Digital System Design
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Lecture - 06
Boolean Algebra

Today, we start discussion on our Boolean algebra.

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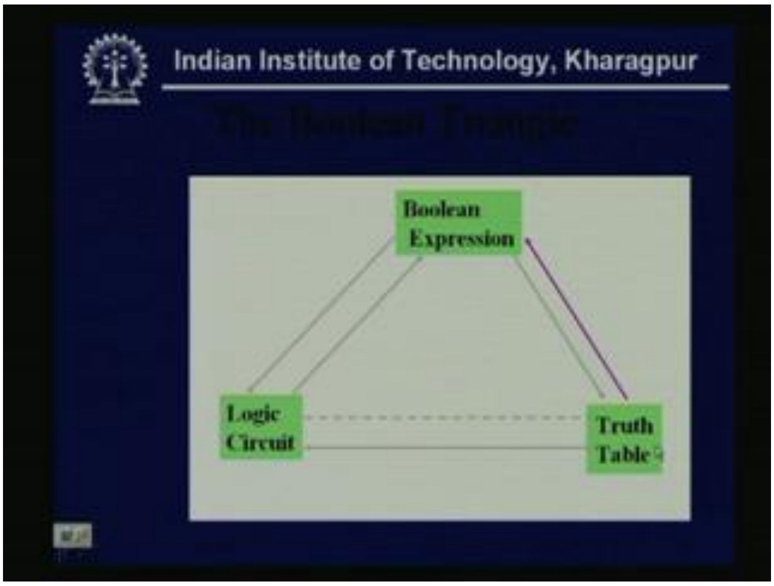

So, last day, we have discussed, that the design of digital circuits, I am just to design the digital circuits, what are the basic needs, that we have discussed in the introductory class. So, if we recapitulate quickly, the summary of the last days lecture, that we found that, three things that logic circuits, truth table or Boolean expressions, that any one of this thing is necessary. Now, last day we have seen some, Boolean expressions, that means, given digital design to be built up.

First, we have identified, say some of the expressions or the, if the truth tables, we have seen that, how the truth table can be built up from the Boolean expressions and again from the Boolean expression, the logical circuit can also be built up. Now, some logic circuit, if the logic circuit is given, from there the Boolean expression can also be evaluated. Similarly, if the logic circuit is given, the truth table can be constructed, if the truth table is given, then the logic circuit can be design from the truth table.

So, today, will see that, how this Boolean expression can be built up or what do you mean, actually these expressions are based on the Boolean algebra. So, first today will see, that what do you mean by this Boolean algebra, how it is the basic building block of the digital circuit design. So, from this diagram, that will be starting our discussion on the Boolean algebra.

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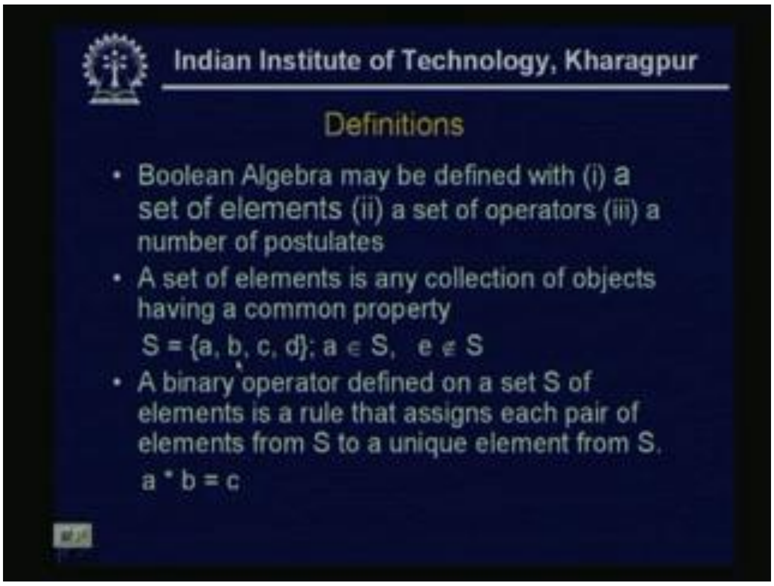

First we see, that, what do you mean by this algebra, so Boolean algebra can be defined with a set of elements, a set of operators and in number of postulates, so these are the three basic ingredients of the algebra. Now, what are the elements, a set of elements is any collection of objects, having a common property, this a definition of set, say S is a set and this is some a, b, c, d, these are some, some variables. Later will see, what type of variables these are and this, now a, b, c, d, we are calling, these are the elements.

So, a belongs S and see e is not belonging to S, because e is not as the, in the set S, so these are my, a, b, c, d are my elements, a set of operators. A binary operator, defined on a set S of elements is a rule that assigns each pair of elements from S, to a unique element from S. So, that means, a, b are the two elements, in the set S, so if we take, a and b, you take an operator star, so a star b, if you compute, we are getting the element c, which is again and element in S. So, then, this star is defined an operator on S, now a set of such operators, defined on S, that we have taking as the set of operators and third is the number of postulates.

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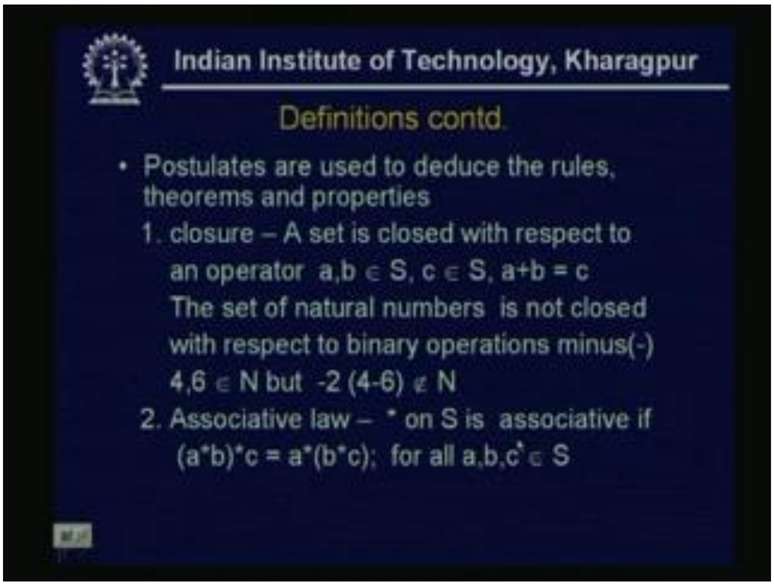

So, postulates are used, to deduce the rules, often we call, that these are the nothing but axioms, so there some theorems and properties. Now the basic, when you define, mainly we are define, a algebra, so when we have define a mathematical systems, we need this three things and in between, in these, within this three, the postulates are the main one, that is the rules defined on the set. Now, what are these rules or this postulates, first thing is the closure, so this we can tell, this is the property.

