Digital Systems Design
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Lecture - 09
Boolean Function Minimization

We will continue the discussion on Boolean Function Minimization. Last day we introduced the concept of Karnaugh map which is nothing but, a graphically representation of the Boolean expression. And using this Karnaugh map how we can reduce the Boolean expression we can minimize the Boolean function we are discussing that, so today, we start that Karnaugh map simplifications.

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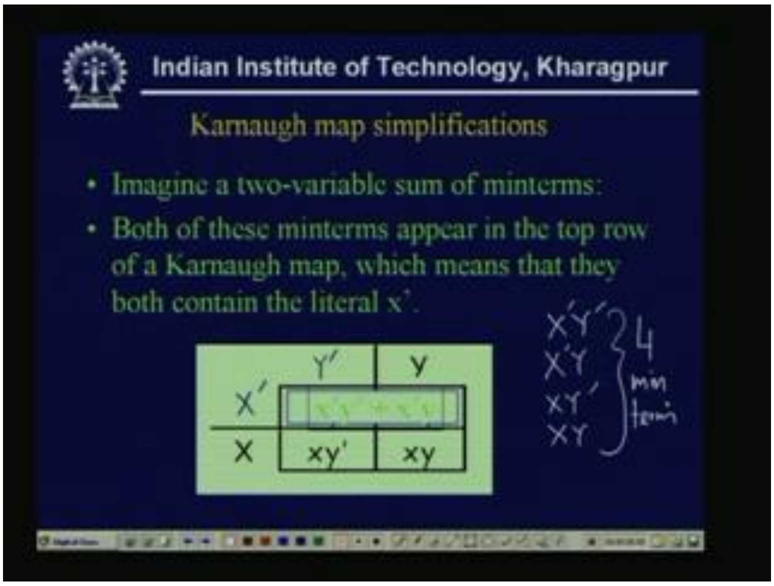

Now, first we imagine a two variable some of minterms, see this is one example that here x and y are the two variables. Now, this is the Karnaugh map of this now here, see that the x both of this minterms appear in the top row of a Karnaugh map, which means that they both contain literal x dash means, the x dash y dash plus x dash y. Now, here see this is the left hand side the variable is x that means, the this 1 is denoted by x dash.

So, the first row for the first row the literal represents is x dash similarly, the first column the literal is y dash, so the for the first element of the first row represents, x dash y dash. So, the minterms are the first 1 is x dash y dash, now the second element of the first row is x dash y. Now, we consider the second row, so the second row for the second row the literal is x, so this is x y dash represents the first element of the second row and similarly, the second element of the second row is x y where the second column is represents the y the second row represents x. So, these are the four these are the four minterms. Now, the first row is x dash y dash plus x dash y.

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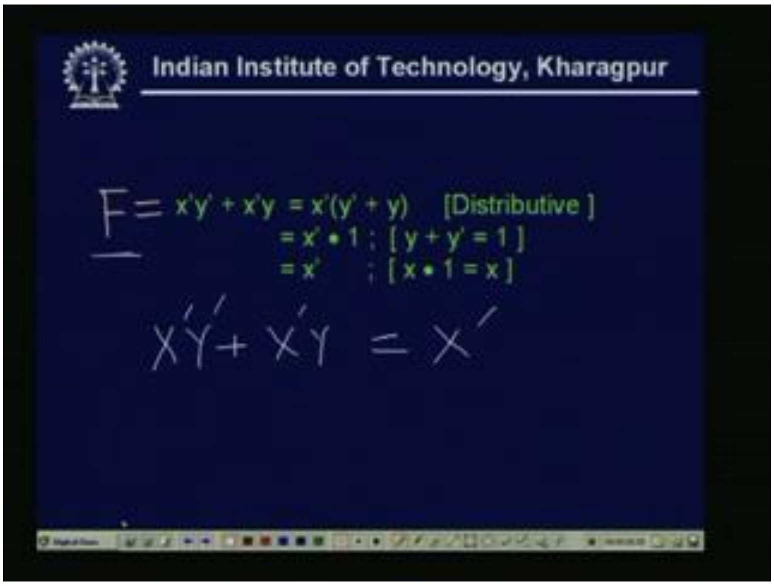

So, this is my function see the function is the say, F equal to x dash y dash plus x dash y, so this is my function F. Now, if I take common that x dash this is y dash plus y now we know that, y dash plus y equal to 1 means this is from the theorem, then x dash dot 1 and that is x dash, so the expression becomes x dash. So, the function was function was x dash x dash y dash plus x dash y and that becomes x dash, now if we see that ((Refer Time: 05:05)) from the Karnaugh map, how we can tell that this is x dash.

See for the first row for the first row the literal represents is x dash, now the function is function consists of two minterms and that is the first element of the first row, and the second element of the first row and this is a 2 by 2 table. So, both the minterms of the first row exist in the function, so, if we see the both the elements that means, my function F equal to x dash y dash plus x dash y this is the first row, now as it is the whole row that means, the other literals y its complemented as well as uncomplemented literal, both exist in the function see y dash and y.

So, they will nullify and only x dash will be there, so from the table I can easily see by seeing the thing I can tell that it will be reduced to x dash that means, if I take the first row then as y y dash both exist in the function or the minterms of the function then only x dash will be only x dash will be there. So, what we have or what we have received or what we had just now, computed by applying the theorems and the postulates that x dash y dash plus x dash y equal to x dash we can easily get that thing from the Karnaugh map also.

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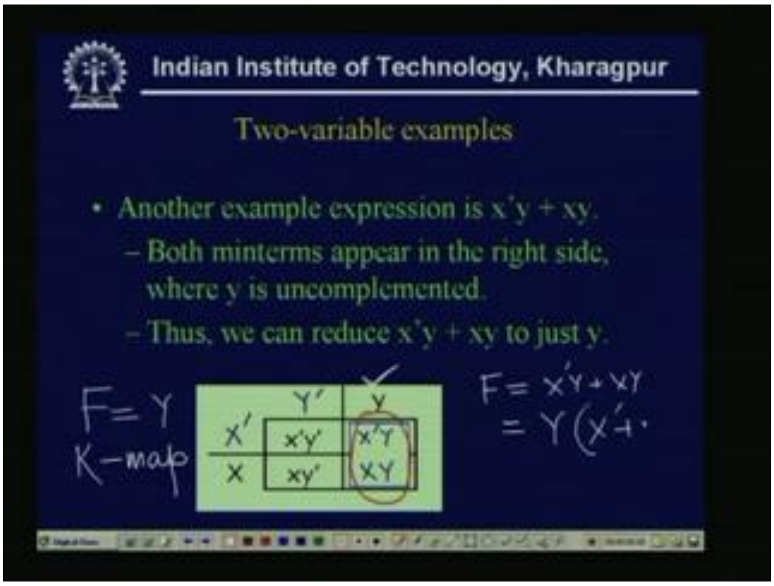

Now, take another two variables examples, say this is x dash y plus x y, so again if we mark the thing say again that first row is the literal x dash first column is y dash, so the this is a for 2 variable this map is a 2 by 2, 2 by 2 table or 2 by 2 map. Then the first element is x dash y dash obviously, the second element of the first row x dash, so this is my x dash y and second element of the second row this is x y this is x y.

So, if my function constitutes of these two terms x dash y and x y that means, my function F equal to x dash y plus x y then from the table what it comes, see now from the table that actually the 2 terms exist in the second column where the literal is literal is y, so this is my y. And the this two minterm consists complemented as well as uncomplemented one that means, they will nullify, so only y will be there in the F. So, it should be it should be y that means. my F should be F should by y from the table or from the k map Karnaugh map.

