the entire 8 by 8 block; we will not be able to precisely pin point that in which part of this 8 by 8 block the low frequency component is happening and in which part is the high frequency component is happening, so the local property the localization property is restricted in the sense that we are only within that 8 by 8 block that is all but within that 8 by 8 block there could be variations and we will not be able to find out that where exactly is the high frequency located and where exactly is the low frequency located. The same thing happens for the time domain signals also. Like say for example, the speech signals which we were considering few classes back.

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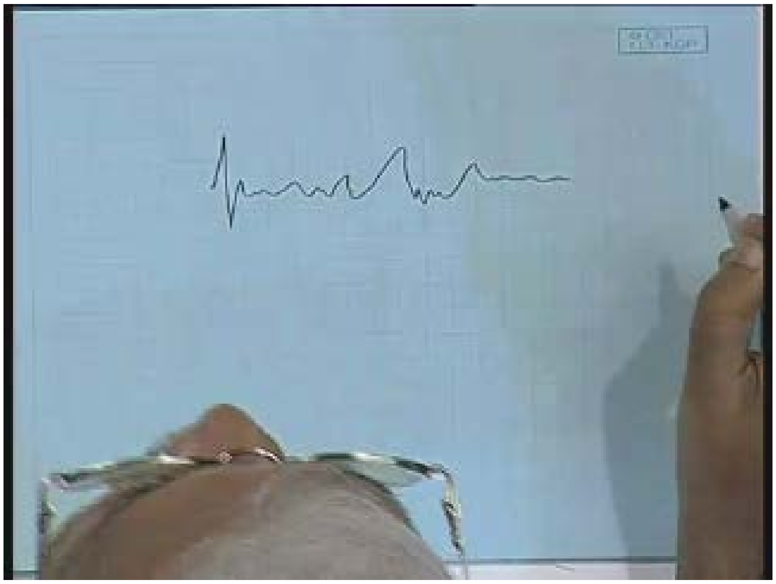

Now you see that supposing we have got a waveform like this and now we want to find out that exactly where the high frequency component has appeared. Say here (Refer Slide Time: 35:06) there is a sharp change, here there is a sharp change, here there is a sharp change, some of the other regions may be uniform. So if I take the FFT of the entire signal I cannot really say that where exactly did the high frequency appear. My spectrum will show something that some high frequency part had appeared but it is not telling me that exactly where it appeared. So what is the alternative; we can go in for shorter and shorter frames and for shorter and shorter frames if we do the analysis, or rather to say if we go in for a short time Fourier transform then we will be able to extract such kind of localization properties better.

But again just see, short time Fourier transform means that you are taking the Fourier series as the basis or rather to say you are taking the trigonometric functions as basic functions and those are the functions that is extending from minus infinity to plus infinity and you are incorporating a truncation a forcible truncation that is what you are doing so as a result of that there is the windowing effects etc which you all must be familiar with so this is what really disturbs the performance to a large extent. So as a result of that we have to go in for some technique where the localization will be better and in this case we are talking about the space frequency localization. Why space frequency; because our signal that we are considering for the images is represented as f of (x, y) it is a two variable x coordinate and y coordinate so x and y those are the spatial signals and we are talking in terms that exactly which spatial frequencies had happened, to know that. So, to know that we go in for a new kind of transformation which is called as the wavelet transform. as the name implies we are not calling it as wave, we are calling it as the wavelet means wavelet means it is a small wave like this

