Digital Voice and Picture Communication

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Lecture -12

Cholesky Decomposition and Durbin's Recursive Approach

Now we continue with what we were discussing in the last class that is to say essentially the computational methodology in order to have a fast computation to obtain the alphas to determine the speech coding parameters. So in that I mean we are going to specifically discuss about the two fast approaches. One is what is due to Cholesky and this is about the square root method in order to determine the coefficients in a fast way and then after Cholesky's decomposition we are going to talk about a recursive approach by Durbin Durbin's recursive approach and this second one that is Durbin's recursive approach is going to tell us about the solution of Toeplitz matrix.

So essentially, since we have to do Toeplitz matrix inversion so that is being done in a very efficient and iterative way. So at first we start with Cholesky's decomposition which we had already introduced; the very basic concept of it was introduced. See, basically we start there with the set of p equations given by summation alpha k phi n(i, k) and that was equal to phi n(i, 0) and k was summed up for k is equal to 1 to p where p is the order of prediction. So we essentially have p such equations from which we have to solve so we have the equivalent matrix equation written as the phi matrix multiplied by the alpha vector and that was equal to psi vector where psi vector was the row vector that is consisting of......... no, no, not the row but rather the column vector which is consisting of this phi n's. So this is how we represent it in terms of matrix.

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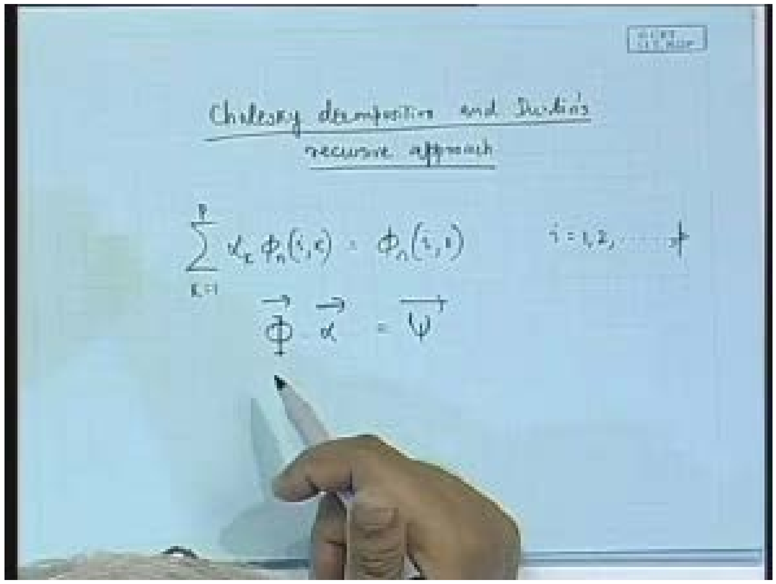

And as per Cholesky's decomposition what we are going to do is that this matrix phi we are going to express as a product V D V transpose where V is the lower triangular matrix whose diagonal elements are all 1s and D is a diagonal matrix having all zero elements outside the diagonals. So now this definition of phi if we put forward........ so in this case V is lower triangular with all diagonal elements 1. So now what we do is that let us solve for the (i, j)th element of this expression.

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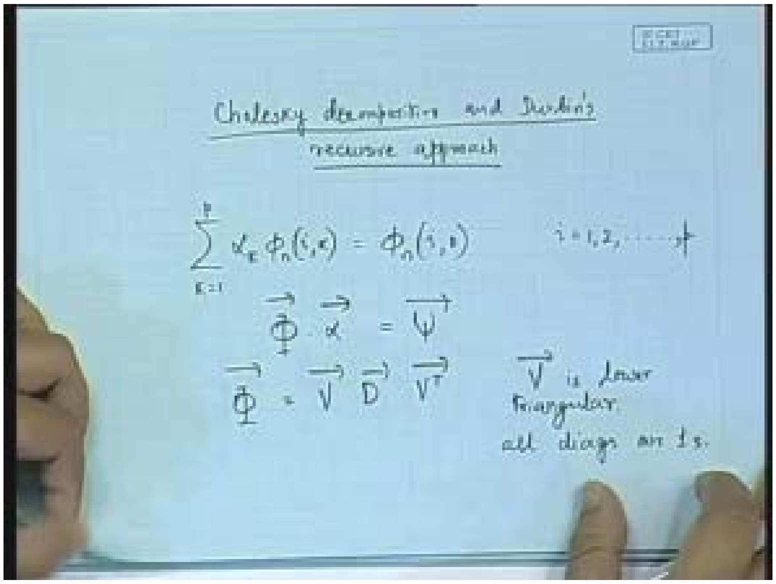

So from this matrix expression we obtain the (i, j)th term and the (i, j)th term can be expressed like this. that we can write as phi n (i, j); this can be expressed from this matrix equation as: k is equal to 1 to j and then we can write V ik d k V jk and in this case the index the summation index j that goes from 1 to i minus 1 where i is which one? i is the first index that we are putting for the correlation term. So the summation k is from 1 to j and this expression; actually what we can do is that this summation k is equal to 1 to j we can just write it as a sum of two terms; one is that this summation expression only what we can write as: k is equal to 1 let us say if you write up to j minus 1 leaving the jth term; jth term in the summation process then we can write k is equal to 1 to j minus 1 V ik d k V jk as usual plus we are going to have the jth term.

What is the jth term?

For k is equal to j we can write plus V ij and then this is going to be d j and then this is going to be V jj V jj and what is V jj? V jj is nothing but the diagonal element of the V matrix which by our choice of the matrix decomposition we have made V jj as equal to 1 which means to say that this additional extra term becomes V ij into d j only. So we can write it as V ij d j, this can be expressed as phi of n (i comma j) minus summation k is equal to 1 to j minus 1 V ik d k V jk for j lying between 1 to i minus 1 as usual what we had done. And then for the diagonal elements we can write down from this expression itself that when we put this as the phi n(i, i) so for the diagonal elements we are going to write it as phi n(i, i) and that is going to be........... that it will be I mean, this expression is going to be the summation k is equal to 1 to i minus 1 if I write the minus term.

First is that phi n is equal to summation k is equal to 1 to i V ik d k V jk V ik just this expression (Refer Slide Time: 8:03) just this expression and I am substituting i in place of j. So having that V ik remains same, d k remains same, V jk becomes V ik and the index of summation becomes k is equal to 1 to i.

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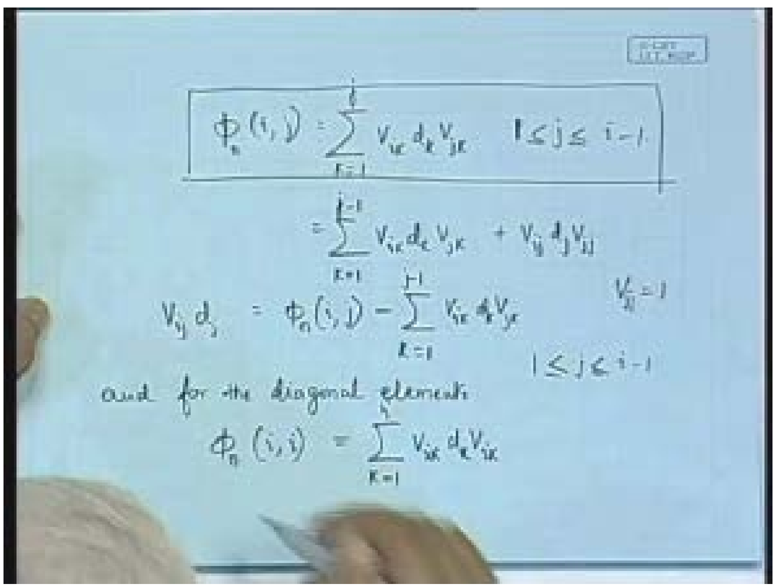

Or equivalently, from this expression what we can write? If we take this one (Refer Slide Time: 8:28) if we take this one then it becomes V ii into d i V ii is 1 then we can write as d i; the left hand side becomes d i and d i becomes equal to phi n(i, i) minus this becomes k is equal to 1 to i minus 1.

