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**Lecture - 07**
**Boolean Algebra (Contd.)**

Today, we will continue the discussion on Boolean Algebra. But before that first we see the answers of the last 2 lectures quiz.

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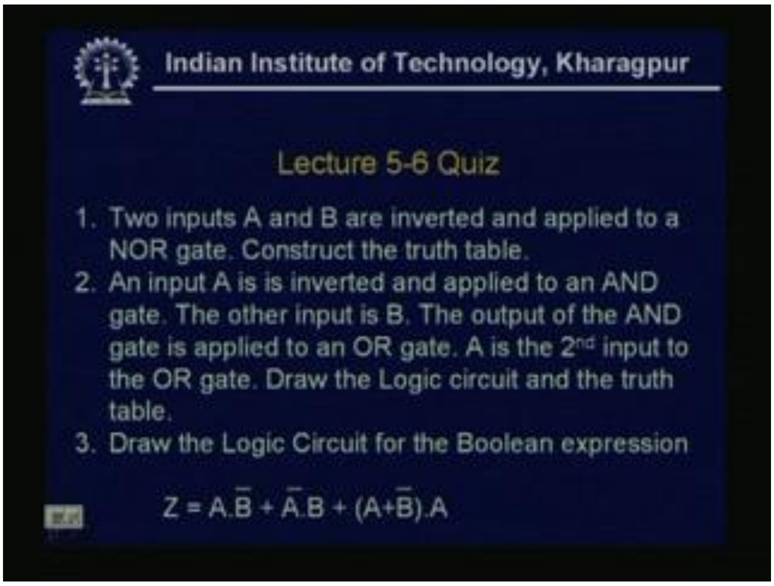


The lecture 5 and 6 quiz, so the first question was the two inputs A and B are inverted and applied to a NOR gate construct the truth table.

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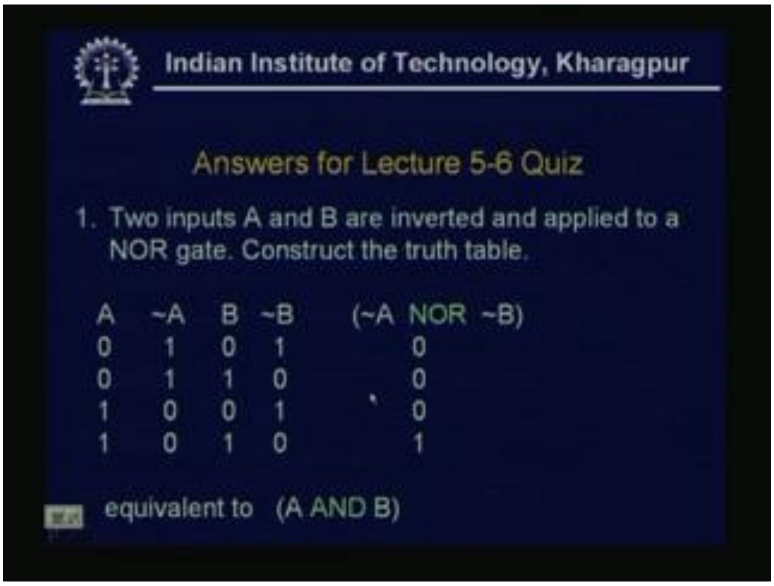


Now, A and B are inverted, so if A takes the values 0 0 1 1 this is complement of A 1 negation of A will be 1 1 0 0 it is inverted. Similarly, if B has the value 0 1 0 1 then negation B will be 1 0 1 0. Now, what will be the negation A and non-negation B, because these are the two inputs. See that, if negation A means 1 and negation B is 1 if we take OR 1 plus 1 then it will be 1 then complement of that will be 0. So, 1 plus is 1 1 complement of that becomes 0.

Similarly, 1 plus 0 it is 1 complement of that 0, 0 plus 1 1 complement of that means, NOR is 0, 0 plus 0 that is the OR of A and B negation A and negation B is 0 complement of that becomes 1. Now, if we ignore negation A negation B and see only A B and our output, then we will see that actually this is nothing, but the AND because 0 0 0 0 1 0 1 0 0 1 1 1, so this is nothing but, the A and B. So, negation A NOR negation B is equivalent to A to input AND gate, see our second question.

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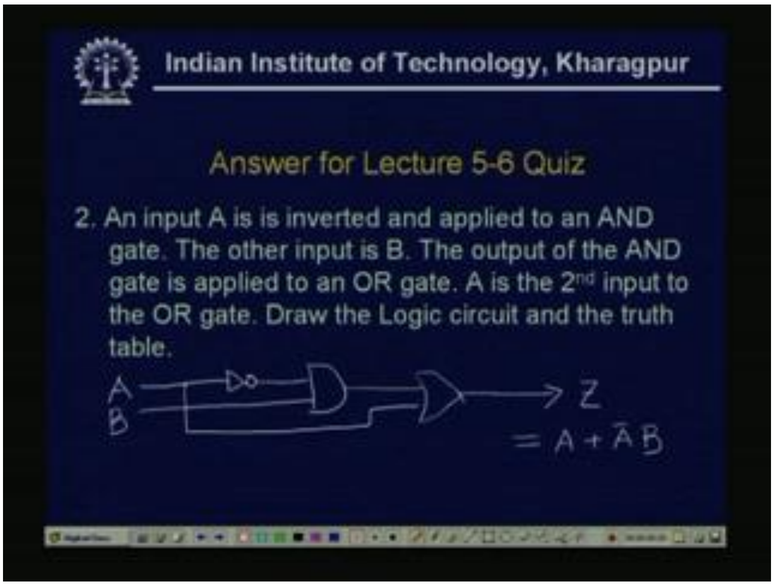


Second question that an input A is inverted and applied to an AND gate the other input is B the output of the AND gate is apply to an OR gate A is the second input to the OR gate draw the logic circuit and the truth table. So, first we see the logic circuit say input A is inverted take the input A it is inverted, so this my input A it is inverted and applied to an AND gate. So, this is my one of the AND gate. The other input is B, so this is my other input, the output of the AND gate is applied to an OR gate, so this is my OR gate.

A is the second input to the OR gate, so A is the second input to the OR gate draw the logic circuit, so this is Z the output is this now, see then what is this Z, Z is nothing, but first this is A bar B. So, this OR gate 1 input is A. So, A plus this is A bar B, so this is my logic circuit, then what will be my truth table.

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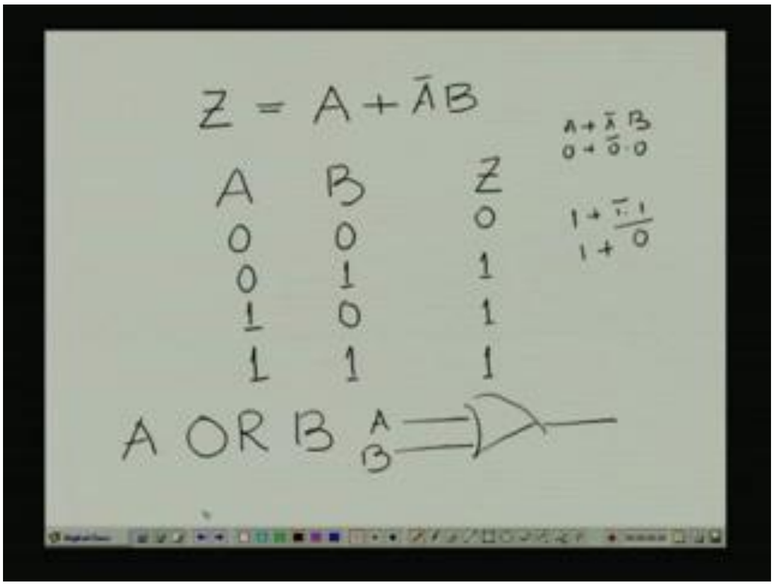


We see now, the truth table see what we have see that output Z is nothing, but A plus A bar B. So if we draw the truth table these are the my two inputs A B and this is my output Z. I take all possible values of A B 0 0 0 1 1 0 1 1 then what will be the value, say this is A bar means 0 bar, so A bar A plus A bar B, I take one example that 0 plus 0 bar dot 0 means this is 1 dot 0 0 0 plus 0. Similarly, it will be 1 this is 1 and then again if it is that means, for 1 it will be 1 plus 1 bar 1, so this becomes 0, but these becomes 1 plus 0 see this is 1.

