Digital System Design
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Lecture - 03
Digital Logic - I

Today, we will start discussing on Digital Logic part one. But before that I want to revisit a quick summary of the previous 2 lectures.

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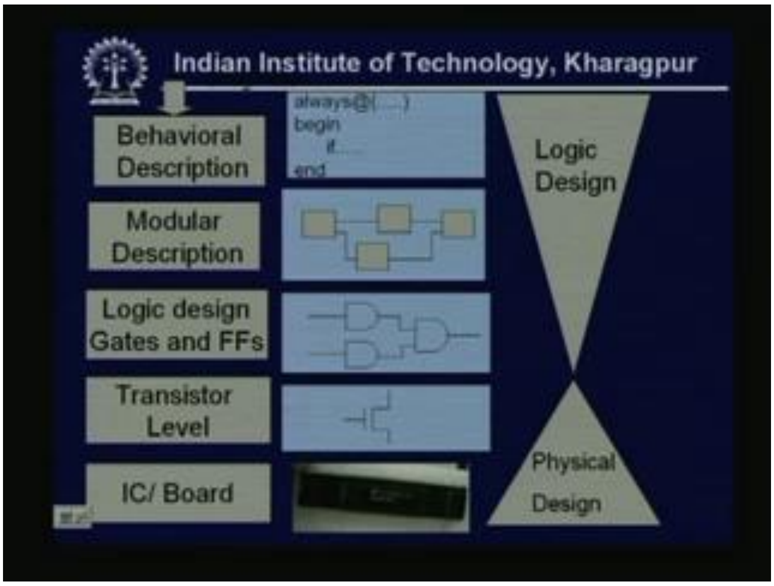

Mainly the previous 2 lectures I gave a introduction of digital system design, what do we mean by the digital design. So, starting from the design idea, that if I want the digital IC's, so first it should be the behavioral description means, that given the problem how I describe the behavior of my problem. Normally, now a days the CAD tools are used for that purposes, the mainly for the size and the complexity of the circuits.

And this is that varies small snap shots how the today's hardware description languages that a lower VHDL the look likes. So, this is you we can tell that behavioral description, description by hardware description languages, then the CAD tool will partition the whole system and some modules it will be generated from there. That means, the original system is partition into to some modules and we call the second phase is modular description.

So, just some block diagram level it was being shown, so how what do we mean by module, we have described that thing or what do you mean by modular design, what are the different type of approaches we have taken, say that top down design approach or bottom up design approach, so we have discussed that part. Then, the logic design synthesis the gates and flip flops the third phase, so once the modules are defined. Then on each module again it is partition into some subsystems or sub modules and this is nothing but, a switched off gates, flip flops, so the third phase is given by that.

So, what we can tell that starting from the now problem of the design idea, when we reach the logic design steps, this we are calling the logic design phases. And next the each gate are again represented by some transistors; that means, what we can tell, now the gate is a smaller subsystem and again it is partition into smaller systems; that means, the transistors. So, the transistor level module and this is shown in the fourth step.

And next and the last we can tell that actual digital IC's; that means, on a piece of silicon that actual transistors are fabricated and we get our final IC that is shown in the last step. So, starting from that transistor description or synthesis of the system using transistors and to get a digital IC, we call this is my physical design. That actual physical electronic equipments whether it is a transistor or whether it is piece of silicon that we have achieved, so mainly this is my aspects of our this digital system design class.

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Today we start the, the digital logic that mainly the topics will cover that basics of digital computer systems. Then, understanding the decimal numbers, mainly the number system we will read in this class, so binary, octal, hexadecimal and different binary coded numbers. Say, BCD numbers that we will read and then the conversion between different number systems, so today we will start discussing on the number systems first.

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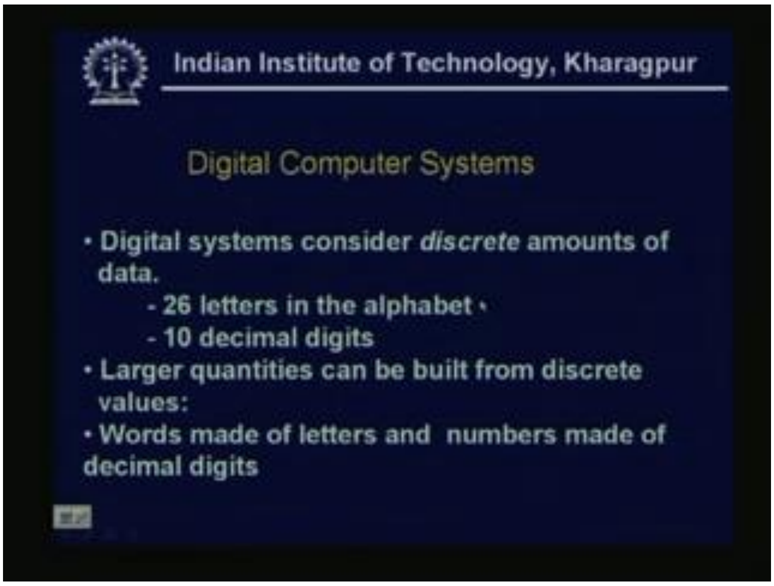

So, first the digital again the digital computer systems, that it consider the discrete amounts of data say 26 letters in the alphabet as an example or say our 10 decimal digits 0 to 9. Then, larger quantities can be built from discrete values, like say words, say computer that see again I can this is a letter o is a letter, m is a letter. So, each can be a these letters, these are some discrete non numerical letters, then the 10 decimal numbers, if I take a huge numbers say 9, 5, 6, 3, 2, 1, 0, then each one each decimal digit this is a discrete number.

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Again that computers operate on binary values 0 and 1, now why this binary are used because, it is easy to represent a electrically. Already in the introductory classes, we have discussed this part, then they say voltage and current, say if it is a power supply, which is generating 5 volt supply. Then say 0 to 2 we have just marked that as a 0 and say 2 to 5 we have not that as a 1, so these voltages and currents we can easily represent by 0 and 1. Now, this can be implemented using circuits and create the building blocks of modern computers, so this is the main basics of the computer systems.

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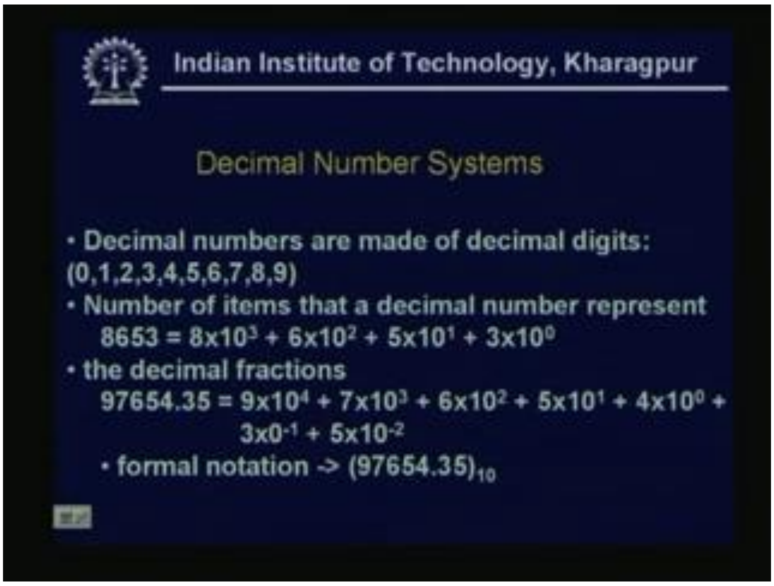

So, before starting the number system as the real word is more conversant with the decimal number system. So, quickly we go through the thing and then the what is a similarity of developing number system or the binary number system, we will see that thing. All of you know that this decimal numbers are made of 10 digits, normally the concept come from that 10 fingers of our hand and that is why it is restricted to 10 numbers.