So just see, we can write first that d i becomes equal to phi n(i, i) minus summation k is equal to 1 to............ what is the summation limit? i minus 1 i minus 1 and then what it will becomes?

Just j gets replaced by i so it becomes V ik square so it becomes V ik square into d k and then this equation actually gives us something very useful.

Actually if we write down the matrix expression; if we just break up the matrix expression what we had written; means the original matrix expression phi expressed in terms of this and noting that all the diagonal elements of this V matrix they are equal to 1 and if we multiply, in that case we will be observing that the phi (1, 1) term that becomes equal to d 1 (Refer Slide Time: 10:07). So noting that condition if we put that phi (1, 1) becomes equal to d 1 in that case using d 1 we will be able to compute the terms beyond d 1 that means to say that for d 2 d 3 etc we can make use of this so what we can do is that we can compute this for i greater than or equal to 2 with the condition that d 1 becomes equal to phi n(1, 1). This d 1 is equal to phi n(1, 1) is taken as the boundary condition and using this boundary condition it will be possible for us to compute this equation for i is equal to 2, 3,....... etc onwards.

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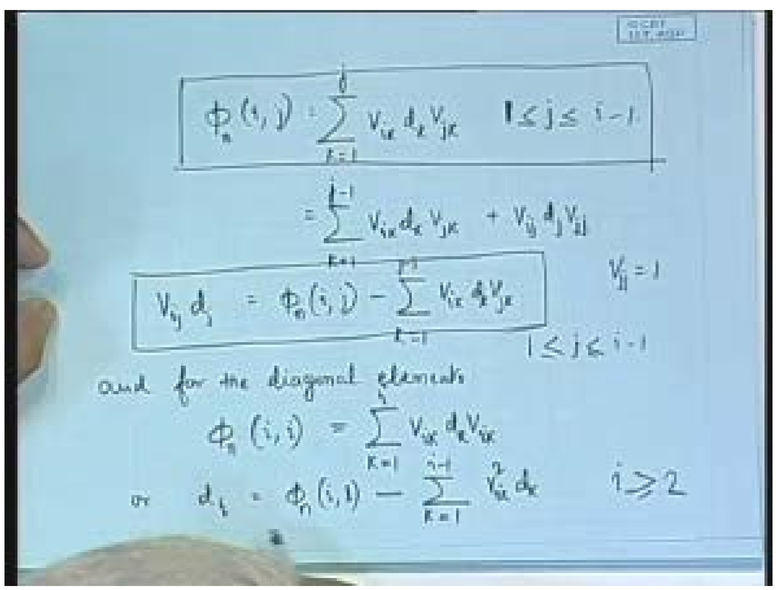

Therefore see, when we put d is equal to 2 what happens? Just see, then d 2 becomes equal to phi n(2, 2). Now phi n(2, 2) is something that we can compute because we are going to compute the correlation values anyway. So phi n(2, 2) will be determined from our speech samples and this minus k is equal to 1, 2 whenever we are putting i is equal to 2 in that case it is only one term that results with k is equal to 1. So using that we have only d 1 term; d 1 term only will be required and d 1 is already known to us; d 1 is nothing but phi n(1, 1).

Hence, when d 2 is known accordingly then we already know d 1 and d 2 and using the knowledge of d 1 and d 2 we can now compute d 3 because to compute d 3 the summation will be k is equal to 1 to now it will be 2, so there will be two terms; one is d 1 term and the other is d 2 term, we know both d 1 and d 3 terms so we will be able to compute d 3. So this is the way we can iteratively find out the values of the diagonal elements. So this way it will permit us to know the diagonal elements and once the diagonal elements are known you can make use of this equation in order to determine the elements of the V matrix.

If V matrix is known the diagonal elements are known then what you are knowing is that in this basic matrix equation you are knowing V D and V transpose is obviously known and by knowing this V and D it will really permit us to do an efficient matrix inversion for this phi because from this expression what are we supposed to do? We are supposed to solve for alpha which means to say that alpha will be nothing but phi inverse into psi.

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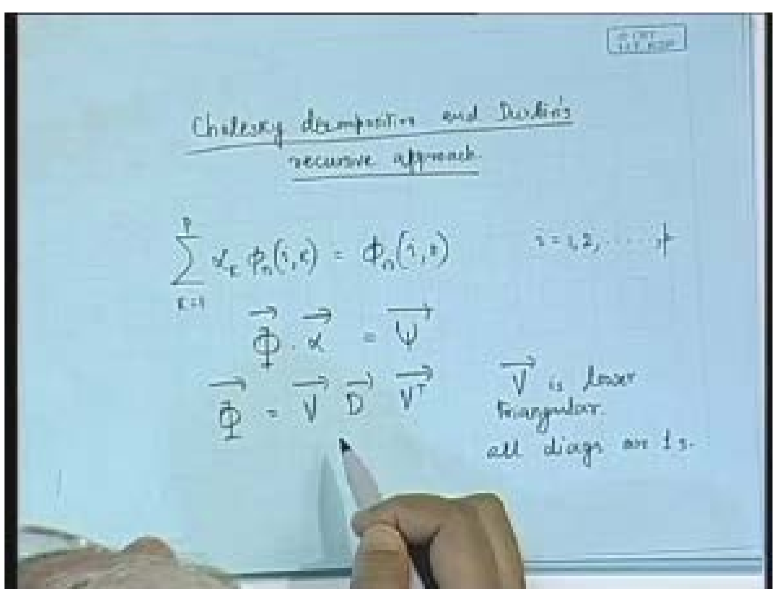

So to compute phi inverse, phi inverse will be expressed in terms of this V D V transpose and once the elements of this V matrix, d matrix these are known this matrix inversion can be efficiently solved. I just want to give you an exercise which you can try out and that is to say that you consider an example with p is equal to 4. So consider........ so this is an exercise for you, so consider an example with p is equal to 4, so fourth order predictor we mean, so fourth order predictor means what is the size of our matrix? Matrix is going to be of 4 by 4 size. So, with p is equal to 4 and instead of writing the matrix elements as phi n(i, j) so matrix element phi (i, j) you just write it in a simpler way and matrix elements phi n(i, j) simply write it as........ just drop this n because this n is implied write it as phi (i, j) and then you can express the elements of V matrix and d matrix in terms of phi (i, j)'s so you will get a feel of the decomposition. We know that decomposition is not something which will be very difficult for us.

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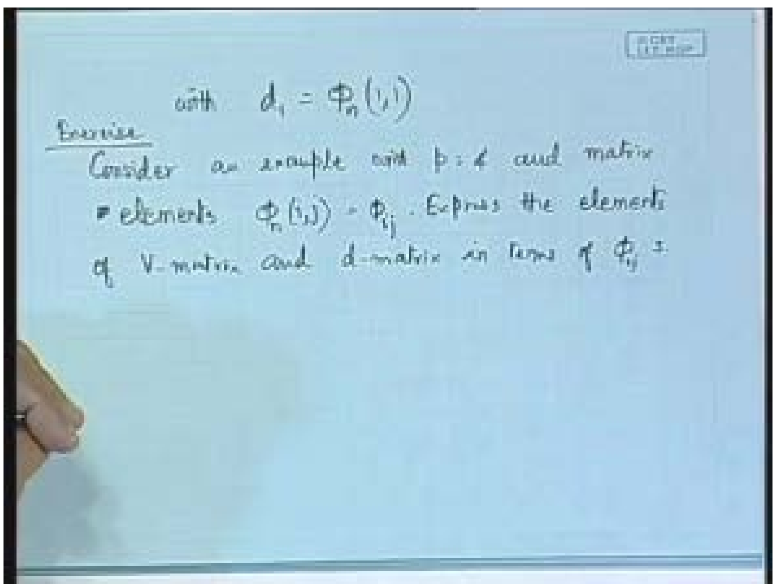