Now, see what is actually this two truth table is 0 plus 0 0 0 plus 1 1 1 plus 0 1 1 plus, so if we consider the OR gate that means this nothing but, a OR that means, A OR B or the simple to input OR gate whose inputs are A B. So, the previous page that the circuit we have got the logic that is nothing, but that is equivalent to a two input OR gate.

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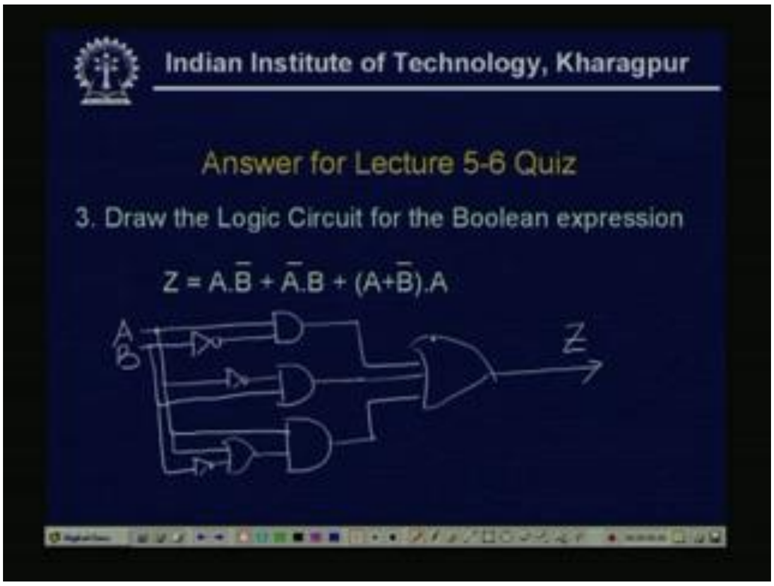


Now, our third question, was the draw the logic circuit for the Boolean expression Z equal to A B bar plus A bar B plus A plus B bar dot A. Now, if we draw the circuit then what will be the thing see first A then again there two inputs A and B. So, this is my A 1 to input AND gate now, this is B bar means B is complemented, so 1 inverter I put NOT gate this is my B, so this is A B bar. Now, A bar B I take another AND gate then this is inverted B is the another input, so this is my this AND gate is the for the second expression A bar B.

Now, another is A plus B bar dot A, so I need another AND gate whose 1 of the input is A, another is the see A plus B bar means this is the output of a two input OR. So, if I put a two input OR then this is the output, what is the input of two input OR, 1 is the A itself another is the B complement. So, I can take B complement from directly from here or I can take B and I complement that and this, so these are the three inputs.

Now, if 1 three input OR gate I draw and, so this is my logic circuit for this Boolean expression. Now, we start our today's lecture we continue the Boolean algebra.

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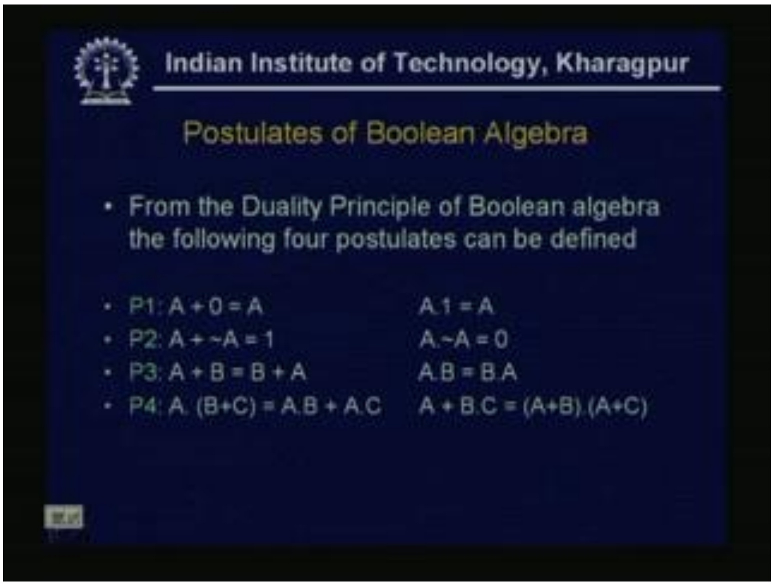


Now, last day all ready we mentioned or dealt the postulates of Boolean algebra, again if I summarize the last days thing that from the duality principle of Boolean algebra the following four postulates we have defined. What is that, say if A is a Boolean variable then A plus 0 is A, A plus negation and means complemented a is 1, A plus B is B plus A commutative, A dot B plus C is A dot B plus A dot C.

Now, the counter what for this four postulates are if this plus means all ready we have defined this is nothing, but a OR. So, this OR can be replaced by AND then this is A dot 1 equal A this is A dot negation A equal to 0, A dot B equal B dot A and A plus B dot C is A plus B dot A plus C. So now, today we define some of the theorems the Boolean theorems of Boolean algebra.

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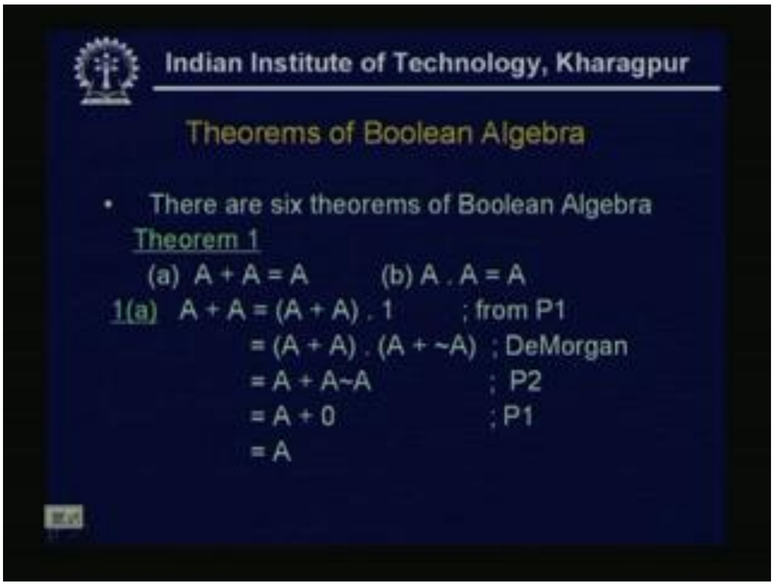


And mainly they are based on these open axioms of the postulates what we have defined. Now, the first theorem we see that this is A plus A is A similarly, the counter part is the A dot A is A. So, how we can tell now, just to prove this thing or see the validity of this theorem we will use the postulates what all ready we have defined. See A plus A is A plus A dot 1 from the first postulate now. I can easily replace by A plus negation A, now, if we apply the postulate two that means, my or distributive postulate then this A plus A dot negation A.

Now, from postulate one that means, the counter part of postulate 1 that we know that A negation A is nothing, but 0. So, this part the second part become 0, so this becomes A, so, A plus A equal to A.