Now, see if I apply my postulates and theorems then if I take common as y is common here, so this is will be x dash plus x and we know x dash plus x is nothing, but 1, so this will be y dot 1 equal to y. So, form the applying the theorems or postulates we are also getting F equal to y and here also from k map, easily I can see that it is F equal to y, so x dash y plus xy reduced to y.

Now, what about that x dash y dash plus x dash y plus x y. Now, I take another example, where that table consists of this is again x and this is my y dash.

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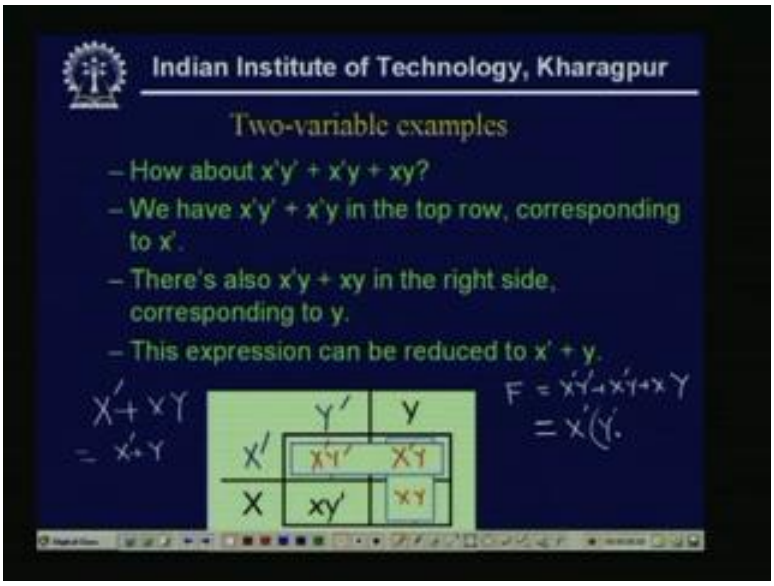

Now, my function consists of my function consists of, F equal to x dash y plus x dash y dash plus x dash y plus x y, now what are the terms from the in the map where it will be. So, we have x dash y dash plus x dash y in the top row corresponding to x dash, so called this is corresponding to x dash, so if I write this is my x dash y dash this is my x dash y and another term remains that is x y, so the another term is the second element of the second row and that is x y.

See the first second row corresponds to literal x and the second column corresponds to literal y, so the second element of the second row will be x y. So, this will be my now how it will be reduced, say first from the table if I reduce from the table, say first we see if it is from table. So, if it is from table if I consider only the first row, so all ready we have seen that from the first row what will be the reduced function it will be simply x dash, now, another variable is there that is x y that is x y.

Now, x dash plus x y is nothing, but x dash plus y from the theorem now if from the from the theorems and postulates, if I take the x dash common then it is y dash plus y plus x y. So, this is nothing x dash, because y dash plus y is 1, so this is x dash plus x y and this is nothing, but x dash plus y. So, simply by seeing the by noticing the Karnaugh map I can easily reduce the function, now we see the three variable Karnaugh map.

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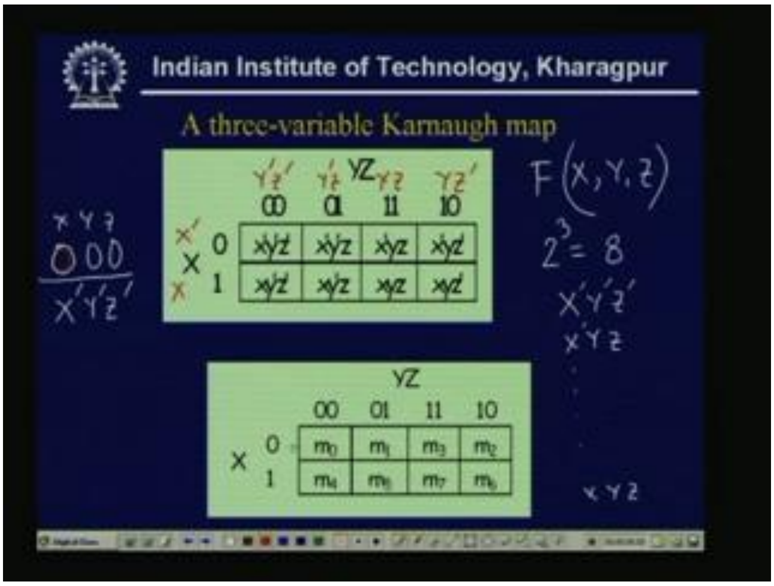

See that x y z the function is see the function is of three variable and we write in this way that is the function F of three variable x y z. Now, as it is a three variable, so what will be number of minterms what will be the number of minterms, the minterms will be 2 to the power 3 equal to 8 minterms. So, already we have discussed this will be x dash y dash z dash then x x dash y dash z like all eight terms will be there x y z.

So, first thing my table should contain the Karnaugh map should contain eight rooms or eight elements. So now, I draw the map say the row represents only 1 variable x and that is 0 and 1. Now, the column represents column represents the 2 two other variables y z. Now, for 2 variables the there will be four possible 2 square means 4 possible combinations or what the y z value can take, y z can be 0 0 y can be 0 z can be 1, so this is 0 1 y can be 1 z can be 1 1, 1 and y can be 1 z can be 0.

So, in this order 0 0 0 1 one 1 0 and this is the convention normally we take and this is 0 1. Now, if it is 0 0 0 I think we can remember our minterm table and the sum of minterm tables, where we have represented that if it is 0 0 0 x y z value then we represent this thing as this will be complemented 1 x dash y dash z dash. So, the as x is 0 this is x dash or we can we can write also, that this will be this will be x dash and this is actually x. Similarly, this 0 0 means this is y dash z dash this is y dash z, because the from the sum of minterm tables we know that if it is the value is 0 that means, it is a complemented 1 and if it is 1 then it means it is a uncomplemented.

So, y z and this is y z dash, now we take the what will be the element, this is the first row when we are we are considering that means, x dash remain same. So, for the all the elements of the first row that minterm in every minterm x dash must be there, so x dash see the first element x dash x dash x dash should be there. Now, the what the column is there that I will add so that means, for the first column y dash z dash. So, it is x dash y dash z dash second column it is y dash z, so x dash y dash z third column it is y z, so it is x dash y z fourth column it is y z dash, so it is x dash y z dash.

For the second row it is x, so for the whole second row all the elements of the second row that the x literal must exist and similarly, it will be x y dash z dash x y dash z x y z x y z dash. So, now if I remember the minterm notation so that means, the first element is actually 0 0 0 means it is the m 0 similarly, it is 0 0 1 means m 1 it is 0 1 1 means m 3 it is 0 1 0 means m 2.

For the second row it is 1 0 0 means 4, 1 0 1 means 5, 1 1 1 means m 7, 1 1 0 means m 6, so this is my k map picture. Now, with this ordering, so we have seen that there are some ordering there in the map.