And then, these are some notation say 0 to 9, no, this number of items that decimal number represent, say 1 decimal number is 8653. So, normally this gives a positional value; that means, all of we know this is the unit 3 is the place of the unit 5 is place in 10'th position 6 is placed in 100'th position and 8 is placed in 1000. So, 8 into 10 to the power 3 means 1000 plus 6 into 10 square 5 into 10 to the power 1 and 3 into 10 to the power 0, so this 8653 this is the number of items, actually the decimal number represents by 8653.

Now, if it is a fraction then how we can actually quantify this thing, similarly that 97654, so the position of 9 is if we start from here that 0, 1, 2, 3, 4 then 9 is a 4'th position. So, 9 into 10 to the power 4, then 7 into 10 to the power 3, 6 into 10 square 5 into 10 to the power 1, 4 into 10 to the power 0. Now, this should be minus 1 because, already I have reached 0'th position, so this is minus 1; that means, position of 3 is minus 1, so this is 3 into 10 to the power minus 1.

So, this should be the 3 into 10 to the power minus 1, then 5 into 10 to the power minus 2 and in formal notation we can write that 97654.35, when there number system is 10, now this 10 we call we define this as a base or radix. So, one thing from here it is from our common concept of decimal number system, that every number system has one base, here the base is 10. That means, that in decimal number 10 base means 10 distinct digits 0 to 9 are used to represent a number system.

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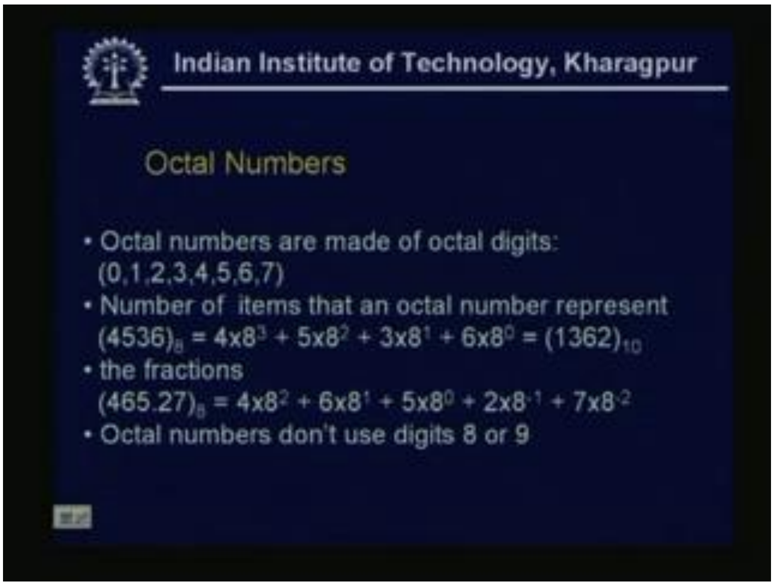

So, if we see now the octal numbers, so for the octal numbers the base is 8, so for this octal number system that 0 to 7, this 8 digits are used in the system to represent a number. So, the number of items that an octal number represent it should be that base is 8 and the positions are same, so if it is 4536 then this is 0'th position. Because, this is a there is no fraction here, this is 0'th position, this is 3 is the in first position means 1, 5 is in second position 2 and the 4 is in position 3 place value.

So, 4 into that it should be 4 into 8 to the power 3 because, base is 8, 5 into 8 square 3 into 8 to the power 1, 6 into 8 to the power 0. And if I do this thing if I do this computation, if I multiply this and then we do this addition I will be getting 1362 in our decimal system. So, 4536 in octal system is nothing but, the 1362; that means, 1362 in our normal decimal system, now how do we compute the fractions, so fraction say 465.27 in when the base is 8.

So, that integer portion again that will be 4 into 8 square plus 6 into 8 to the power 1 5 into 8 to the power 0, then it should be place value should be decremented. That means, 2 into 8 to the power minus 1, 7 into 8 to the power minus 2 and if we compute this thing, then we will get the decimal equivalent of the 465.27 in base 8. Now, one thing I notice that this octal numbers do not use digits 8 or 9, so only 0 to 7 this 8 numbers are used that is why the base is 8.

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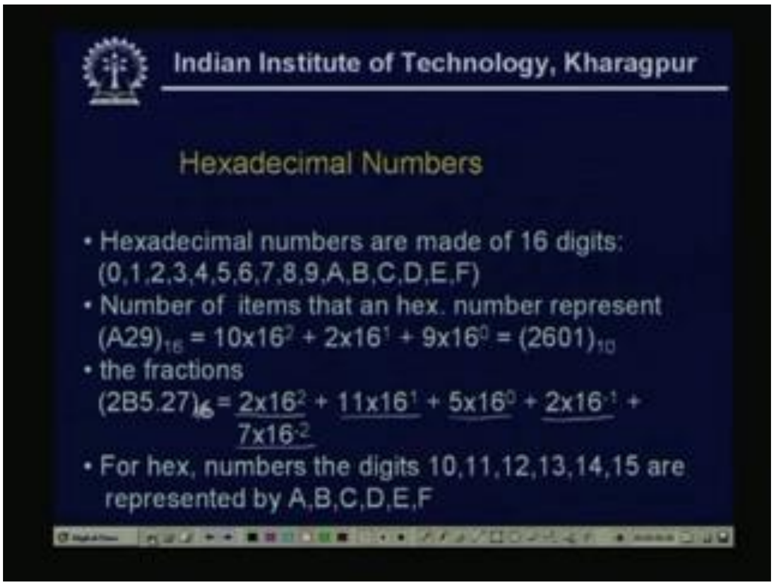

Now, similarly I have hexadecimal numbers, hexadecimal numbers are made of 16 digits. So, this should be 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, then 10, 11, 12, 13, 14, 15, so 0 to 15 these will be are the 16 numbers.

