**Digital Voice and Picture Communication**

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**Lecture - 11**

# Computational Aspects of LPC Parameters

In our last lecture, we were talking about the introduction to the LPC the linear predictive coding and we had shown that how one can relate the linear predictive coding parameters with the speech model that we had studied earlier. And we had seen that, basically the LPC parameters or the linear predictive coding parameters, they follow from the speech model essentially. Towards the end of the class we had developed a relationship which goes like this. It refers that summation k is equal to 1 to p alpha k and then we had summation x of n minus j into x of n minus k which was equal to the summation n x of n x of n minus j where x of n happens to be the samples.

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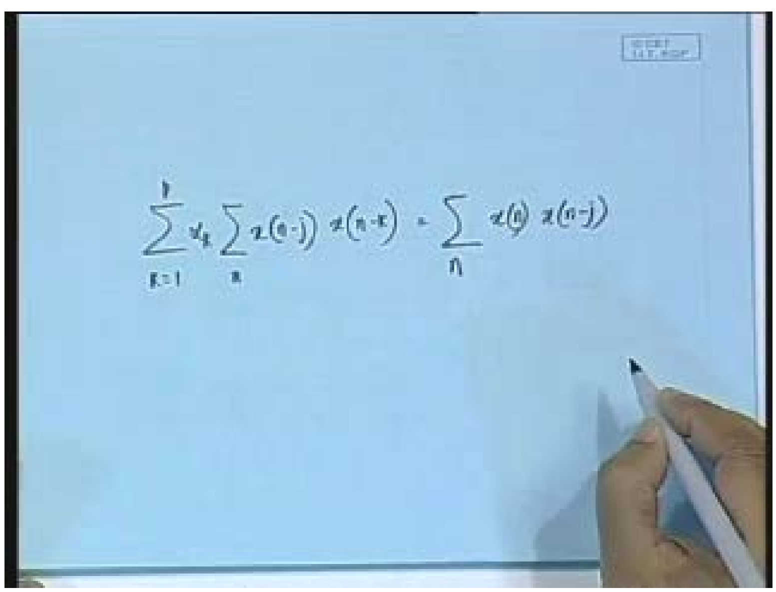

Then we also have that this index j was varying from 1 to p and what is p; p is the order of prediction; means up to how many pass samples are we taking for prediction and alpha k is where the coefficients which we want to determine. So essentially in this we have got a set of p equations and we have also got p unknowns. What are the unknowns? The unknown quantities are alpha 1 to alpha p. So, what we are going to discuss in this lecture is the computational aspects of this LPC parameters. So today's lecture will be on the computational aspects of LPC parameters. By LPC parameters we mean these alpha k's what we need to solve.

Now basically there are two very popular approaches or two major approaches to the computational aspect of LPC parameters. Because, as you understand that basically the problem is posed like this that there are p equations and there are p unknowns. So if we have to go in for the normal matrix inversion methodology it may be computationally really a great troublesome for us because all that one has to do is to compute these parameters, I mean, this matrix inversion and all these techniques we have to perform within one frame time because we are fixing up this alpha k's only for one frame time which can be between 10 milliseconds to 25 milliseconds, let us say, as a figure of merit let us say, I mean as just a crude figure let us say that 20 milliseconds of time so 20 milliseconds of time computation of all these is not very easy so that is why we have to go in for some specialized approaches to the computation of these LPC parameters and this is what we are going to discuss.

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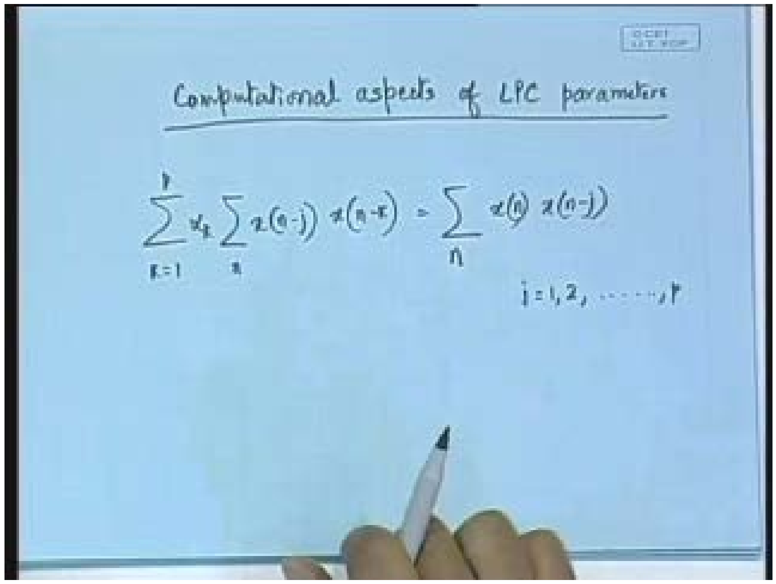


Now, for us, in the equation that we described in the last class x(n) were the samples. So we took the sample to be the x (n's). Now today, before we begin our analyses we just make some change of notation which I would like to tell you right now. See, what we are considering essentially is that we will consider the segments of speech. So segment of speech we will be denoting by s and we will say, s suffix n where suffix n basically refers to that the segment is considered from the point n. So n is the starting of the segment and from the starting the index or rather the offset beyond that will be denoted by m. So S n(m) when we write, this will basically represent the segment of the speech. So we are going to have some change of notation that means to say that instead of writing it as n we should write it as m because the n notation we are reserving for defining that where the segment starts and the index of summation instead of n we will write it as m and then with that with that difference instead of writing in terms of x if we write it in terms of s, that is same thing; and then instead of j's as an index we just use i so we use i in place of j, i in place of j, and if we do that then the same equation what we derived last time could be rewritten as the summation k is equal to 1 to p alpha k, (Refer Slide Time: 6:31) this k we are keeping as the index for the LPC parameters and this summation as I have just told you that should be over m and then instead of x we are writing S n and instead of n we are writing as m so it is m and instead of j we are writing i so S n(m minus i) and then the next product will be S n and what it will be it will be m minus k and this will be equal to the summation over m. Now here we have to write S n(m) multiplied by S n(m minus i) and we have to and we still have three equation so i index is between 1 to p.

Because there are because the i ranges within 1 to p we have got essentially p equations. So now the focus will be on the solution of this p equation so that we can determine the alpha k's.

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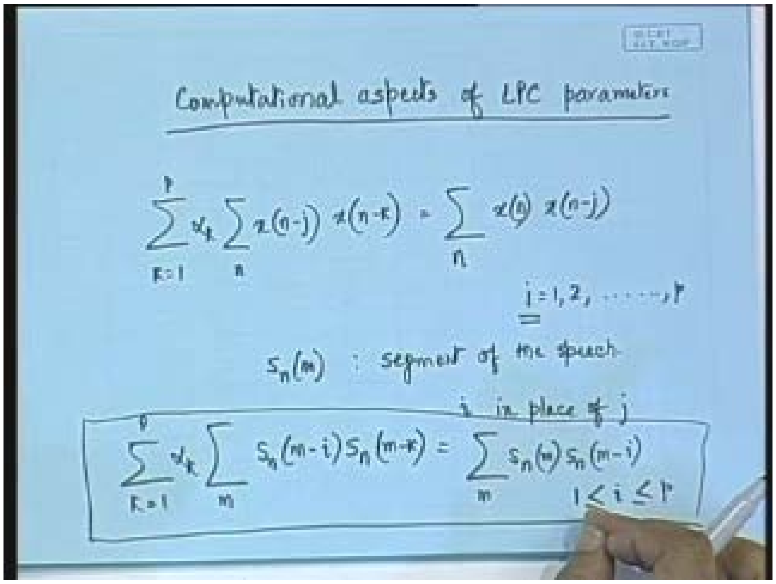


Now we just define a quantity which is nothing but the autocorrelation function. Now you just note one thing that we are writing summation over m which means to say that explicitly we are not specifying the limits of the summation. We have kept the limit open-ended. We will specifically put the limits when we go in to the specific approach of computation. Right now let us leave with an open-ended limits of m in the summation and we define that the quantity summation over m S n(m minus i) multiplied by S n(m minus k) this is what we have on the left hand side of this equation.

