Digital System Design
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Lecture - 10
Boolean Function Minimization (Contd.)

Today, we read some more things on Boolean function minimization.

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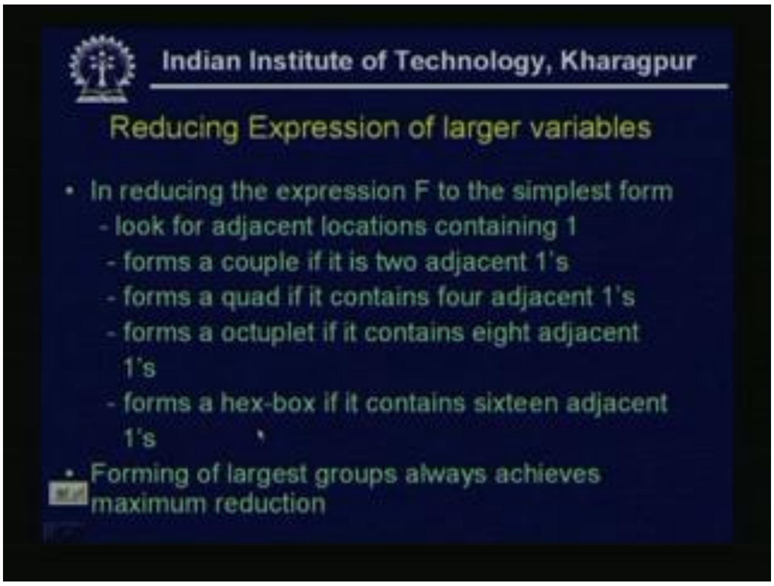

If I quickly summarize the portion, that already we have read in reducing the expression using K map, that if we draw one K map for a given expression. Then first we see the adjacency of 1's present in the K map. Then we try to form the couple means 2 adjacent ones quad 4 adjacent 1, octuplet the 8 adjacent 1 or a hex-box means which contains a 16 adjacent. And always we will try to form a largest grouped because largest grouped always achieves maximum reduction.

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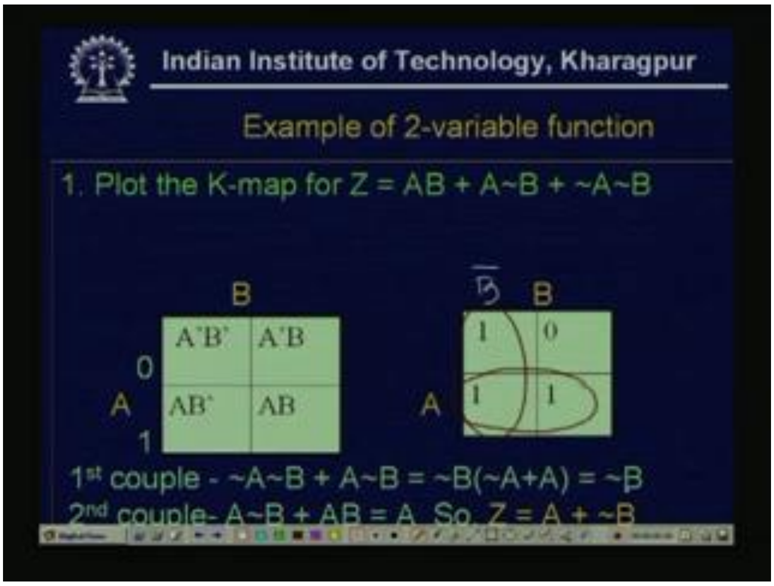

We have seen last day one two variable function depend if we apply the rule of reduction, then this is one example that Z is A B A B bar plus A bar B bar, then what we have seen that the there are three minterms, so there will be three 1's in the K map. So, the first couple will be the one the first column the first column as well as another couple is the second row. So, the first couple will be A bar B bar, because that first column represents that thing that A bar B bar plus A B bar and that will be nothing but, B bar, which is which represents the first column that B bar.

Similarly, the second couple is the nothing, but second row A, so this is reduced to A plus B bar. Last day we have seen that thing, now today we see the how this rule is applicable for three variable function.

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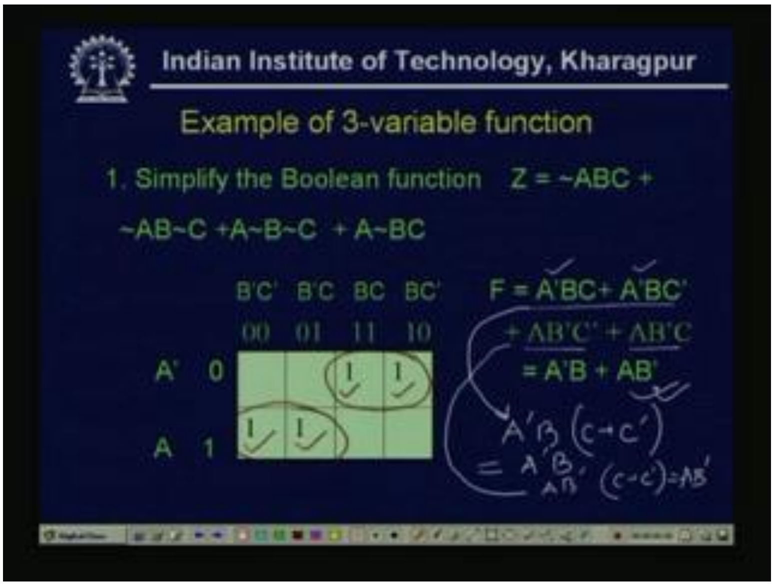

We take one 3 variable function, now function is that A bar B C plus A bar B C bar plus A B bar C bar plus A B bar C. Now, if we draw the Karnaugh map see here there will be 2 rows and four columns because it is a 3 variable, so it has 2 to the power 3 means 8 minterms. Now, in these out of these 8 minterms there are only 4 minterms present in this particular function, what are these 4 minterms, so one is A bar B C, so in the table this is A bar and this is B C, so this is 1, so this is one A bar B C.