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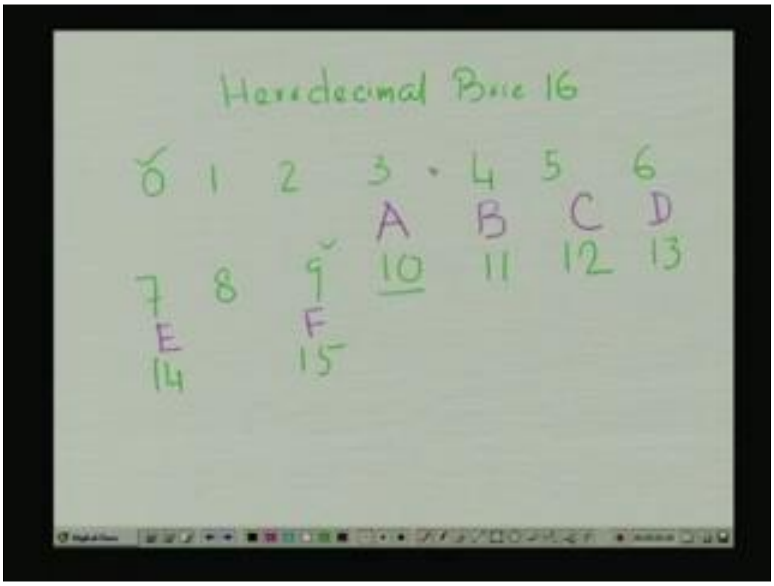

Now, see that as already that if I take that thing say if it is 16 numbers I take, so hexadecimal the base is 16. So, there should be 16 distinct numbers say 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 and 15, now from 0 to 9 these are all distinct. So, I will keep as it is. Now, see here 10 means again this is a repetition; that means, again the 1 and 0 these two values are used, so this is represented as A; that means, A means in hex or hexadecimal number system it is 10.

Similarly, this is B means 11, C means 12, D means 13, E means 14 and F means our 15. So, 0 to 9 and then A, B, C, D, E, F these are the 16 distinct numbers that are used in our hexadecimal numbers ((Refer Time: 16:48)). So, now number of items that an hex number represent, this is a similar only here the base is 16, so I am taking one number A29. So, our A means actually it is in decimal system it is 10, so this is 10 into 16 square because, that place value of A is 2, if I start from here it will be 0, 1, 2, so 10 into 16 square plus 2 into 16 to the power 1, 9 into 16 to the power 0.

And if I do this multiplication and addition then I will be getting 2601 in our decimal number systems. And the fraction can be treated as the same way, that we have handle the octal and the decimal, see if the number is 2B5.27 it should be in 16, so the base should be in hexadecimal, the base should be 16. Then it will be the place value of 2 is 2; that means, it should be 2 into 16 square, then the place value of B is 1, so it will 11 into 16 to the power 1, then 5 into 16 to the power 0.

And now after the decimal point, that place value is minus 1, so this is 2 into 16 to the power minus 1 as 7 means this is 7 into the 16 to the power minus 2. So, if we do this computation, we will be getting the number in the decimal number systems.

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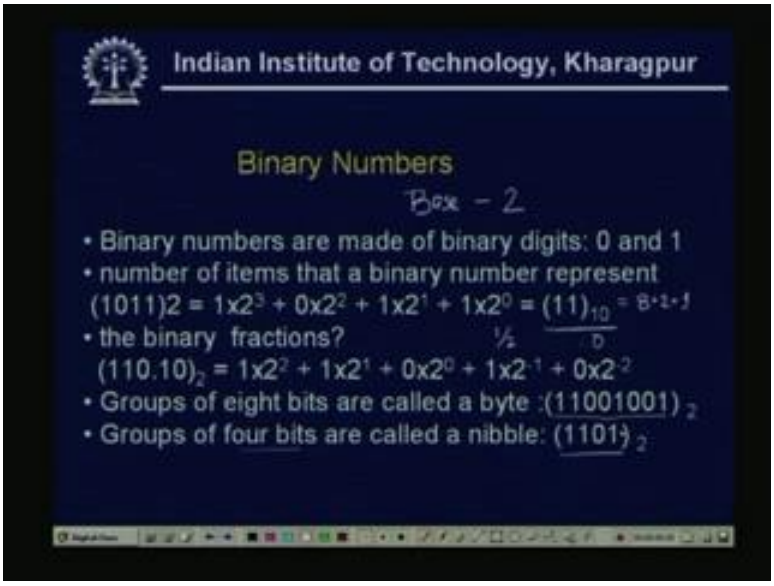

Now, we take our binary number systems because, already I several time I mentioned that in digital design that will be mainly using the binary numbers. And in the introduction also, also I mention that the mainly binary numbers are used for computer application and particularly for the digital process. So, binary numbers are made of binary digits, means 2 only 0 and 1; that means, here the base is 2, so here our binary number systems our base is 2.

So, number of items that a binary number represent as the digits has only to 1011 in base 2 it will be 1 into 2 to the power 3, 0 into 2 square 1 into 2 to the power 1 plus 1 into 2 to the power 0. And if we compute this thing say 2 to the power 3 is 8 this is 0 multiplied by 2 square means 0, so this will be 2, 8 2 plus 2 10, so this will be my numbers that 8 plus 2 plus 1. That means, my 11 and decimal number systems, now how I get the binary fractions again in the similar way.

That 1 into 2 square here 1 into 2 to the power 1, 0 into 2 to the power 0 plus after decimal point it is 1 into 2 to the power minus 1, means this is equal to half 1 by 2 this is equal to it is multiplied by 0, so it is of no many it is 0. So, this will give my decimal number, now this groups of 8 bits are called a byte, in binary number systems the 8 bits are called a byte. And groups of 4 bits it is called nibble, so if we generalize now that our number systems.

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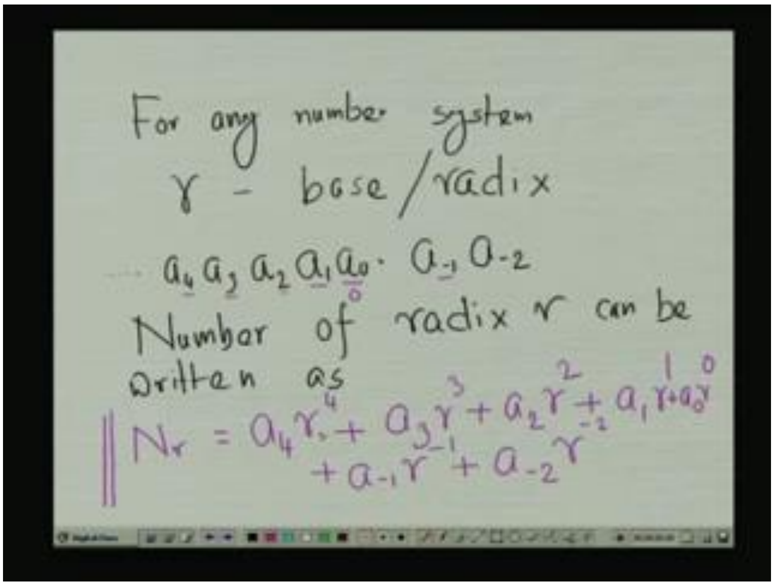

See that how we can first thing for any number system, say for any number system I need a base, say that r is the base or called radix. So, say I have one number in it is that number system whose base is a 0, a 1 base is r say a 2, a 3, a 4 and so on and after decimal it has some a minus 1, a minus 2, etcetera. Then, the number of radix r can be written as say I am representing this by N r and this should be, say if we start from a 4; that means, a position is here the position is 0, it is one already 2, 3, 4.