So, a set is closed with respect to an operator a, b belongs to S, c also belongs to S, such that, a plus b equal to c. That means, we are taking three elements from S, a, b, c and one operator start here, one operator plus a, so a plus b, the result a plus b is given, is the c, which is again, one element in S. Then we are telling, that this is a closure property, the set of, set it, take an example, the set of natural numbers is not closed, with respect to a binary operations, minus.

So, if I take, the operation or the operator as minus, then say, binary minus, then 4, 6 are the 2 elements, say a equal to 4 and b equal to 6, then 4 minus 6 is minus 2, but minus 2 is not an element in N, the natural numbers. So the natural number N, with respect to the binary operation minus, is not closed, so closure property does not hold, if we take the operator as the binary minus, if we take the operator binary plus, then see. We take this, see we are taking a operator, see we are taking a binary operator plus positive.

See then, next rule is the associative law, then we take one operator star on S and it, it will be call associative, if, that a star b star c. That means, the operator is operated first on the two elements, a and b and then again the operator is operated on the result of, a and b and c is called a associative, if it holds, if it is equal to the a star, b star c, means the, see, the left hand side, the operator is first operated on a and b. Then, the result we get, that would be treated as one operator as c, the another element of S is the another operand.

And then, the operator is again treated on this, the result we get, if it is equal to, that if the operator operates on the b and c first and then, the operator is again operated on the other element a. And the result of b star c, if these two results are equal, then we are calling that, this rule is associative.

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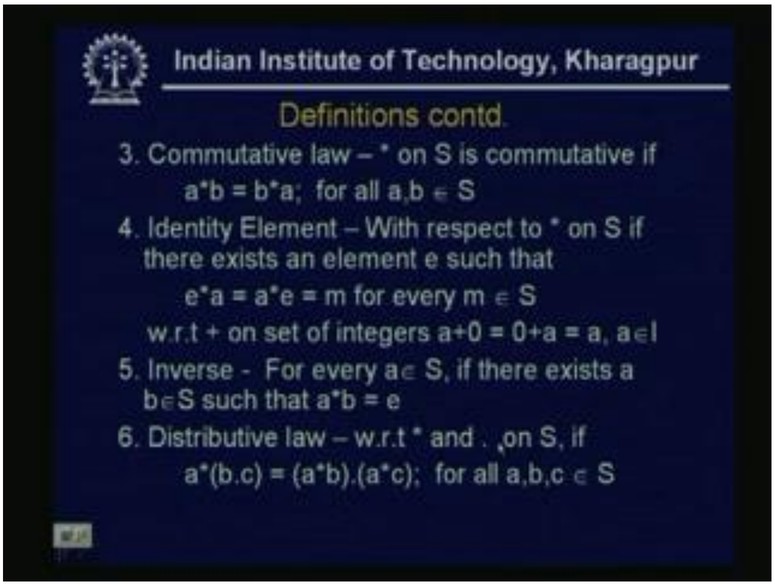

Now, another rule is commutative, again we have taking the same operator star and the same set, S, that means, the star operator on S is commutative, if the a star b equal to b star a, for all a, b belongs to S. Now, one concept of identity element, what this is identity element, again we are taking that same operator star on S, now, if there exist an element e, such that, e star a, equal to a star e. That means, the operator is taking two operands, the e first and then the one element a and a first

And then the same element e and the results are same, equal to same for every m belongs to S. That means m is again the result, is again one element in the same set S, then we are calling that, with respect to the operator star, e is the identity element of the set S. Now, if you take one simple example, say our plus, addition operator, so with respect to addition operator, on set of integers.

So, say a is one integer, then a plus 0, we know that, it will be 0 plus a and this is nothing but a and a belongs to that set, set of integers, so this set is denoted by I, capital I, so a belongs to I, then the 0 is the additive identity of the set of integers, I. Now, similar to that identity element, in another concept is there, it is called the inverse element, again for every element, a belongs to S. If there exists a, b another elements belongs to S, such that a star b equal to e, means if the operator star, say this is a multiplication.

Then, if it operates on two operand a and b and this gives a, the identify element, just now we have define the identify element, if give, the result is a identity element, then that any one of this element is the inverse of the other element. Another rule is that, distributive law, so for the distributive law, we take the two operators together, say star as well as dot on S. Now, if the a star, b dot c equal to, a star b dot a star c, for all a, b, c belongs to S.

Then, we call this is the, the distributive law holds, with respect to the two operators, star and dot, on the set S. Now see, here the rule is, we take two operators, star and dot and we take three elements on the set S, now, so three elements are a, b, c, we take to two elements b, c that is operated by the operator dot, now the result is operated, by the another operator star and the operand is the third operand, means the third element. The result we get, if that is equal to a star b dot a star c.

What does it means, that means, the c the last operator star, that is treated in this way, that is it, a operates or star operates on a and b, star operates on a and c and then the two results are being operated by the operator dot. Then, if these two results are same, then we are calling, these are the distributive law.

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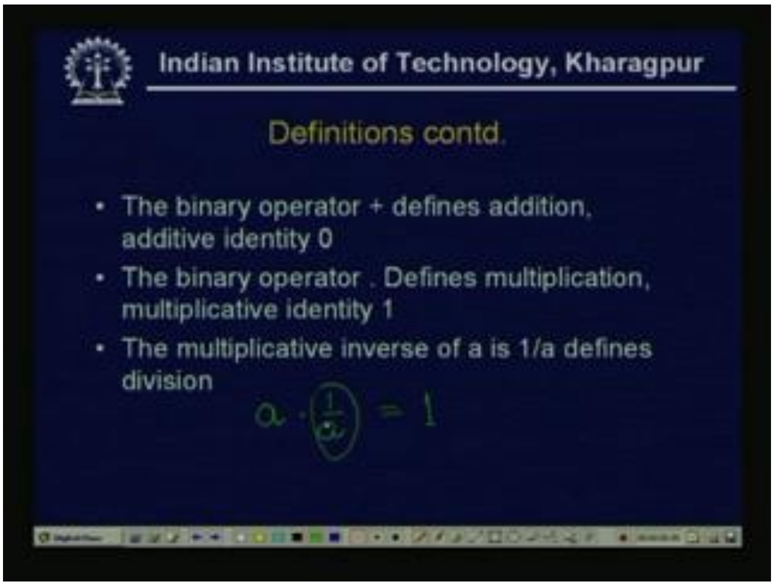

Now, if, so we take the two operators, plus and dot on the set S, then we can summarize, that with respect to our identity elements, that if the binary operator is plus, that means, it defines addition, then the additive identity is 0 and if we take dot as the operator, which defines the multiplication, then the multiplicative identity is 1. Now, if we take one element a on S, then the inverse of a is 1 by a and if the operator is dot, then a dot 1 by a equal to 1, because as already I mention that multiplicative identity is 1. Then, we can define that, 1 by a as the inverse of a, which defines the division actually.