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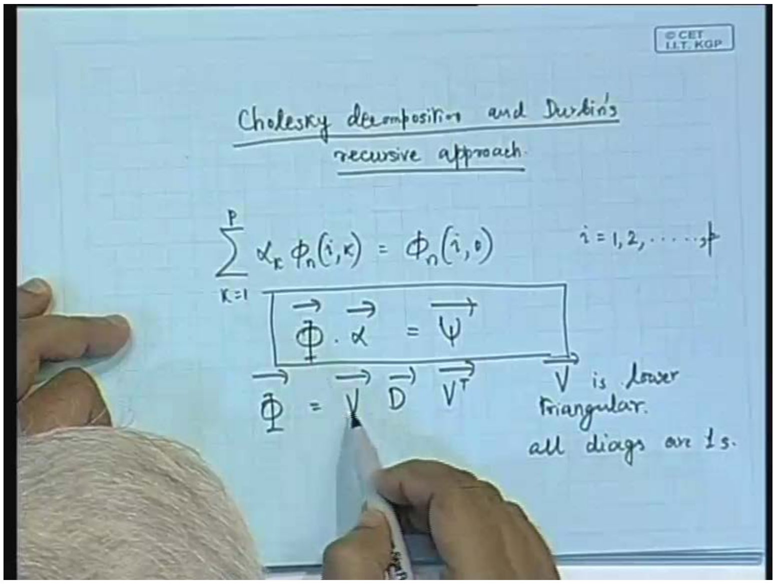

Now let us go back to our original equation phi alpha is equal to psi, we are talking of this original equation. And in this original equation now in place of phi we are going to substitute this V D V transpose. If we do that then what are we going to get? We are going to get V D V transpose alpha and that is equal to psi. So we can write this expression as, we can write it as V Y V terms Y matrix and that is equal to the psi. In this case what we are doing is that we are writing this D V transpose alpha this we are writing as the Y matrix which means to say that we can write V transpose alpha is going to be nothing but D inverse into Y. So ultimately what we have to do is to solve for this alpha.

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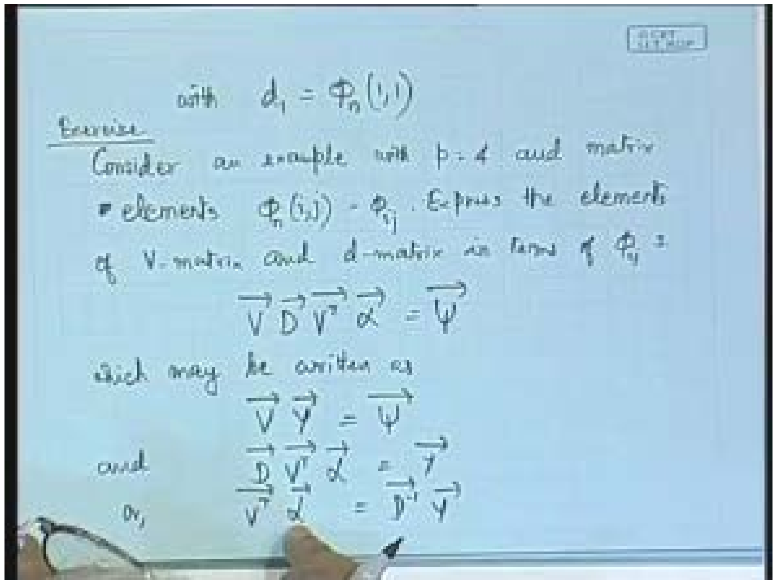

Let us see that whether writing this expression as V transpose alpha is equal to D inverse Y really helps us in getting any efficient solution or not. Now let us just have a look at this equation. Better that we number the equations. Let us say that we call this first basic equation we number as equation 1, then this phi is equal to V D V transpose what we are doing after the decomposition, this we are writing as the equation number 2 and then let us number this one as equation 3 and then this we write as equation 4 and this one we will be calling as equation 5, this one as equation 6.

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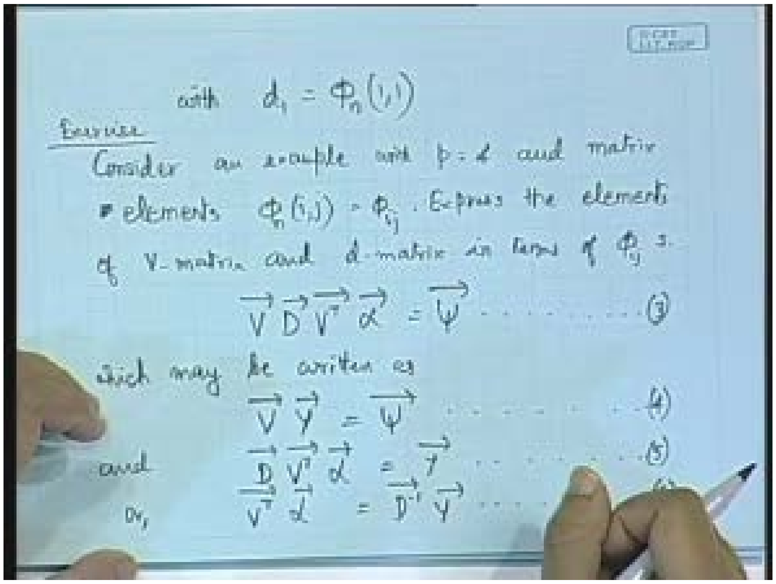

Now, from equation 4 the matrix equation what we get this can be written; again when we solve for the (i, j)th element we should be able to write it this way. By the way what is the nature of this Y matrix? Is it a matrix or is it a vector; what you feel? This is a vector because after all what happens is that this D V transpose that is a matrix and alpha is a vector so ultimately it just makes in to a p element vector. So essentially what we want to do is to write down the Y ith element and the Y ith element this can be written from the equation 4 as....................... So, from equation 4, from equation 4 we will be obtaining that Y i is equal to psi i minus summation j is equal to 1 to i minus 1 V ij Yj and i greater than i lying between p and 2. Now here this equation we call as 7.

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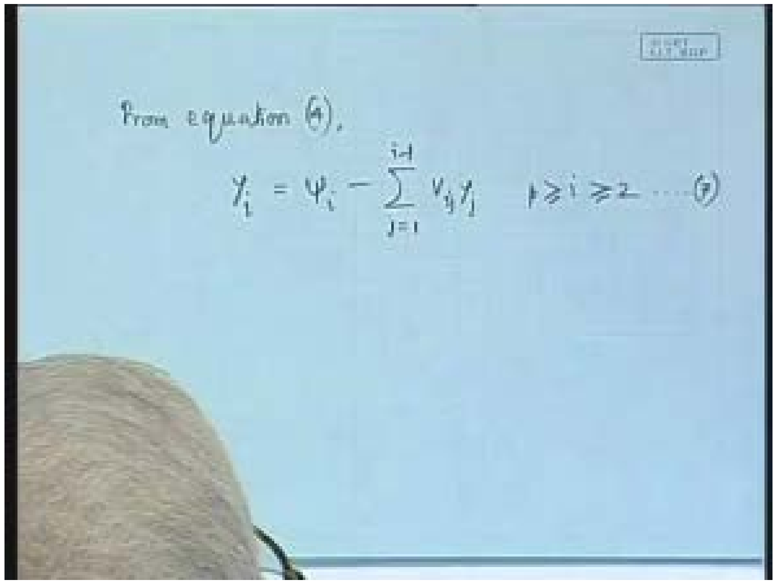

Now you know that how we have obtained it. We must have obtained it in a very similar way we have obtained in a very similar way. Essentially what you can write directly from equation 4 is that you could have written the equation 4's ith term as psi i could have been written as the summation j is equal to 1 to i and then we would have written it as V ij Y j. This would have been the summation from j is equal to 1 to i. And like in the previous way we are just going to split this summation into two terms; one is: j is equal to 1 to i minus 1 V ij Y j plus then the (i, i) term which will be V ii Y i and this V ii Y i is nothing but Y i. So that is what we are writing in equation 7. But just see; writing like this really helps us; how? Because now Y i we are writing in terms of Y j and if we do a recursion from i varying from 2 to p in this case what we are essentially doing is that yes, let us say that when we put i is equal to 2 in that case just j is equal to 1 only one term there in this summation so it will be V (1, 1). Again you have an initial condition so leave aside this because this is how we are going to deduce it. So, from this we can write with an initial condition. This follows from the matrix expression itself.