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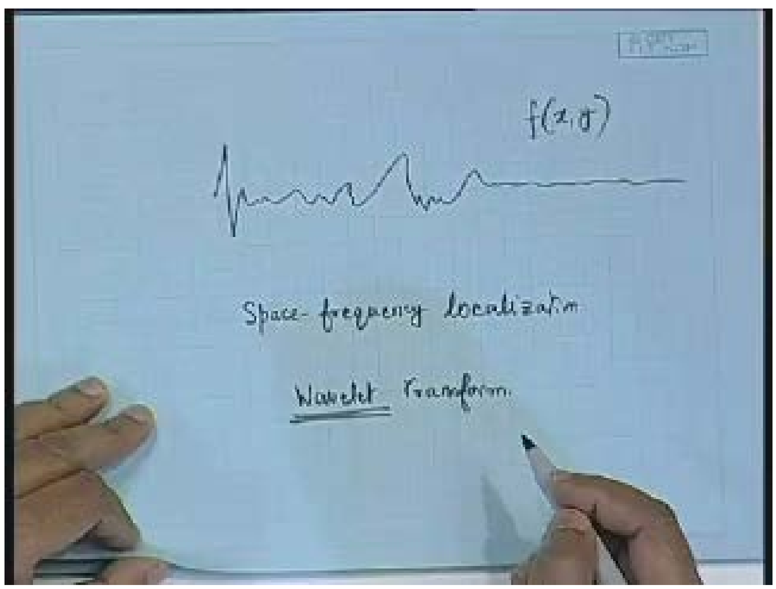

This is just one cycle of sign wave but the actual sign wave is like this going to infinity starting from minus infinity. So this is a wave.

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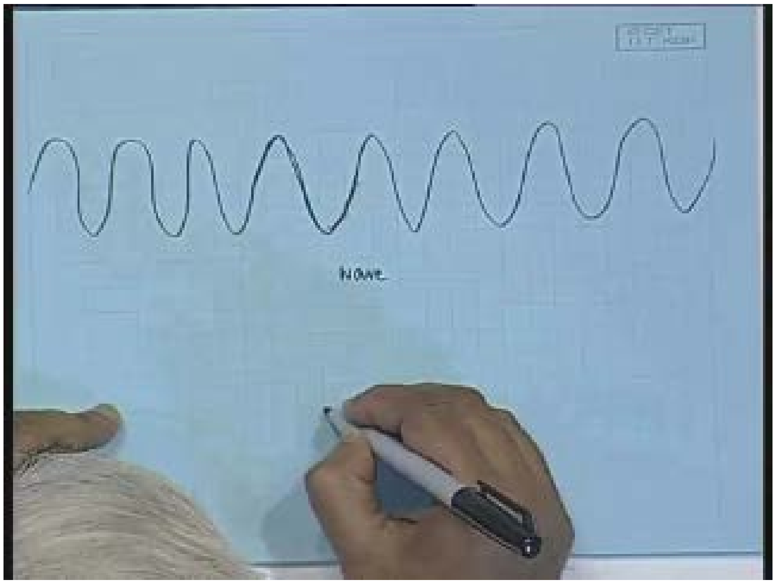

But if I consider a function like this (Refer Slide Time: 38:24) and symmetrically obtain here what you find is that this is not a wave anymore, this is a small wave or a wavelet as if to say that on a lake containing still water you just drop a pebble and after dropping the pebble the ripples that go through that ripples are definitely going to die down after certain distance so you see that the oscillations are dying down and otherwise the responses are oscillatory in nature. So this is a wavelet. So this is instead of using the waves if we use wavelets as our analysis. But how the theory of the wavelet follows; so, to understand that let us go to some aspects of Mathematics.