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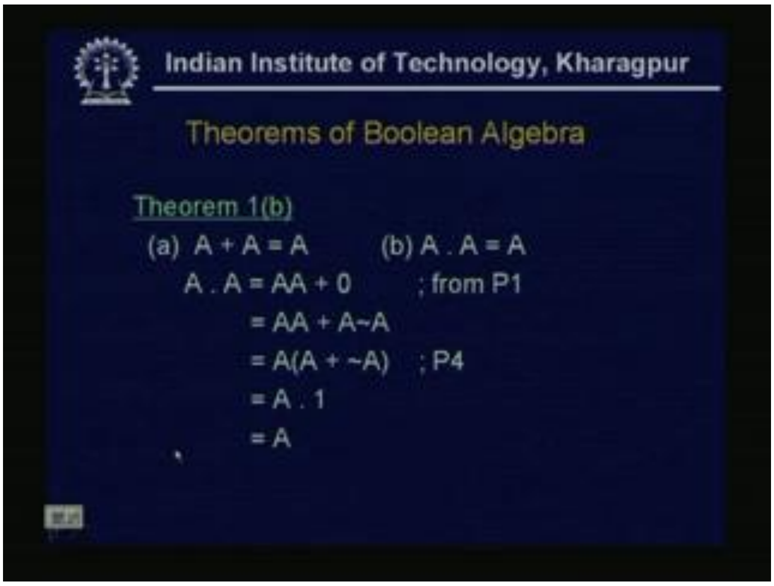


Now, if the plus is replaced by dot A dot A, so A dot A is A plus 0 this is 0 we have defined that additive identity. So, from postulate 1 we have added 0, so this is now 0 we can define a negation A just now, we have of used from postulate two. Then if I take the common then A is A plus negation A now see A plus negation is A is nothing, but 1. So, this is A dot 1 and this is A because, from the postulate 1 A dot 1 is A, so A dot A is nothing, but A, so this is my first theorem of Boolean algebra.

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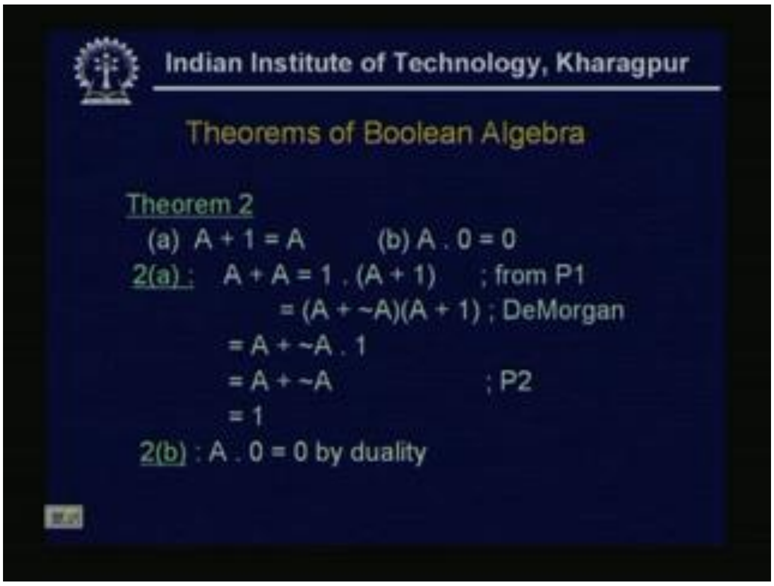


We will see the 2nd theorem, second theorem tells that A plus 1 equal to A and A dot 0 equal to 0 now, A plus A equal to 1 dot A plus 1 equal to A plus negation A because 1 is nothing, but A plus negation A and as it is we keep A plus 1. Now, if we take or is distribute that thing, then A this is A then negation A dot 1, now negation A dot 1 is nothing, but negation A, so A plus negation A is1. Similarly, that this is A dot 0 equal to 0 by duality because, duality means that dot is plus is replaced by dot and thus by using the similar way we can prove that this thing.

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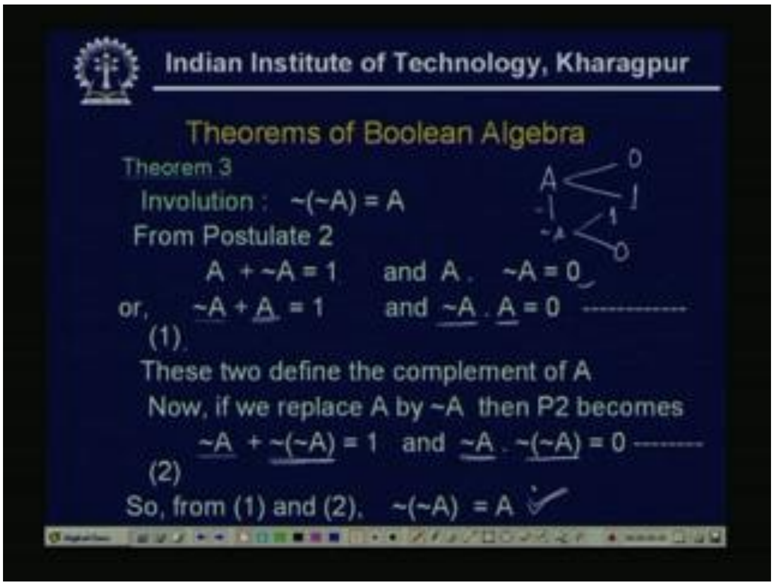


Now, theorem 3 is the involution what is the involution property, that negation of negation A means if I take 1 Boolean variable and twice we can take that complement twice then we will get the variable itself. Say A is a Boolean variable first we take 1 complement of A and negation of A, again I take the complement of this thing means the complement of complement of A we will get the variable itself means A. Now, from postulate two we have all ready read that A plus negation A is 1.

See if my A is 0, so 0 plus 1 is 1 if my A is 1, because it is a Boolean variable it can take only two values either 0 or 1, so if it is 1 then it will be 1 plus 0 1. Now, A plus negation A is 1 and A dot negation A is 0 this is from my postulate two. Now, or if I write negation A plus A just change this thing this is equal to 1 and negation dot A equal to 0 I mark this as a equation 1, now these two define the complement of A.

Now, if we replace A by negation A, so in these expression if I replace because see that A is a A is a Boolean variable, so A can take two values either 0 or 1. Now, if I take the complement then I will be taking negation A and what will be the values, if A is 0 then negation A is 1, if A is 1 then negation is 0. See that means, if A is the Boolean variable negation is A is also a Boolean variable, so I can easily replace A by negation A.

Then what will be the postulate two that negation A plus A plus negation A, now A is replaced by negation A, negation of negation A equal to 1. And here, A means negation A dot this negation is negation A is replaced by negation A, so negation of negation A equal to 0 I mark this as equation 2, so from equation 1 and equation 2 what we get, that say this is equation 1 was negation A plus A equal to 1 here, equation 2 this is negation A plus negation of negation A equal to 1, so this means A and negation of negation A is same.

Similarly, from the counter part or that if the operator is dot, then this part is negation A same here the right hand side is A or the next term is A and here this is negation of negation A then this equal to 0. So, from equation 1 and 2 what we see that A if we replace A by these then actually this is same, so that means, from equation 1 and 2 negation of negation A is A itself this property is called involution.