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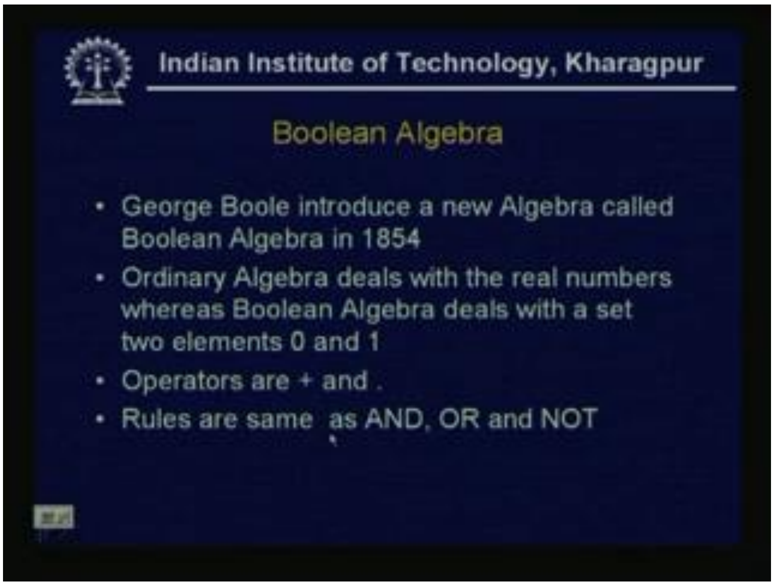

So, if we now concentrated on the Boolean algebra, so far we have define, that general algebra and if we consider the Boolean algebra, we can define like that, first the George Boole introduced this new algebra called the Boolean algebra in 1854. Now, the ordinary algebra deals with the real numbers, means say, the set is a infinite set consist of a infinite number of elements. Now, here in Boolean algebra, it deals with a set of two elements, that is 0 and 1.

The operators are, two operators plus and dot and rules are same as AND, OR and NOT, already we have introduce that, the logical AND, OR and the NOT. So, how we have define that our initial algebra, that ((Refer Time: 18:36)) it is a set of elements, a set of operators, a number of postulates. Now, here the set of elements is only a set of two 0 and 1, set of operators, two operators we have taking plus and dot, a number of postulates and these are the postulates we discussed, that are same.

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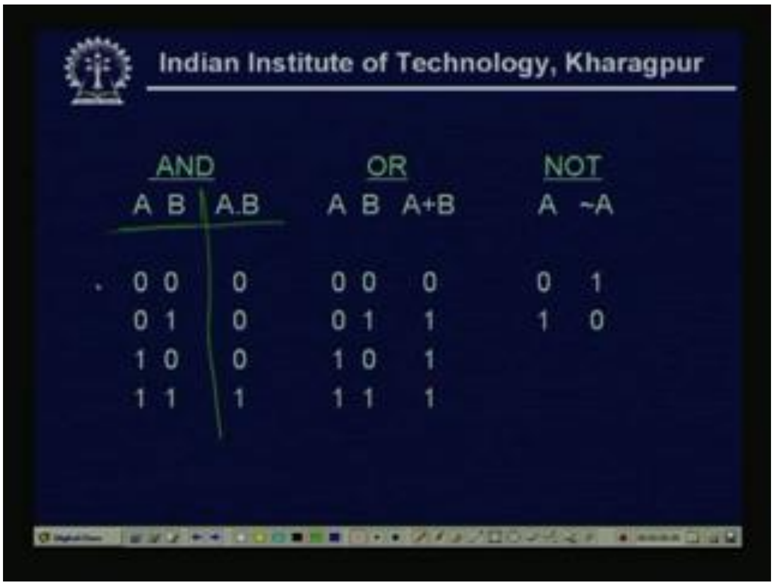

And you remember that AND, OR, NOT, again if we take, now from the concept of the Boolean algebra, see these are a, b are the elements, now the operator, say a dot b, this is similar to the our AND gate, that means, this is the truth table of AND gate. Now, if we take a plus b, the plus is the operator and again the two OR elements are A,B, then thus it is similar, the input output relationship, this is similar to that of OR gate and another we have taking, that if you take only one element A and the complement of A.

Then, this are, this is the nothing but the similar to NOT gate, so just now we have defined, the algebra, the set of, if the set of elements A, B are the element, two elements, the set of operators that are plus and dot and the rules. Just now we have discussed and we just formed this A dot B, A plus B and negation A or the complement of A, then we are getting actually the N gate, OR gate and NOT gate.

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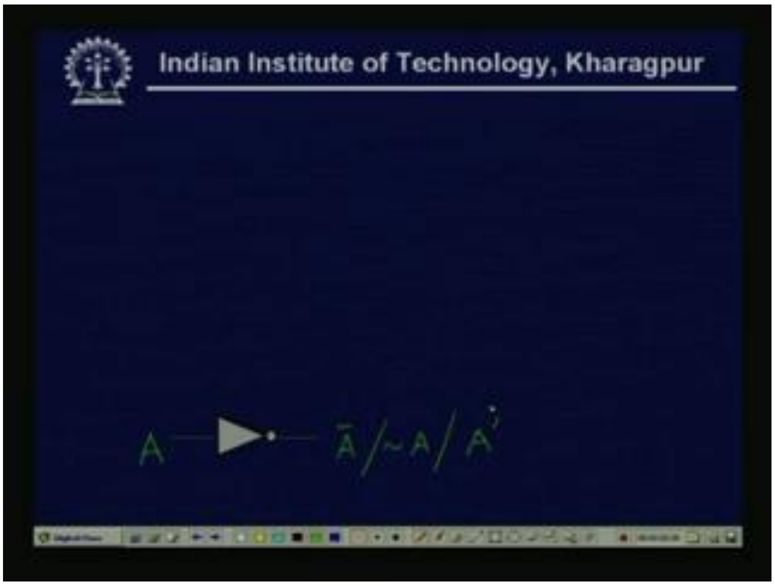

So, I remember that, this A and this complement, you can define like that, so this is, if this is A, then actually this is, that A complement or we write negation A, sometimes we write here.