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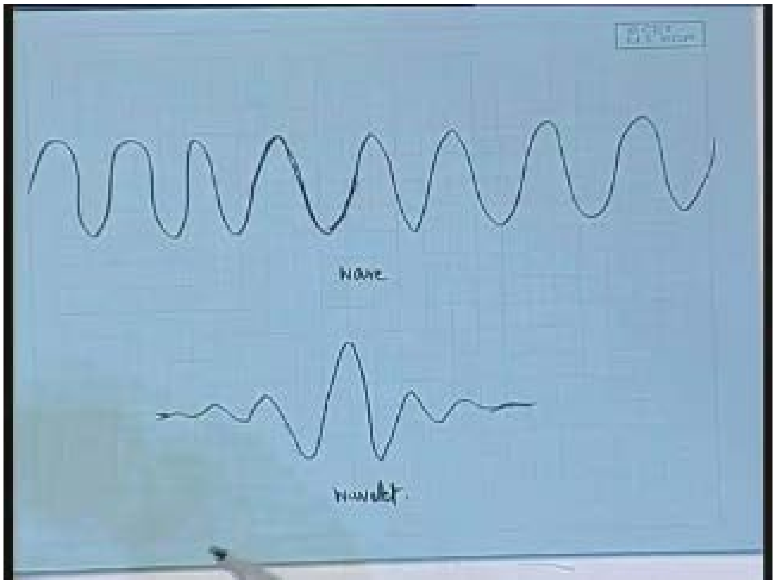

What we have is that lets say let us first develop the theory with one dimensional signal and then it will be its two dimensional expansion will be extension will be quite logical. so let us say that if we have a function f(x) defined in the real space it is possible for us to express this real valued function in terms of an expansion function like this that we can we can express it as sum as a product of the coefficients alpha k times phi k of x and it will be summed up over k. Now in this case k is an integer index of summation. So what I mean to say is that the real valued function f(x) will be approximated by a series summation of a set of functions given by phi k's. So when I have phi when I have k is equal to 1 it is alpha 1 phi 1; when I go in for k is equal to 2 it is alpha 2 phi 2 and so on.

Now how many terms I keep in this series is again............... we have to decide based on how close the approximation is going to be. Now this set of functions phi k's we realized using a scaled and shifted version of some basic function. Since I mentioned scaled and shifted these functions will have two basic parameters which are associated with it: one is to what extent it is scaled and the other is to what extent it is shifted. So there will be two variables that will be associated with it and we call those variables as phi the set I am writing phi and as suffix I am writing two quantities r, r with the argument (x); x continues to be the argument but these two parameters r and s what are they; r is called as the scaling parameter, r is called as the scaling parameter and s is called as the shift parameter and both these r and s this r as well as this s they belong to the integer space Z.

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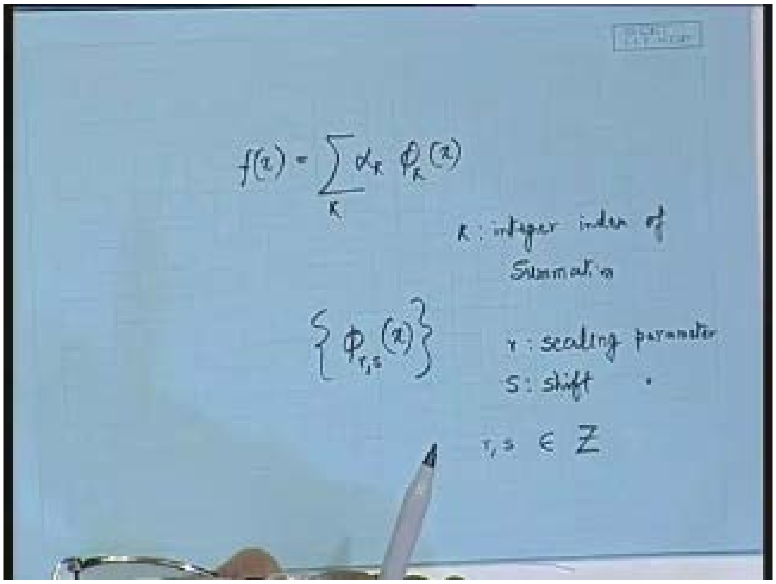

Thus, we have to consider a set of functions given by phi r, s and how are these functions interrelated to each other, we define their inter relationship like this that phi r(s) actually is defined as phi r(s) of x is given by 2 to the power r by 2 phi of 2 to the power r(x) minus s. This is the basic definition of these functions that we are considering: phi r, s is equal to 2 the power r by 2 is this. Do not worry about the mathematical expression; we are not going into anything abstract.