Now what is the initial condition?

The initial condition is that Y 1 is equal to psi 1. So if Y 1 is known to you in that case you just substitute Y 1 over here so j is equal to 1 so when we have i is equal to 2 in that case Y 2 this can be determined in terms of Y 1 because we need psi 2; psi 2 is determined because we are always measuring the correlation value so psi 2 is determine minus summation j is equal to 1 and this is going to be, this V i one term or rather to say we will be obtaining V 2 1 when i is equal to 2 we will be obtaining V 2 1 into Y 1 and Y 1 in this expression will be known to you because of this initial condition so you get Y 2. So writing like this essentially permits again another recursive relation to determine Y i. so this way we will be knowing the composition of this Y matrix elements.

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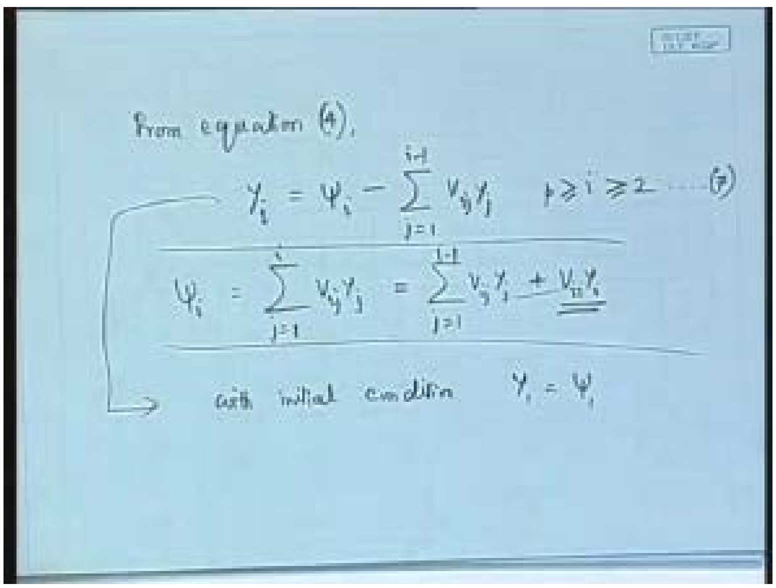

So having solved for Y equation 6 can be written as so equation 6 is this much. This is our equation 6 (Refer Slide Time: 22:35) so Y matrix elements are known and in fact we can directly write down this as a recursion for alpha. So what we obtain........ so having solved for Y the equation 6 can be solved recursively and I will just go over to the next page to the write down the recursive expression; so solve recursively for what for alpha.

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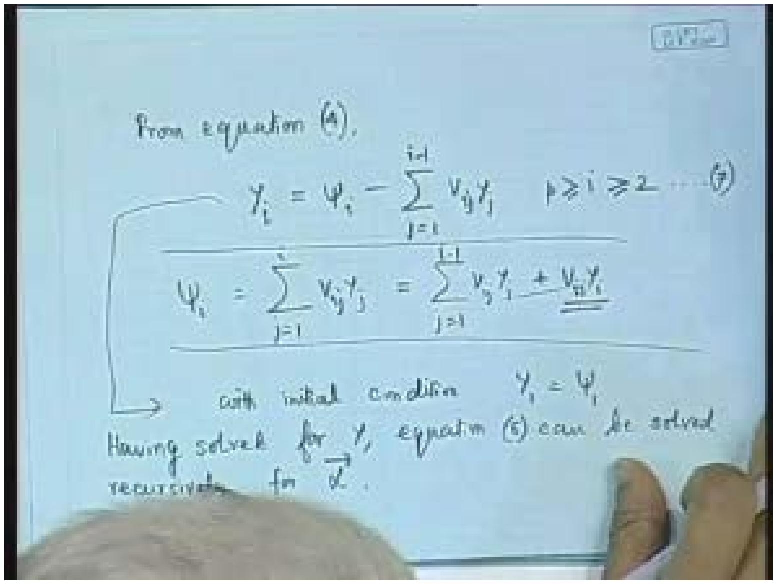

So we are going to write the recursive relation as alpha i and that is going to be y i upon d i minus summation it will be written as j is equal to i plus 1 to p V j alpha j and in this case i has to vary from 1 to p minus 1 and the initial condition that we have to put here is, the initial condition will be alpha p equal to y p upon d p.

Now you see one beauty. Our initial condition so far used to be in terms of the first element. But in this case the initial condition is with respect to pth that is to say the last element. So just see, if we are putting alpha as........ if we are knowing alpha p only first then what we can do is that we can determine. I mean, by putting the value of i to be equal to p minus 1 we can solve for alpha p minus 1 and to solve for alpha p minus 1 when you put i is equal to p minus 1 what results in the summation process is there is only one term with j is equal to p; so it will be V p into alpha p and alpha p is known to you from the initial condition.

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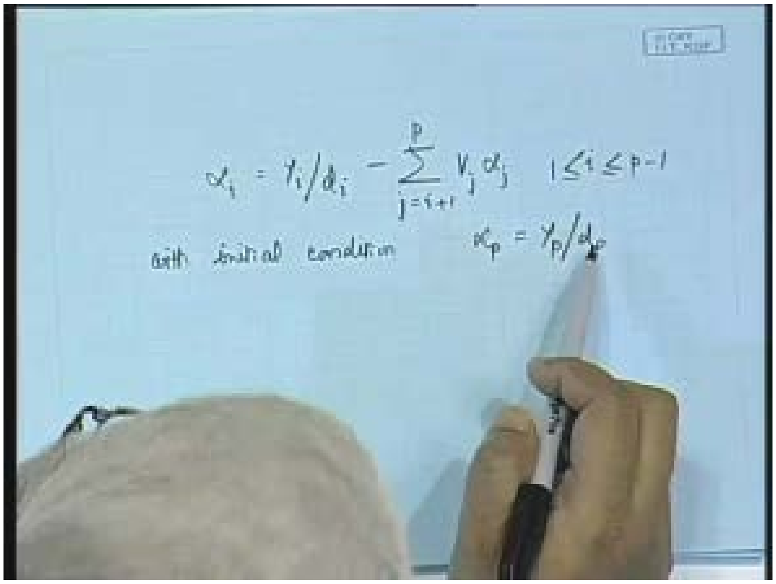

Therefore, having known alpha p you will be able to solve for alpha p minus 1; and having known alpha p and alpha p minus 1 you should be able to solve alpha p minus 2. So in this case also it is a recursion but only difference is that this is going to be a backward recursion. So in this case......... yes please, any question? [Conversation between Student and Professor – Not audible ((00:25:44 min))] V j yeah just a minute yeah yeah yeah correct correct correct this this will be this will be V ji (Refer Slide Time: 25:58) yes, thank you for finding it out.