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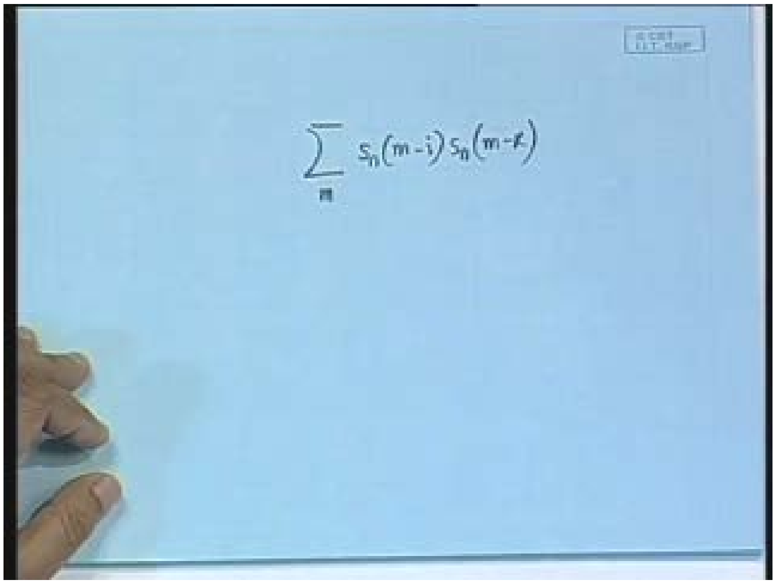


Thus on the left hand side we have this S n(m minus i) into S n(m minus k). So this summation quantity of S n(m minus i) n minus k (m minus k) this we are calling as phi suffix n and then we have to write this as (i comma k); and what is this? This is equal to nothing but the autocorrelation function. So here you can say that i is a sort of an initial offset from the starting and then what is k? k will be the lag. So essentially the lag in this case is going to be (i minus k). In fact we are going to show that, I mean just a change of variable. But let us introduce this notation that phi n i k the autocorrelation we define this way and in that case in terms of this equation, (Refer Slide Time: 9:49) this equation whatever we had written we can just rewrite with the autocorrelation function notation.

Hence, we can define it. So the same equation we just rewrite as summation k is equal to 1 to p alpha k phi n (i, k) so this is phi n (i, k) what we have on the left hand side of the equation is phi n (i, k) by our definition itself and on the right hand side what we have? We have summation over m S n(m) S n(m minus i). So (m minus i) remains the same so this is one and the same as saying that it is summation over m S n (m minus i) into S n(m). So S n(m) when we said is k is equal to 0. Therefore, if we put i and then we put for........ I mean, in place of k which has put k is equal to 0 or rather we can write it as i 0. So this we can equate to phi suffix n (i comma 0). So this so here i will be from 1 to p. Again p equation; so it is the same equation, we are we have just written in terms of........ rather than writing as summation of S n(m minus i) S n(m minus k) of that form we are writing it in the autocorrelation form.

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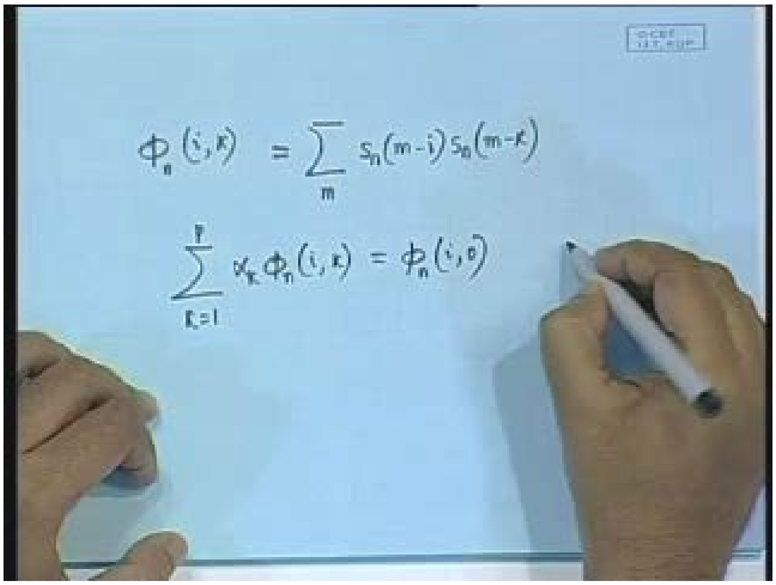


Now, let us keep this equation and then what we are doing in order to determine these parameters alpha k's are that we are taking the mean square error and then we are differentiating that with respect to alpha i's and then we are equating that to zero and then determining these alphas. So that is how we approach. So it is from the minimization of the error consideration.

So the minimum mean square error, I mean, if we put this condition and we substitute this into the error expression then the minimum mean square prediction error; so the minimum mean square prediction error this will be given by E n which is equal to the summation over m S n square m this minus summation k is equal to 1 to p alpha k summation over m S n(m) S n(m minus k) which is equal to............ so this S n square m how we can write in terms of phi? This is [Conversation between Student and Professor – Not audible ((00:12:51 min))] phi n (0, 0); so this is phi n (0, 0) and this we already know that this is nothing but the summation k is equal to 1 to p alpha k phi n (i, k) phi n not not (i, k) in this case (0, k) correct phi n (0, k). So this is the expression for the minimum mean square prediction error.

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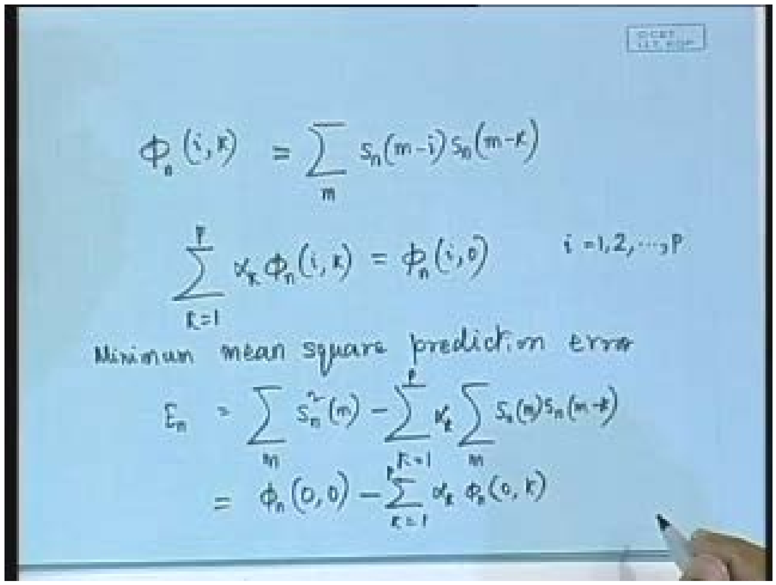


Now let us remember this and we now go over to one of the computational approaches to the solution of this alpha k's and this approach we will be describing as the autocorrelation approach. In fact there are two approaches two very popular approaches: one is called as the autocorrelation based and the other is what is called as the covariance based. We will first talk about the autocorrelation based. In fact the two are related. But you will see that you will first understand that what is the difference in these two approaches. So first let us understand the autocorrelation method.