Now, A bar B C bar see A bar B C bar, so this is another minterm, then A B bar C bar A B bar C bar means the 3rd minterm, 4th minterm is A B bar C, so this is another min term. Now, if we try to find out the couples or quads you see here only two couples exist, so this is one couple this is one couple and this is another couple. Now, if we just see the how the first couple reduces, because the first couple for the first couple the row is row represents the literal A compliment means A dash.

And the two columns are B C and B C bar see C and C bar are the complimented as well as the uncomplimented one exist, but B is fixed. So, the thing will be A bar B or in other way if I draw or if I write the minterms of the first couple these will be A bar B C this is one min term A bar B C. And the second couple this is A bar BC A bar B C bar then if I from these two if I take A bar B common A bar B.

So, from here if I take A bar common, then C plus C bar C plus C bar is nothing, but 0, so this will be A bar B similarly, the other couple this becomes and that A B bar C bar A B bar C bar and A B bar C. So, if we take the second couple this becomes from here, if we take common the A B bar then it will be C plus C bar and this will equal to A B bar, so I get A bar B plus A B bar.

Now, from here also I can directly tell from the Kernel map I can directly tell that this will be this 4 terms will be reduced to this 2 min terms, because the first couple reduces to the a compliment as B is fixed. So, A bar B for the second couple the second row represents the literal A and for the first 2 columns. See this is 0 0 and 0 1 means that B bar is fixed, B bar is fixed for the first 2 columns, where as C exist both in complimented and uncomplimented form, so it will nullify. So, from the 2 first 2 columns B bar will be there and the for the secondary it will be A. So, it will reduces to A B bar. So, in that way we can directly that it will be A B bar.

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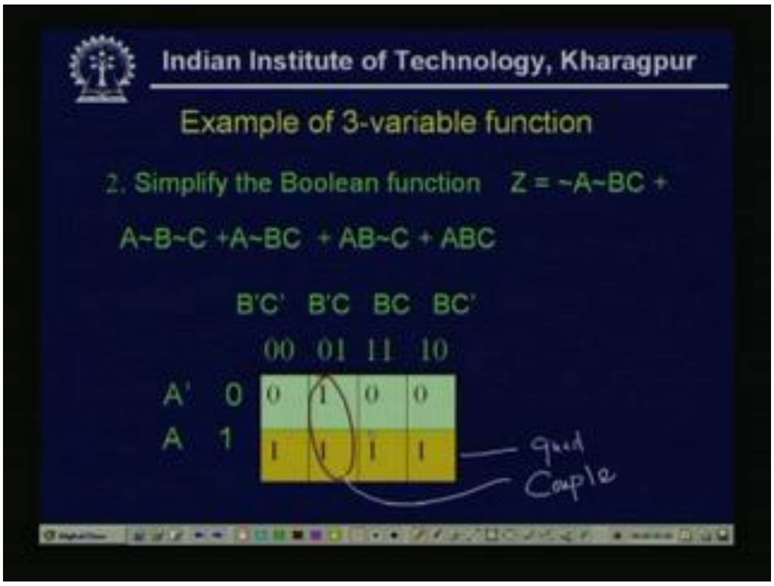

Now, we take another three variable function that A bar B bar C plus A B bar C bar plus A B bar C plus A B C bar plus A B C. Now, if we here there are 1, 2, 3, 4, 5 minterms, so all min terms we put in the K map and the 5 minterms will be… So, this is my A bar B bar C this is my A B C bar A B bar C A B C A B C bar, now see here we are getting one here we are getting 1 quad. So, this is this is 1 quad. And we are getting another couple this is one couple, so this is 1, so in this K map one couple exist and 1 quad exist. So, what will be the quad reduces to that.

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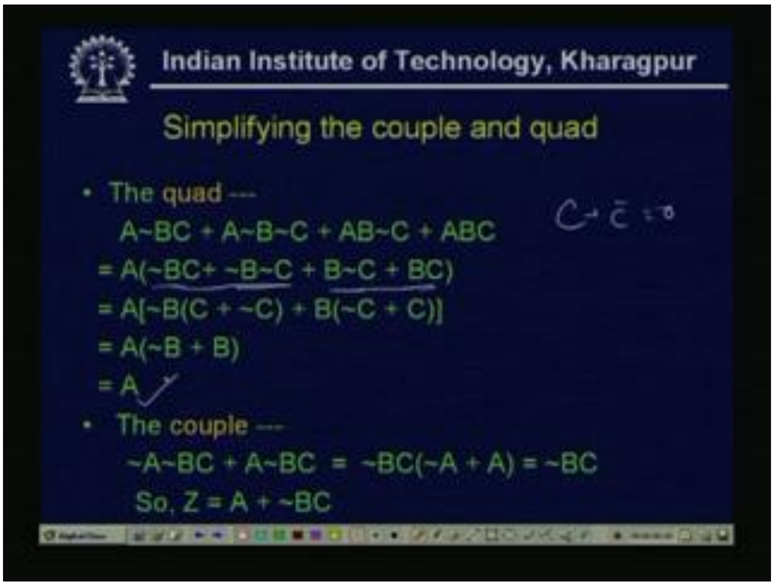

See the quad will be the second row, so second row means this 4 minterms, so from here if I take the first we notice that each of the min term contains A the literal A. So, I take A common then it will be the term will be A then B bar C then B B bar C bar B C bar B C. Now, see the from the inner expression if the we consider the first 2 minterms then that B bar is common and then we take C plus C bar. Similarly, from the last 2 minterms B is common, so this will be C bar plus C, now C plus C bar is 0, so A B bar plus B and again B plus B bar is zero, so the quad reduces to A.

Now, see that from the map itself I can easily tell that this quad means A, because see that B bar C bar B bar C B C B B C bar, so all the complimented and uncomplimented of the two literals B and C exist in the quad, so they will nullify. So, simply that for the second row of the literal that means, A that will be the result, so this quad reduces to A now here from the couple that it will be only, because it is a second column that means, A bar and A will nullify.