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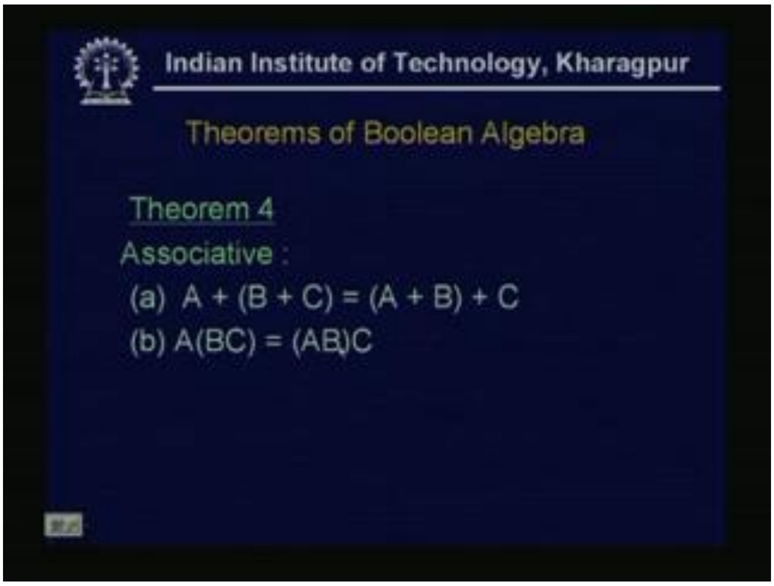


Now theorem 4, these are associative all ready we have seen that property the commutative when we have taken the two Boolean variables that A plus B is B plus A. Now, if it is three variable we take that means, A plus B plus C is if we take the operator operating on the first two variable A plus B and then on C plus C it will be same. Similarly, if plus is replaced by dot then A dot B dot C equal to A dot B dot C, so the associative property tells that if we take three Boolean variables and the order of the operator.

That means if in the left hand side the operator is operating on the on B and C the last two operands or the variables and then it is operated on the first variable the result will be same. If we take the order of the operators in different way that means, the operators is first operated on the first two variable and then on the last it will be same, and this is same as if the plus is replaced by dot.

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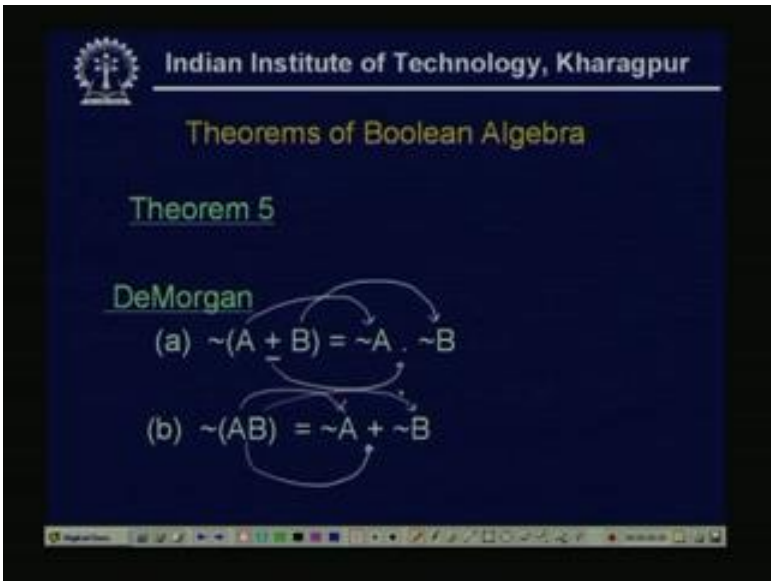


Theorem 5 all ready we discussed in the last class, but again if we summarize this is nothing, but DeMorgan's law. So, DeMorgan's law that if plus is replaced by dot then the Boolean variable will be replaced by the complement of that or negation of that. So, that means, that if say we are taking negation of A plus B the two variables then this is same as that of negation A dot negation B see that.

Here, operator was plus, so first this plus is replaced by this dot and the A and B operators A is replaced by negation A and B is replaced by negation B. Similarly, if I take the left hand side the dot operator that means, this dot this is replaced by plus and if A is replaced by negation A and B is replaced by negation B, then there they will give the same output. So, this is nothing, but our DeMorgan's law when we are taking only or the it is operating on two variables.

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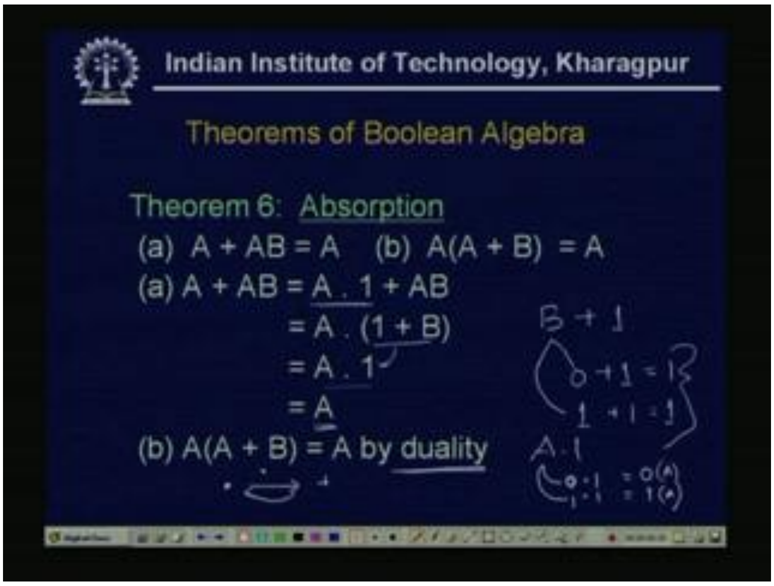


Now, theorem 6 is the A B sorption, that see A plus A B equal to A and A dot A plus B equal to A see how we can tell this expression is true, see first if you see the first part A plus A B what is the A plus A B . See A plus A B if I take A, A means A dot 1 from the postulate 1 and A B as it is taken, then if I take A common then this will be A dot 1 plus B. Now, B plus 1 B plus 1 is nothing, but 1 how again by truth table like can see that thing say B plus 1 I am telling this will be 1 why, so B can take two values either 0 or 1.

So, if it is 0 plus 1 see this will be 1 or if it is 1 plus 1 this will be 1, so we can easily check that actually, 1 plus B is B itself 1 itself 1 plus B is 1. So, we replace now, again if it is A dot 1 now this will be A. Similarly, I can tell that A dot 1 again A can take two values 0 or 1, so if it is 0 dot 1 means this is 0 means A what was A and 1 dot 1 means 1 that means, A value what was A, so actually A dot 1 is A itself.

Now, if we replace for the B part to be truth to be proved, if we replace the plus by dot and the dot by plus that means, by duality principle means if we exchange the dot by plus and the plus by dot then we will be getting the same result. So, these are my 6 theorems of Boolean algebra.

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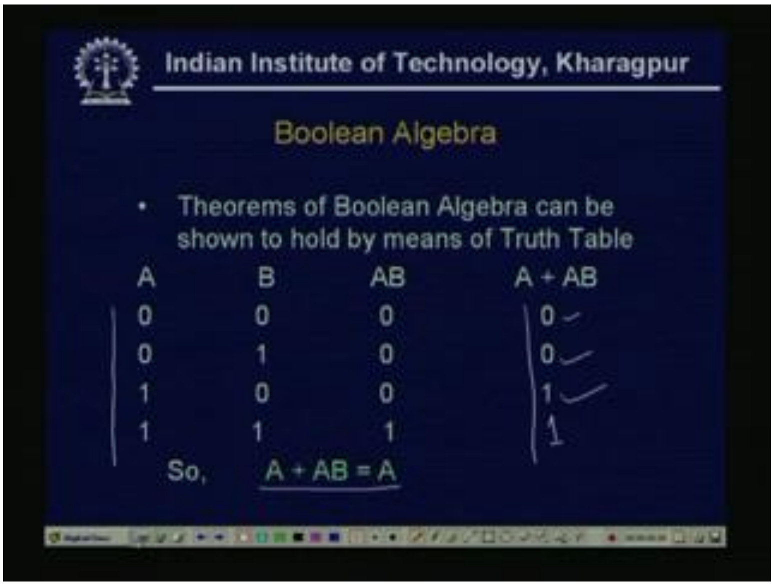


Now, just as I mentioned that this Boolean algebra theorems can also be checked by truth table. Just now we have seen for one simple example, that 1 plus B equal to 1 or A dot 1 equal to A. Similarly, if I take say for the theorem 6 what we have told that A plus A B equal to A. See if I draw the truth table then A can take four values actually two there will be four combinations because there are two variables. So, this will be two variables that means, and every anyone can take 0 or 1, so it will be 2 to the power 2 means there will be 4 combinations.