In the autocorrelation method what we do is that we first of all we are taking a segment. Now, are we taking an infinite segment? Certainly we cannot. Although the...... I mean, in this definition we have put forward, mind you, you must have noted that we have said phi n we did not say r of n because we had earlier introduced that we will be calling the short time autocorrelation as r of n and phi n will be the long time autocorrelation. But we have written the long time autocorrelation expression. But actually speaking whenever we are computing it is a finite segment that is what we have to take which means to say that we will be multiplying the segment that we are taking by some wind wing function.

Therefore, we define our S n(m) this sequence itself we define as the original sequence s of m. We say s of m plus n into w of m where m lies between 0 to n minus 1. So w(m) as you already know is our wind wing function. And if we take a short time segment like this then we can define the error E of n to be the summation m is equal to 0 to N plus p minus 1 into e n square m. Now please be careful about this limit, mind you. That means to say that the wind wing that we have performed is over n samples; 0 to capital N minus 1. But just note that just now the e n expression what we have written takes its limit from m is equal to 0 to N plus p minus 1. So there is an N plus p term which have come about and why is this N plus p coming in? Because again, whenever we are taking the autocorrelation in that case as you know that because it will be from p samples we are predicting so if we give a lag of p we need to have these N plus p minus 1 those many things we require.

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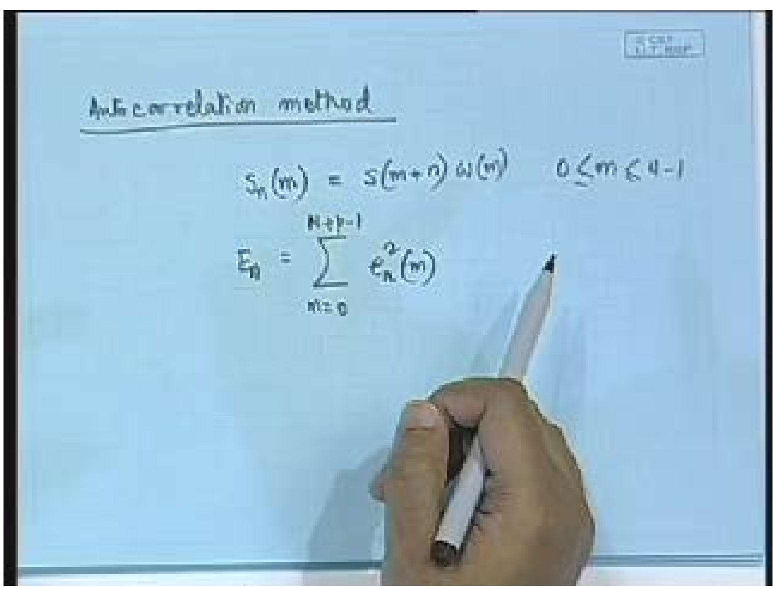


Now one point that that you should note at this stage is that always the prediction what we are making will not be a good prediction at least in the initial part and in the end part. Why? Because you see that, at the beginning of the segment, when a segment starts how are you predicting the initial samples from the past p samples? Now past p samples are what? Past p samples are zeros because the window has started which means to say that whatever past samples we require the past samples as zeros. So, the present sample within the windowed part is being predicted from zeros so it is not a good prediction so our initial prediction will not be very correct. Again at the end what we are doing; beyond our windowed part what we are doing; we are having the sample values as zeros but we are predicting from the windowed samples which are non-zeros, so from non-zero samples we are predicting the zeros. So the solution for that is that we use a window which is not an abrupt window like a rectangular window but some kind of a smoothening effect at the two boundaries and how we can have it; by having hand windows, hamming windows something like that where there is a tapering, there is a tapering at both the ends.

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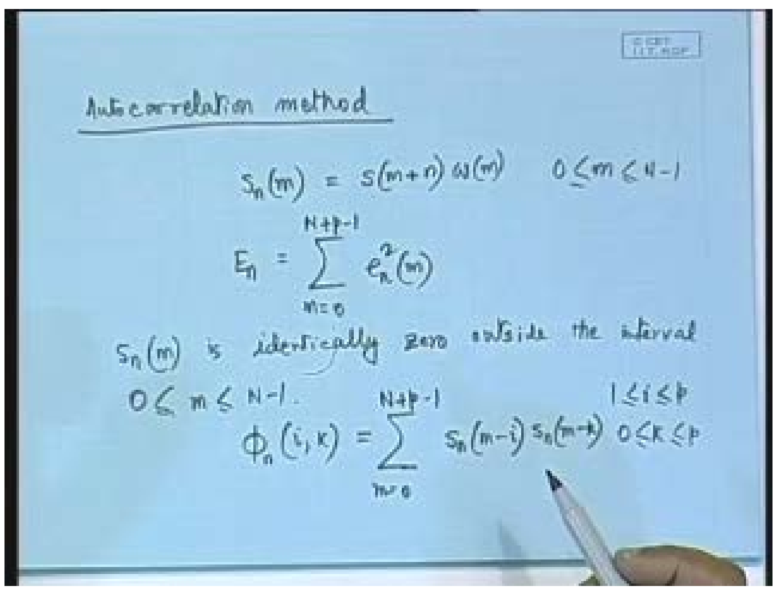


Now with this definition of the error that is what we are doing that the error is defined from m is equal to 0 to N plus p minus 1; again we just go over to our definition of phi n (i, k) so what is the our phi n (i, k); since so what we are doing is that we are assuming that S n(m) is identically zero outside the interval m lying between 0 to N minus 1. So we just assume that S n(m) is identically zero outside the interval, we just placed here outside the interval m lying between 0 and capital N minus 1.