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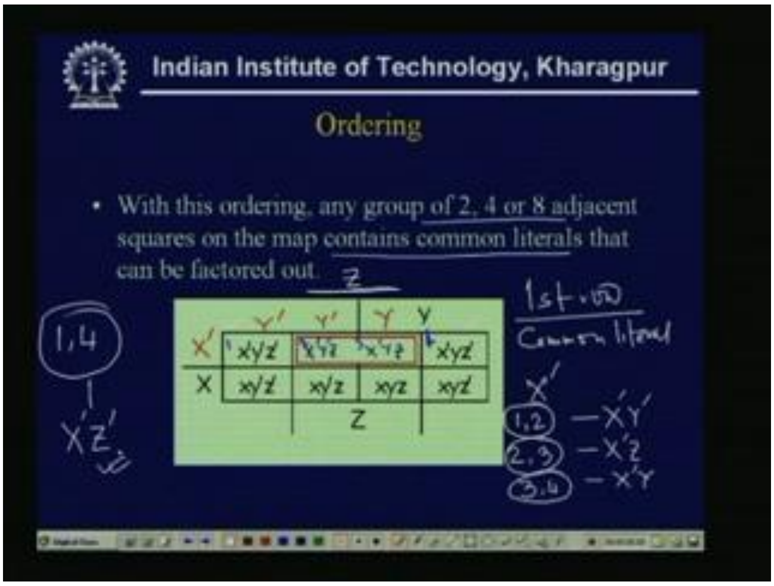

Any group of 2, 4 or 8 adjacent squares on the map contains common literals that can be factored out, see if I consider say this is my this is my x dash this is my x dash and these are my see here this is y common this is y is common here and here y dash is common. So, now see that if I consider this middle two adjacent minterms then it will be it will be x dash y dash z and it is x dash y z.

So, see for these actually z remains constant; that means, for these 2 element for these 2 element z remains constant x dash constant and z is also constant whereas, for these 2 element that x dash y z and x dash y z dash x dash remains constant and y remains constant. Similarly, the first 2 terms if I see x dash is there because all are in first row and y dash remain same, so that means, what just now we mentioned that with these group of 2, 4 or 8, 2, 4 or 8 adjacent 6 squares the it must contains some common literals.

So, for the first row, so for the first row the common literal common literal is x dash, and if I take a group then if I if I make this as a the first element 1 this is my first element say this is my 1 say this is my 2, 3 and 4. Then what we can tell for 1 2 that means, if I to take this these as a group 1 2 then x dash is for all the elements all the minterms x dash will be there and then y dash will be there. Similarly, if it is 2 three then x dash is at it is there and z is there common similarly for three four three four group x dash and y is the common literal.

Now, see that if I take 1 and four if I take 1 and four as a group means this 1 1 and this is 4, then x dash is as it is common and z dash is also common, so these are the some common minterms I am getting, so easily it can be factored. That means from the common if common literals exist 2 minterms the then the some the common literals I can take out from the both the minterms, so it can be factored, now we see that how we can do this grouping.

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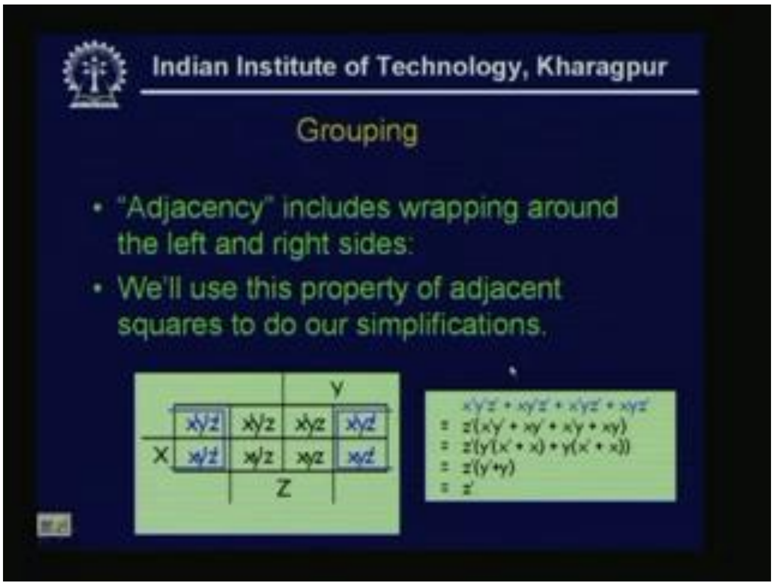

So, keeping this thing in mind that the adjacent just now what we have seen the adjacent 2 minterms always contains some common literal that can be 1 that can be more than 1. So, keeping this thing in mind now we will group 2 or more than 2 minterms how, so we call that adjacency because we are considering only the adjacent minterms, so adjacency includes wrapping around the left and right sides.

Just this is one example that means, as if we are considering the extreme right hand side minterms they are actually adjacent to the extreme left hand side minterms means as if they are wrapped around. That though the table is kept as a flat thing, but if we fold this thing if we fold the table in a cylindrical manner then as if the left hand column the rightmost column and the leftmost column they are adjacent.

So, we can group them together or we can treat them as a adjacent 1, so we will use this property of adjacent squares to do our simplifications. So, say for this particular example, if I take the this 4 as the adjacent groups and we see whether the common what are the common literals or whether we can reduce that thing or not that means, say I have 1 function that x dash y dash z dash the function consists of…

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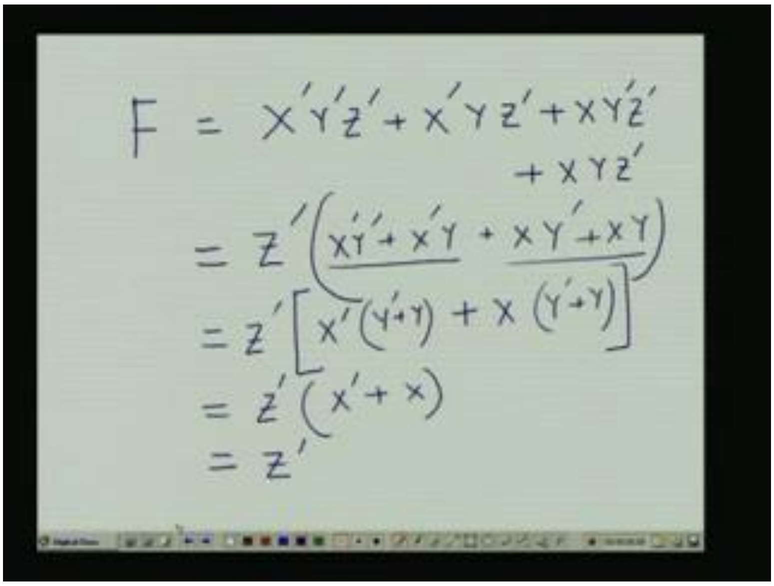

So, I have 1 function if that consists of x dash y dash z dash plus, the last term means x dash y z dash plus the x y dash z dash plus x y z dash see again that thing. So, this is x dash, y dash, z dash, x dash, y z dash similarly this is x y dash, z dash, because the for the second row x is the common literal and the last term will be x y z dash, now how we can reduce that thing, so how we can reduce.

See first what we are seeing in all the terms z dash is common, so I take z dash common then this is x dash y dash plus x dash y plus x y dash plus x y, now see whether I can reduce more see the in the first 2 terms x dash is common, so take common y dash plus y the last 2 terms x is common, so this is y dash plus y. Now, we know y dash plus y is one. So, this become z dash x dash plus x, so this becomes only z dash see, so the this four minterms reduces to four minterms reduces to z dash only, so this four minterms reduces 2 only z dash.

Now, what we can tell that means, if we can group four adjacent cells if we can group four adjacent cell then it always reduced to only 1 variable if the function is a three variable function just now what we have seen. So, again we are taking some example of k map simplification.