So, it should be a 4 into r to the power 4 plus a 3 into r to the power 3 plus a 2 into r to the power 2 plus a 1 into r to the power 1 plus a 0 into r to the power 0, this is my integer portion. Now, what will be my the fraction after decimal point, so it will be a minus 1 r to the power minus 1 because, the place value is now minus 1, then plus a to the power a minus 2 r to the power minus 2, so this is my general representation of a number system.

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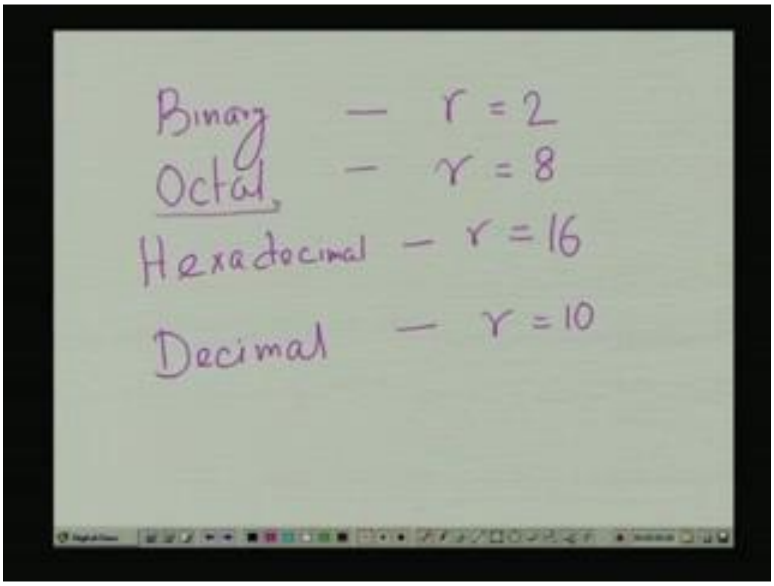

And so far we have discussed that for binary number systems that radix is 2, for octal that radix is 8, for hexadecimal radix is 16 and our known decimal that is our real word works in when the radix is 10. So, normally these are the four different number systems that are mainly used in digital design, now another thing is already we have noticed that mainly the binary are very suitable for that digital design.

So, that means, the even the number is in the octal number system or hexadecimal number system or in decimal number system, if somehow I can convert that thing to the our binary number system. Then obviously, it will be very efficient and advantageous for the implementation of digital logic, so now we see that how this conversion between two number systems can we can do.

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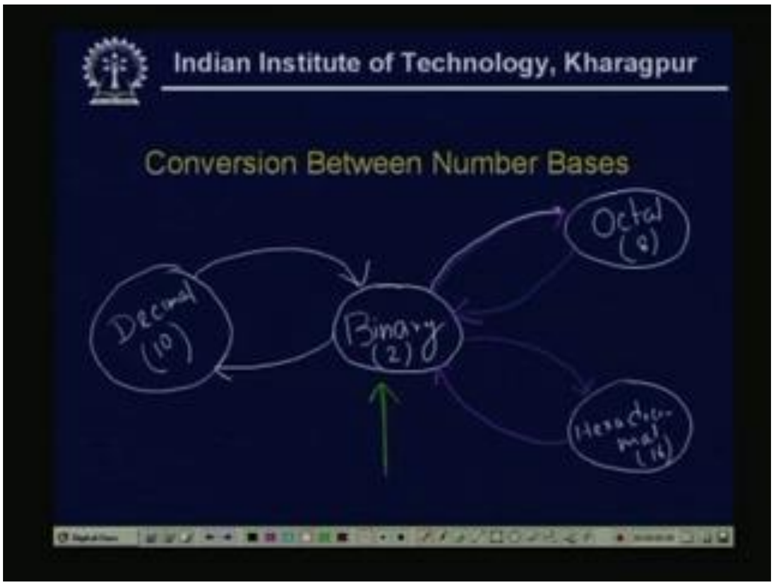

See, so what we have seen that we have four such number systems, our main where we concentrate or what is that main thing we want that is our binary number system, where the radix is 2. Now, our most common one that our digital world or the our real world walks, that is our decimal number system where the radix is 10. Just now we have read another two, one is called the our octal number system, where the base is 8 and the hexadecimal where base is 16.

So, conversion means I can do decimal to binary, that means given a decimal number the it I can convert it to binary and the vice versa. That means, if the binary number is given then its decimal it is also possible, now say octal if it is the similar thing will happen that binary to octal it is possible and octal to binary is possible. Similarly, the binary to hexadecimal is possible, hexadecimal to binary is possible, and again we will see that actually inter conversion between decimal to octal or decimal to hexadecimal, that is also possible.

But, here our main emphasis is on the binary systems and that is why we are more interested to be converted the or three decimal systems to the binary or from binary to other three. So now, we will see that actually how this can be converted.

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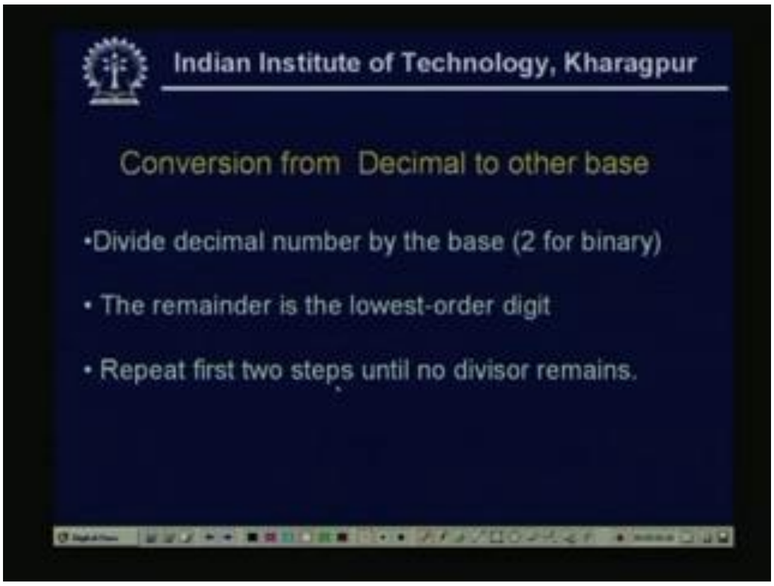

So, first thing is that we think that conversion from decimal to any other base, see first what we will do that this is a general principle or general rule I can tell, that define the decimal number by the base. That means, base means here the number systems that I want to convert from decimal number systems, see if I one decimal to binary; that means, my base is 2. So, that remainder is the lowest order digit and repeat the first two steps until no devisor remains. So, this is the general rule of our better you can tell this is the my general rule of conversion from decimal to any other number systems only the base will be changed.