So, 0 0 0 1 0 1 1 what will A B then 0 0 0 0 1 0 1 0 0 1 dot 1 1, what will be A plus A B , see that 0 plus 0 this is A this is A B this is 0, 0 plus 0 this is 0 1 plus 0 this is 1, 1 plus 1 this will be 1. So, by truth table also by drawing the truth table also we can see that see A plus A B A plus A B A 0 0 1 1 see these are 0 0 1 1 and see our A is also 0 0 1 1. So, actually A is equal to A plus A B, so what we have proved by the help of the postulates that also we can check by drawing the truth table also.

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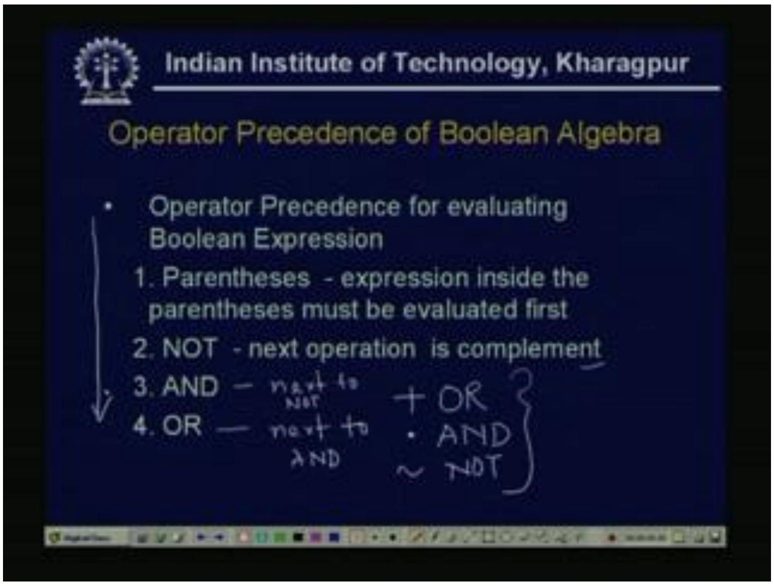


Now, the operator precedence of Boolean algebra, so just now what we have seen that actually our plus is nothing, but our OR dot is actually. When we are taking this is my AND and negation is our or the complement this is the NOT in digital results. Now, in Boolean algebra these operator precedence's for evaluation the Boolean expression first is the parenthesis that means, expression inside the parentheses must be evaluated first. NOT is the next operation and that is the complement, the next we will do the AND this is the next to NOT and OR is the next to AND. So, these are the precedence's in this order the operator precedence's when we will evaluate one Boolean expression.

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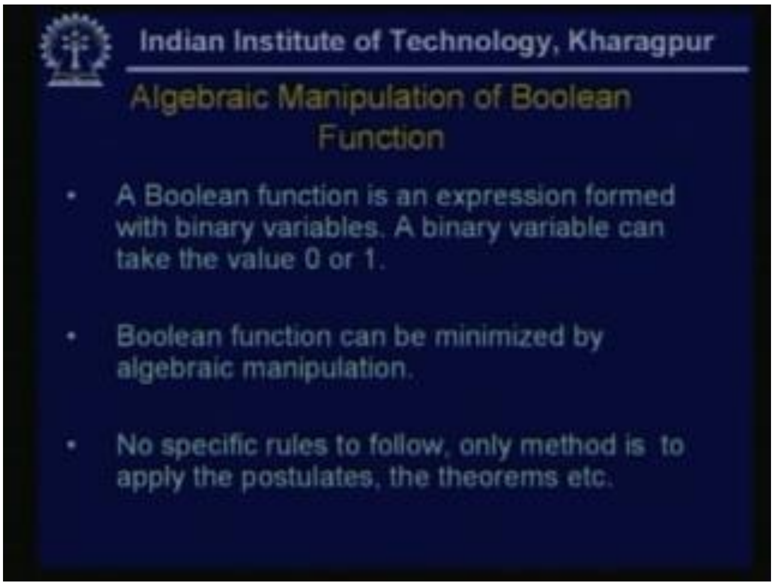

Now, a algebraic manipulation of Boolean functions, so a Boolean function is an expression formed with binary variables a binary variable can take the value 0 or 1. Boolean function can be minimized by algebraic manipulation and no specific rules to follow though we have read the postulates and the theorem. So, when we want to minimize or when we will be minimizing the Boolean expression we will do the algebraic manipulation. And these algebraic manipulation consists of the postulates and the theorems, but this is on that cut and try method cut and trial. There is no specific rules that which 1 will be followed when, so this we will we will see next.

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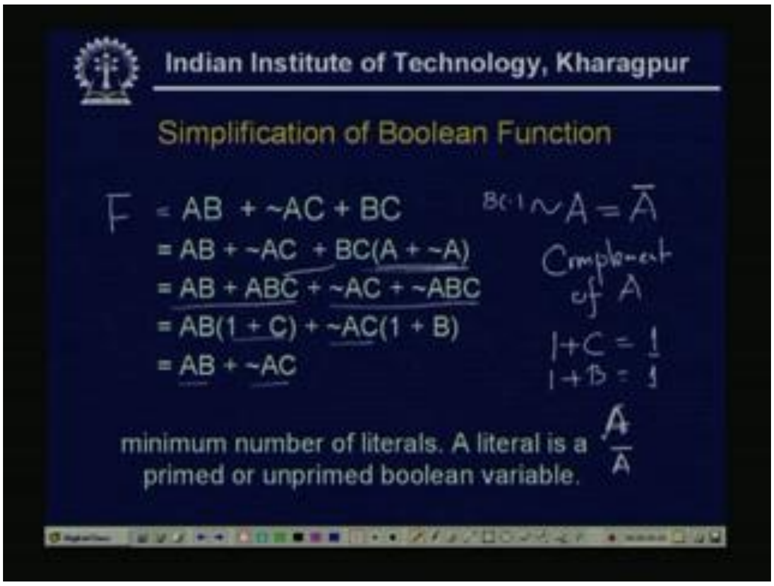


See I want to minimize or I am telling the simplification of some Boolean function, say I have 1 Boolean function A B plus A bar C mainly the negation AC. So, on negation A and this is same as, A bar these are all complement of A, so if I have 1 function say F equal to A B A bar C BC how we can simplify this thing. Now, I have I know the 4 postulates and the 6 theorems, now I have to apply this to simplify or to minimize this expression.

See A B as it is I have kept A bar C and now this BC I have kept BC as if this is 1 this is BC see I am writing BC dot 1, so 1 is replaced by A plus negation A from our postulate A plus negation A is nothing, but 1. Now, so this is A B now this if I distribute this thing that means, this is the distributive law this is BC dot A plus BC negation A, so BCA means A B C that I have taken here and this is negation A B C next term and negation AC means A bar C that will compare.

Now, from here if I take common A B this will be A B into 1 plus C plus here if I take common negation AC from this two term and then 1 plus B. Now, we know 1 plus C is nothing, but 1 plus C is 1 all ready we have seen from the our postulate and similarly 1 plus B is nothing, but 1, so this is A B plus A bar C. So, see there are three terms here A B A bar C BC and that is actually now, becomes A B plus A bar C there is no BC term as if BC is redundant in the expression, so this becomes A B plus A bar C.

And this is the minimum number of literals a literal we can define as if a primed or unprimed Boolean variable, means either it is complemented variable A or it is a it is normal uncomplemented or if it is a complemented variable. So, literally we are telling this is a literal, so this is a minimum number literals.