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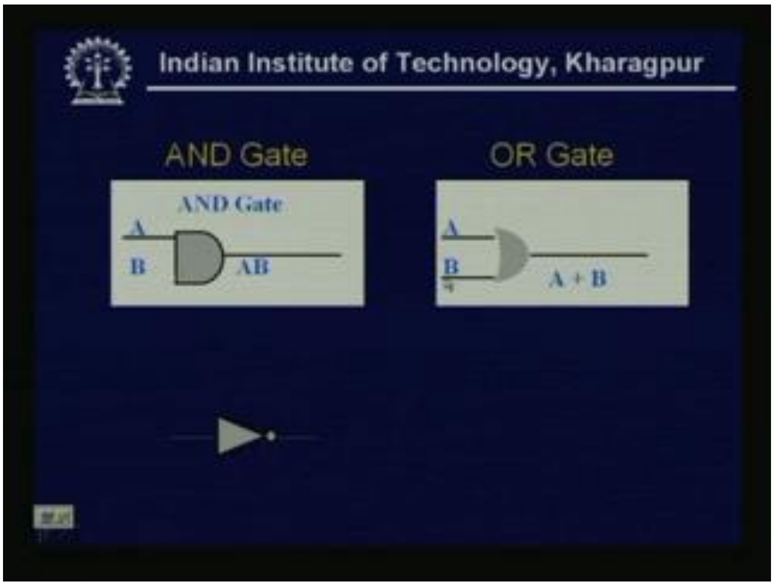

Now, you see the AND gate and now, OR gate, so again these are same now, another input B and this is my OR gate. So, mainly these three things, we will be discussing.

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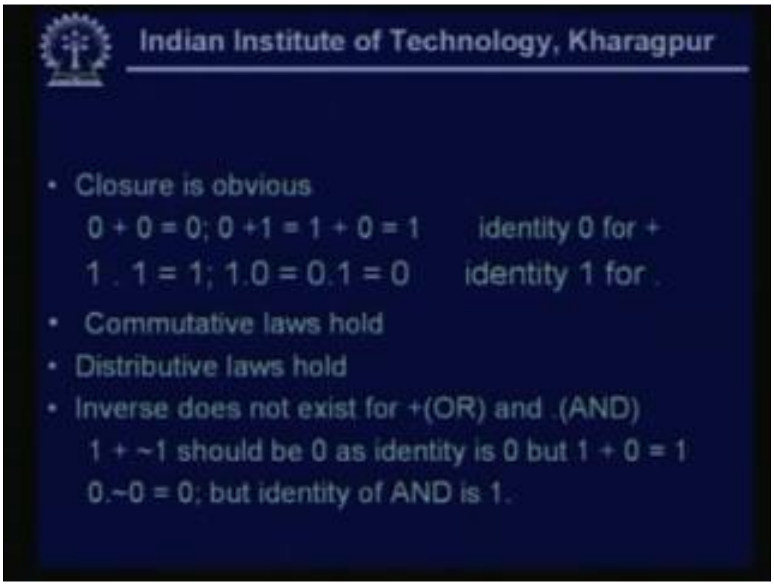

Now, you see how the rules are applicable, so just now, we have discuss the rules as the, the properties or the postulates we have mentioned, that one, first one is the closure. So, if we take the plus and dot as the operators, then closure is very obvious, see that, here the two elements are only 0 and 1, so 0 plus 0, it is defined as 0, 0 plus 1, it is 1, 1 plus 0, it is to as . Similarly 1 dot 1, if we take the, now the operator as dot, 1 dot 1 is 1, 1 dot 0 is 0 and 0 dot 1 is 0.

Now see, the results are also either 0 or 1, so this is the, the closure property is hold, it holds, now here, the identity is 0 for plus and the identity is 1 for dot. Now, the commutative law holds, if this is very obvious, from that, if it is, if we take 0 plus 1, is equal to 1 plus 0 and this is 1. Similarly, if we take dot, then 0 dot 1, equal to 1 dot 0 equal to 0, so commutative law holds, distributive laws hold, will be seeing this property later.

Now, the inverse exist, see that, if we take again the operator plus and the element as 0, then 0 plus complement of 0, means 1, so 0 plus 1 is 1 and similarly, 1 plus complement of 1 is 0. Here see, if we take the 0 and dot as the operator and 0 as the element, then 0 dot complement means 1, complement of 0 means 1, so this equal to 0, similarly 1 dot complement of 1, this equal to 0. So we see, that here actually the inverse exist, now, for the distributive law, see, distributive law to be hold.

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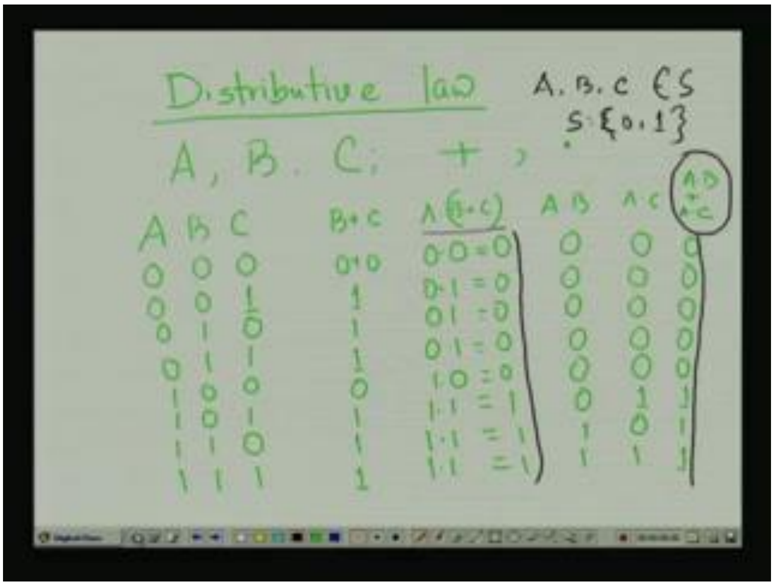

See, for distributive law, we take three elements, say the three elements are A, B and C and the two operators are plus and dot. Now, say I am taking A, B, C; B plus C, say A dot, B plus C, I am doing A dot B, A dot C and A dot B, plus A dot C. Now see, we take that all the values of the A, B, C, then B plus C is 0 plus 0, 0 then 0 dot 0 equal to 0, here it is A dot B, this is also 0, dot C is also 0, then this is 0 plus 0,0. Now, if it do 0,0,1, similarly, so this is 0 plus 1,1, A dot B plus C this is 0, this is 0, this is 0 and this is again 0.