So, only the column that means, B bar C the min term B bar C will be the min terms for the couple. Now, we see if we reduce that thing whether it comes or not see that the couple that A bar B bar C and A B A bar A B bar C. So, from there B bar C is common and A plus A bar and that means, that B bar C. So, which is nothing but, the column the second column, so Z is the quad reduction of quad A and the reduction of couple B bar C, so this will be A plus B bar C.

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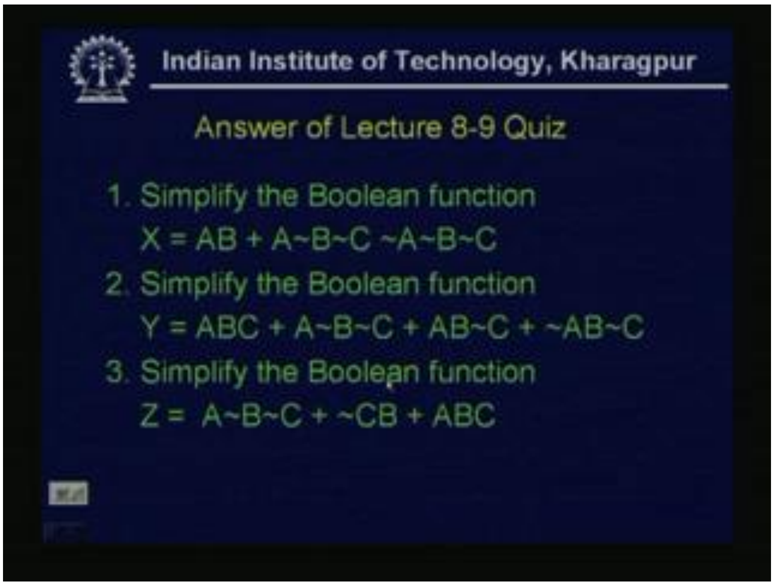

So, now we see the last day's answers of the quiz, the three questions we have given and mainly that is the simplification of the Boolean function. Now, this is the all the three questions are nothing, but the three variable function reduction, so the just now we have discussed the same way if we see the first one.

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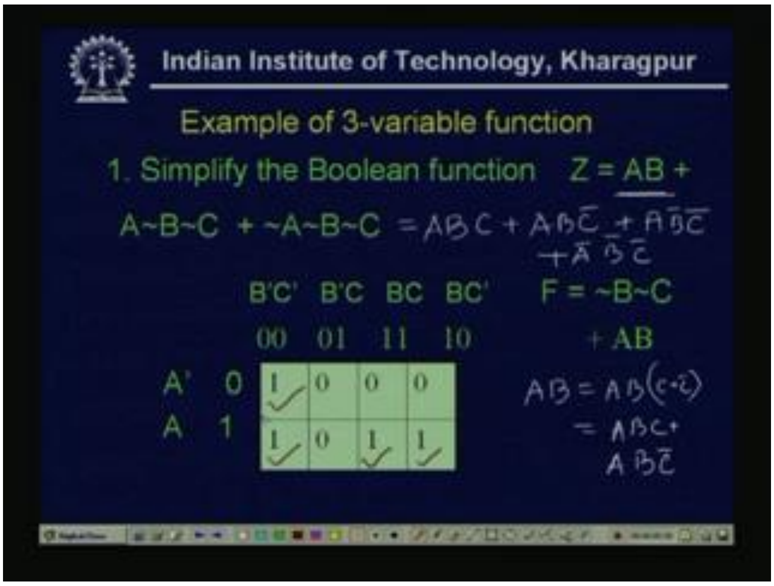

The first one is the A B plus A B bar C bar plus A bar B bar C bar, now one thing we notice here, see that here this is the three variable function, but the 1st minterm A B here one literal is absent. Now, if I introduce the literal which is absent there that means, A B becomes A B is A B into C plus C bar means this becomes A B C plus A B C bar. So, that is why now these three variable function where 3 minterms are there now it has been extended to 4 minterms that means, A B C plus A B C bar plus A B bar C bar plus A bar B bar C bar.

Now, see this is actually initially, we are not reducing rather we are extending that thing, but this will help as already we have discussed this will help the drawing of K map or just putting the 1's in K map. So, now we put, now we can easily put the four 1's. So, this will be the A B C this is my this is my A B C, so this is 1 is for the A B C, now A B C bar this one is for A B C bar. Now, this is for A B bar C bar and only one is left A bar B bar C bar, so these are the 4 minterms.

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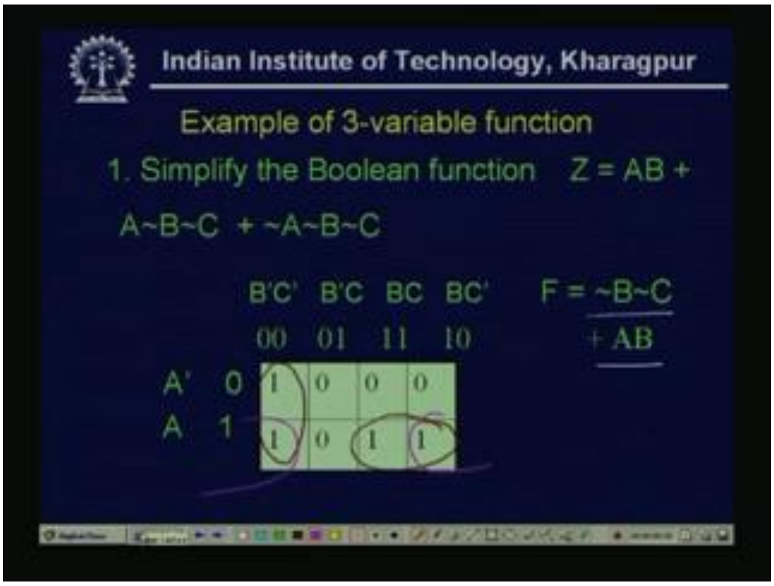

Now, if we draw the couple what will be the thing, if we draw the couple then this is one couple identified and this is another couple identified. So, what will be from the K map itself it the first column represent B bar C bar and so, this is B bar C bar for the first column. And the other couple is the C the for second row it will be A and B C and B C bar means only B. So, that will be A B, now just notice that according to the rule there can be another couple.