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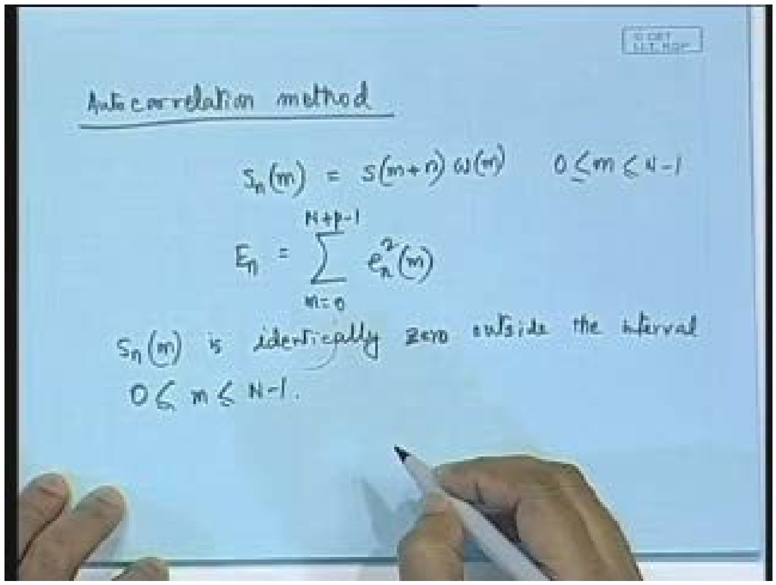


Then we can show that our phi n (i, k) our original definition of the autocorrelation and what we were writing last time? Just now, whenever I was writing phi n (i, k) look that I have kept the limit of the summation open-ended; I will get the summation m but then wrote here S n(m minus i) S n(m minus k). Now this S n(m) itself is restricted that it is zero outside the range of m lying between 0 and capital N minus 1. So then whatever infinite summation; if I had intended to write an infinite summation then the infinite summation now effectively gets truncated to a finite summation and that finite summation will be nothing but m is equal to 0 to N plus p minus 1; N plus p minus 1 and then we have to write the same thing that is S n(m minus i) S n(m minus k) and in this expression we have to say that how many such (i, k's) are possible.

now i we can vary between 1 and p and k; k is for the predictions so k can also assume a value of 0 so k lies between 0 and p. this is the equation. And now this phi n (i, k) expression what has come from the definition, this can now be rewritten in a little bit of difference form in the sense that we have what; we have S n(m minus i) S n (m minus k) so let us write it in a write it with the change of variable so that we want to write with S n (m) multiplied by something. So what we have to do; we now have to make just the substitution like this. So, by substitution we can express the phi n (i, k) phi n (i, k) that becomes equal to the summation of m is equal to m is equal to 0 to N minus 1 minus (i minus k) S n(m) into S n(m) plus (i minus k) and then in this case this phi n (i, k) is nothing but the short time autocorrelation function.

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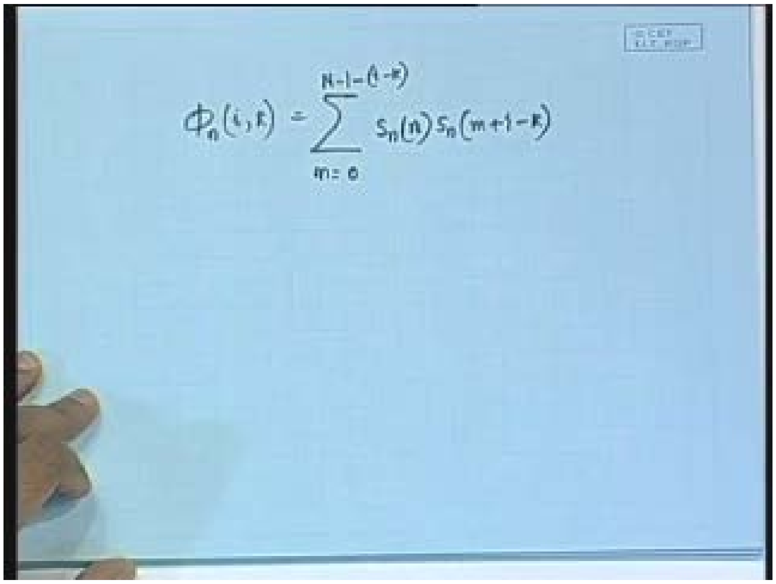


So phi n (i, k) now becomes a short time autocorrelation function which is evaluated at the point i minus, I mean with a lag of (i minus k). So I can write down that phi n (i, k) instead of phi n (i, k) I can write it as R n (i minus k) where; what is R n(k)? R n(k) is nothing but the summation m is equal to 0 to n minus 1 minus k S n(m) into S n(m plus k). So with this definition just instead of in place of k we put (i minus k) and then this equation results (Refer Slide Time: 24:15). So this equation is effectively R n (i minus k). So phi n (i, k) becomes R n (i minus k).

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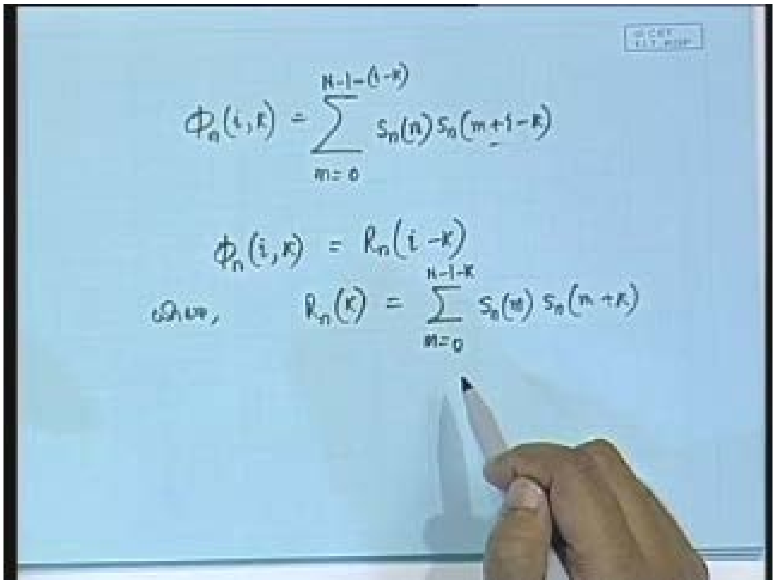


Now again you just note one point that this R n(k) you already know that R n(k) is going to be an even function; the autocorrelation function. So we can write down that phi n (i, k) now we can write down in this form: R(n) and we can say mod (i minus k). So i is equal to 1, 2,........ upto p and k is equal to 0, 1,........ up to p. So this is a good equation that we have obtained.

And having obtained that we can now try to substitute this into our original equation means what we have to solve exactly. So now let us let us go back to this equation. Hence we now we are required to solve this: alpha k phi n this equation we have to solve (Refer Slide Time: 25:33). So now, in this equation you just put, in place of phi n (i, k) you just put R n (i minus k) under mode and what is going to be phi n (i, 0) phi n (i, 0) is nothing but R n R n i. So I can rewrite the equation; this particular equation i can rewrite as the summation k is equal to 1 to p alpha k into R n mod (i minus k) is equal to R n i and how many such equations are there?

[Conversation between Student and Professor – Not audible ((00:26:26 min))] Same, p equations. So till now, just note that till now we have not solved the equation; we are back again to the same equation. Whatever was our starting point we are back again to the same equation but just writing in a different form that is all. So this 1 less than or equal to i less than or equal to p. And mind you this equation is no different from our starting point equation of this (Refer Slide

Time: 27:02) so we still need to solve our alpha k's; only thing is that this summation we have written in terms of the autocorrelation, short time autocorrelation functions now in this form.