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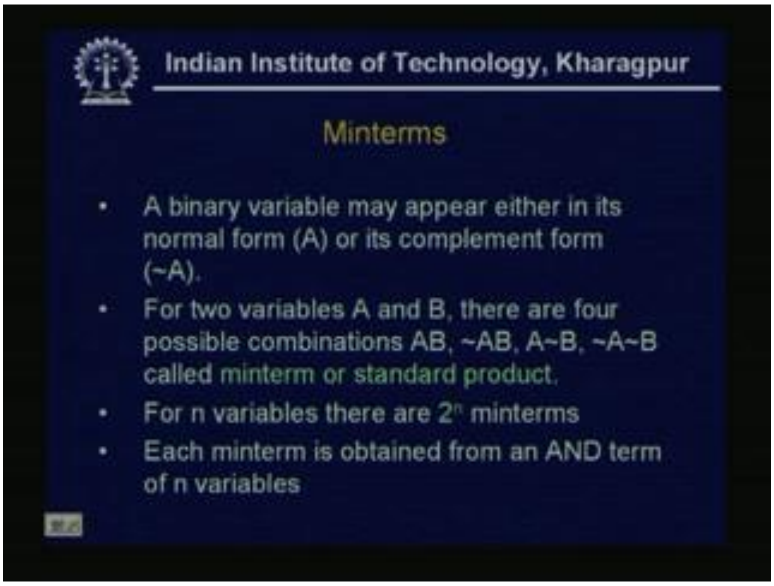


Now, we see the canonical and the and standard forms, before we define the canonical and standard forms first we define the mean terms, means terms and maxterms. So, a binary variable may appear either in its normal form just now, I mentioned A or its complement form negation A. For two variables A and B there are four possible combinations A B negation A B, A negation B negation A negation B.

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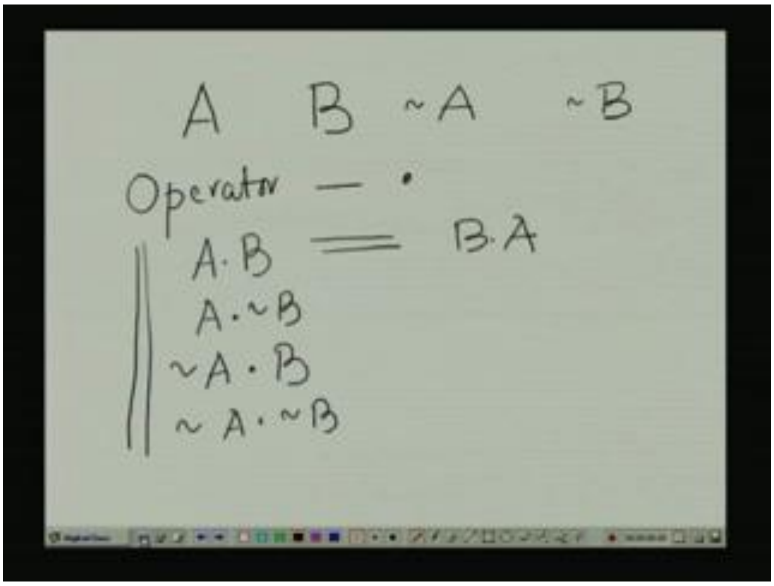


See if I take two variables what I if I take two Boolean variables, then either the A can B negation A also, B can also B negation B or complement also. Now, as if I have four such variables, so what will be the combination if I take the operator as the dot then it will be see A dot B A dot negation B. Now, if I take negation A it will be negation A B , or it will be negation A dot negation B.

See A B this is same as B dot A, I know all other combinations are same, so these are the four combinations I can get, so this is written in the second point that for two variables A and B there are four possible combinations A B, A B then A bar B, A B bar, A bar, B bar now these are called minterms or standard product. So, what is mainly the product terms or if the operator is a dot operator, then the variables operated by that operator is called the minterms, now for n variables there are 2 to the power n minterms.

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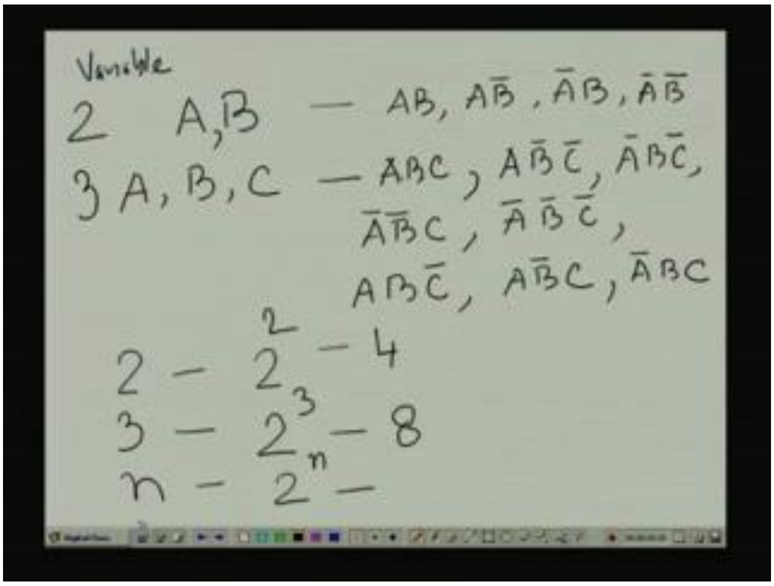


See just now, what we have seen say for A there are two variables A and B there are A B then A B bar, A bar B this is same as that of negation and A bar B bar. Similarly, if it is three variable this is variable two, if it is three variable then A B C what will be the product terms, see this will be A B C then A B C means A dot B dot C. So I am telling simply A B C, this is A B bar C bar, this will be A bar BC, A bar BC bar, then it will be A bar B bar C, it will be A bar B bar C bar, then it will be A B C bar, it will be A B bar C, another is left A B bar C bar, A B C bar, A B bar C, then A bar BC bar and A bar BC.

See there are eight terms these are the terms, so this is nothing, but that if it is two variables then it was if it was for two variables, it was 2 square, If it is three variables the product terms where 2 to the power cube that means, there are four terms here it is eight terms. So, if it is n variables this will be 2 to the power n such product terms will be there or the minterms are 2 to the power n minterms will be there, so for each for n variables there are two to the power n minterms. Now, each minterm is obtained from an AND term of n variables as just now I have shown.

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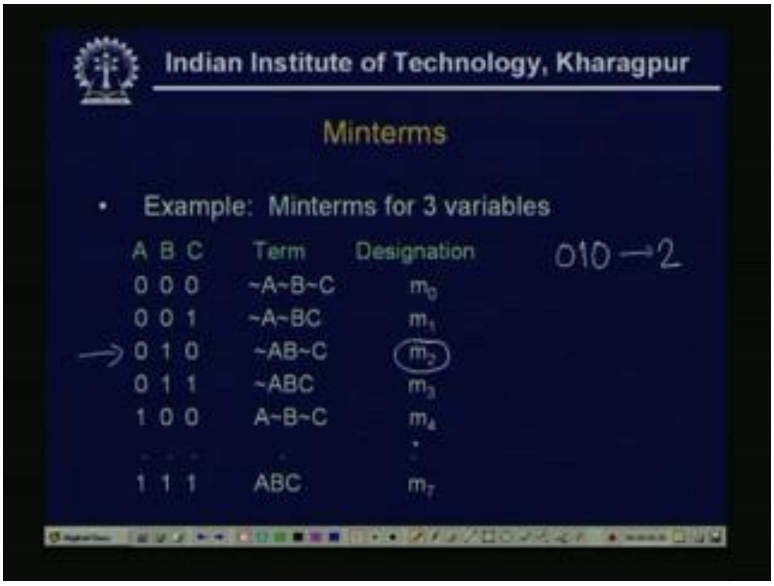


Now, we take that a terms or normally what is the conventionally how we designate this minterms, see these are the three variables A B C just now I have shown these are the terms. So, if it is three variables and we know that they can take either 0 or 1 values then this A B C values are 0 0 0 0 0 1 0 1 0 0 1 1 like 1 1 1 all 2 to the power 3 all 8 possible combinations are there all 0 0 0 to 1 1 1. Now, that there is some convention that normally that 0 means it is a complemented variable, so A equal to 0 here means this is denoted by the term is this is negation A or complemented A.