See this can be say if I take this one and this one then this can be another couple, but see this one is already included in this couple and this one is already in could included in this couple. So, we have just ignored this thing and it will, it is giving that a reduced one, later again we will consider this thing in details.

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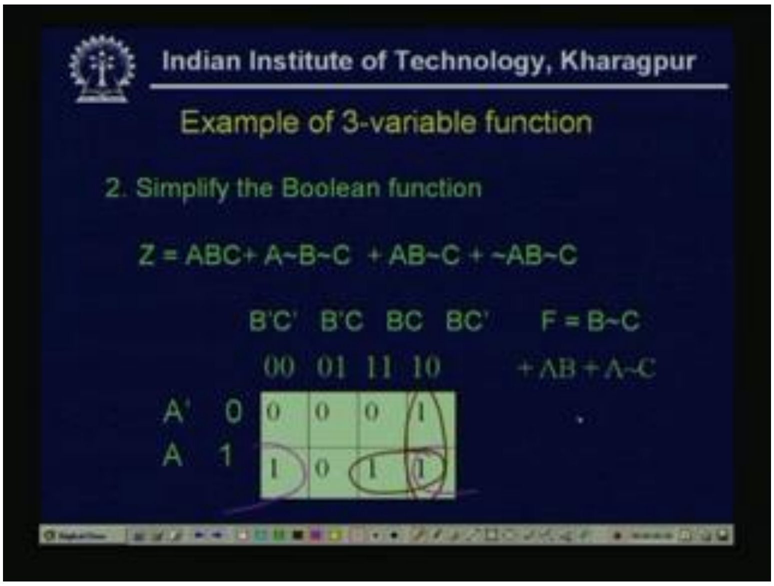

So the second example, the second question of the quiz this was a three variable function and here it was the function is given this, so again the same way, because here all the minterms consists the three variables. So, immediately we will give the 1's for each a minterm and we will try to find out the couple, now we see here that the couples are this is one couple, this is one couple and see this one is again left, so we will put that this is and this 1. So, there will be 3 minterms reduce terms, but again for the three variable literal all reduce to two variable and similarly, we can the third one, third one is also you can do in that way.

Now, already we have seen the 3 variable that 3 variable function, now we see that how they work for the four variable function. Now, first we see one four variable function, See the four variables are A B C D.

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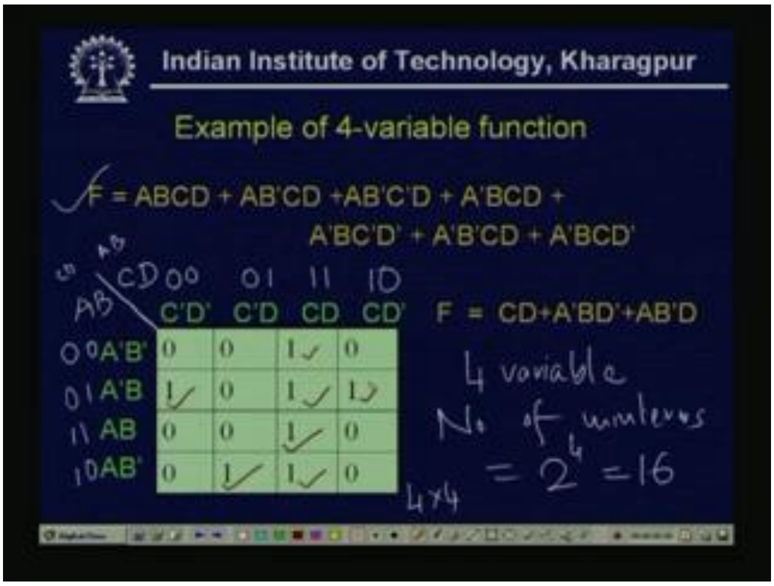

And this is one function given that F is A B C D plus A B bar CD A B bar C bar D plus A bar B C D A bar B C bar D bar A bar B bar C D and A bar B C D bar, now as here that it is a 4 variable, it is a 4 variable. So, number of minterms will be four variable functions the number of minterms is 2 to the power of 4 equal to 16. So, we the K map size will be 4 by 4 means 16 and we divide this as it column, we are giving A B these 2 variables and the rows we are giving A B and columns we are giving C D that can be vice versa also.

So, for this particular example we have taken A B or C D, this can be this can be C D this can be A B, but for this example we have taken this. So, for all possible combinations of A B that will be A bar B bar A bar B A B and A B bar means, these are we can write in different way also this can be 0 0 0 1 1 1 or 1 0. Similarly, this C D value can be 0 0 0 1 1 1 1 0. So, now the all for all the minterms, because these are the 7 minterms existing the function F, so we put 1.

So, for A B C D now one by one we see that how we are giving, so A B C D. So, this is A B the third row and third column means, this is my term this is my term A B C D, see A B dash C D, so A B dash means fourth row and C D is the third column. Now, A B dash C dash D again it is the A B dash means the fourth row and C dash D is the second column. So, this will be thing then A dash B CD, so it is the second column and the sec third or second row and third column.

Now, A dash B C dash D dash, so for A dash B this is the second row which represents, second row and C dash D dash means the first column, so this is the 5th minterm. And A dash B dash C D means, A dash B dash is the first row and C D is the third column A dash B C D dash. So A dash B is my second row and C D dash is the fourth column. So, all the minterms are included, now we try to find out we try to find out the couple or the a quad.