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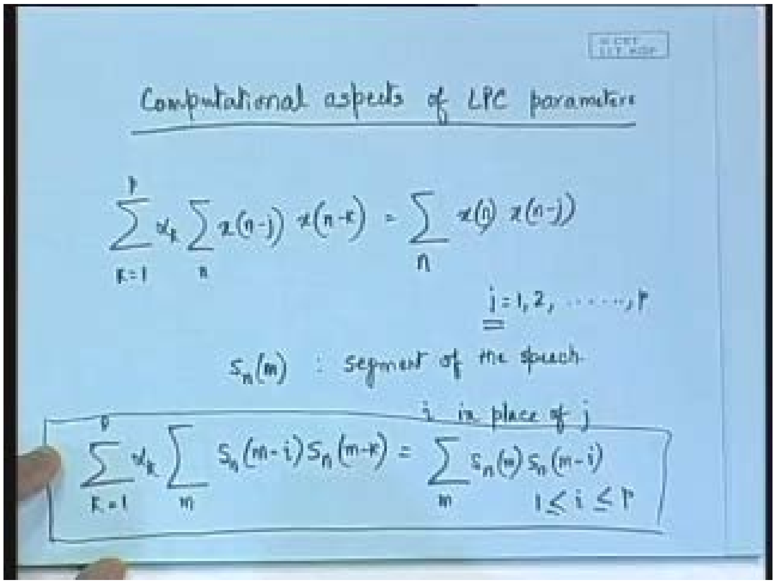


Now, to solve this equation let us break it up. So after all what we are going to do? Let us take a particular i let us say that we take i is equal to 1 and now we write down for the summation terms. So what will be there? So, if we if we are going to write down for R n (1) we have to just substitute i is equal to 1 over here so what are we going to write on the left hand side? Alpha 1 R n; R n it will be mod of 1 minus 1 because i is 1 so this equation we are writing for i is equal to 1and that means to say R n of 0.

What will be the second term of the solution second term of the left hand side? Alpha 2 times R n and what is mod of (i minus k) now? Now k has become 2 and what is i i is still 1 I mean this equation we are writing for i is equal to 1. So i is 1, k is 2 so R n mod of minus 1 so this means to say it is R n (1). Third term alpha 3 R n 2 R n 2 and plus so on.

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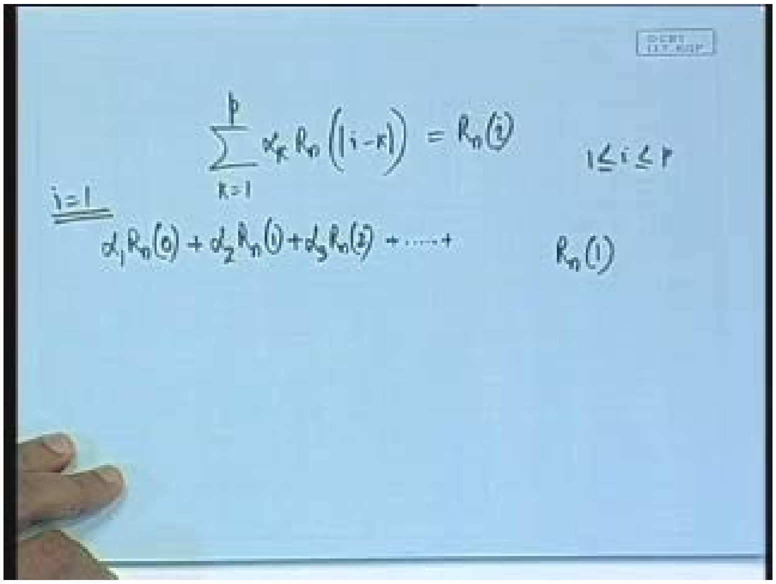


And if you want me to write down the last term the last term is going to be for k is equal to p and i is still 1 so it is 1 minus mod of 1 minus p which is nothing but p minus 1. So I can say that alpha 3 R n p minus 1. So this whole thing, these summation series is going to be equated to R n (1).

Now we make i is equal to 2. So we compute R n (2). So R n (2) will be equal to what? Again alpha 1 we have to start with; this time what are we going to write? Alpha 1 R n this time we have to write 1 because i is 2 and k starts with 1 so 2 minus 1 1 so alpha 1 R n 1.

What will be the second term? Alpha 2
[Conversation between Student and Professor – Not audible ((00:29:59 min))] R n (0); what will be the third term? Alpha 3 R n (1) plus so on. What is going to be the last term? No, this is not alpha......... alpha p last term; coefficient is alpha p. So this is alpha p R n (p minus 2)
[Conversation between Student and Professor – Not audible ((00:30:20 min))] Symmetry is definite but you will see another interesting property of symmetry. This is....... I mean as some of you must have already noted that this is resulting in a matrix equation. so I just write down i is equal to 3 which you can now understand very easily, it is going to be alpha 1 R n (2) plus alpha

2 R n (1) plus alpha 3 R n (0) plus and so on and it will be alpha p R n this time it is going to be (p minus 3) which is going to be R n ( (3) and so on we go on and the last one will be alpha 1 R n (p minus 1) plus alpha 2 R n (p minus 2) plus alpha 3 R n (p minus 3) plus so on and the last term will be alpha p R n (0) which will be equal to R n (p) because the last expression we have to substitute i is equal to p.

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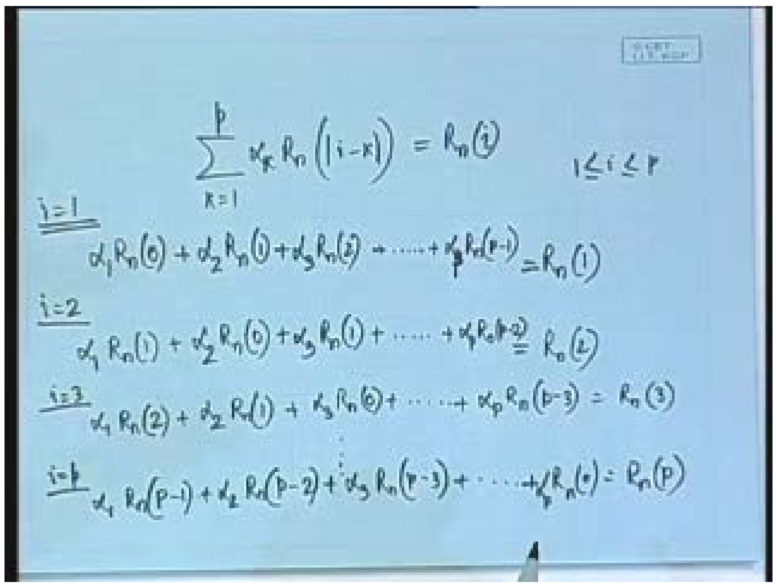

Therefore, now we have got p equation. So we have explicitly written down the p equations. So as people have already noticed that it is going to be a matrix form of representation. So now let us represent this set of p equations into matrix and what results is something like this that we can write down a matrix and this matrix will have elements R n(0) R n(1) R n(2) up to R n(p minus 1); next row will be R n(1) R n(0) R n(1) up to R n(p minus 2) R n(2) R n(1) R n(0); last term R n(p minus 3) last row is going to be R n(p minus 1) R n(p minus 2) R n(p minus 3) and so on up to R n(0) and this will be this matrix........ this is a matrix of what size? p by p; p by p, so this is a p by p matrix and the p by p matrix will be multiplied by a column vector so that column vector will have elements; what we want; so we want to solve for this alpha 1 alpha 2 alpha 3 up to alpha p's and this is available here in the form of a column vector and this will be equal to what? Another column vector and what is this column vector, what we have written down on the right hand side? So R n(1) R n(2) R n(3) up to R n(p) (Refer Slide Time: 33:55) so this will be R n(1) R n(2) sorry R n(2) R n(3) up to R n(p). So this is nothing but; same thing, whatever we had written is summation form we have written in a matrix form now. And one of the participants here have already pointed out that it results in a symmetric matrix; indeed so because you just see R n(1) R n(1) is symmetric about the diagram; R n(2) here R n(2) here R n(1) here R n(1) here.