Similarly, if we just complete this full table, then this is 1 plus 0, 1, again a same thing and 0, this is 0, this is 0, so again 1. Now, the plus C equal to again 1, but A dot B, A dot B plus C is again 0, this is 0, this is 0, this is 0, now 1, 0,0, again B plus C is 0, A dot B plus C equal to 0, A dot B, A dot C and this is 0. Now, then 1 0 1, so B plus C is 1, now A dot B plus C, this becomes 1, now, A dot B, 1 dot 0 and this is zero, but A dot C is 1, so this becomes 0 plus 1, 1.

So 1, 1, 0 again B plus C is 1, A dot B plus C is 1, A dot B is 1, A dot C is 0, this is 1, last term is 1, 1, 1, so this is again B plus C, 1, A dot B plus C, 1, then A dot B is 1, A dot C is 1, 1 plus 1 is 1. Now see, that, if we do the comparison, then this A dot B plus C, see this, these values that are, there are five 0s, three 1s, similarly, from here, we will be seeing five 0s and three 1s. That means, A dot B plus C, equal to A dot B plus A dot C, so the distributive law holds.

And see, here this plus and dot operator, that we have define from the algebraic point of view, this is similar to our OR and the AND operator. So, the last day if we discussed that, some Boolean expression, so now we can tell, this A dot B plus C or A dot B plus A dot C, this are again the two Boolean expressions. And it has, today what we have seen, actually these expressions are we evaluated from the Boolean algebraic concept, that, were that A, B, C these values, are can take only the 0, 1 case.. That means, these A, B, C, this are belongs to S, where this S is S belongs to actually 0 or 1, the two values, S equal to either 0 or 1.

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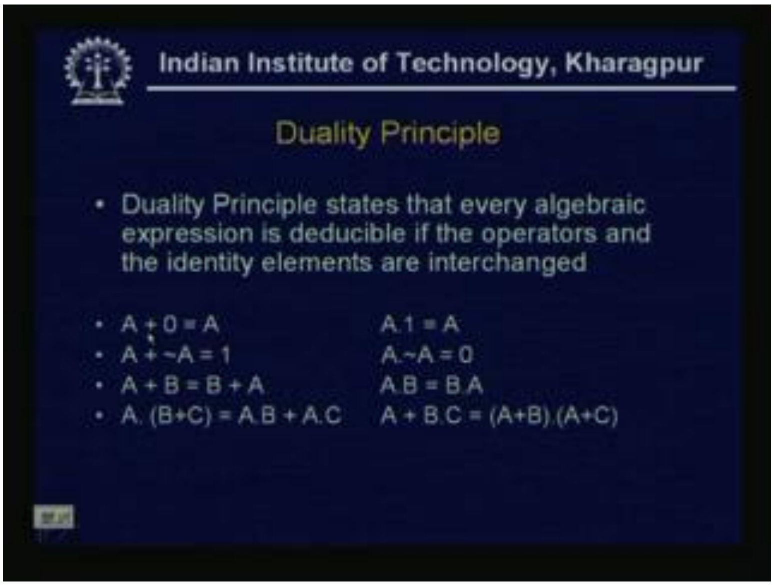

Now, we see that, how this Boolean expressions can be developed from this algebraic concepts. First we tell, that one very important, from principle that is the duality principle, the duality principle states that every algebraic expression is deducible. If the operators and the identity elements are interchanged, so here that operators are again the plus and dot. So if we take, see A plus 0, this is equal to A, because 0 is the identity element, additive identity.

Now, if we replace the plus by dot and 0 by 1, we will be getting the same thing, duality principles states that, so see from the first expression A plus 0 equal to A, plus is replaced by dot, 0 is replaced by 1, so A dot 1 equal to A. Similarly, the second expression A plus, complement of A equal to 1, so plus is again replaced by dot, negation A dot, negation A equal to 0. So, 1 is inverted, A plus B equal to B plus A. So, similarly A dot B equal to B dot A , now A dot B plus C equal to A dot B, plus A dot C.

So, if we now replace, so this first dot is replaced by plus and the second operator, this plus is replace by dot, so the left hand side of the expression becomes A plus B dot C and the right hand side of the expression becomes A plus B, dot A plus C. So, this is the, mainly the duality principle, if we consider only the postulates, basic postulates of the Boolean algebra, then it shows this type of property.

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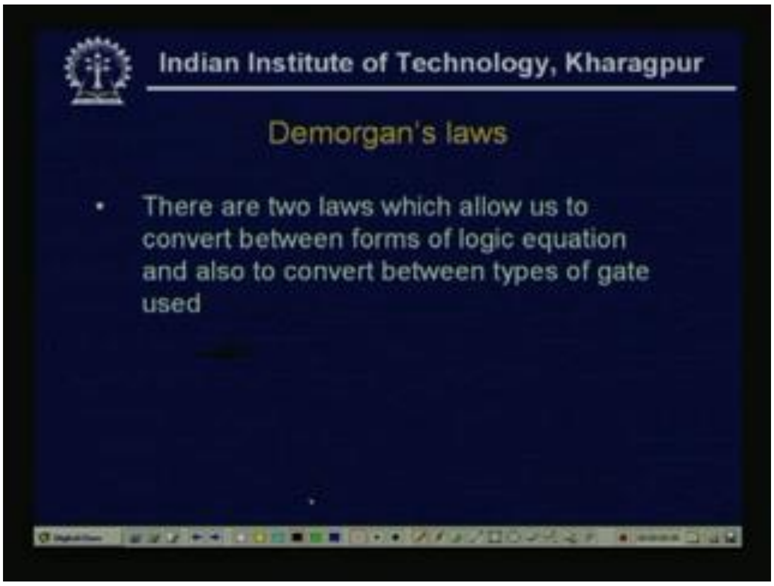

Now, one very important law, we call that Demorgan's law, that would, now there are two laws, which allow us to convert between forms of logic equation and also to convert between types of gate used. See just now, what we have seen, that, here that actually the plus is replaced by dot, so what we can tell, that if we see that, a plus is, plus is nothing but your OR, OR operator, so, see this OR plus is, OR operator and dot is AND operator, so this two operations.

I can simply, tell that these are two input AND gate and two input, this is two input OR gate and this is two input AND gate and this 0 and 1, see there, first is convert between forms of logic equation and also to convert between types of gate used. That means, the plus is replaced by dot means, nothing but the conversion of between two types of gate, AND and OR. So, what we can tell, that at plus,((Refer Time: 35:31)) is this, plus is the and OR operator and dot is the AND operator, now, we see the laws.