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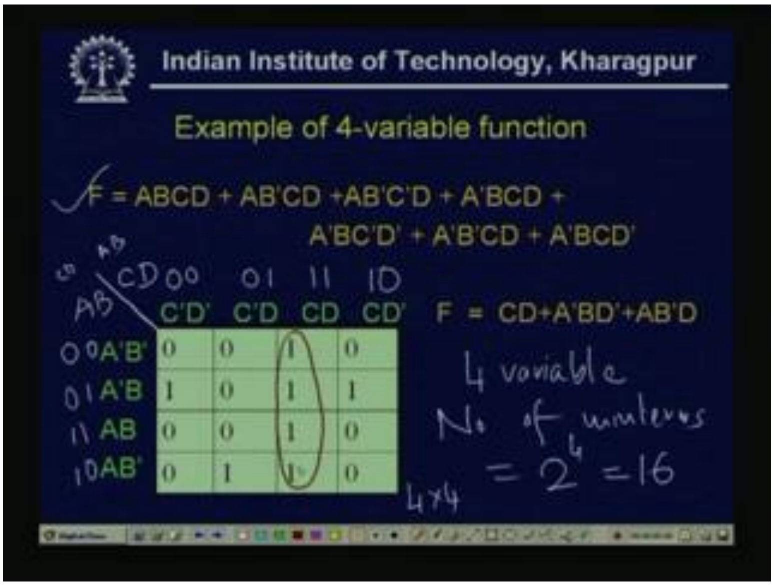

Now, first we have seen there exist 1 quad here this is 1 quad, now see here there is another couple and another couple I can, if I fold I can put this is and these are one couple. So, two couples and 1 quad exist in the four variable function K map.

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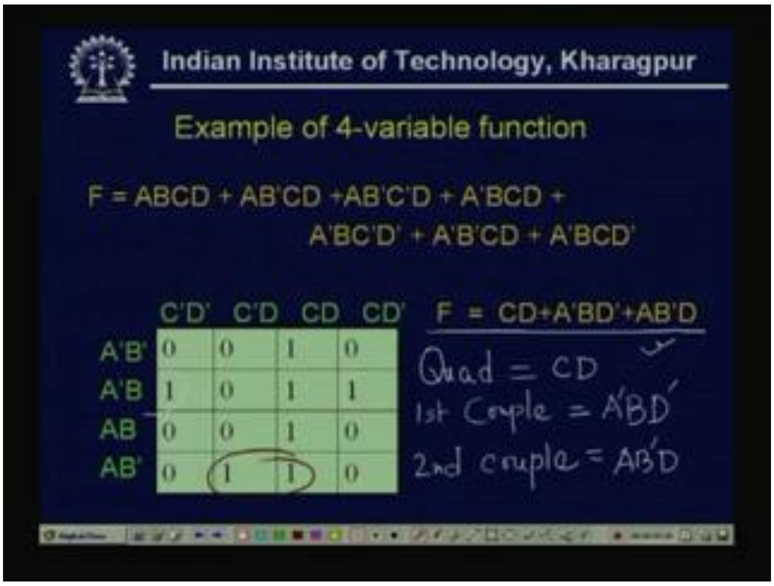

Now, we see that, so what will be the what if I this is my C this is from the K map the quad will be the quad will be simply the C D, because that all the four columns exist that means, all combinations of A B exist and that is why all complimented and uncomplimented literals they will cancel each other. So, it will be simply the third column now the first couple, see if I take this one as the first couple then it will be reduces to, because A bar B is A bar B that is common for the both the min terms for both ones or and these 2 adjacent 1's that C is common.

And D and D bar will cancel, so A bar B C or no, so this will be not A bar B C this will be A bar A bar A bar B and actually here this is the couple this one and this one, so for this these 2 adjacent D bar is common A bar B D bar. Now, for the second couple, second couple means second couple is this 1. So, for the second couple that row is A B bar that will be there, because both the adjacent one they exist in the same row, and for the 2 adjacent column see that C and C bar varies that means, they will cancel and it will be D. So, these are the CD A bar B D bar A B bar D, now see this is my reduced function that F equal to CD plus A bar B D bar plus A B bar D. So, this is my reduced function.

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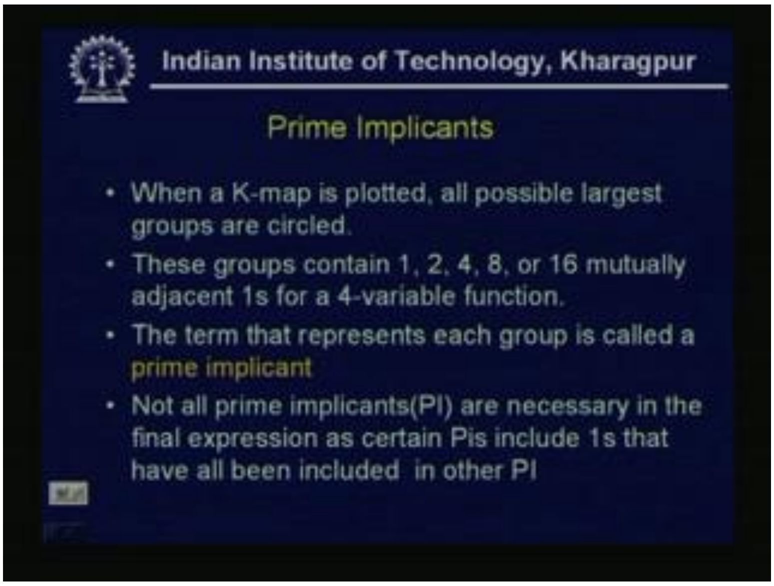

Now, we read another thing that is called the prime implicants, now when a K map is plotted all possible largest groups are circled that already we have mentioned in the rule. Now, these groups contain 1, 2, 4, 8 or 16 mutually adjacent 1's for a 4 variable function. Now, the term that represents each group is called a prime implicant. So, just now the example we have seen they are these are the prime implicants, because for this quad C D is the reduced minterm.