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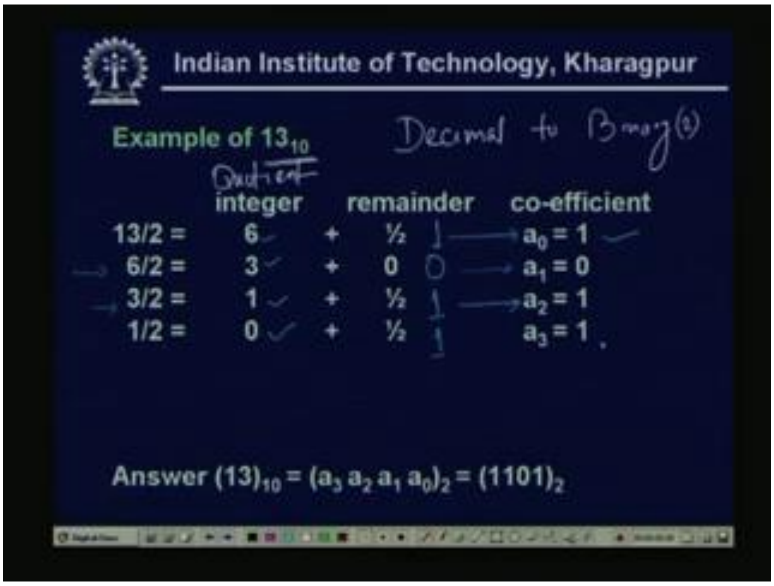

First we take one example, ones a very simple example say I have decimal value 13, so say this is my 13 when the base is 10, I want to convert that thing. So, first I will divide my I want to convert from decimal to binary, so my binary means base is 2, so 13 divided by 2. So, the integer portion; that means, my quotient I get this is actually my quotient I get 6 a remainder is 1, so if I write the remainder again, here my remainder should be 1.

And see that I am writing this remainder as if the coefficient say 0 equal to 1, now already I got that my question becomes 6. So, next step I am taking the 6 and I will repeat the definition, so this will be 6 divided by 2, so quotient is 3 reminder is 0. So, the again this is 0 I am taking that the coefficient should be a 1 is 0, now that question becomes 3, so again 3 divided by 2 in a next step. So, this becomes in the question becomes 1 and now the remainder becomes 1, so this is my next coefficient is 1.

Now, that quotient is or that number becomes 1, so again 1 divided by 2 now my question becomes 0 because, 1 is less than 2. So, my reminder is 1 and I stop because, my it will be stop when the quotient become 0, so this is my last coefficient that is 1 and I will be taking this coefficient because, this is my a 3. So, in reverse order; that means, a 3, a 2, a 1, is 0 this is my a 3, a 2, a 1, a 0, so this is 1101, so now if you want to check, so far the what we have done.

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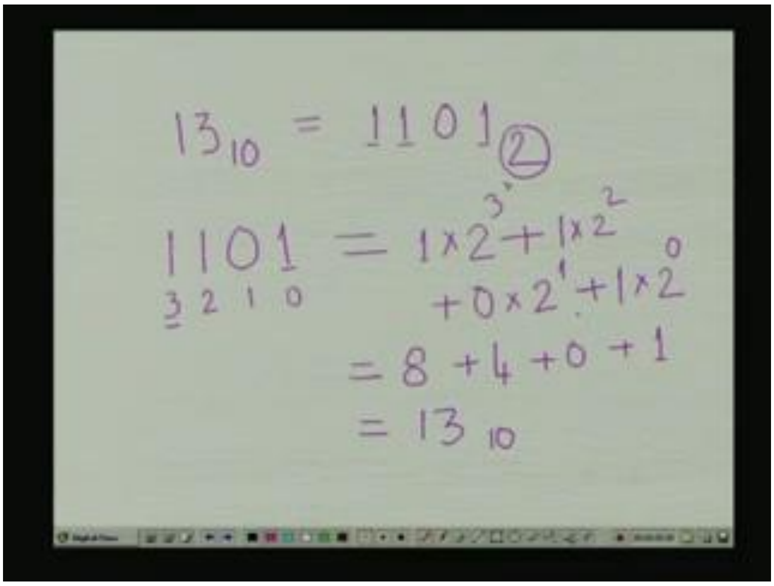

So, what I get, I get that my binary representation of 13 binary representation of that 13 in decimal system I got 1101. So, if I see that what is that 1101 in decimal, if I want to check, then see this is my 0'th position, this is my first 1 position, 2 and 3 this is in base 2. So, what will be that thing, this 1 into this is 2 to the power 3 because, place value is 3, then, 1 coefficient is 1 into 2 to the power 2, then plus 0 into 2 to the power 1 plus 1 into 2 to the power 0.

So, what will be the value, then it is becomes 8, then 4 this is 0 multiplied by 2 then this is 1 into 2 to the power 0 means 1. So, this becomes 13 in 10 decimal is it, so it is verifying the result.

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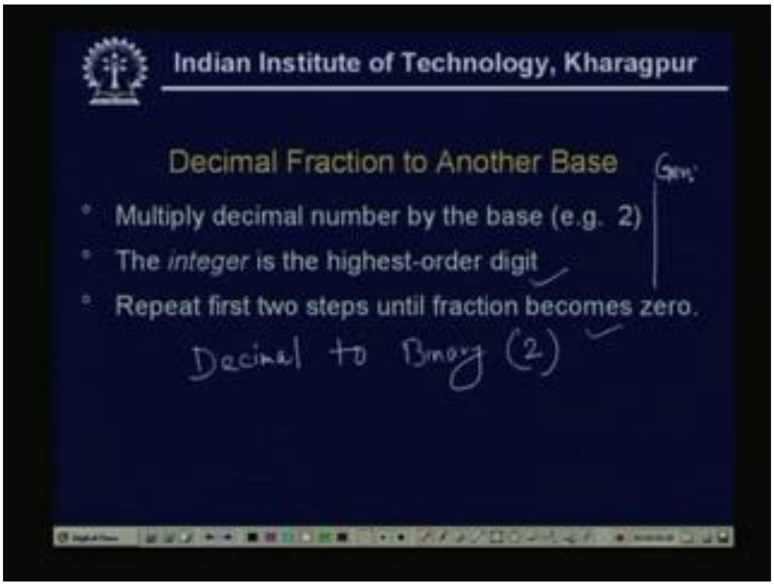

Now, decimal fraction to another this, so just now what we have seen that how the if it is a decimal integer number, then how it is change to the any other base. Now, if it is fraction; that means, after decimal point, then instead of division we will be doing multiplication. So, what will be the general rule, the general rule is that multiply decimal number by the base again I am considering that this is for say for decimal to any other number system say binary then this is for 2.

So, we will be multiplying the integer is the highest order digit, so the integer I will get I will keep that thing, now here it is the highest order digit. That means, a reverse one and repeat the first two steps until fraction becomes zero. So, this is my general rule of converting the decimal fractions to any other number systems or either it can be binary, octal or hexadecimal.