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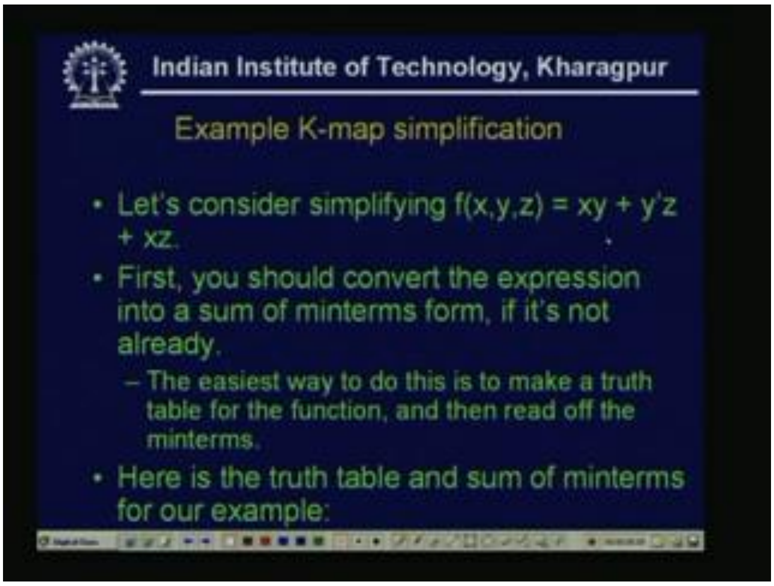

So, this is again a three variable function and the function is x y plus y dash z plus x z, so first we should convert the expression into a sum of minterms forms if it is not all ready. The easiest way to do this is to make a truth table for the function and then read off the minterms, so see how we can convert that thing, so first we will form the read table truth table see this is the truth table for the function given.

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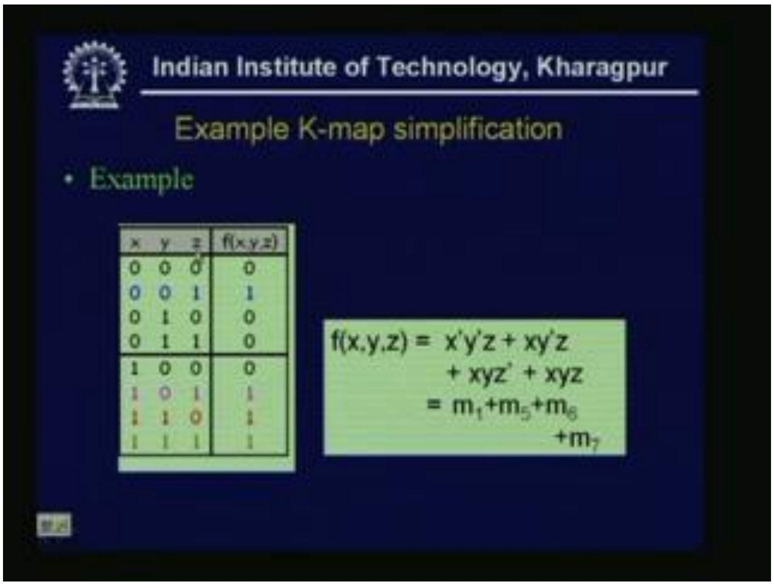

So, as this is a three variable function, so 2 to the power 3 or 8 combinations of a variables are possible all 0 0 0 to 1 1 1, now what will be the function, the function will be x dash y dash z, so x dash y dash z for x dash y dash z the value is 1. See this is the function is x dash y dash z, so for x dash 0 0 1 it is 1. X y dash z means 1 0 1 see 1 0 one; that means, for m five minterm that is 1 then, x y z dash means 11 0 6, so m 6 term is 1, 1 1 1 means m 7 that it is 1, so that means, my in the truth table m 1, m 5, m 6, m 7 terms will be 1.

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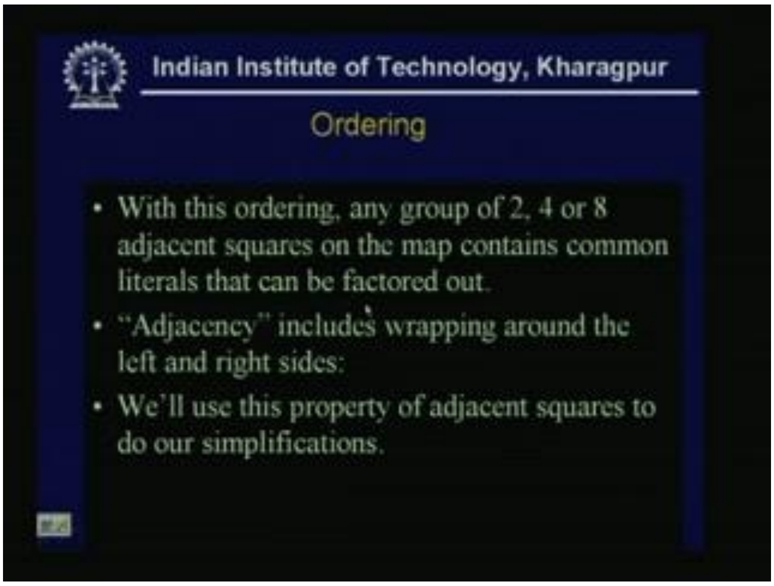

Now, with this ordering any group of 2, 4 or 8 adjacent squares on the map contains common literals that can be factored out. So, now as we know that for this minterms that means, just for this example m 1, m 5, m 6, m 7 this is 1 then what I will be doing…Now, I can draw a Karnaugh map and I will put 1 for the minterms, which exist in the function. And now we will form the adjacent or we will select the adjacent 2, 4 or 8 such minterms, now adjacency includes wrapping around and the left and right sides.

Aready we have seen that when we are considering or when we are selecting the adjacent minterms, then we will consider the wrap around also as if the table Karnaugh map can be folded. And, so that the extreme leftmost column is adjacent to the extreme rightmost column, so we will this property of adjacent squares to do our simplifications, now I see how we are doing the ordering.

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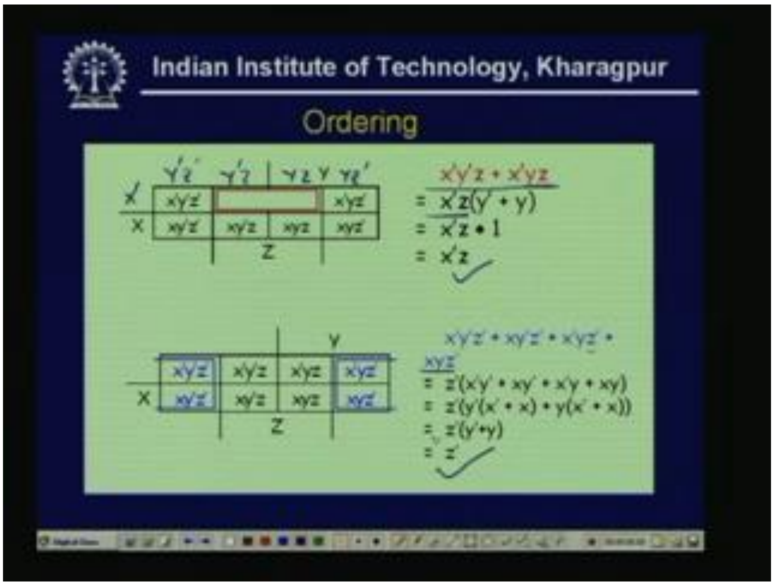

See I take one example of three variables, so I put the x dash literal here and this is for y z terms, so for this is y dash, z dash this is my 0 1 means y dash z this is y z and this is y z dash. Now, see if I take the two adjacent minterms in the first row what are this this is marked as the this red line, so this is x dash y dash z plus x dash y z this 2. Now, as they are adjacent, so there must be some common literal all ready we have seen and from here also we are seeing that the common literal is x dash z.