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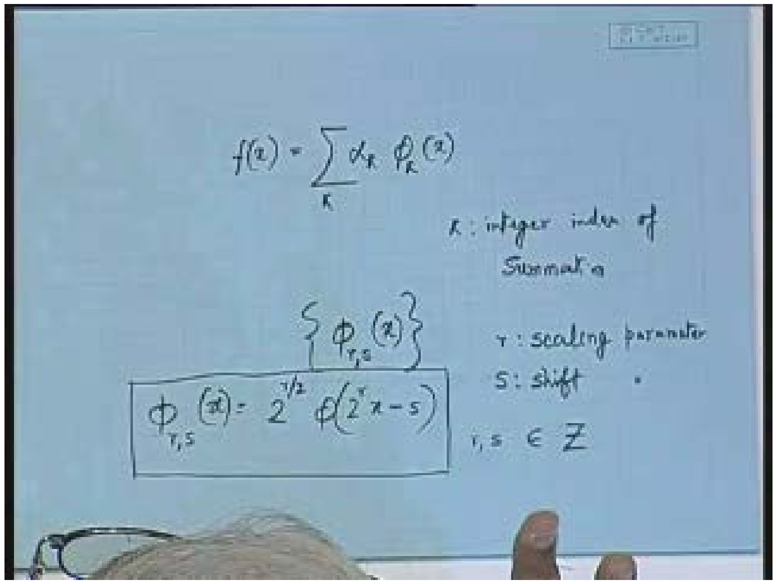

Just see that let us try interpreting it in a very nice way. Let us say we put r is equal to 0 and we put s is equal to 0 then what results; then what results is phi 0, 0 x becomes equal to 2 to the power 0 and 2 to the power 0 means 1 and phi 2 to the power 0 so here x, s is also 0, so phi 0, 0 x is nothing but phi x. So what we are considering as a basic function is this phi of x, let us say this is our starting point.

Now if we want to obtain......... let us say instead of phi 0, 0 x if I want to obtain phi 1, 0 x how do I realize that? To have phi 1, 0 x I have to put r is equal to 1 and s is equal to 0. So phi 1, 0 x I have to write as 2 and here r is equal to 1 so it is root 2 into phi of 2 x that is all. Means whatever we consider as phi of x, instead of phi of x we are now obtaining phi of 2 x. So say that if our phi 0, 0 x is a function like this let us define a function.

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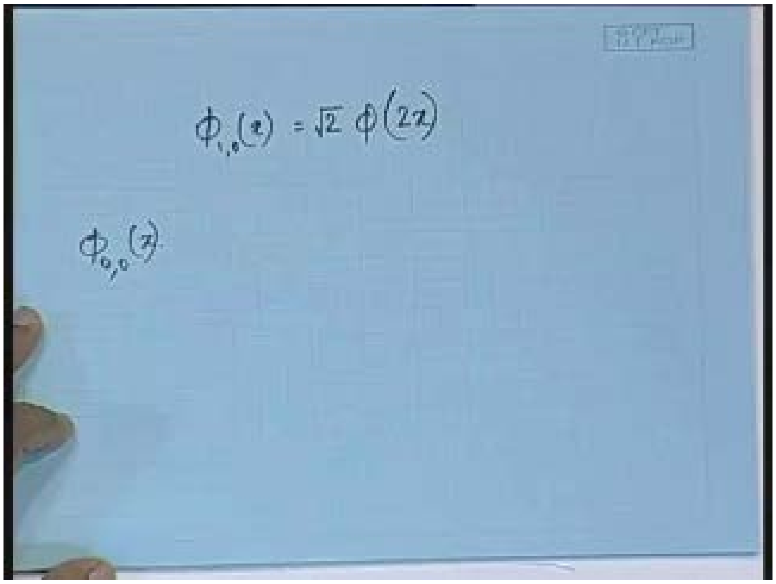

In fact it is a Haar function that we are defining; say phi (x) we define as 1 for x lying between 0 and 1 for x lying between 0 and 1 and it is equal to 0 otherwise which means to say that the plot of the function would be like this that say this axis if I draw x and this axis I draw phi of x in that case the function would look like this say this is x is equal to 0 and let's say that this is x is equal to 1 (Refer Slide Time: 46:18) this is x is equal to 2. So all these smaller grids are 0.25 distance apart and if this equal to 1, say this is equal to 1 then this will be the nature of the phi x function.