Then for one couple that the A bar B D bar is the reduced minterm and for another couple this is minterm, so the this terms the three terms they represents each group two couples and one part, so they are called the prime implicant. Now, what will see or what we will notice that not all prime implicants are necessary in the final expression. As, certain prime implicants include 1's that have already been included in other P I, because we have identified all the groups whether it is a couple or quad or for larger variables whether it is a octuplet.

Now, there exist some ones which actually is included in a couple or it may included the same one may included in a quad So, then what will happen that 1 or the contribution of one is actually twice it is in one part only or it is in one couple also. So, the third point is that that now all prime implicants are necessary now we will see that method, so that it can be reduced further.

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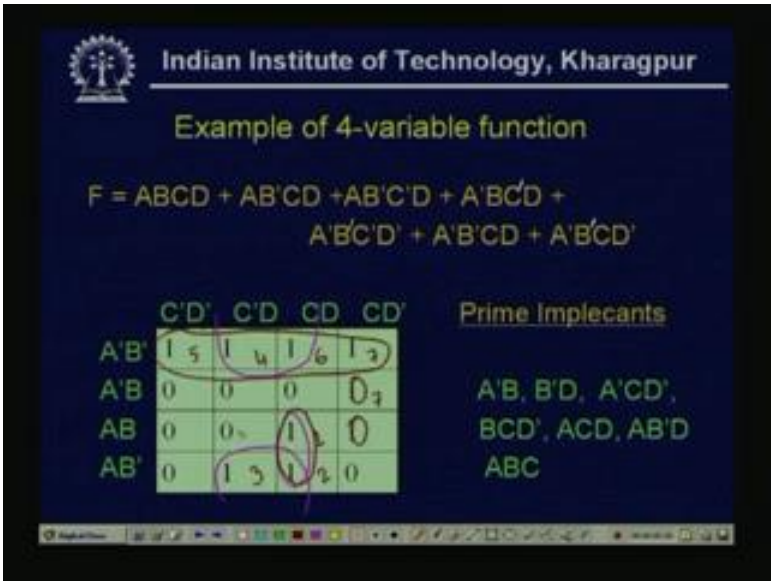

Now, again we take one four variable function, now see that this four variable function given that if that there are 7 minterms, so 1, 2, 3, 4, 5, 6, 7 minterms, now we see that what are the minterms see that A B C D, A B C D this is 1 minterm. So, I am giving that as if this is my this is my first minterm 1, A B bar C D A B bar C D this is my second min term A B bar C bar D A B bar C bar D this is my third min term.

Then A B bar C D again it has come, because already it is their, so A B bar C bar D take this one as A bar A bar C bar, so this is my this is my fourth min term. Now, A bar B C bar D bar 5th minterm, so A bar B C bar D bar A bar B A bar B C bar this is A bar B this one is A bar B C bar D bar. Now, this will be A bar B C D just we put, so this will be A bar B C bar D bar A bar B bar A bar B bar C bar D bar, so this is my 1, 2, 3, 4 this is my fifth minterm.

Then sixth is A bar B A bar B bar C D this is my sixth one and seventh one, seventh one is A bar B C D bar seventh one is A bar B C D bar, so this will be my seventh one. So, I will putting this as 0 this as 0, so now what will be the prime implicants. See if I take if I take this as the if I form a want form a group or a quad group then I am taking this is A bar B bar A bar B bar C D bar.

So, I am putting I am putting A bar B bar C C D bar, I am taking this as the 0 this I am taking 0, so this is my seventh minterm. Now, see that for this the prime implicants are see one is the first this is 1 quad this is 1 quad. So, for this the prime implicant is if I write the prime implicants.

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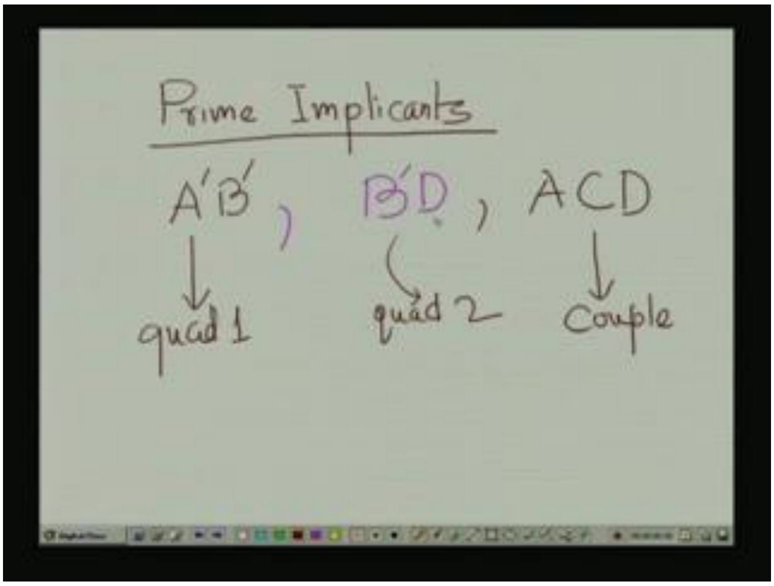

So, for the one is that A bar B bar because the this is nothing, but the first row now similarly, if I group say this is one this is one see another quad can be there. So, if I take see this can be a. If I fold, this can be a another quad, now for this quad what will be the minterms or what will be the reduced term from the K map. See that for the 2 adjacent design 2 adjacent columns C bar D and C D, so D is common, so I put D.

Now, they in the first row and the fourth row, so in between first row and fourth row B bar is common and A A bar will cancel, so this will be B bar. So, another prime implicant will be B bar D, now see another one is left. So, I kept to form another, this is another couple I can do, so this is one this is one couple. So, for these two couple this as they are in the same column, so C D must be there and A must be there, so A C D these will be these will be A C D.

So, this is for my this is for my quad one this is for my quad one this is for my quad 2 and this is for my couple one couple exist. So, these are also these three are the prime implicants, so these are some example of the prime implicants.