So, x dash z we take common and it will be y dash plus y, y dash plus y is 1, so this is x dash z, so that means, this 2 see this two minterms are reduced to x dash z. Now, all ready we have seen this example that if it is this four adjacent minterms, if I take that x dash, y dash, z dash, x dash, y dash, z x dash, x y dash, z dash means the first column and the last column and we will be seeing all ready we have seen it is minimize to z dash.

So, if it is a three variable function, if it is a three variable function the two adjacent minterms reduced to a minterm contains, two literals and for a three variable function four adjacent minterms are reduced to a term which contains only 1 literal. So, they are reducing in this way, now we see that why we why do we need the unsimplifying expressions.

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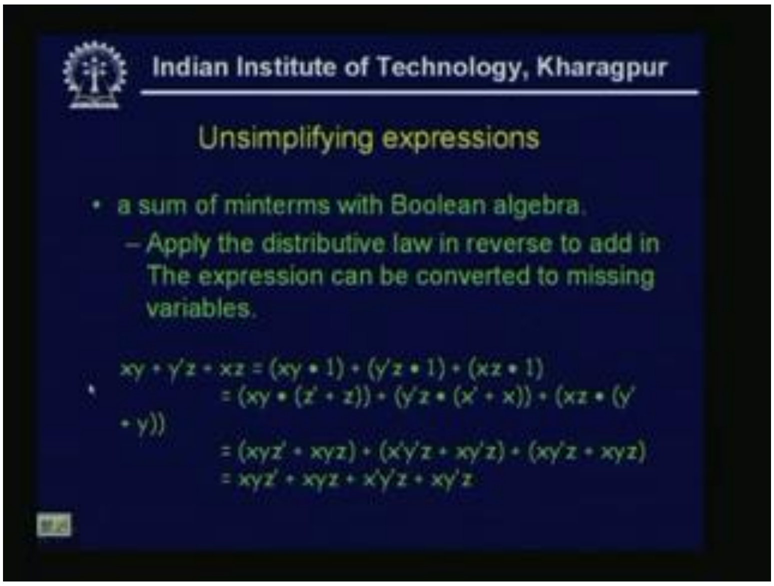

Many time when you use the Karnaugh maps we will need thing, see a sum of minterms with Boolean algebra apply the distributive law in reverse to add and the expression can be converted to missing variables. Sometimes, that even it is a three variable function, but the function contains a minterms, where two literal exist, so if the other literal does not exist what does it mean, we see what does it mean.

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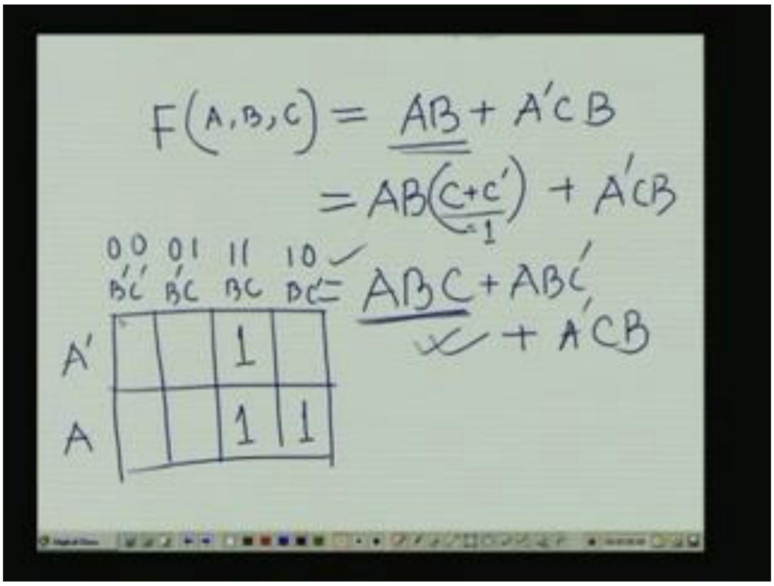

See what say one function F this is a A B C the three variable function, now it consists say A B plus A bar C B, so this two terms, now this what does it means A B, because it is a three variable function it means that one literal C that does not exist in the A B. So, I can easily introduce A B in such a way that C plus C bar, because A B means A B dot 1 and 1, I can always replace by C plus C bar and the remaining thing is there A dash C B.

So, this actually becomes A B C plus A B C bar plus A bar C B, so this is a some of product forms and where all the minterms consist the all the variables. And this is absolutely necessary when we are making the truth table, because in the truth table or in the Karnaugh map which elements will be 1. That means, if I draw a Karnaugh map now say we are we are drawing a Karnaugh map say we are drawing a Karnaugh map.

This is three variable, so see this is A dash and this is my A and this is my B dash C dash this is my B dash C B C and B C dash, so this is 0 0 0 1 1 and 1 0. So, that means, A B C see A B C exist, A B C means here this is a 1 1 A B C dash. So, A B C dash means the this is my A B C and A dash C B as A dash is there. So, it will be in the first row and C B means this, so this will be 1.

So that means, if I can represent my function in this form that means, the sum of minterms or what we have defined as a canonical forms the sum of minterms forms, where all the minterm contains all the literals then I can easily make my Karnaugh map like this. So, if I now take this example say 1 example we take that example is that x y x y plus y dash z plus x z.

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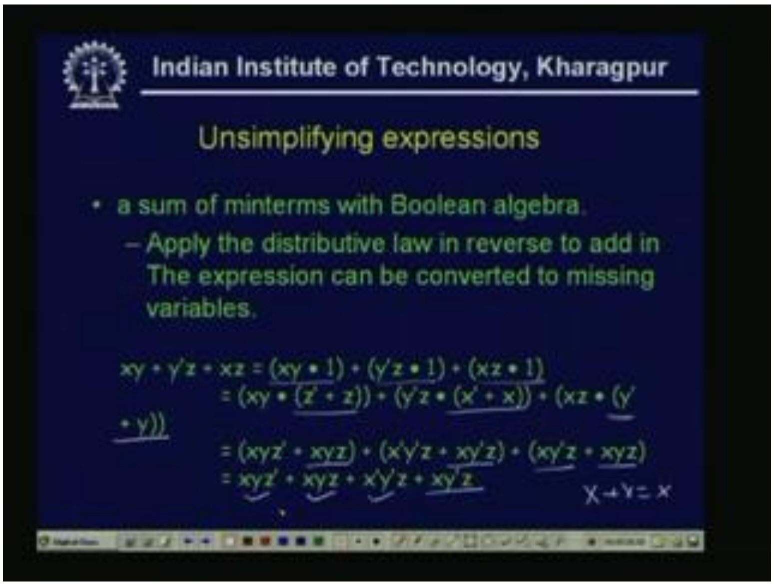

Now, how I can this is a three variable function, this is a three variable function, so if I want to make that thing that all the minterms should contains all the variables then x x y I can put x y dot 1 similarly, y dash z is y dash z 1 x z dot 1. Now, that see here the first term z is missing the literal z missing, so 1 I can replace z dash plus z similarly. In the second term x is missing, so I can introduce x dash plus x as 1 and here the 1 is replaced by y dash plus y, because that y literal was missing in the third term.