Similarly, that B this is B equal to 0 means this is negation B and C equal to 0 means this is negation C. Now, it is 0 0 1, so this is negation A negation B C only C this is uncomplemented, 0 1 0 that means, negation A and negation C ((Refer Time: 44:14)) B only. Similarly, 1 1 1 means that is simply A B C and these are the designation or normally it is written like that this is the minterms, that is small m and this is the 0th term, means that say m 0 this m 1, m 2, m 3.

So, now you observe these subscript 0, 1, 2, 3, 4 up to 7 this is nothing, but the decimal equivalent of the value A B C is taking. See if I take this 1 0 1 0, so 0 1 0 means the decimal is 2 see actually, this I am giving that m 2 this is my m 2. So, the subscript is the decimal equivalent of the value A B C is taking and this way we represent the minterms So, my m 7 is nothing, but my A B C and normally this is the convention that, but 1 can take the totally reverse 1, means the 0 means the uncomplemented, 1 means the complemented, but normally this is the convention and that in this class we will follow the convention.

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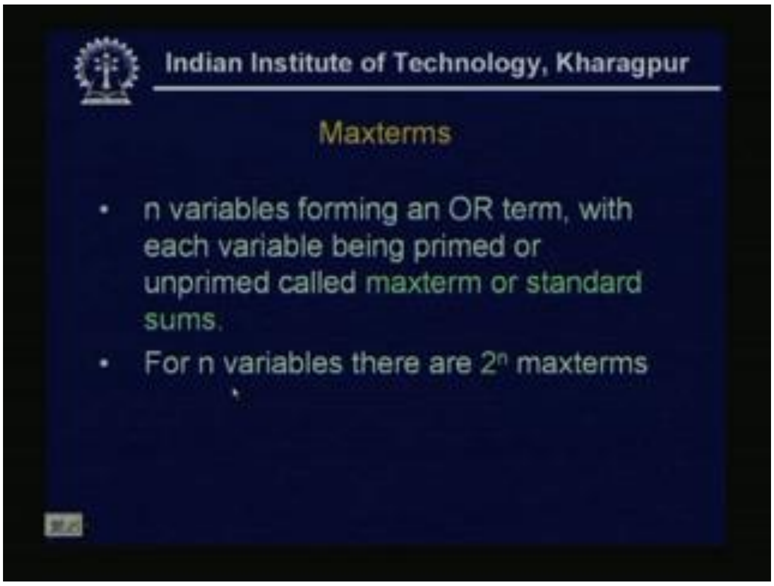


Now, we define similarly the maxterms, so n variables forming an OR term with each variable being primed and unprimed called maxterms or standard sums. In the minterm what we have seen that the operator was AND dot and the terms are minterms are defined that, the each variable being primed or unprimed means complemented or uncomplemented they are operator by the dot operator. Here, the dot is replaced by plus means AND is replaced OR and similarly, the terms are defined as the sum or the maxterm, here also that for n variable there are 2 to the power n maxterms.

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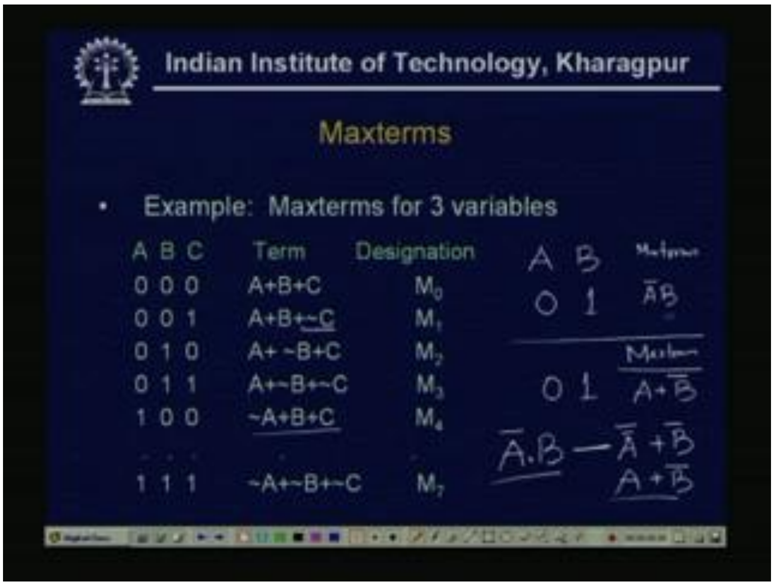


See again the same similar type of example we can see for the three variables, here this is again it takes the eight combinations for three variables 2 to the power of 3 8 and see the all dots are replaced by plus. So, this is A plus B plus C this is A plus B negation C, now observe 1 thing that here in the minterms that 0 we have defined as complemented A and 1 is the uncomplemented 1. Here we are taking the reverse 1 again this is the convention that in what we have taken in the minterm the reverse we are taking in the maxterm.

That means that 0 is the uncomplemented 1 A plus B plus C, here A plus B plus C bar this my negation C bar similarly, say if I take 1 0 0 means this is A bar plus B plus C A bar plus B plus C. See why that if I take and before that I complete this again designation is that. Similarly, the subscripts are the decimal equivalent of the Boolean value that A B C is taking and this is normally, define denoted by the M that M 0, M 1, M 2, M 3, M 7. Now, see why the reverse convention is maintained, see if I take two values A and B, so I am taking 0 A is taking 0 B is 1.

Now, for the minterm what will be my minterm it will be that 0 means it will be A bar and this is B. Now, again if my A B is 0 1 and if I consider the maxterm then it will be according to this convention it will be A plus B bar, now see that from using DeMorgan's law actually, A bar B is that if I replace dot by plus then the variables will be replaced by the complemented 1, so this will be A bar and this will be B bar. Now, again from the involution theorem that complement of complement A is nothing, but A itself. So, this is

A and this will be B bar, so which is nothing our maxterm A plus B bar and minterm is A bar B.

So, that is why the reverse convention is taken that means, in the minterm 0 is treated as the if A equal to 0, then is 0 treated as the complemented 1 in the maxterm the 0 will be treated as the uncomplemented one.

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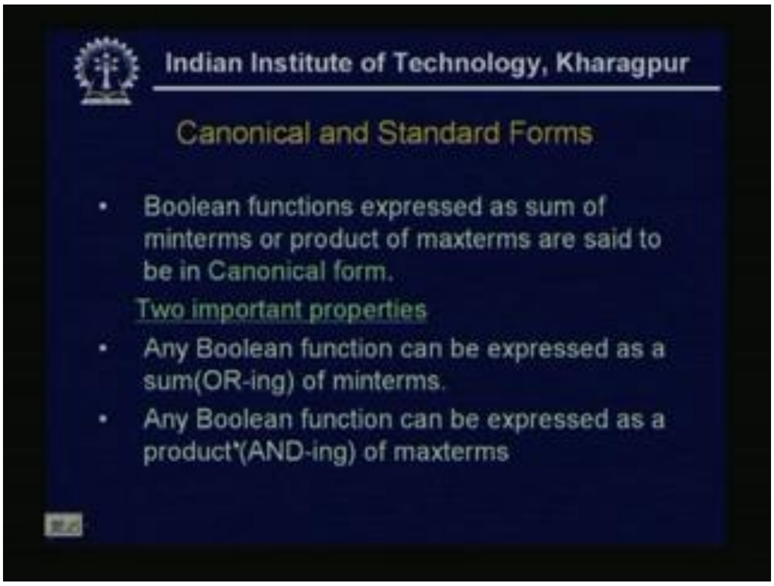


Now, we defined we have all ready defined the maxterms and minterm, so now, we can define that what do you mean by canonical and standard forms, See the Boolean functions expressed as sum of minterms or product of maxterms are said to be canonical form. What do you mean, that see if I have 1 expression and the terms are that terms are either related by the sum of minterms means, they are minterms means they are product product terms and they are related by plus sum.