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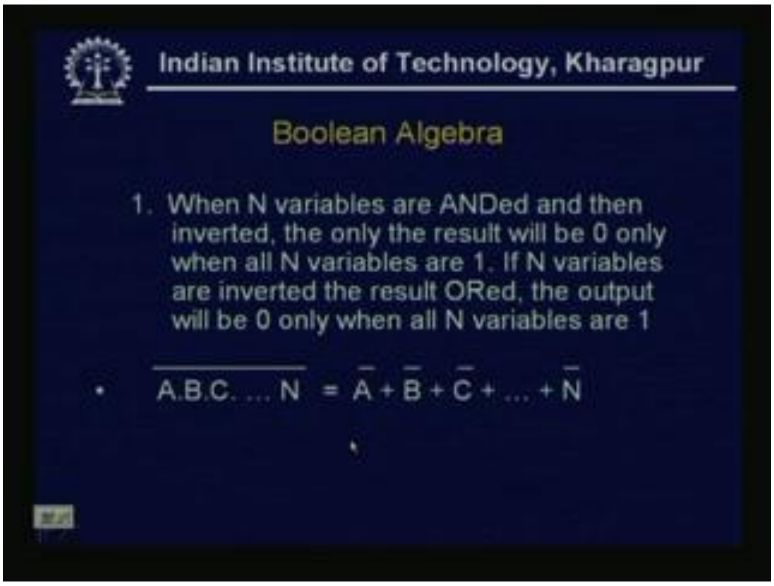

The first one, that when N variables are AND and then inverted, the only result will be 0, when all N variables are 1. If N variables are inverted, the result and the result OR, the output will be 0, only when all N variables are 1, what does it mean, see I am taking, if I write the expression, I am taking Ns variables A, B, C up to N, these are N such variables, now, first they are, when N variable AND, so they are AND, so this dot means, these are AND operation is being done.

Now, the whole thing is inverted, that means, some complement of the result is taken. Now the result will be, that, if every variable, every, each of N variable are inverted and then it is O Red, that means, A bar, it is inverted means A bar plus B bar plus C bar plus N bar and then it is taking OR, then this is same.

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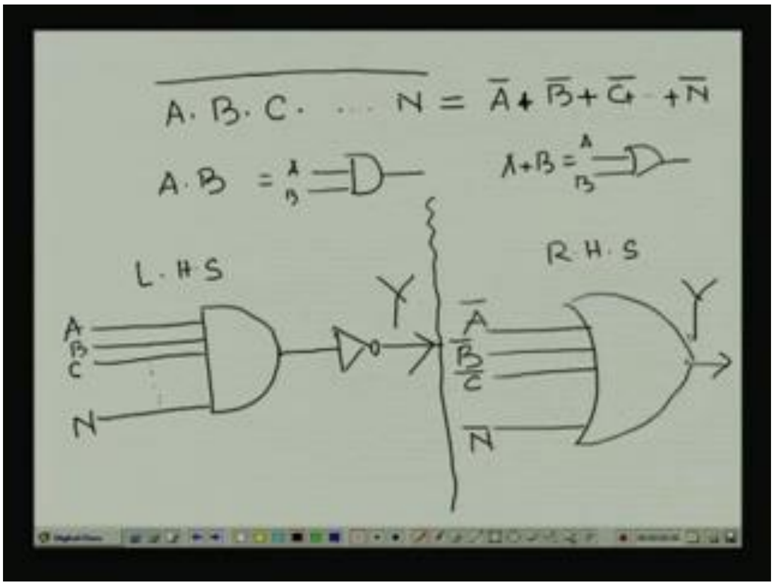

Now, if we take the thing, say what is written, that A, then B, it is ANDed, C up to say, N such variables. And then, the whole thing is inverted, then the result will be, it is told that result will be, if each of the variable is inverted, that means, say A bar, dot B bar, dot C bar, then it is dot N bar and all the dots will be replace by plus. So, this will be replaced by, see that if we draw, say this is nothing but a dot means, what we call, that A dot B is nothing but a two input AND gate, this is a two input AND gate, A, B.

And this plus means, that A plus B, this is a two input OR gate, this is two input OR gate, so now you, if we extend the operator, for N such variables and we write this thing. Then see this, then left hand side will be, say I am taking a N input AND gate A, B, C, dot, dot up to N. So, this is the N input AND gate, then the output is inverted, so I put one inverter here, so this is my, the logic circuits of the Boolean expression, represented by the left hand side, what will be the right hand side.

See, if I take one, N input OR gate, then the input will be, inverted of the variables or the elements, that are used in the left hand, see in the left hand side, the variables are A, B, C up to N, here all are converted. So, this is, so if, so this, this is my output Y, this will be my output Y, now this are, this two are same, so this, in this way we can convert the logic gates. That means that, actually here the, how the AND gate is converted to the OR gate, that rule is mention by the Demorgan's first rule.

Now, we thus see the second rule.

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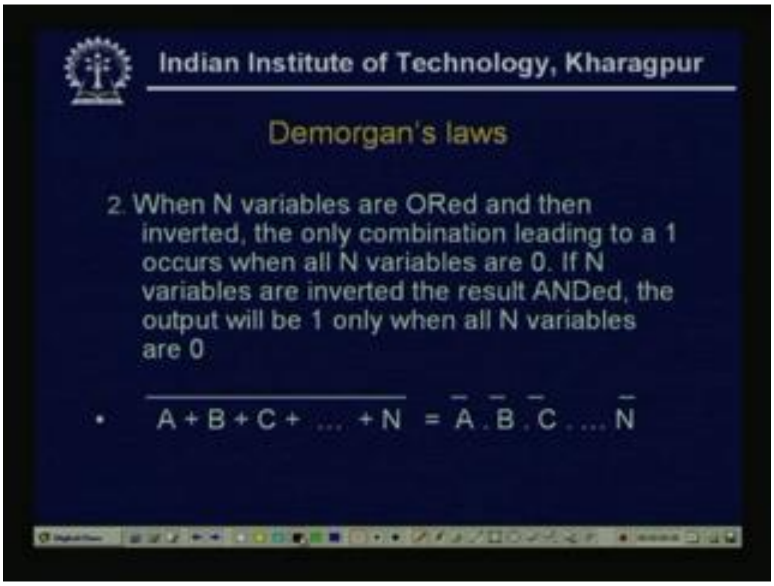

See when, N variables are ORed, totally reversing, now here the N variables are first ORed, that means, the OR operation on the ((Refer Time: 42:26)) N, OR and the x and the, the only combination leading to a 1 occurs, when all N variables are 0. So, again this is, then it is inverted and what do you mean by this term, the only combination leading to a 1. See, if we do that, first we describe that rule ((Refer Time: 43:15)), that A plus B plus C plus N, if it is inverted.

Then it will be similar, as if each variable is convert, converted and then it is ANDed with each other and the rule says, that N variables are ORed and then inverted and if N variables are inverted, the result is ANDed, the output will be 1, only when a all N variables are 0. Later we, first we explain this term.