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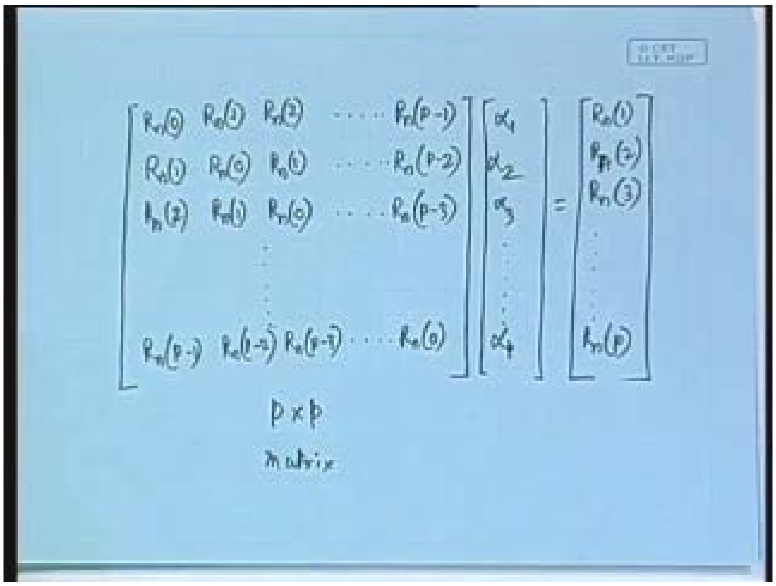


Other than the symmetry you notice yet another interesting property of this matrix. Just travel along the diagonal, what are you seeing? All the diagonal elements are the same. Travel along any diagonal; take R n(1) R n(1) R n(1) and here it ends with R n(1). instead of doing that you take R n(2) you travel along R n(2) you will find you will always find R n(2); you find along R n(1) you will always travel along R n(1) you will end up in R n(1). So the diagonal elements are the same. So it is not only symmetric but the diagonal elements are the same.

Can anybody tell here that what is this matrix known as? Anybody dealt with this matrix? [Conversation between student and Professor: (35:32)] Yes, the correct answer has already come. This is called as the Toeplitz matrix Toeplitz matrix very interesting matrix. so it is not just symmetric but it has got a Toeplitz but in Toeplitz matrix the beauty is that very nice solution techniques alternative solution techniques are available already for Toeplitz matrix and we can make use of that in order to solve for this alpha 1 to alpha p's because after all what we are supposed to do here is, after this you just write down in a matrix, you premultiply both this this whole matrix equation by the inverse of this matrix, so what you get? you get alpha 1 to alpha p as a vector on the left hand side and on the right hand side you get inverse of this matrix times this.

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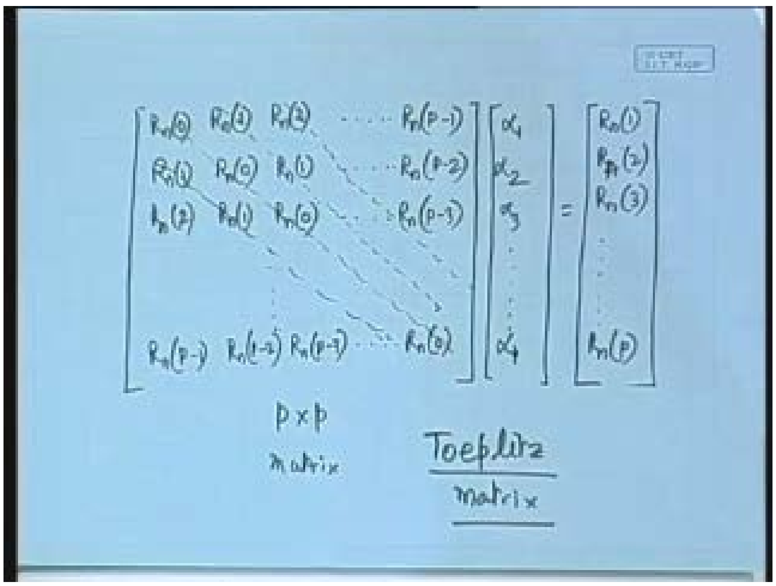


Now correlation the autocorrelation values, you can compute. You just look at the speech samples; you compute the autocorrelation value that is not a problem. Everything here is known; all the elements are also known; elements of this matrix; but only thing is that we want a good technique for carrying out the inversion. And because of this Toeplitz matrix property this will really help us.

Now the solution of this equation using the Toeplitz matrix solution approach we will show shortly may not be in this class. But in this class what I want to do further is that we will now explore the second technique of solution. One approach is this, this approach is called as the autocorrelation method and now I will introduce you to the second approach which we will be describing as the covariance method. So we will now treat the covariance method and then I promise to come back again to solve for this Toeplitz matrix part in order to make it based on the autocorrelation approach.

So now I will deal with the covariance method. The covariance method of solution, essentially it does almost the same thing but you will be seeing that there the definition of the error is little different that is why the index of the summation looks different.

Did you notice one thing that when we are talking about the autocorrelation approach; where did we talk of? We talked off that during our pitch deduction. Now there we had said that whenever you were taking the short time segment the problem happens that whenever you are giving some lags to it the overlap period is becoming less that is why ideally we wanted that if we could have a have an autocorrelation with an extended sequence multiplied by the original windowed sequence or rather convert with the original sequence in that case essentially it is not the autocorrelation any more but as if to say it is the covariance with the extended sequence. So we call we that time called that approach as the modified autocorrelation based approach for the pitch detection.

Now we are going to do something very similar over here. What we are going to do is that we will now define our e(n) that is to say the error to be the summation m is equal t 0 to n minus 1 of e squared n(m). Now this is very interesting. That means to say that, we want that, for all this n samples the error signals must be computable. This is this is different from our error definition that we had used last time. Just have a look at what definition we had given earlier.

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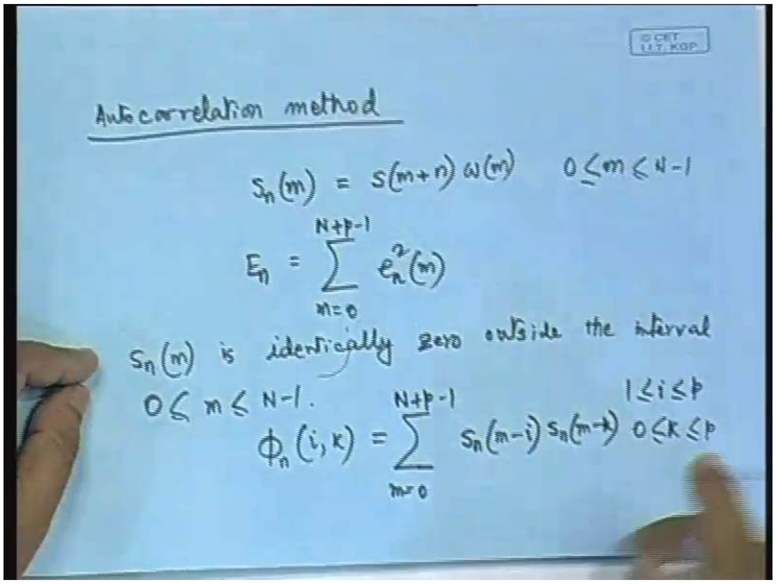


For the autocorrelation method we had given the definition as E(n) equal to E n square (m) but m is equal to 0 to n plus p minus 1 whereas in this case we are giving m is equal to 0 to n minus 1. So just note that the limit is different. But the limit difference makes a lot of difference in this expression. Although deceptively it will appear that as if to say that we are redoing the same thing.