And maxterms means they are plus terms and they are the product or they are related by the product, then it will in the canonical form. So, if I take 1 simple example what will be the thing.

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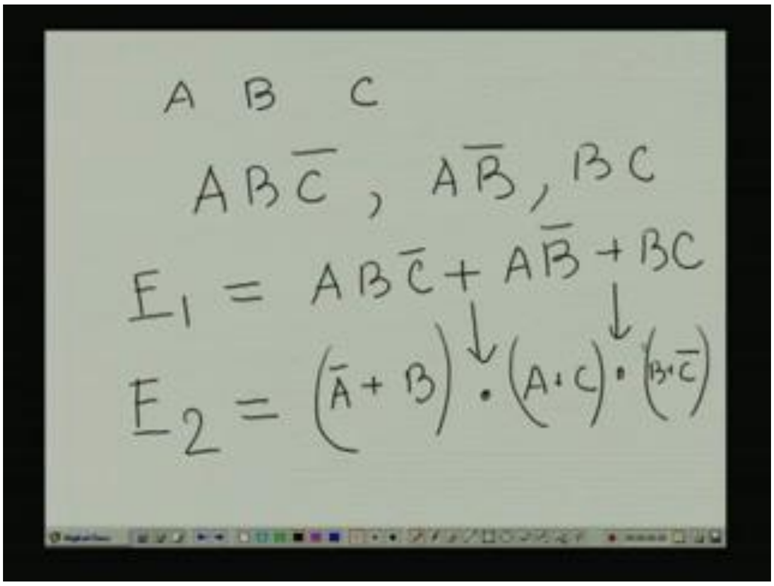


See sum of the minterms are if I take three variables A B C sum terms are say A B C bar say A B itself or A B bar and take say B C, so these are the three minterms. Now, if some expressions say I am taking E 1 equal to A B C bar plus, see these are the sum of product form or sum of minterms this is 1 canonical form. Now, say again A B C are terms now if I can some other expression E 2 where this plus actually, is replaced by dot this plus is replaced by dot.

Say here I am taking say A bar plus B A plus C and say B plus C bar same technique step with the then again this is the canonical form. So, either this is sum of minterms or product of this is the product of maxterms, ((Refer Time: 53:47)) so any Boolean function can be expressed as a sum OR-ing of minterms. Now these are the two very important properties given any function that always can be expressed as the sum of minterms.

Similarly, any Boolean function can be expressed as product of maxterms that means, either OR-ing or AND-ing and obviously, it will come from the DeMorgan's law because or that duality principle whatever you call, so these are the very two very important properties that normally we utilize. Now, today we will here we end this lecture again we will continue that next day, but sum of the quiz question based on this lecture we see, these are lecture seven quiz.

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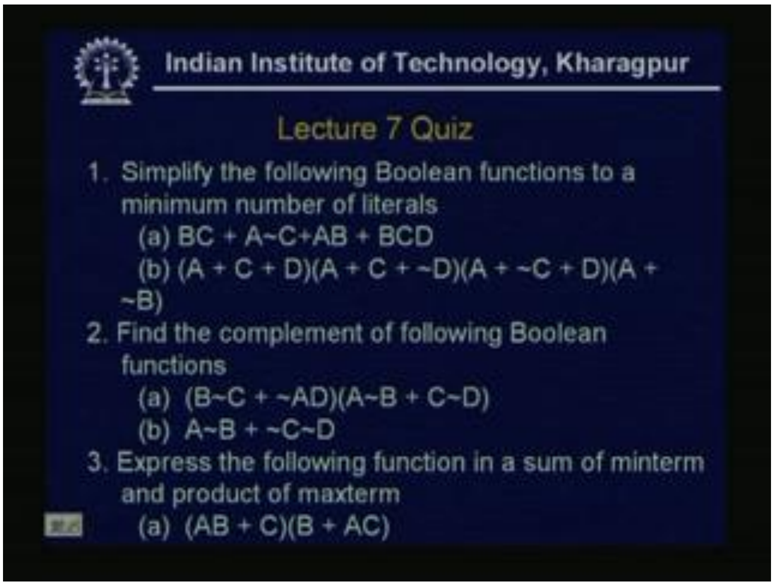


That simplify the following Boolean functions to a minimum number of literals, that means this is nothing, but the minimization of the Boolean expressions using the postulates and the theorems this is B C plus A bar A C bar plus A B plus B C D. And another expression is A plus C plus D, A plus C plus D bar, A plus C bar plus D, A plus B bar. So, just note that this is the sum of minterms and the next one, I have given is the product of maxterms and we have to simplify this thing.

The second question is, find the complement of the following Boolean functions, we have take the complement and again when we will be taking the complement the we will use the DeMorgan's law, because complement and uncomplemented means normally that DeMorgan theorem that comes into picture. So, you can use this thing, term is B C bar plus A bar D plus A B bar plus CD bar, see normally this is same that complement I have mentioned all ready that negation A and these are same A bar.

So, actually these expression or the say if say these expression means this is A B bar plus C bar D bar, so this two are same notation complemented normally this is this denotes a complement of A or negation of A. So, this is A B bar plus C bar D bar again we have to take the complement of this expression. Another is the third one is the, express the following function is sum of minterm and product of maxterm.

As just now I mentioned that ((Refer Time: 57:24)) the two important properties that any Boolean function can be expressed as a sum of minterms or product of maxterms always it is true. So, based on this two properties that I have given this ((Refer Time: 57:40)) question that express the following function as a sum of minterm, because this is a given a product of maxterms. Again it is not a same as it is product of maxterm, because see A B is A B is a minterm AC is a minterm.

So, again this is a mixed this is a normal form a standard form this is a standard form I have to we have to write it is the this type of equation. That means, it will be a sum of only sum of minterms in that way we have to do that thing.

# Digital Systems Design

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## Lecture - 08
## Boolean Function Minimization

Today, we will read how to minimize the Boolean function, but before that first we will discuss the answers of the previous lectures quiz.

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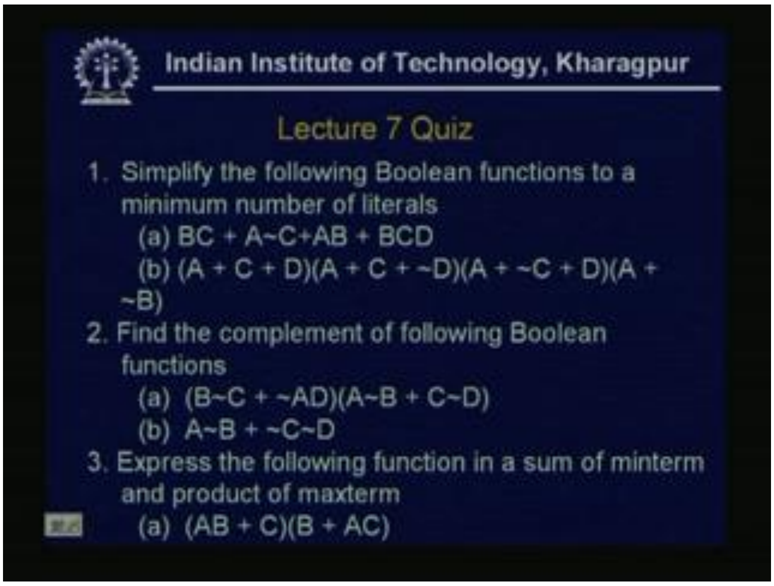


Ah the first question was to simplify the following Boolean functions to a minimum number of literals, first 1 plus BC plus AC bar plus A B plus B.