Now this this phi x I can also call as phi 0, 0 (x). Now how am I going to draw phi 1, 0 (x) first thing is that its amplitude has become root 2 times and second thing is that instead of phi x this is phi 2 x so phi 2 x means that it is scaled by a factor of 2 which means to say that it is compressed by a factor of 2 which means to say that I have to draw in the same scale; let me use a different color; so first of all that its extent will be only up to 0.5 and the second factor is that its amplitude will be root 2 or 1.414 so this red is something that represents phi 1, 0 (x).

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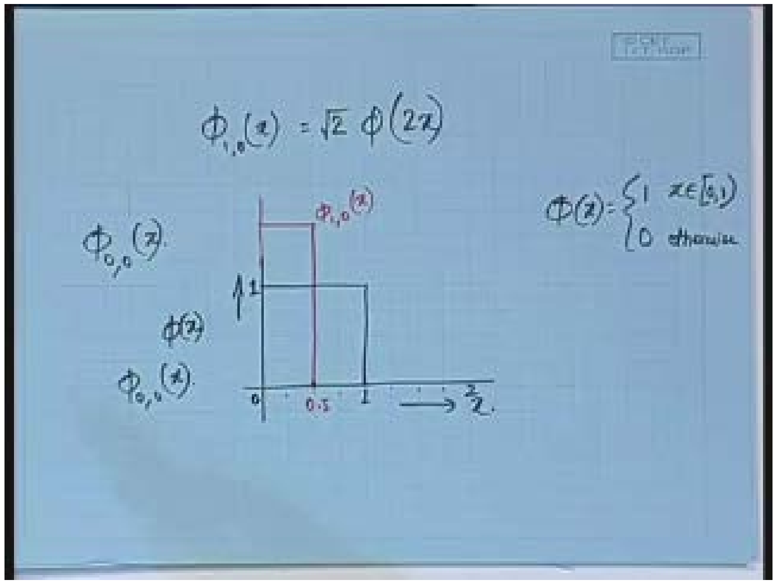

Now, supposing I want to know that what should be my phi 1, 3 (x) in that case you take this phi 1, 0 (x) because amplitude decided by this 2 to the power r by 2 this is the amplitude (Refer Slide Time: 48:09). So 2 to the power r by 2 so r continues to remain as 1 so amplitude will be fixed at root 2 but whenever i want phi 1, 3 (x) that means to say that my shift parameter has become 3. So shift parameter now by what extent does it shift? By 3, three units from here which means to say that this is the first one, this is unit one, this is unit two and unit two according to this scale (Refer Slide Time: 48:44) and this one is unit three.

Therefore, I must draw my function like this that this is 1.5 and then here I will be drawing and this will be up to the extent of 2; this will be from 1.5 to 2 because its width is also not changing, its width continues to be 0.5 because of this phi 2 x. In fact what will be the function of phi 1, 3 (x); if you write it phi 1, 3 (x) in terms of the Maths it will be written as root 2 phi (2 x minus 3) so its width is going to be 0.5 as before so this green one will be our phi 1, 3 (x).