So now in this case the index i proceeds backward from i is equal to p minus 1 to i is equal to 1. Now, whatever we wanted to solve can be written. So we can now solve for all the values of alpha, this is what we were looking for. So this is the first approach that is to say the covariance approach. Of course we will discuss the covariance approach little later because first we had talked about the autocorrelation approach and then we had talked about the covariance approach.

So the covariance approach, the iterative solution for alpha would be like this. so essentially what we did it may appear that as if to say that we have complicated the problem because the initial matrix equations looks so simple phi alpha is equal to psi but it is the inversion which would have been the most troublesome part of it. But what we did just to summarize, what we did was to break up the phi matrix into a decomposed product of V D V transpose, so we are just multiplying it with the triangular matrix lower triangular matrix and its transpose and in between we are having the diagonal matrix, solve for the elements of the V matrix and d matrix and having obtained that we are then ultimately through a recursion process we are going to determine the value of the alpha.

It can be numerically determined. Only thing is that there will be involvement of some multiplications in the process. So as a result of that some computation will be involved and assuming that p is generally not too high that is why the order of the matrix will not be excessively large. We talk about p in general but what can be a typical practical value of p; p is equal to 3, p is equal to 4 like that because we have seen........... it is experimentally also determined that beyond a predictor order higher than 4 virtually contributes to nothing extra because the past samples are not really......... I mean, those many past samples are really not needed. Up to the first three or four past samples that is good enough for the prediction process. This is one very effective approach. And now we are going to talk about Durbin's recursive solution corresponding to the autocorrelation method.

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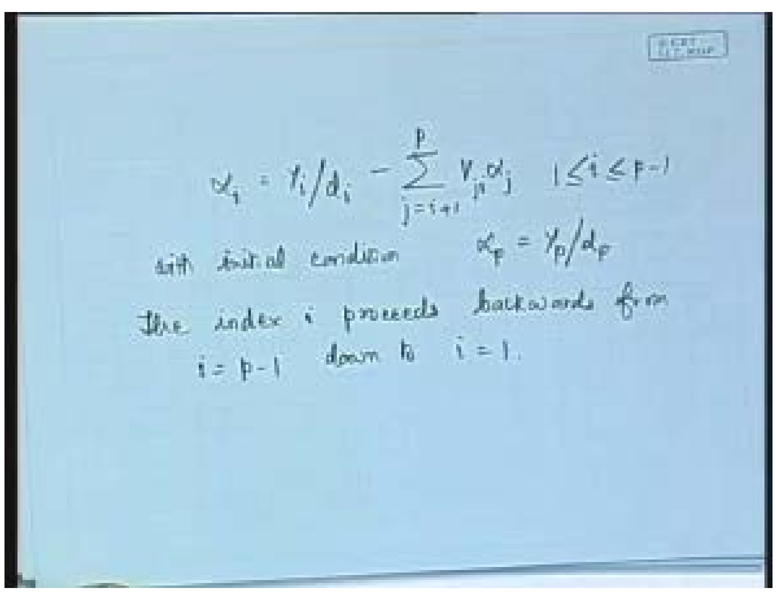

We are going to talk about Durbin's recursion as applicable to the autocorrelation equations. And in Durbin's approach what we do is that we first begin with the basic equation. Now the basic equation is also a set of p equations as before, we know that. So what we know is that summation k is equal to 1 to p alpha k R n (mod of i minus k) and that was equal to R n(i) and this is i varying from 1 to p. we did not name the equations so let us number let us number let us continue to number those equations.

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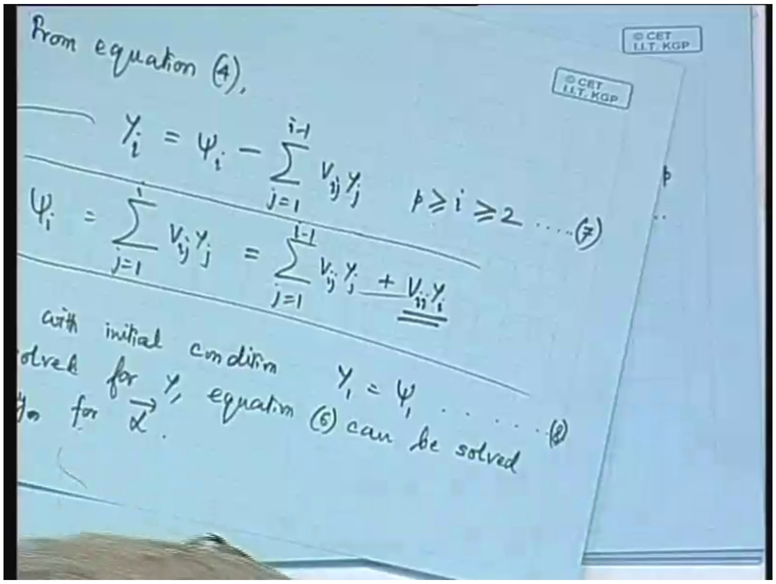

This one we called as 7 and then this one is only a derivation to obtain this so that is not a problem. And initial condition we put as equation number 8 for the earlier one.

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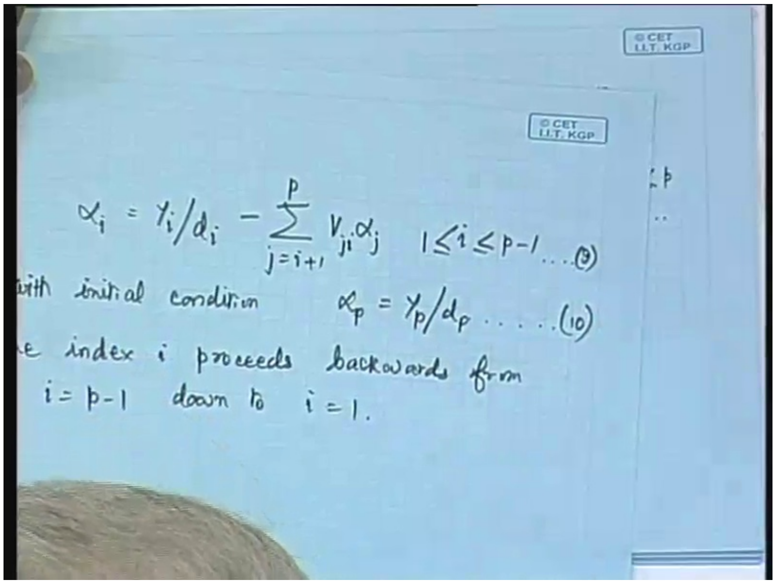

Then this alpha recursion, alpha recursion we call as the equation number 9, so this we call as equation number 9 and the initial condition of alpha we call as equation 10. With this, this particular autocorrelation equation what we are writing becomes number 11.

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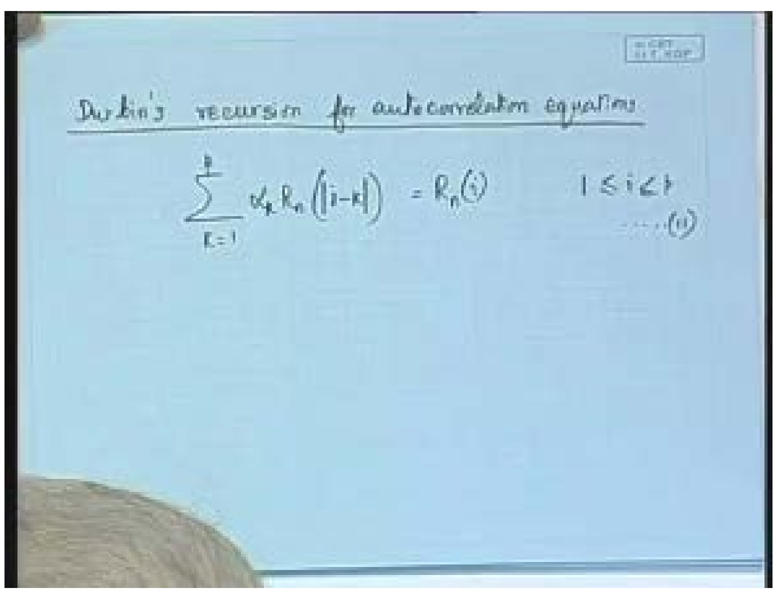

Now what we are going to do in the case of Durbin's recursive procedure is that, we are not going into the actual derivation of the iteration. Because after all there is a matrix inversion which will be needed that you can very clearly understand. Because basically what we are having on the left hand side is a matrix of this R n elements and because of this R n (mod of i minus k) it results in the Toeplitz matrix what we had already discussed in the last class. So essentially it is the inversion of the Toeplitz matrix and if we follow the inversion steps then there is a very beautiful alternative methodology that results and what we can essentially write down is that this steps of iteration will be written in terms of the error terms, the prediction error because after all what we are doing is that we are predicting the function using the past sample so there is going to be a prediction error; and if we just take some initial prediction error the initial prediction error is going to be that of R(0) and then one can iteratively solve and get the solutions for the alphas ultimately because ultimately we will be interested in solving for alphas in an efficient iterative way.