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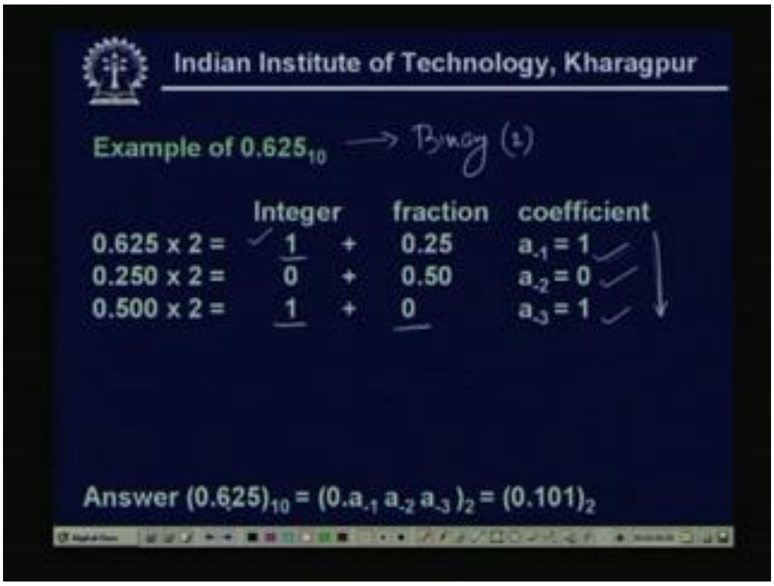

See one example first, so now I am taking that 0.625, this is in a decimal number systems. So, what was the rule that rule is that multiplied it should be multiplied by the base, so here we are taking that this two binary this two binary means 2. So, 0.625 into 2 what we will be getting, we will be getting if we multiply this thing, we will be getting 1.25, see this integer is 1 and the current fraction value is 0.25.

So, this is the coefficient is now this is the highest, that is why that a minus 1, that minus 1'th coefficient minus the position value is minus 1'th and the coefficient value is 1, this is the this integer I got. Now, I take the current fraction; that means, 0.25, so 0.250 again I multiplying by 2, so it would be 0.50, so integer is 0 current fraction 0.5 so my a minus 2 means the second position coefficient is 0.

Now, again 0.50 into 2 because, I have to continue until I get my fraction to be 0, so here it is 0.50 again now if I multiply I get a integer 1 and now my fraction become 0. So, the coefficient is 1, now I will take in this order that 101 not in the reverse order, that when we have taken that we have converted the decimal integer to binary, we have taken the reversal. Now, we are taking that the correct order that 101. So, 0.625 means that 0 is that actually before decimal this is 0, then a minus 1, a minus 2, a minus 3 and these are nothing but, 101, so this is 0.101 to in base 2 now this is my conversion.

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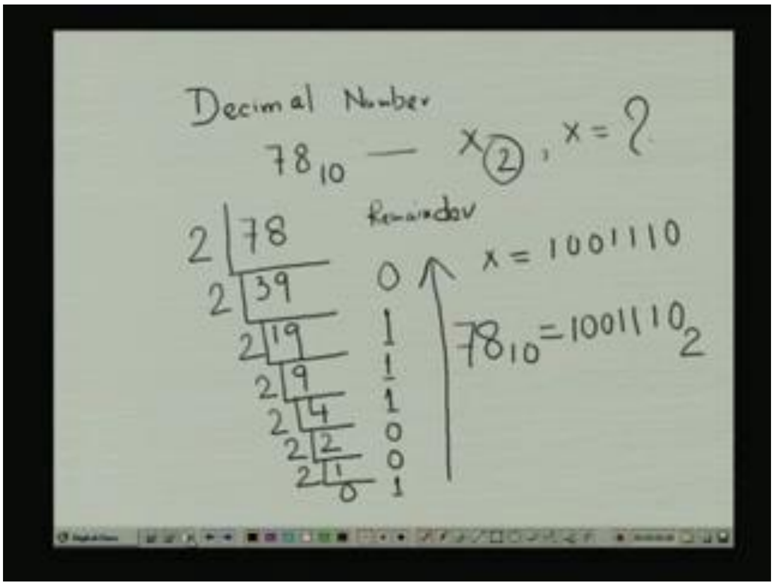

So, now we take we do one example see we are taking now one decimal value, first we see the some decimal value, some decimal number say 7, 8, 9, 10 of course, say 78 when the base is 10. So, first how we will I want that this to be some it should be converted by some value in when the in binary system and I want to know what is my value of x. So, what was the general rule because, this is pure integer value, so I will divide by 2 because, in binary system the base is 2.

So, I will divide that thing by 2, what will be the value that this is 39, my remainder is 0, so I am keeping the remainder same and this is my remainder. Now, this is my quotient again I am dividing by 2, so this is 19 again my remainder is 1, again I am dividing by 2, 9, 18 remainder is 1 again I am dividing by 2 remainder is 1, again I am dividing by 2 remainder is 0, I am dividing by 2 remainder is 0, I am dividing by 2 now my quotient become 0 and this remainder is 1.

Now, what I will be doing as it is as integer I will be taking in reverse ((Refer Time: 43:57)) my x is 1001110. So, it means that 78 in decimal number system is same as that of it is converted to 1001110 in binary number system, so this is if I do in this decimal.

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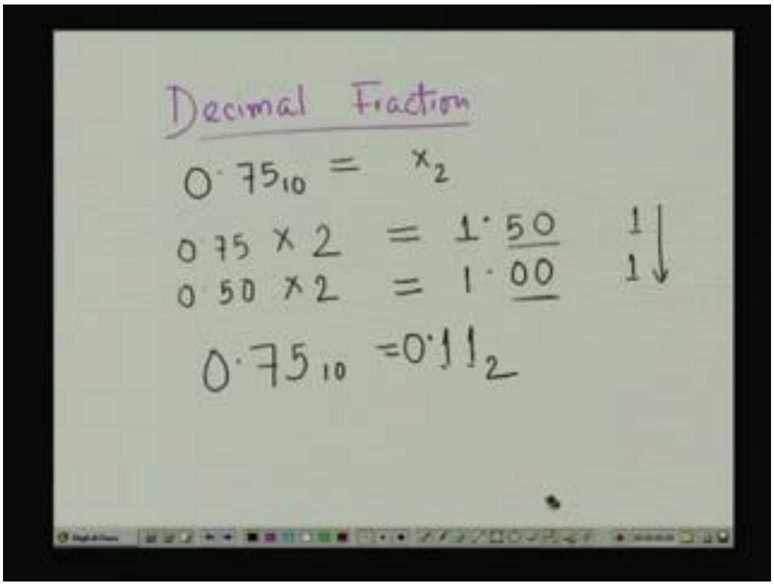

Now, if I take one say fraction, say I am taking some decimal fraction now I am taking some decimal fraction. Say I am taking 0.75 and again I want that some binary value, now what is the rule, now here the rule is the some multiplication by the base here the base value is 2. So, what I will be doing 0.75 into 2, what will be the thing, so 1.50, so my integer is 1 and fraction is 0.50, so 0.50 again into 2, so this will be I know that this will be 1.00. So, this becomes again the 1 integer value is 1 and we will stop because, my fraction becomes 0.

Now, I will take in normal direction; that means, first I got 1 then I got 1, so it will be in the in this direction. So, 0.75 10 will be 11; that means, that this will be 0 means 0, so this should be 0.11 2, so we have seen that how the decimal number to binary that can be converted. See, now I am taking see if I want decimal to octal because, when we have a given the general load, that my base can be anything because, the from decimal number system to any other conversion binary or any other number system I can convert.