Now, if I apply the distributive law, so what will happen x y z dash x y z y dash z x dash y dash z x z y dash, now from here all x y z see there are 2 x y z, so it will be here. Then x y z dash, so x y z dash is here now, x dash y dash z this is here and x y z dash there are two terms here, so there are mapped to because, x plus x is x here x plus x is x. So, applying this rule it is simplified to this actually further we can do that simplification.

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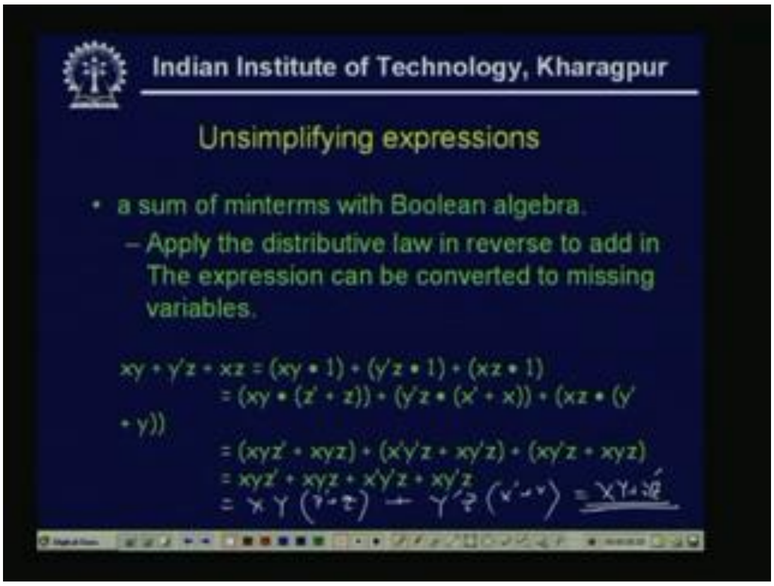

We can do the further simplification here that we can take x y from here we can take the x y common say x y then z dash plus z, here we can take y dash z is common then it can be x dash plus x. So, what will happen this becomes x y plus y dash z, so it is reduced to x y plus y dash z.

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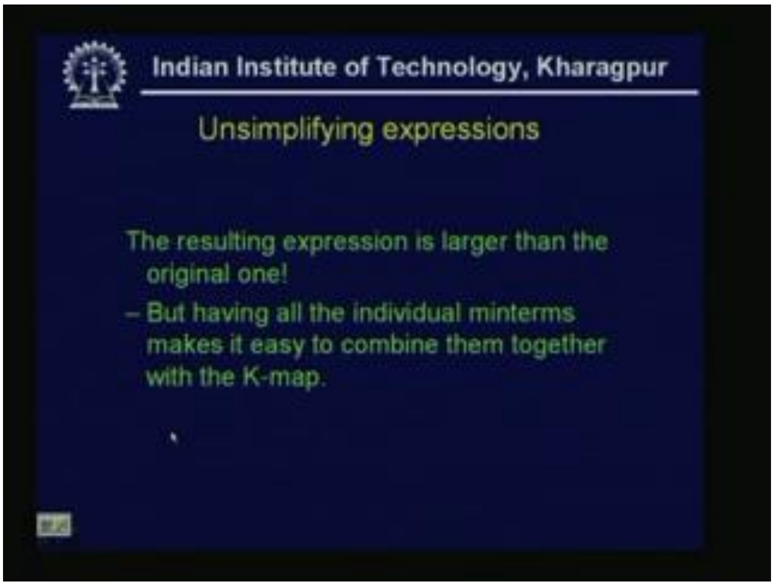

So, the resulting expression is larger than the original one sometimes we are intentionally we are doing that thing. If some literal is absent in minterm, we are introducing that minterm, so that we can easily form our Karnaugh map, but having all the individual minterms making it easy to combine them together with the k map. And this is the reason that why we are this is a reason, that why we are introducing the literals which are absent in one minterm and that is why the canonical form is very important, now the k map example.

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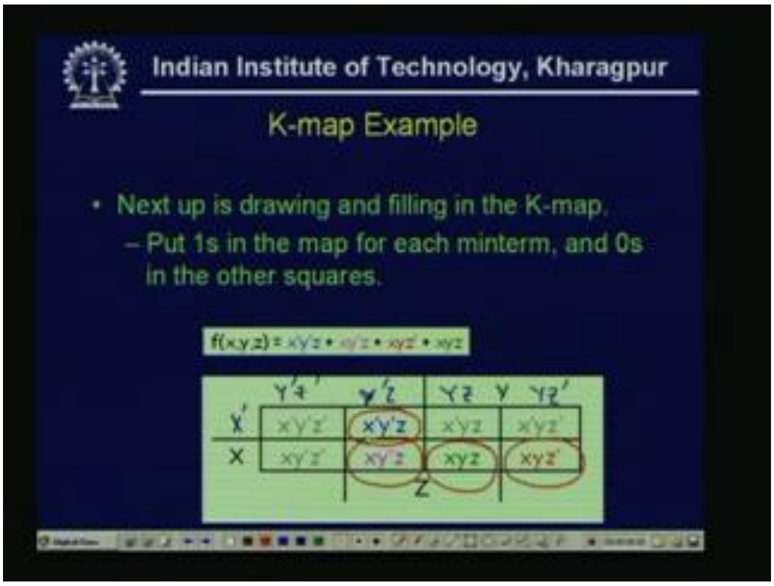

Again we are taking that three variable example, so put ones in the map for each minterm and zeros in the other squares that means, if I take a 1 function say here the function is x dash y dash z plus x y z x y dash z x y z dash plus x y z, so I will put 1, where this minterms exist. So, see that this is a x dash, so x dash and this is my y dash z dash this is my y dash z y dash z this is my y z and this is this is my y z dash.

So, x dash y dash z x dash y dash z means this is the minterm exit, x y dash z x y dash z means this is another minterm which exists in the function. X y z dash see this is x y z dash this minterm exist and x y z this is the minterm exit, so I can put 1 for them.

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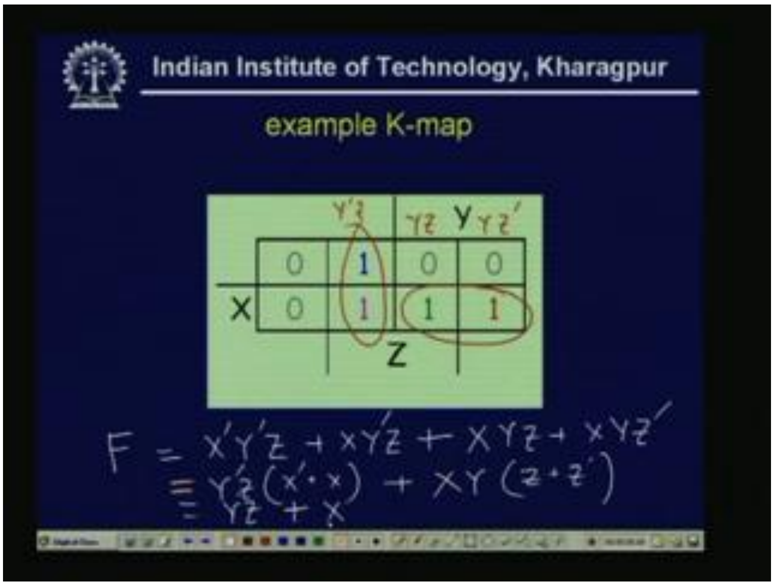

See I have I have putted this thing I have put a that this as the 1 this as 1 this 1 there are four minterms and I have put a it that thing there. So, all ready what we have seen now we, if I can take the adjacent two that means, see these two are adjacent these two are these two are adjacent and these two are adjacent cells now, easily this can be reduced. We can see now itself see for this two that what we can see that F is F equal to this is x dash y dash z plus xy dash z.