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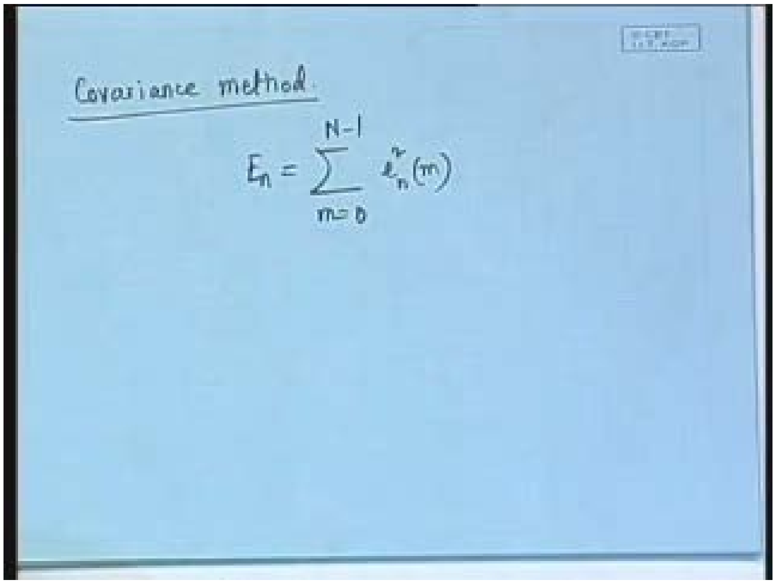


Now if we have this then our phi n of (i, k) phi n of (i, k) now becomes the summation m is equal to 0 to n minus 1 S n (m minus 1) into S n (m minus k) and this what we have to do for i is equal to 1 to i lying between 1 to p; k lying between 0 and p. And if we change the index of summation in that case we can write down phi n (i, k) to be equal to the summation m is equal to minus i to (n minus i minus 1) S n(m) into S n(m) plus i minus k i 1 to p k 0 to p or we can also write down as phi n of (i, k) which was equal to summation m is equal to minus k to n minus k minus p into S n(m) S n(m plus k minus i); again i between 1 to p k between 0 to p.

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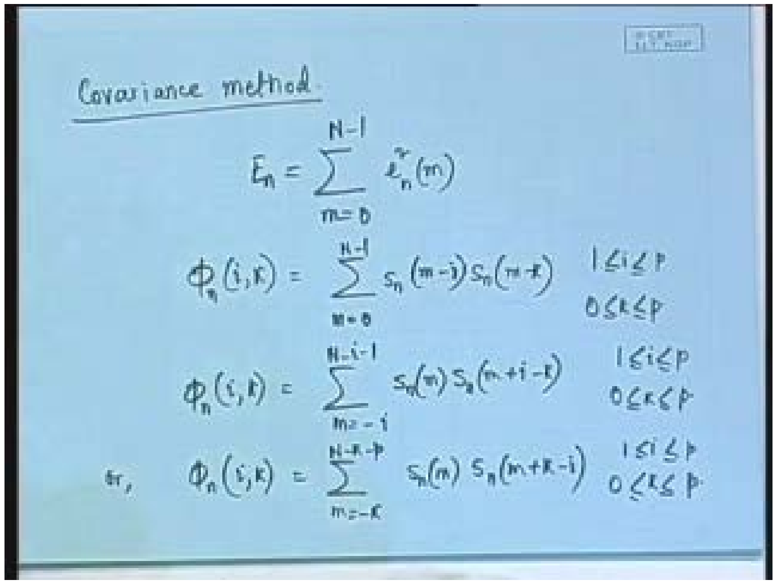


Now again you just see that last time we had a similar definition. But what did we say m is equal to 0 to (n minus i minus 1) and why did we do that because we had assumed that S n(m) is 0 outside the interval. We had said that S n(m) is zero outside the interval m varying from 0 to n minus 1.

Now if we have to compute our phi n (i, k) by this expression where we want the limits of summation to go from 0 to n minus 1 then in order to solve in order to have this phi n of (i, k) we must have an.......... I mean, this is equivalently an approach like this that is what we have to follow and in that case we cannot want we we cannot have the S n(m) values to be zeros outside this interval. Either before that either before zero you make the samples up to index minus i or minus k you make them as zeros or otherwise you just extend it up to n minus 1 so that you really get the product or the correlation up to that state. So this basically calls for the extension of the sequence because now we need values of S n(m) in the interval p to n minus 1. So we need we need S n(m) in the interval m between p and n minus 1.

now if So by having this limits of e(n) now we can write down the equation that is to say our original equation involving the alpha k's and the summation of product terms which we are calling as the autocorrelation term. So alpha k's times the autocorrelation terms equated to another autocorrelation expression, the same thing remains. But now in this case we are going to write it as the summation the set of set of equations now become k is equal to 1 to p alpha k phi n (i, k) which is equal to phi n (i, 0) for i is equal to 1 to p and now we can have........... again it is a set of p equations but looks very similar. But in this case we will not be able to equate it to the R n terms but rather we have to keep it to the phi n terms and the kind of matrix that results is like this. So this will be phi n(1, 1) phi n (1, 2), I mean, you just expand this equation I am not writing or explaining because this is quite obvious phi n(1, p) that means to say all the first row elements then I write the second row elements phi n(2, 1) phi n(2, 2) up to phi n(2, p) and the last one I am writing directly phi n(p, 1) the pth row phi n(p, 2) and it ends with phi n(p, p) so this is also a p by p matrix what I wrote and here (Refer Slide Time: 46:57) as usual it will be alpha 1 up to alpha p the column vector and then we have to write here again this phi n it will start with (1, 0) in the first row it will be phi n(2, 0) and last will be phi n(p, 0) so this is the matrix equation that we have.