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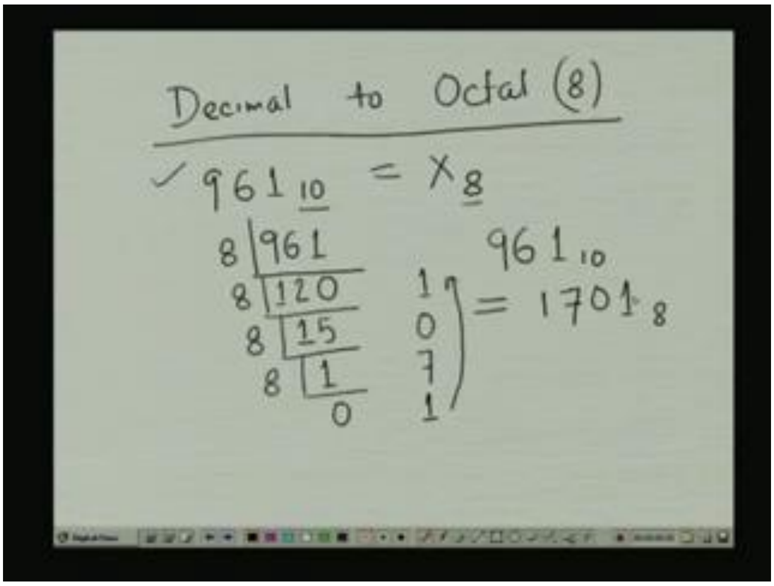

If I want, so now, we see that how we can convert the decimal number system to octal, now decimal number system means the base is 10 and octal means the base is 8. So, if I take an example, say 961 then in decimal number system, what it will be in octal means base is 8. So, what was our general rule that was the division and division by base, so again I am taking that number 961 I am dividing that thing by my base 8 So, it will be 120 the quotient and the remainder is 1 again I am taking the division I am doing. So, quotient is 15 remainder is 0 again division now the remainder is 7, again I do the division, so question become 0 it will be 1. And now if I take a reverse direction, so 961 in decimal value it will be 1701 in octal system on the similar way I can do the fraction also.

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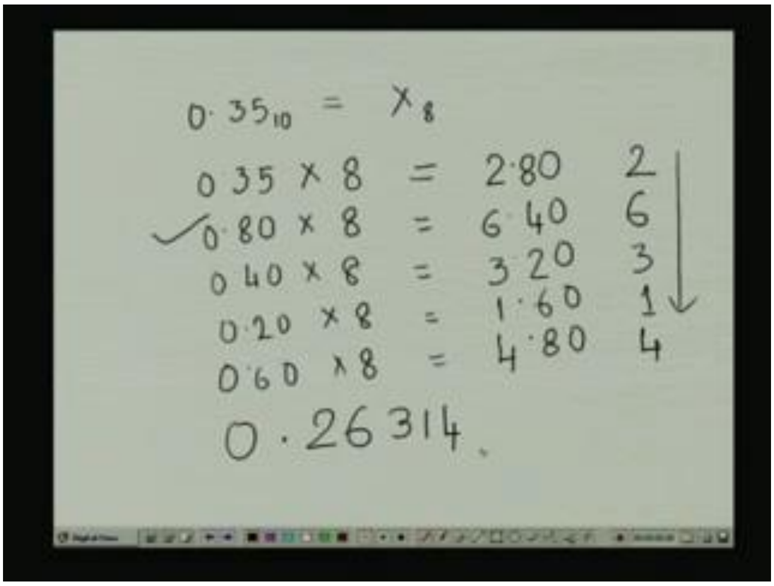

Let us take one example, so if it is the see some decimal we take that 0.35 and then what will be doing, again I want some x in octal. So, now rule is multiplication, so 0.35 again this will be multiplied by 8 and what will be the value, this will be 2.80, so my integer becomes 2. Now, this decimal is again it will be multiplied by 8, so this will be 6.40, then again this will be 40 into 8, this becomes 3.20 and it will be, so on, say again if I take 0. 60 and then it will be see again that point 80 has come back.

So; that means, from here it will be a repetition, so now if I take in the same direction it will be 26 and 0.26314 and again it should come that 6314 should be repeated. So, this is my that 0.35 in our 10 decimal number system, so in that way that from decimal system to any other number systems I can easily convert.

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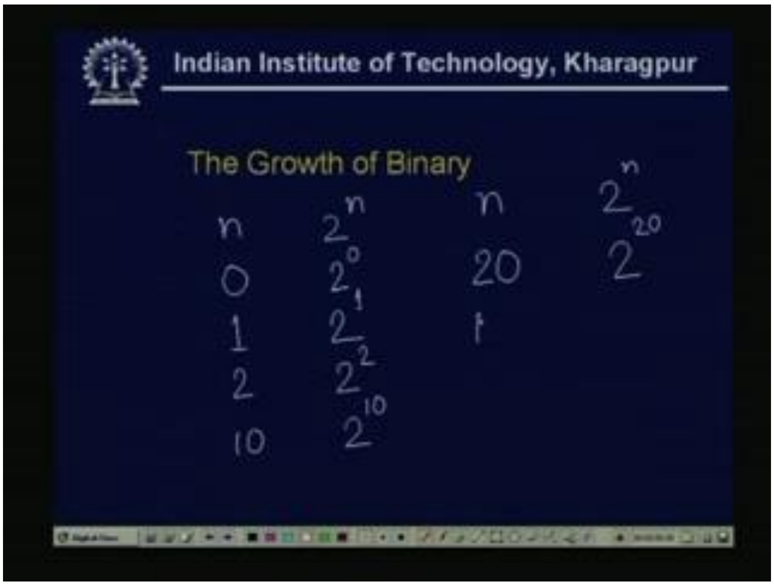

Now, again if we come back to the binary and the our decimal system because, our main emphasis is the binary; that means, how the we can convert the numbers from any other number system to binary number systems. So, if I consider that the number should be seen in this way, just if I form a table if it takes n and what would be the value of 2 to the power n. Means, say if my value is 0 then it will be some 2 to the power 0, if my value is 1 then it should be 2 to the power 1, if it is 2 then it will be 2 square, if it is say 10 then it will be 2 to the power 10.

Now, say if it is 20 say if my n value is; that means, if my 10 value is 20, then it should be 2 to the power 20. If it is 40 or say then it will be 2 to the power 40 like that it will be going the binary numbers will be going.

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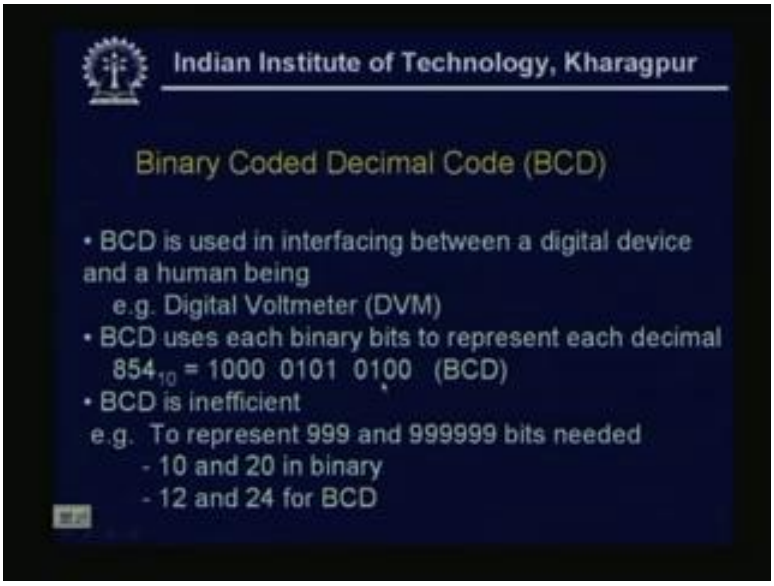

Now, another is a binary coded decimal code, it is called that BCD, the BCD is used for applications where the it is interfacing between digital device and the human being. Say that digital voltmeter and the BCD uses each binary bits to represent each decimal, see that if we take one example this is a 854 in decimal number system. Then each decimal number is represented as a 4 bit binary numbers; that means 8 means it is a 100 just now the way we have converted, if I convert that thing to 8, then it will be 1000, the 5 means it will be 1010 and then 4 means it is a 0100.