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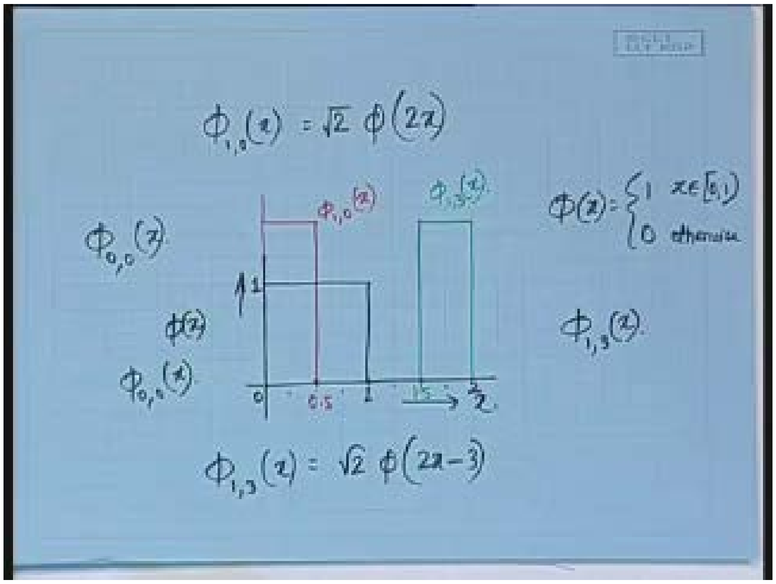

What you are seeing is that we are having scaled and shifted versions of this basic function which is phi 0, 0. So defining a basic function phi 0, 0 we can obtain its scaled and shifted versions of the functions.

Now what it means to say............. now let us look at another aspect of it that phi 0, 0 was this but phi 1, 0 we obtained this. Now if I ask a question that can I use this phi 0, 0 function in order to analyze phi 1, 0? Should the answer be yes or no?

Just repeat the question again; can I use this function (Refer Slide Time: 50:55) to approximate this function? Not possible; why? Is because its width is 1 whereas its width is 0.5 so I cannot use this function to analyze this one; this red one. But if ask in the other way that can I use this function (Refer Slide Time: 51:24) to approximate this one? Possible; what am I going to do; very simple; I scale this by 1 by root 2 and then what I do I create a shifted version of the scaled function by one unit and I adopt these two, so I can approximate phi 0, 0 as............... just tell me that whether am I writing correctly or not phi 0, 0 (x) I can obtain as 1 by root 2 phi 1, 0 (x) plus 1 by root 2 phi 1, 1 (x) because phi 1, 1 (x) is going to be this one so add this with this so you are going to get this function.

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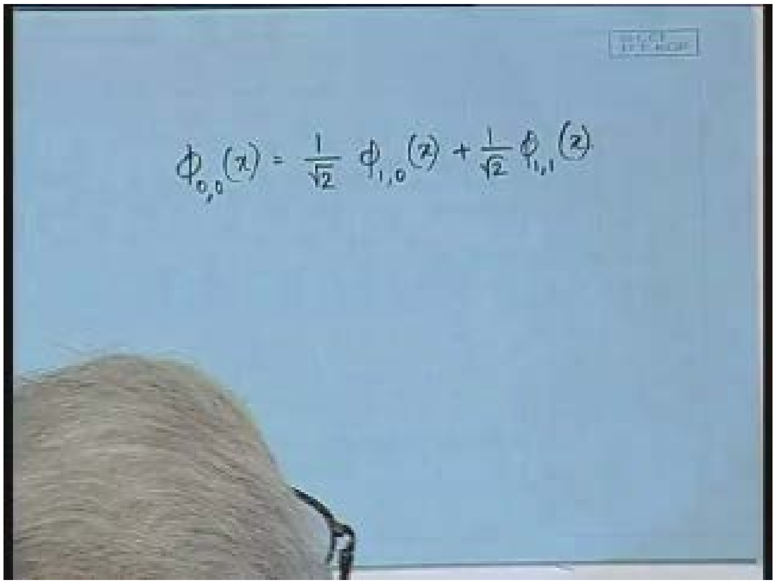

Hence, phi 0, 0 you are now realizing as a series summation of the higher functions: phi 1, 0; phi 1, 1 this thing. Now this has become very simple because only these two terms will be sufficient to compose the signal phi 0, 0. At other places; I mean, instead of asking you to approximate this function if I had asked you to let us say approximate a function like this in that case you would have used the other higher versions of scaled and shifted versions and added them together. That means to say that more number of terms would have been possible.