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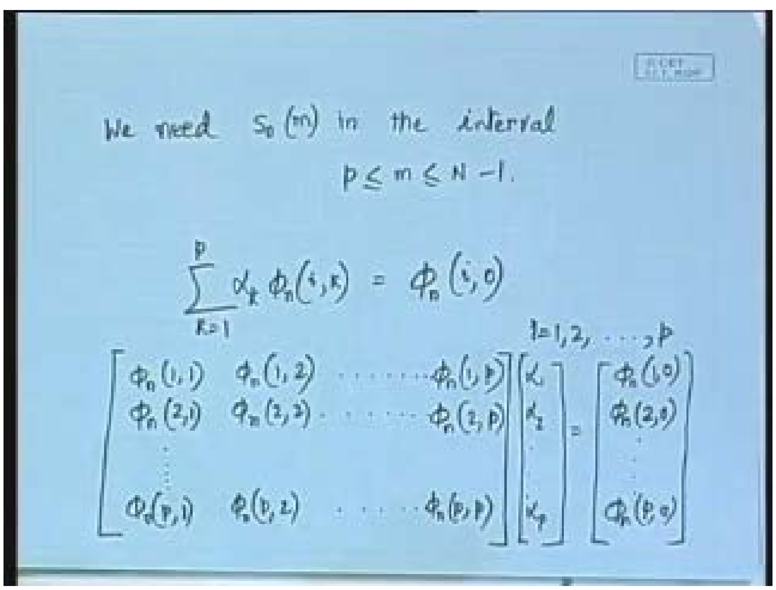


Now just can you tell me that can any property result out of this matrix? Is it symmetric? Yes or no?

it is symmetric because phi (i, k) phi n (i, k) is same as that of phi n (k, i) it does not matter, I mean by the basic definition itself it does not matter whether I interchange this order of (i and k); instead of (i and k) if I write (k and i) it is going to remain the same. So phi n(1, 2) is going to be phi n(2, 1) so it is symmetric about the diagonal; only thing is that it is not Toeplitz but it is a symmetric matrix. There should be good techniques for its inversion also; may not be like the Toeplitz matrix but even inversion of this should be possible and in fact we first show you the solution of this, may be in this class if I do not have the time I will continue with the approach in the next one; I will just basically introduce the solution approach.

In fact this equation what I have written down, I can write it down in the form of a matrix equation or a matrix equation; for that I just use a different color pen let me use a red color pen to give you the matrix equation.

So this is phi, phi times alpha is equal to psi. This is the matrix equation representation of what we have written down over here. Now only thing to notice is that here what I write by phi is a phi matrix; alpha this red alpha is nothing but the alpha as the column vector so I should better write this as alpha vector and this one, this is another column vector this phi n (0, 0) so this also is a vector, so this is a vector, this is a vector and by the matrix notation I should also write down this as...... and this a matrix not a vector exactly but phi alpha is equal to psi this is the basic matrix equation which we have to solve. Our objective will be now to solve this alpha.

But to solve this alpha we have to make a manageable form of this phi. Phi as I said that phi is a symmetric matrix, that property is known and in fact phi this phi matrix will be represented in this form that it will be a V matrix multiplied by the D matrix multiplied by V transpose. So I was actually dictated by one of the participants so you must have known this approach very easily. So let me ask that what is that approach? This V, what is what is what is D first? [Conversation between Student and Professor – Not audible ((00:50:54 min))] D is a diagonal matrix and what is V? V is triangular matrix; no, so V is a triangular matrix so this is the upper triangular matrix that is what we have to do. So now what results basically is that this will be referred to as the square root approach. I mean, if I express the phi matrix in this form then that method is called.......... this decomposition form is known as the Cholesky decomposition. So this is referred to as Cholesky decomposition solution of the covariance method or this is also known as the square root method. This is popularly known as the square root method. Sorry, V is actually the lower triangular matrix so V is the lower triangular matrix and in fact this is a lower triangular matrix whose diagonal elements are all 1s, so here the diagonal element are all 1s so we will be making use of this phi is equal to this in order to solve this basic equation.

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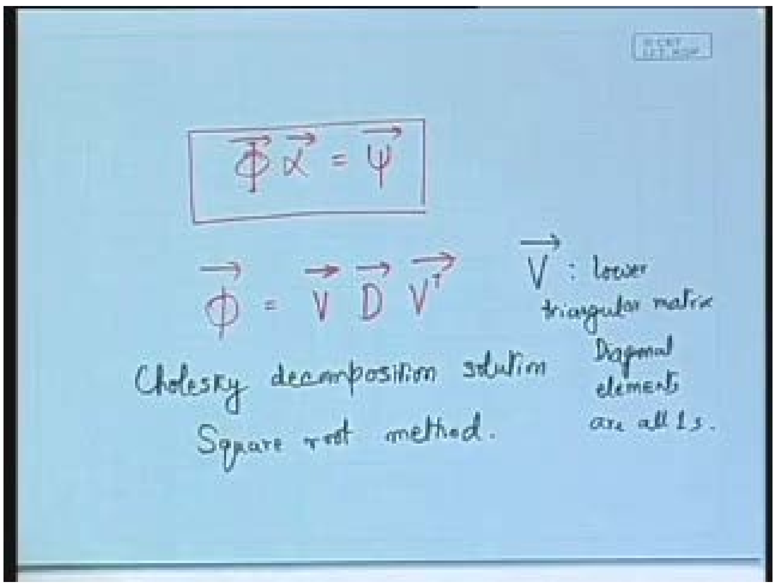


In fact, before we approach a solution of this first let us try to express this phi in this manageable form. So, if we can somehow solve for the elements of this V matrix and the D matrix that should help us because from using this V matrix and D matrix the solution of this alpha vector would become much easier. So, to solve for the V matrix and D matrix elements what we will the approach that we will be taking is that we will equate the (i, j)th element of this phi matrix with the (i, j)th element of this product matrix and if we just equate the (i, j)th element then the (i, j)th element equation would look like: phi n (i, j) mind you, now there is no more of vector nor matrix notation, now I am back again to the scalar.

So phi n (i, j) is equal to summation k is equal to 1 to j V ik into d k that is the diagonal matrix element into V jk and this j lies between 1 to (i minus 1). So this is the (i, j)th element of the phi matrix being equated to the corresponding part of the product matrix that is VD, V transpose matrix what we have written down it is equivalent form (i, j)th element is being written here and that will give us a starting point to solve for the elements of this V matrix and the elements of the D matrix.

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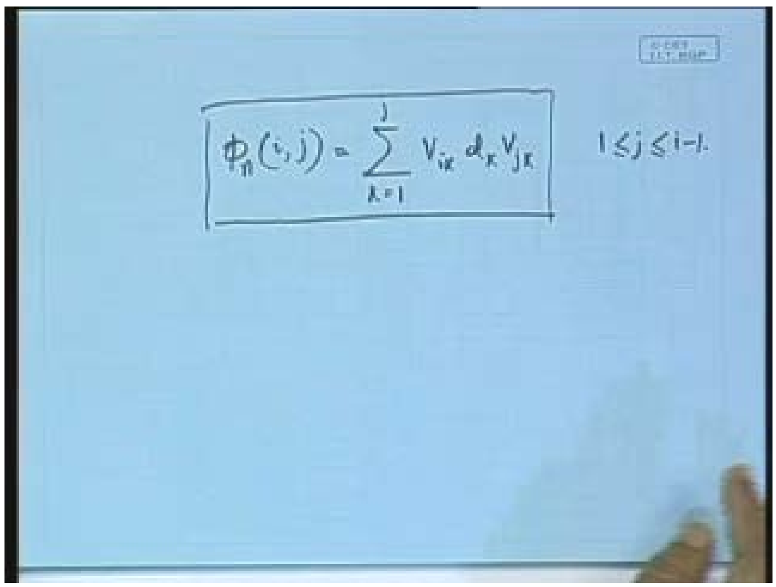


Once we solve for the elements of the V matrix and the D matrix, rest of the solution for alpha would become pretty straightforward. This is what we will be showing you in the next class and also this will be followed by the methodology for autocorrelation method solution; means where we have to do the inversion of the Toeplitz matrix. Thank you for today.