So, this will be the BCD code, now another one thing is this BCD is very inefficient why, see if I take this example 999, then 10 bits are needed if I convert it into or I represent into a binary number system. But, 12 bits are needed if I convert it into a BCD number system, see that 2 bits are extra and if the number increases, then it is actually it becomes 20 to 24. So, this is not BCD is not always efficient, so binary is very efficient in that sense.

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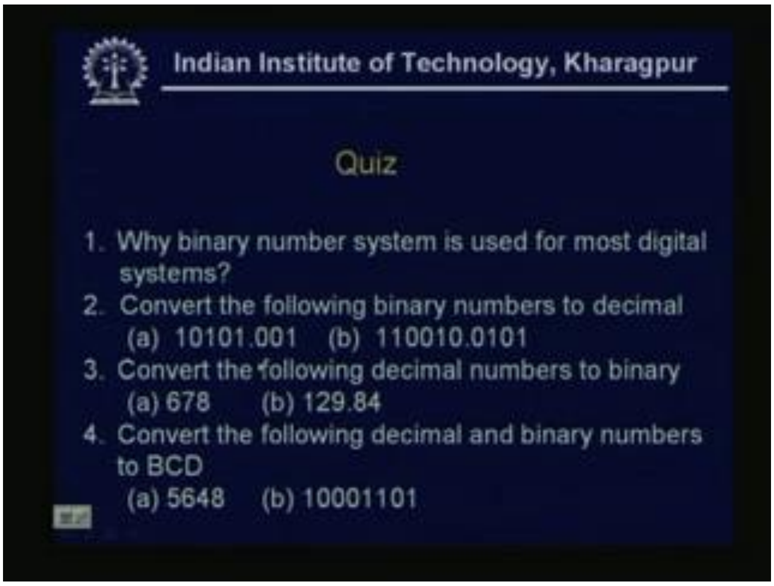

Now, we stop here and I give some small questions that quiz, that very simple questions that why mainly based on this today's discussing and why binary number system is use for most digital systems. And then some of the conversions a this is just for practice, that convert the following binary numbers to decimals because, here already there in base 2 and I have to do the reverse form.

Again here it is a binary to decimal and convert the following decimal numbers to binary, that this 3 and convert the following decimal and binary numbers to BCD. So, I will stop today's class and this again digital number systems, we will be continued in the next class. Before starting the today's lecture first we quickly go through the latest lectures answers of the quiz, the first question was why binary number system is used for most digital system.

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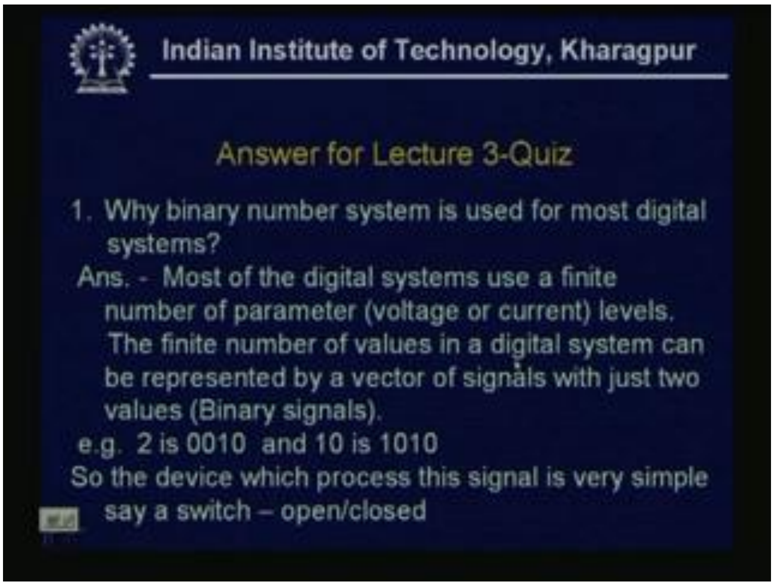

Now, as we know the most of the digital systems use discrete number of parameter levels, see the voltage levels or current levels. And the finite number of values in a digital system can be represented by a vector of signals with just two values, this is a binary signals means that 2 that can be represented by a string of 0 and 1 0010 or 10 that is 1010. And the device which process the signal is very simple to implement, say a switch either 0 or 1 means switch on or switch off or open or closed. So, that is why that mainly binary number system is used for more most of the digital systems.

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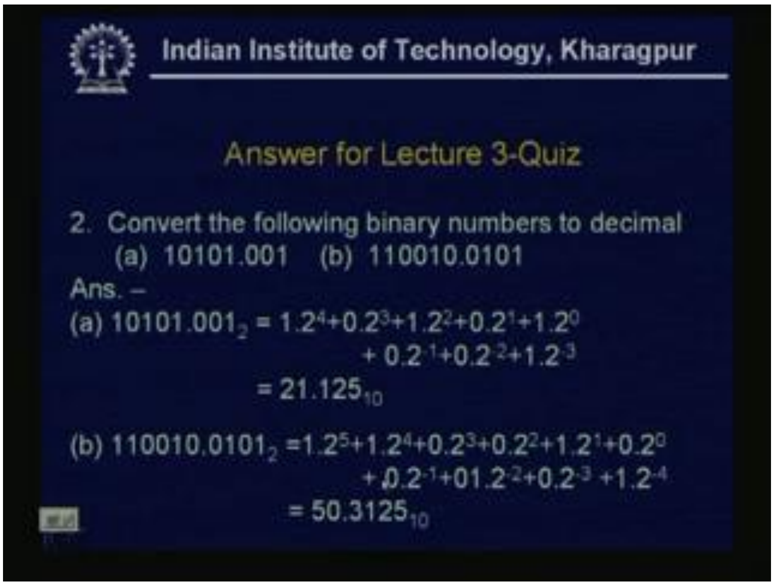

Second question was the conversion the following binary numbers to decimal, so as here the binary radix is 2. So, using the formula we did this should be 10101 means 1 into 2 to the power 4, 4 is the plus value, similarly 0 into 2 to the power 3 plus 1 into 2 square 0 into 2 to the power 1 1 into 2 to the power 0. And the non integer part 0 into 2 to the power minus 1 0 into 2 to the power minus 2 1 into 2 to the power.