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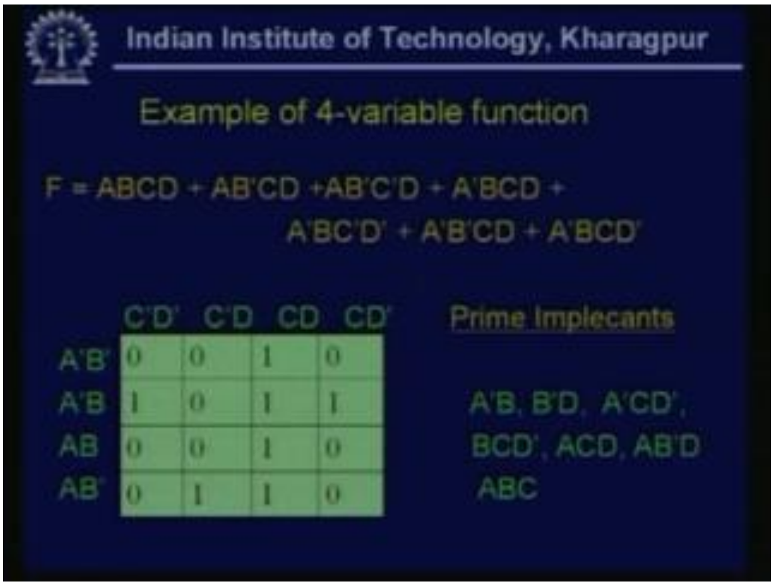

Now, if I take another example then this where the prime implicants are say that the previous page we have seen that if I take one example where the prime implicants are, these are the prime implicants that A dash B B dash D A dash C D D D dash B C D dash A C D A B dash D and A B C, so these are the prime implicants. Now, if we reduce these terms then what will happen. So, if I take this example where we see that how many quads and this is one quad, now say this is 1 quad this is one quad.

Now, what we can do there can be many couples see here we can take these two these, two as the couple, because this one is left we can take we can take this one is the couple and we can take this one is the another couple. Now, see if I take this particular example then this A dash B or actually this is A dash B dash A dash B dash is the for the first quad, then this is for the C C dash D on C D that means, D and B dash.

So, for this quad it is B dash D for this quad that folded or wrap around this is B dash D. Now, which one is A dash C dash is A dash C D D C D dash, see C D dash means the last column and A A dash is fixed. That means, for this couple this couple this is A dash D C D dash. Now, B C D dash C D dash means again the last column and see the second row and the third row the B is common. So, this is the couple this is the couple this couple for this couple it is it is B C dash.

Then A C D, A C D means which one A is this must be the third row and fourth row. So and C D is the, so another couple is there this is another couple B C D. Now, A B dash D which one is 1 A B dash D is the A B dash is the fourth row and D is D does this one mean that, here another couple can be found and another is A B C. So, A B C means this is my A B and C means. So, this one is the couple, now see that means, here for this particular example there are 2 quads and five couples exist.

Now, as already we have seen that there are many ones that are already or that are included in more than one group that can be a quad or that can be a couple, so there can be many redundant thing, so all these are prime implicants, because they represent one group. Now, only we will keep which are this these are not all necessary for the function, because there are many are many are redundant that one exist in more than one group, so these are not all necessary, so now how we can reduce that thing.

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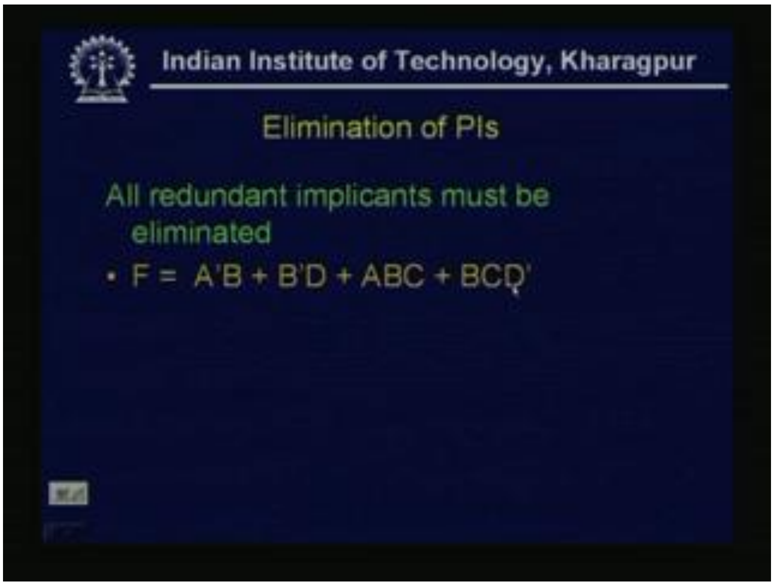

So, this from this seven terms all redundant implicants must be eliminated, now how we can eliminate this redundant terms. Again we see that thing that that means, now what we will be doing that, if I already consider 1 1 and if that forms a quad or couple. Then I will not consider again unless its adjacent 1 exist there what does it mean? See that again for this particular example say now again we will regroup that thing.

Now, see I have this is one simply this is 1 quad for this quad this is the term A dash B dash. Now, I will form this quad which one that wrap around fashion, because always the this can be a couple. But if possible I will always find out the largest possible group, so that is why I will not take the couple rather I will take the quad. Now, this two they will reduces to they will reduces this quad they reduces to the B bar, B bar D. So, this the B bar D now see that earlier there are many couples we consider, but now I will we will not consider this couples, say only three ones are left.

So, what I can easily do, I can form, because these one's are already considered. So, I will not take these as a couple rather I will take this one as the couple and I have to, I have to take either this one or this one. Because this one for this one this is actually I have to take, because otherwise this one be left it will not be grouped. So, either I can take this one or either I can take this one. So, for this one this couple this couple it will be A B A B C, so this for this couple it will be A B C.