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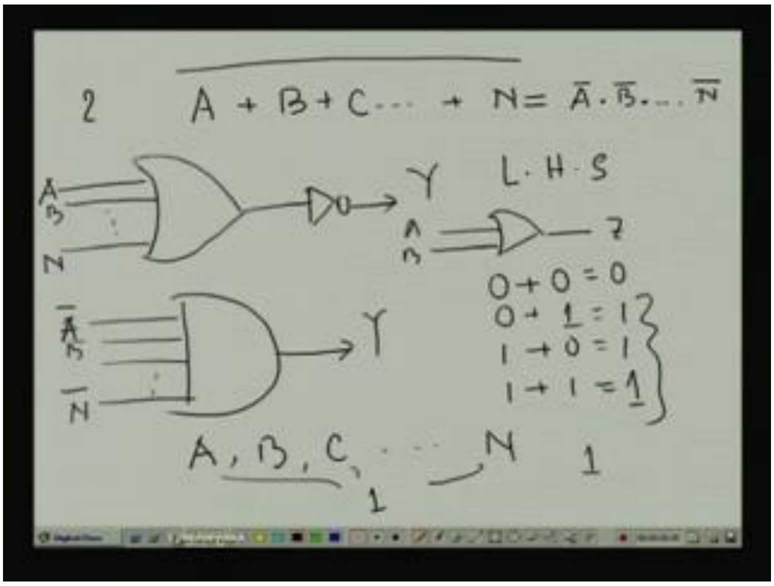

See, the second rule, we call that, again we are taking the, N such variables and there ORed and then it is inverted. Then Demorgan's law tells that, if you, you take each variable, inverted A bar, B bar and then up to N bar and we take, instead of plus operator, we are taking, as if dot operator, then the, it will be the result. So, the first rule tell the reversing, now, what will be the, if we, if we take the logic circuits, then it will be a, it will be the left hand side will be a, N input OR gate, N input OR gate, A, B and it is inverted, I said this my Y.

What will be right hand side, right hand side will be a AND gate, again all, the inputs are inverted, again this will be Y, so this two are same. Now, the first one, we have seen that, how from N gate, the OR gate can be derived here, form the OR gate, how N gate can be derived. So, mainly, that, if the, if we interchange the plus by dots and the each un complemented variable, the complemented or that non inverted elements are inverted or even vice versa, then it will give the same results.

Now, another thing is, that when it will, when the left hand side will produce a 1, see for the second case, that it is, it is a an input OR and what is the two input or, or truth table or truth logic. If it is a two input, we remember that A, it means A plus B, so 0 plus 0 is 0, 0 plus 1 is 1, 1 plus 0 is 1, 1 plus 1 is 1, see that, out of four, that three cases, it will be 1. That means, if any one of the input variable is 1, then the output is 1, so if we now extend this concept, that for N input OR gate, if any one of the N variable is 1, then output will be 1.

So, that means, from this A or from this A, B, C from N, if anyone is, anyone is 1, anyone either A, C or N, 1. Then, it will be, a 1 and left hand to the side, is actually the complement of the whole thing, the N input OR gate, see the results, so it will be 0.

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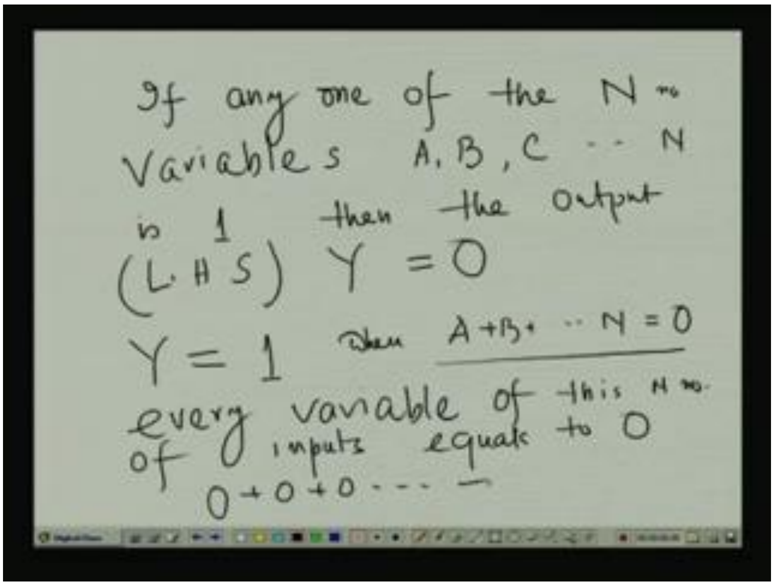

So, that means, that means, that if, if anyone of the N variables A, B, C up to N or if the N number of variables is 1, then the output, actually left hand side output, Y equals to 0. So, when the Y will be 1, so Y will be 1, Y equal to 1, when this A plus B plus N, this value equal to 0, when this value will be 0, when every, when every variable of this N number of inputs equal to 0, equals to 0. Then only, that is 0 plus, 0 plus, 0 plus, 0 then only the output of the, see that output of the OR gate.

Here actually, this output of the OR gate will be 0, when the each of this A, B, N will be 0 and if this value is 0, then Y equal to 1, so now, if we see the results.

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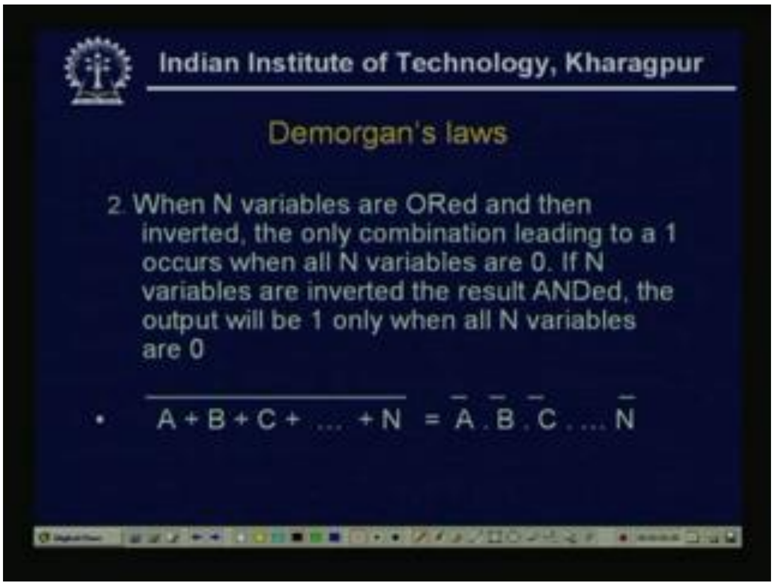