In fact what you can do is that the phi of x can then be in general approximated as the summation of h phi of n h phi of n means some coefficient into root 2 phi 2 x minus n and this should be summed up over n.

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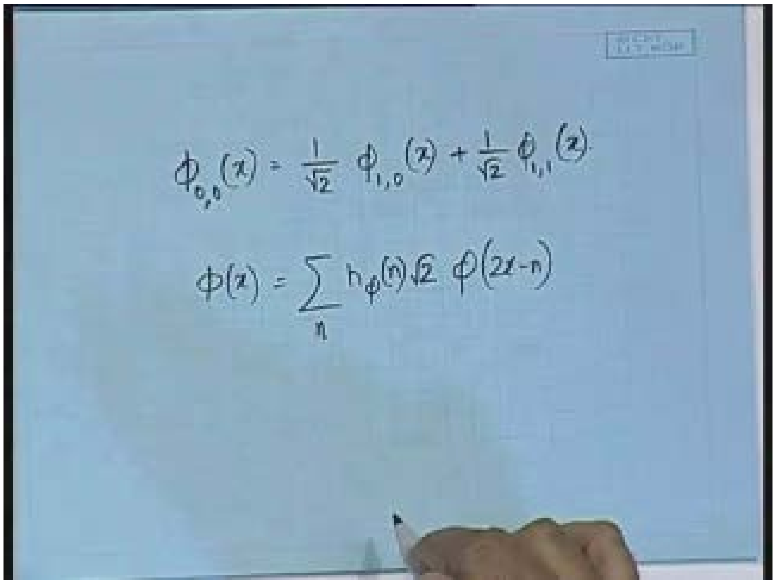

Thus, as a series summation of the higher order wavelet higher order function.................... so sorry because i have not introduced the wavelet yet and this phi functions are called as the scaling functions. Please remember this, in the next class I need to explain this because just writing this may not carry much of a sense to you at this stage but we will be developing the theory in the next class and this function phi x what we designed like this, these are referred to as the scaling functions.

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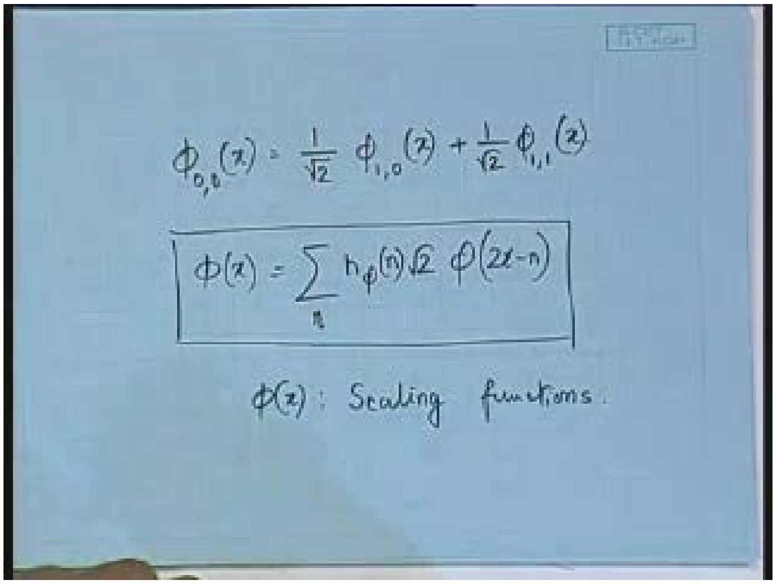

So next class we will elaborate on this idea further and then show that how the images can be compressed using the images can be transformed using the wavelet that we will show in the next class, thank you.