See this is the column this is the column for y dash z, so only y dash z will come here see here only y dash z will come we, if I take y dash z this is x dash plus x and this becomes only y dash z. Now, but another two terms are there, so now we see another 2 terms this is this is the in the row x and this is x y z plus x y z dash now, see for these 2 terms the literal x is common and the y is common, this is y z and this is y z dash, so this is yz dash.

So, for these two minterms y is common and as they are in the second row, so x is common, so if I take if I take x y common then again this is z plus z dash z plus z dash is 0, so this becomes x y, so the function reduces 2 y dash z plus xy. So, how we can made the group how we can make the group.

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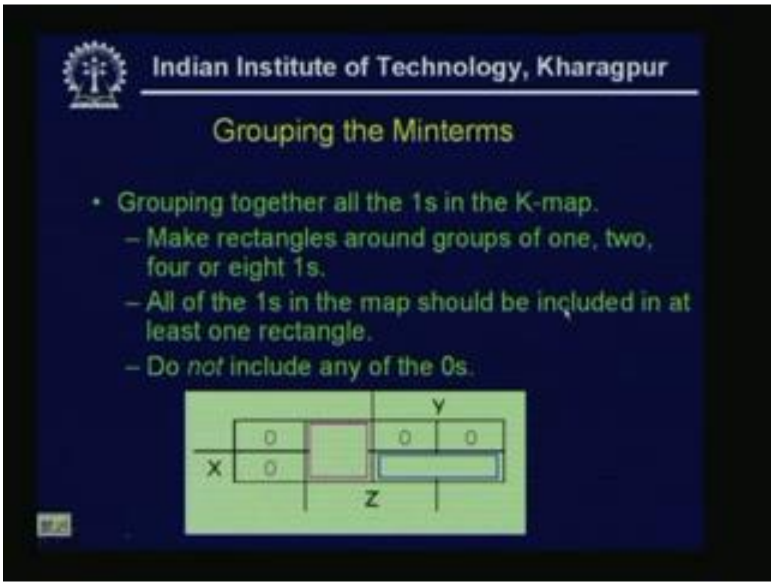

That grouping together all the ones in the k map make rectangles around groups of 1, 2, 4 or 8 1's, so, here there are only two groups I can made each consist of only 2 minterms. All of the ones in the map should be included in at least 1 rectangle, so no one can be left during the grouping and 1 1 can come in more than one group, so do not include any of the zeros, so this is the rule for my grouping the minterms.

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Now, each group corresponds to 1 product term for the simplest result, all ready we have seen that for three variable if we take or we can make a group of two minterms, two adjacent minterms then it reduced to a minterm having two literals. For a three variable function if we can make a group having four adjacent minterms, then it always reduces to a minterm consisting of single literals.

So, each group in this way each group corresponds 1, product term for the simplest result make as few rectangles as possible to minimize the number of products in the final expression. So, as the each group reduces to a minterm, so our aim will be first to reduce the number of groups that means, to reduce and reducing the number of group means always we will try to select the largest group if possible. So, we will find out or we will search for a largest group.

So, make each rectangle as large as possible to minimize the number of literals in each term, all ready we have seen if it is group of four for three variable case if it is a group of four then it is reducing to a single literal. If it is a group of two then it is reducing to a minterm, but it consists of 2 literal, so if the size of the group increases then the reduction is more, so it is all right for rectangles to overlap if that makes them larger that means, that 1 1 can appear in the more than 2 groups.

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Now, each rectangle corresponds to one product term and the product is determined by finding the common literals in that rectangle. And all ready we have seen by example that how the literals are common for one particular column or one particular row.

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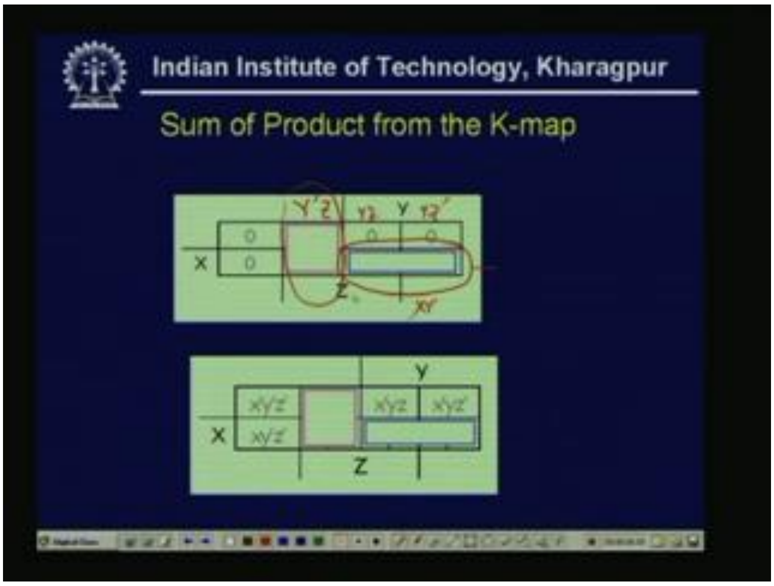

All ready we have seen that for, say for again we are repeating that thing that say for this column for this column, so this is actually for this column y dash z, so for this column it is it always this will reduce to y dash z. Say for second row for this group obviously, x will be there and for this two, which 1 will be common, because it is y z and it is y z dash so that means, y is common, so they reduce to x y.

So, already just know the 1 example we have seen that this type of function reduces to y dash z plus x y. So, only by seeing the k map and by grouping them the I can tell that if the function will reduce to this, now reducing expression of larger variables.

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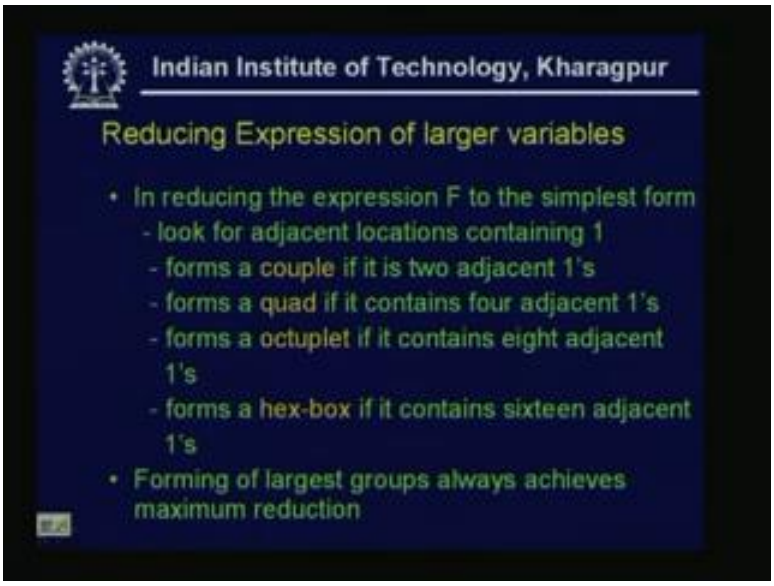

This all today we have seen that for two variables and for three variables this is the rule, now if I summarize the rule, that how to how we can reduce the expression then in reducing expression F to the simplest form first look for adjacent locations containing 1. So, I will draw the k map in the k map some of the elements are 1, which elements are 1 the minterms which exist in the function that particular minterms will be 1 in the k map, so first I look for adjacent 1, then forms a couple if it is two adjacent 1.