That, when N variables are ORed and then inverted, the only combination leading to a 1 occurs, when all N variables are 0, so this is the explanation of the first statement. Now, if N variables are inverted, the result ANDed and the output will be 1, only when all N variables are 0. So, if we see, now the, the previous one, that means,((Refer Time: 51:29)) see right hand side, right handed side is a N input AND gate and all the inputs are inverted variable,

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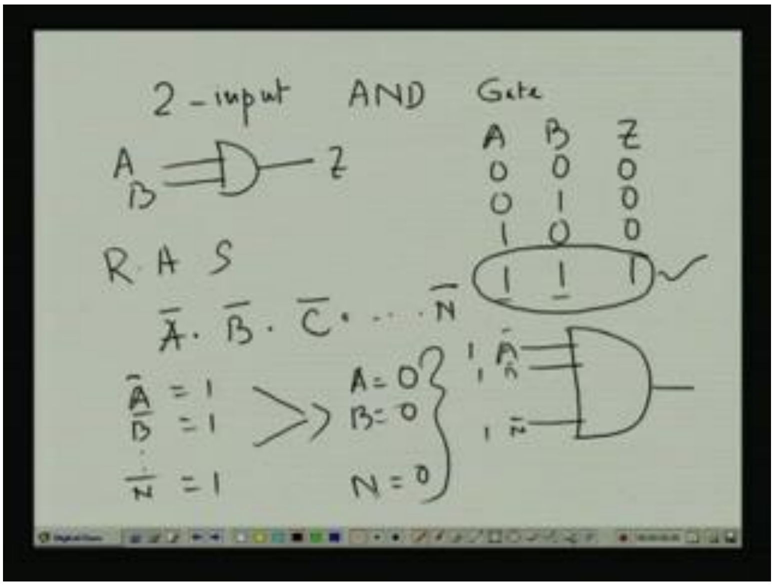

So, if we take the, just like the AND gate, 2 input, 2 input AND gate, what is the logic, so this is A, B, Z and the truth table of this is 0,0,0; 0,1,0; 1, 0, 0 and 1, 1, 1, so this is the truth table. Now, what, what was our right hand side of the second rule, this is A complement, dot B complement, dot C complement, up to N complement. Now, when it will be 1, because we have already seen that our left hand side becomes 1.

So, when it will be 1, only from the two, two input AND gate you have seen, this is the only situation, that when the output becomes 1, what is the situation, that when all the input variables are 1. So, if this is a N input AND gate, this is N input AND gate, then all the input should be 1 and what are the inputs A bar, B bar, N bar, so that means, all of this inputs should be 1. Now, if this becomes 1, that means my A bar should be 1, B bar should be 1, similarly my N bar should be 1.

And what does it implies, these implies my A equal to 0, B equal to 0 and similarly my N equal to 0, that means, all the variables, when all the variables are 0, then only, it generates a one output and this is the only situation of the second rule, Demorgan's rule that when one output will be generated. That means, when all the input variables, it is not inverted of that, non complemented or un complemented variables are 0s. So, if we see((Refer Time: 54:33)) see here, every variable of this N number of input equal to 0.

What we have achieved from the left hand side also and only this time, that my our, that output becomes 1 and similarly, if we see our the first rule, ((Refer Time: 55:01)) that when N variables are ANDed and then it is inverted and it is ANDed first, then it is inverted and it will be same, as that of OR, N input OR of the inverted variables. In this case, it is the, case is reverse, means the result would be 0, only when that every variable or each variable of this N numbers will be 1.

Because only that is the situation of the AND gate, when all the inputs are 1, the output will be 1 and then the, it is a complement, so it will be, it, this becomes 0. So, this becomes 0 and when it is, A is 1, that means, all the variables are 1 means, the each individual term, will be 0 and then 0 plus 0 plus 0 and this is the only, cases of only one case of the N input OR gate, when it generates the 0 output. So, this explains that, when the any, the output of the left hand side will be 0, when all the variables will be 1.

Similarly the, for the right hand side also the output will be 0, when all the variables N variables will be 1. So, today we finish this lectures here, but we see some of the quiz questions for the last classes, see that, for lecture 5 and 6 together, we have giving some of the quiz questions. So, the first question is, the two inputs A and B are inverted and applied to a NOT gate, construct the truth table.

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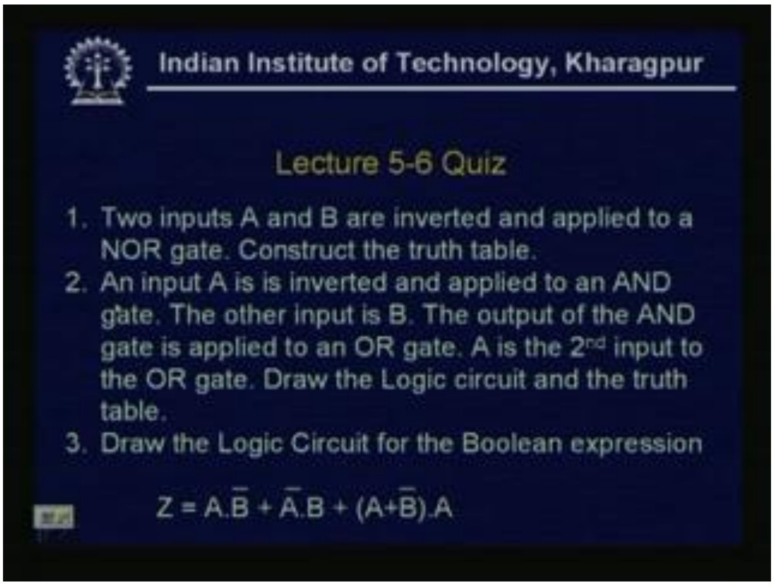

Second question is, an input A is inverted and the applied to an AND gate, the other input is B, the output of the AND gate is applied to an OR gate, A is the second input to the OR gate. Draw the logic circuit and the truth table, question 2 is from the lecture 5, draw the logic circuit for the Boolean expression, Z equal to A dot B bar means B complement, plus A bar dot B, plus A plus B bar, dot A, this can be anyone, we can follow anyone of the lecture 5 and 6, we can follow, in the lecture 5.

I have given the Boolean expression and the truth table and the lecture 6, we have discussed from the Boolean algebraic point of view, so anyone of the concept we can apply, to solve the question 3. So, these are the three quiz questions, so we end the lecture here.

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Lecture - 07
Boolean Algebra (Contd.)

Today will continue the discussion on Boolean algebra, but before that, first we see the answers of the last two lectures quiz, the lecture 5 and 6 quiz, so the first question.