Thus, what we do is that in these steps we will just make some simplified way of writing; we will be dropping this suffix n. So instead of writing it as R n(i) we will be simply writing it as R (i). So what we do is that, the recursive procedure we can write like this. So we write as E the error and the initial estimate or the zeroth estimate of the error so we are going to write it as E and top we write as superscript keeping zero within the parenthesis. So zero within the parenthesis means it is the initial condition. So E(0) is written as R(0) that is the autocorrelation value zero zero which means to say that this is the power expression; so E(0) is R(0) and then K i K i is a quantity which we will be defining iteratively.

Essentially you will be obtaining the K i like this. so K i will be written as R (i) minus summation j is equal to 1 i minus 1 and then it will be alpha j times superscript (i minus 1) or (i minus j) this is one equation so this term we have to divide by E of (i minus 1) and this has to be done for i varying from 1 to p. So what we will do is that this we will call as equation number 12 and this we will call as equation number 13.

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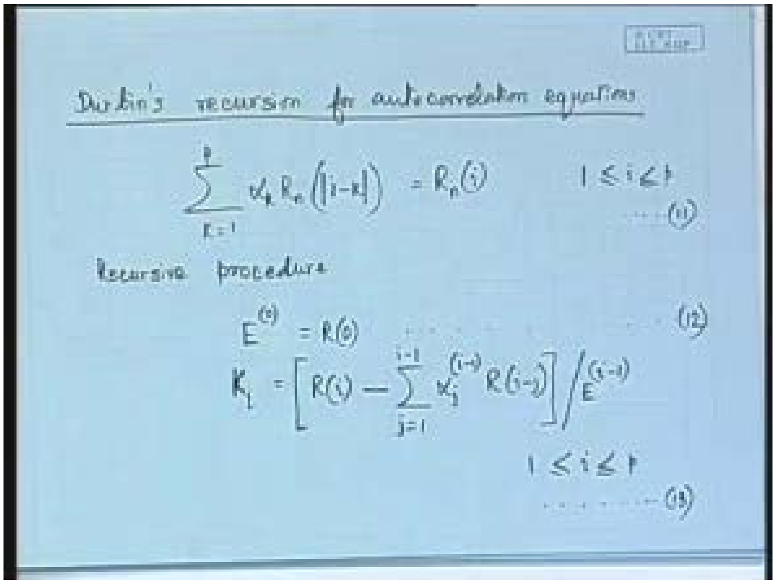

So essentially what we are trying to do in equation number 13 is like this. 13 is going to be an iterative equation, you can see. Let us see that what result when i is equal to 1. if you put i is equal to 1 then you have E (0) over here, then you have alpha j 0, you have got R(0) over here as the first term because the expression is j is equal to 1 and then you will be able to obtain alpha 1. So you just need an initial estimate of alpha............ with the initial estimate of alpha you will be able to obtain the value of K I; so this K i that will be used in order to determine the next set of quantities. In fact what results is that the alpha i estimate that becomes equal to K i. So we can write down, as equation 14 we write down alpha i........... do not worry, I will illustrate that with a very simple example of P is equal to 2 that means to say with just a second-order predictor we will be illustrating this approach. So this will be alpha i (i)th estimate that will be equal to K i. So we can compute K i. having known E(0) we will be able to compute K i; all i's are known to us so all that we need to do is with an initial estimate of this alphas we can solve for this K i and this K i's will be the next estimate of the alpha.

So alpha i will be equal to K i and then alpha j of (i) can be updated from alpha j (i minus 1) minus K i alpha i minus j (i minus 1) superscript which means to say that whatever i minus 1th estimate was there with that one can have (i)th estimate; in this case j varies from 1 to i minus 1.

So this will be called as equation number 14; this will be called as equation number 15 and then from here we can obtain E i as (1 minus K i) square into E (i minus 1) so that becomes the next estimate of E; call it equation 16.

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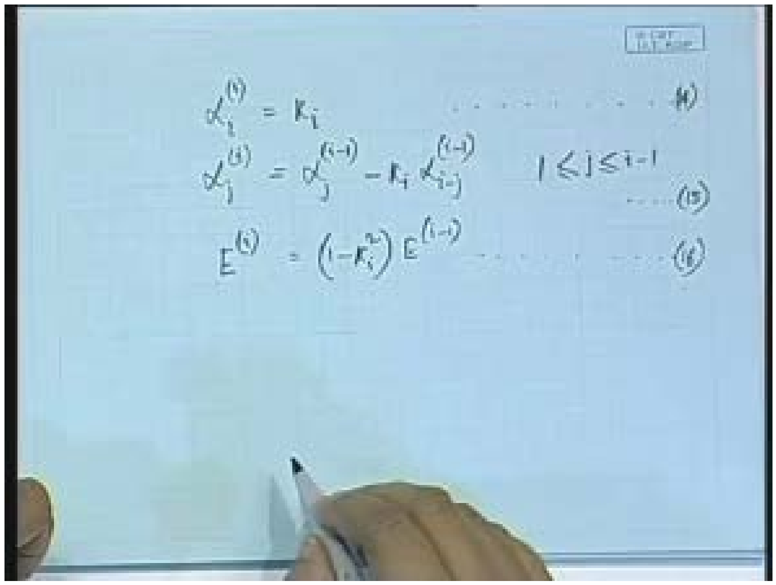

And from the next estimate of E it should be possible for us to go back again to the iteration, that is to say if we now go over to equation 13, 14; equation 13 to 16 if we repeat for all the values of i's up to p in that case we obtain that alpha j that the final estimate of this alpha will be alpha j corresponding to the (p)th iteration. So all that we can say is that the equations 13 to 16 these are solved recursively for i is equal to 1, 2, ... etc up to p and this is the final solution. This we can just simply illustrate with a simple example of P is equal to 2 a second-order predictor.

Therefore, if we take a second-order predictor then from this equation that is from equation number 11 what we have to write is simply in terms of a 2 by 2 matrix for this R and then just a two element vector as alpha 1 and alpha 2 will be for this alpha K and here again we will be having R(1) and R(2) from this what we do is that, for a predictor of order 2; so we just illustrate our algorithm with an example that p is equal to 2 and then our basic matrix equation can be written as R(0) R(0) and this is going to be R(1) and this is R(1). This a Toeplitz matrix definitely because the diagonal elements are all same; this is a symmetry and also the next diagonal which is consisting of one element is R(1). So this is definitely a Toeplitz matrix and then this is alpha 1 alpha 2 and then we have R(1) R(2) and the idea is to solve for this alpha 1 alpha 2.