This couple is defined as a box or as a group having two minterms having two adjacent 1, so couple is a group having two adjacent 1 similarly, quad is a group having four adjacent 1, octuplet contains eight adjacent 1's and the hex box contains sixteen adjacent 1's. And our always our aim is to find out the largest groups if possible because it always achieves the maximum reduction, so this is the overall rule for the simplification of expression of reducing the Boolean expression using Karnaugh map.

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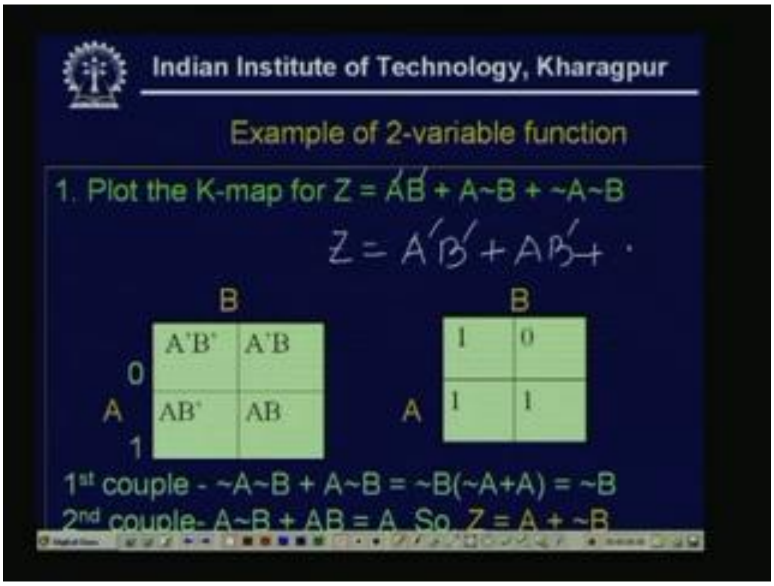

So, again if we now if we apply the rule, just now I mentioned say for two variable it is A B plus A A bar B A B bar A B bar plus A bar B bar, so what these are the Karnaugh map and the minterms exist is A B. So, the minterm will be this is my I just now the first couple will be see this will be the term exist is A bar B bar this will be A bar B bar this is A bar A bar and B bar. So, the term that I can write the function this is a function that Z equal to A bar B bar plus A B bar plus this is my AB the same example I can tell.

So, that I can AB AB plus A bar AB bar plus A bar B bar, so that particular three elements will be 1 this is my A B this is my A bar B bar this is my A B bar. So, what will be the first couple first couple means two adjacent 1, now see here if I plot then actually there will be two couple this is sorry this is one couple and this will be one couple, see here that this one that means, that A B bar that is common in both the couple, but it takes a, so what will be the first couple.

First couple will be A bar B bar plus A B bar and if I take B bar common this will be only B bar, and see, so actual the first column first column represents the first column represents the B bar first column represents the B bar. Similarly, it should be A from the Karnaugh map the second couple should be reduced to A and see the second couple is A B bar plus A B, if I take A then B bar plus B means 0.

So, this becomes A plus B bar that means it is reduces to the literal for the second row A and the literal for the first column B bar. So, it is reduced to A plus B bar now, if it is for a three variable function.

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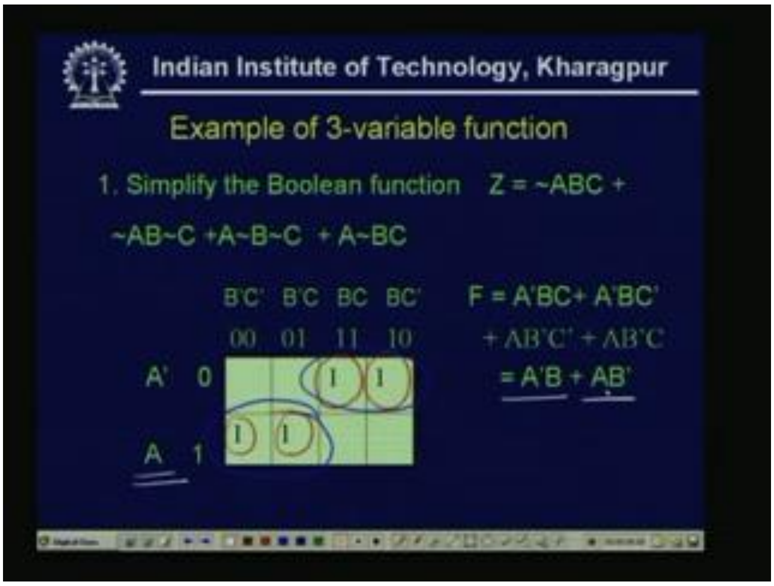

For a three variable function then simplify the Boolean function this A bar B C A bar B C bar A B bar C bar A B bar C. Now, I have to draw the Karnaugh map like that this is A bar A B bar C bar B bar C B C B C bar. So, the minterms exist A bar B C, so A bar B C means A bar B C this is one minterm, A bar B C bar A bar B C bar this is another minterm and the rest minterms are these.

Now, see if I take the couple now if I take the or if I want to form the couple then this will be a couple and this will be a couple. So, the first couple reduces to what will be the first couple reduces to first couple reduces to the first row, A bar A bar and B see here this two term B is common, so A bar B see it will be A bar B.

And the second couple A is common and B bar is common, so A B bar, so it reduces to A B bar. So, from seeing the only noticing the Karnaugh maps and making the groups of two or making the couple or this thing I can easily do that thing, we will finish that here we see that the quiz of this.

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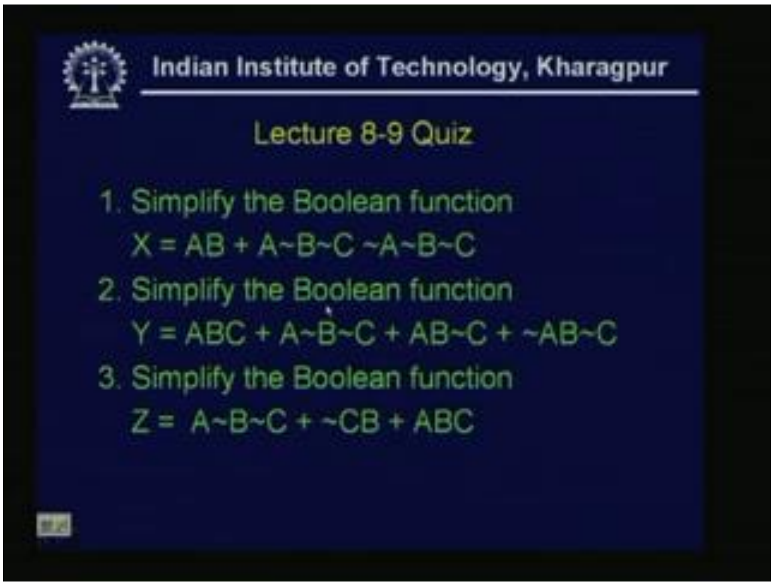

So, these are the three questions for the lecture 8 and nine quiz just there are three simplifications using k maps. The first one x equal to A B plus A B bar plus C bar plus A bar B bar C bar, second one is y equal to A B C plus A B bar C bar plus A B bar A B C bar plus A bar B C bar, and third one is A B bar C bar plus C bar B plus A B C, again we will continue this thing in the next class

Thank you.