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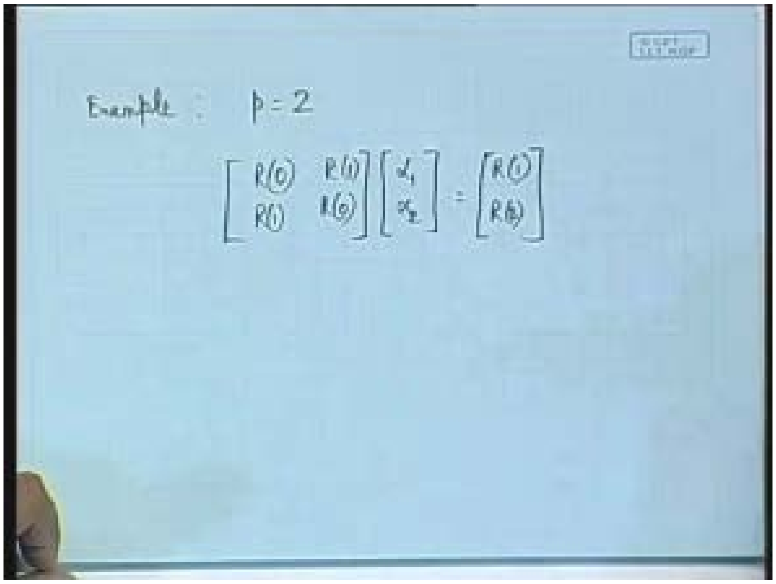

Therefore going by our iterative approach what we presented............ so just have a look at the iterative approach and just verify that what I am writing is correct or not. You just keep in your notebook, you just keep these equations handy; that is to say equation number 12 13 14 15 16. Equation 2 to 16 you just keep; let us call this as 17 (Refer Slide Time: 41:51); so you just keep these equations with you and then follow how we illustrate this example.

What is E(0)?

E(0) is nothing but the R(0). And what is going to be K 1 in this case?

Just see, K 1 means you have to apply equation number 13; you have to apply equation number 13 and there you have to put K is equal to.............? [Conversation between Student and Professor – Not audible ((00:42:23 min))] K is equal to 1, sorry you have put i is equal to 1 which means to say that K 1 will be equal to what?

It will be R 1 minus..... minus what? j is equal to 1 to (0, 0) what does that mean? [Conversation between Student and Professor – Not audible ((00:42:49 min))] Do not do it, do not do the summation; you do not have to compute this summation, it will be simply an R(1) and this divided by E of 0 so it will be R(0) by sorry R(1) by E(0) because this is for i is equal to 1 so it is R(1) divided by R(0) so R(1) by R(0). And having known that now what is it that you are going to have?

In fact this is one beauty of the algorithm may be that I did not explicitly note and tell you this point while I was writing down the equation. You see, yes of course I was telling you that some initial instrument of alphas are needed, that is not needed. In fact what you were doing is that you are only determining this K 1 and while you are computing K 1 you will never be requiring this summation (Refer Slide Time: 43:56) this second summation what we have this summation term will be zero for this so you will always be getting your K 1 expression in this manner and with the K 1 you can have an estimate of the alphas.

Therefore, having known K 1 what is going to be the application of equation? So what is equation 14 going to give us now, tell me? K 1 is known so alpha 1 (1); so alpha 1 (1) is going to be equal to K 1. So we now obtain an estimate of this. This will be K 1, K 1 means the same; it is R(1) upon R(0).

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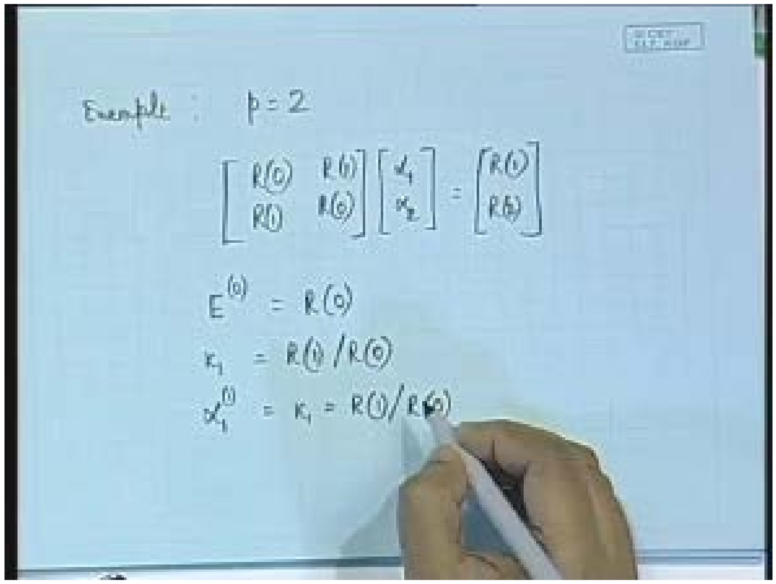

Now what about the alpha j's; because j varies, look at equation 5, equation 15 says that j varies from 1 to i minus 1. So in this case what is our i? i is equal to 1 that means to say that there is no particular solution for this 15 it is meaningful so we cannot obtain i is equal to............ a solution for j is equal to 2 in this case which means to say that say that we have to be satisfied with the alpha 1 estimate only because equation 15 we cannot apply simply. So in that case what we have to do is directly to go over to equation number 16 with i is equal to 1. That means to say that we can now obtain E (1) E (1) in terms of E(0) because if we put i is equal to 1 in that case it is E(0) so E (1) is equal to 1 minus K 1 square into E(0). So let us apply that. We know K 1, K 1 is R(1) by R(0) and the other thing is that E(0), E(0) also we know that it is R(0). So just tell me that what is going to be E (1)? E (1) is equal to.................. [Conversation between Student and Professor – Not audible ((00:46:34 min))] R(0) square minus R(1) square by R(0). R(0) square minus R(1) square upon R(0). So this is our E (1).

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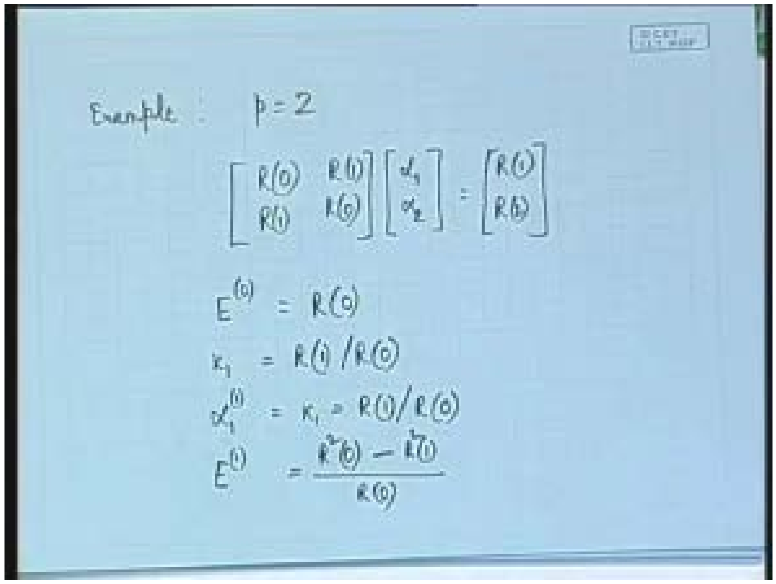

And now what are we going to do in the next step?

[Conversation between Student and Professor – Not audible ((00:46:56 min))] Put K is equal to 2. So now we have to solve for K 2. So K 2 is going to be.........; you can just verify after putting into the expression. Because now with K is equal to 2, just have a look at 13; at equation number 13. Now we are going to solve for K is equal to for i is equal to 2. So K 2 will be equal to R(2) minus the summation will be j is equal to 1, 2; what is i? i is equal to 2 so j is equal to 1 only, so only one term will be remaining corresponding to j is equal to 1 and what is that alpha 1; alpha 1 (1) and alpha 1 (1) we know already; alpha 1 (1) we require, i is equal to 2 and j is equal to 1 so alpha 1 (1) R(1), this upon in this case with i is equal to 2 it is E (1), E (1) which is just now computed.

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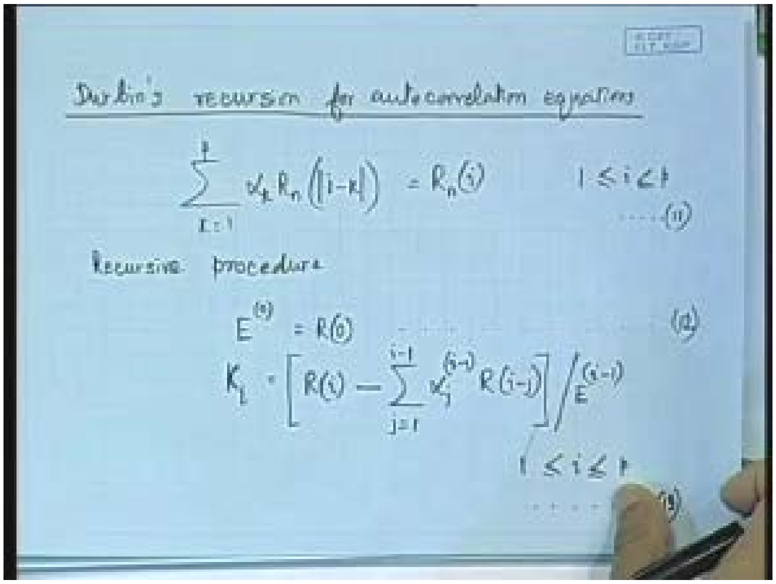

So what we have to do in this case is that we will be obtaining from this K 2 is equal to R(2) R(0) minus R square (1) this upon R square 0 minus R square (1) this is the expression for K 2. And then having known K 2 we will be in a position to have alpha 2 (2); alpha 2 (2) will be but nothing but K 2 then this will be the same: R(2) R(0) minus R square (1) divided by R square (0) minus R square 1, this will be alpha 2.

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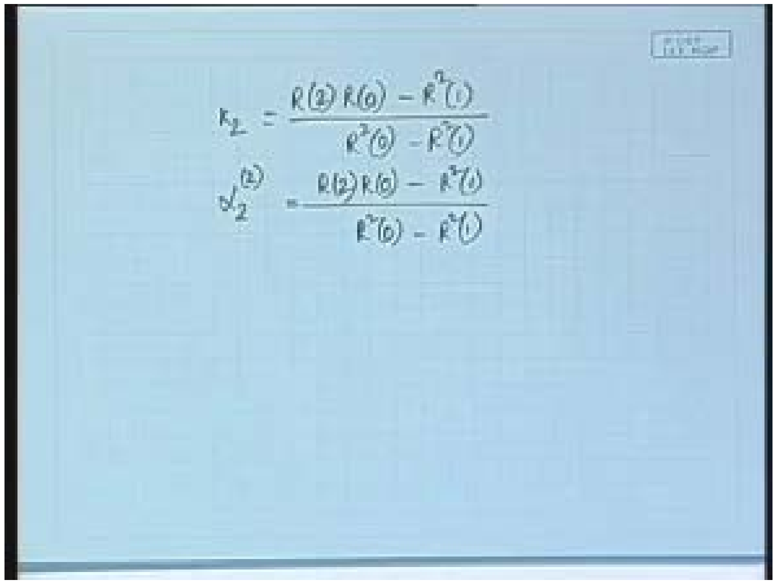

and then we also obtain

Now we will be in a position to apply the other equation which we could not apply in the last iteration that is to say equation number 15. Now we can obtain now we can apply equation number 15 (Refer Slide Time: 49:23) for j is equal to 1 to; for j is equal to 1 we will be able to apply which means to say that alpha 1 with i is equal to 2; so alpha 1s second estimate now we can obtain. So alpha 1s second estimate that becomes equal to R(1) R(0) minus R(1) R(2) divided by R(0) square minus R(1) square.

Then what we have obtained?

We have obtained alpha 2 second iteration, alpha 1 second iteration. Anything more we require? We have to go through another iteration? No need because we have to stop the iteration for i is equal 2 in this case, because it is it is it is only up to p that is what we have to go and we are having p is equal to 2. So p is equal to 2 so whatever are the alpha i's pth estimate that becomes the final estimate of the alphas. Thus, in this case we can say that alpha 1 is equal to alpha 1 superscript 2 and alpha 2 is equal to alpha 2 superscript 2. See, the only point to be noted is that, look alpha 1 for that it is really the second estimate. Now for alpha 2 although the superscript we are writing as 2 but alpha 2 is what we have obtained for the first time.

So what happens is that for a general pth order estimate when we have the terms going from alpha 1 to alpha p, the first iteration will only yield alpha 1; the second iteration will yield alpha 1 and alpha 2; the third iteration will yield alpha 1 alpha 2 alpha 3 so what happens is that alpha 1 is getting modified, is getting updated; alpha 2 is also getting updated but to a lesser extent because alpha 2 we did not have initially and the last term that is to say the alpha p that will get estimated only once, but that is after a long iterative process so that is why the estimate that we are having already for the alpha p's are close to the accurate estimate. So this is one effective computational approach that one can take in order to determine the coefficients this predictor coefficients alphas and this is what we were requiring.

Now these two methodologies are very popular and in fact the two approaches that I discuss today that means to say the Cholesky's decomposition pertaining to the auto pertaining to the covariance method and Durbin's recursion what we described just now pertaining to the autocorrelation method both are very effective techniques and we can summarize that there are two basic steps what you must have observed in these solutions, there are two basic steps: one is to obtain the elements of the correlation matrix. So we have to obtain the phi matrix elements or the R matrix elements by computing the correlation, that is the first step and the second step is the iterative solutions of these p equations.

Now these two are disjointed steps in the sense that you first need the formation of this correlation elements and then you have to apply the iterative solution techniques. You cannot have the second one without the solution of the first one.

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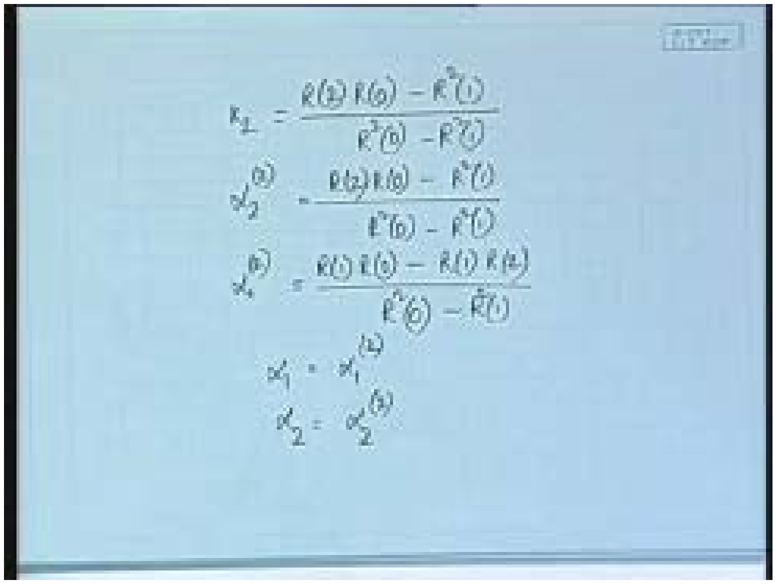

Now this approach these two disjointed approaches can be integrated if we go in for what is called as a lattice method of solution. In that case it is basically a filtering technique, that will be a lattice structure digital filter realization through which we will able to integrate these two steps combinedly and we can obtain the solution for the alphas very efficiently. So this lattice structure of filters and the lattice formulation that we will be taking up in the next lecture; thank you.