And if I take these as the couple these as the couple then we will it will be C D bar and B, so this will be B C D bar, so this will be B C D bar, so which one will be taking, we have taking this one as one this one this is and this. Now, we see that see this is A bar A bar B bar B bar D A B C and B C D bar, so these are reduce to, so elimination of all actually we have eliminated we have eliminated some of the implicants. Only we have consider four prime implicants out of these seven, so three are redundant, so normally we follow this rule otherwise this is always this explores.

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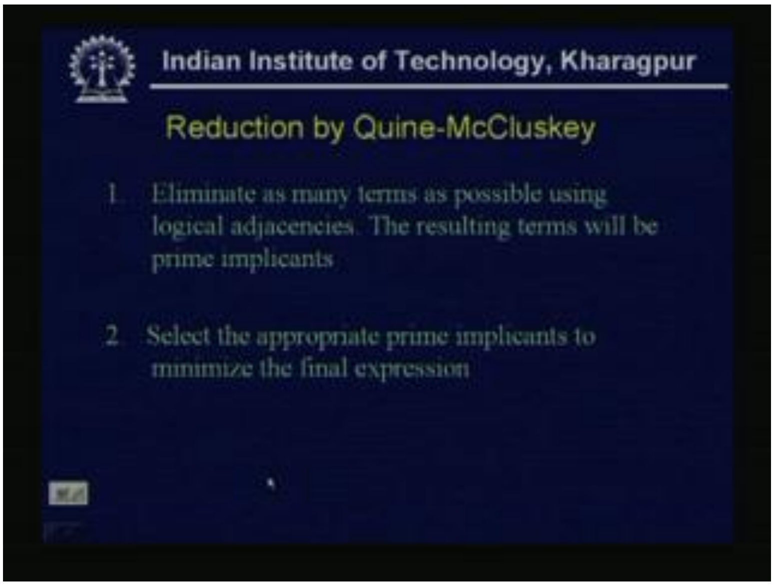

Now, there are some disadvantage of this, so far we have discuss that K map method, see if it is for larger variable what we have seen that for n variable functions the number of min terms will be 2 to the power n. So, say for 4 variable functions the map become, becomes 4 by 4 table, if it is a five variable function then it will be 2 to the power 5 means this will be 32. So, what we have to do that this will be 4 by 8, either I can make or 8 by 4 either I have to give 4 rows and 8 columns or 8 rows and 4 columns, so that the map contains 32 elements.

Now, as the variable increases that 2 to the power n, the number of minterms will also increases and the size of the map or size of the K map table will also increase. Now, if I want to reduce that thing or automatically or by… we can write some computer program then graphically. It will be very tough for say ten such ten variables, because for 10 variables e to the power, 2 to the power 10, then it will be 1 0 2 4 is the number of minterms. And we have to that size of the tables will be such that some m by n m rows and n columns then it will be m by n.

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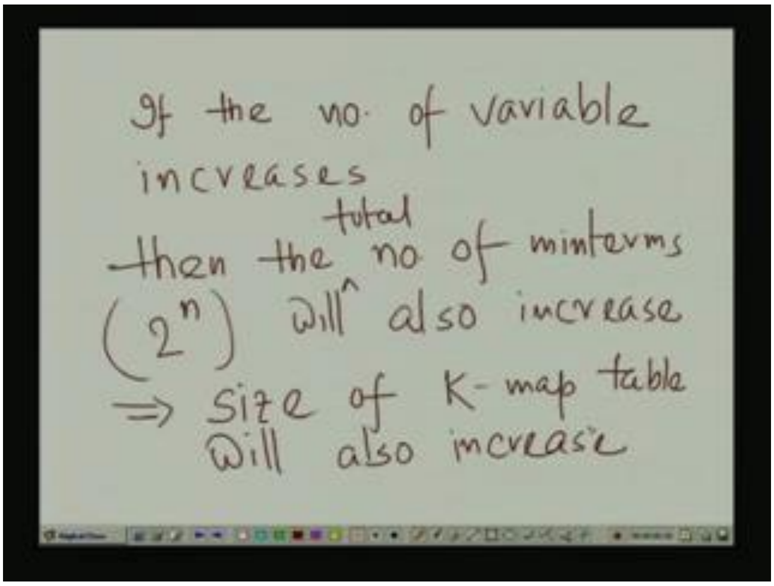

So, just what we are telling if the variable increases or the number of variables, if the number of variable increases, then the min number of minterms or total number of minterms that is 2 to the power n will also increase. That means which implies that the size of K map table will also increase, now say if we take one example that…

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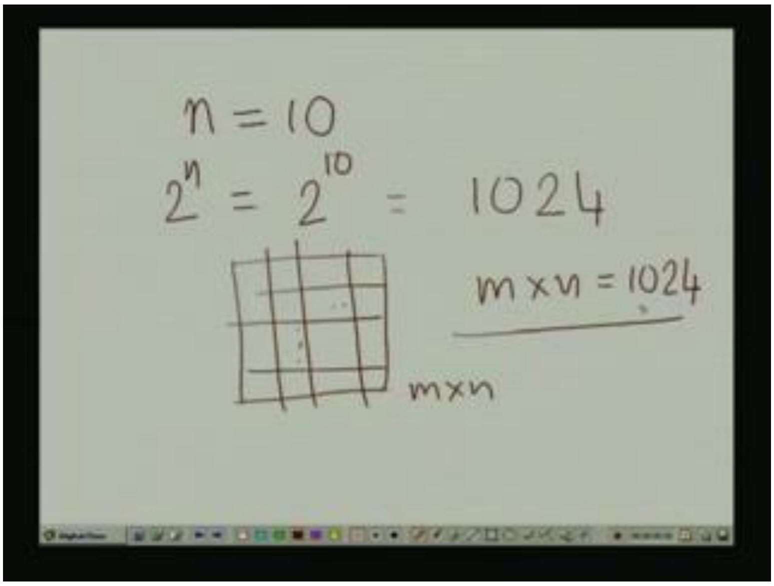

If say number of variables n equal to 10, then we have to that 2 to the power n is 2 to the power 10 and it will be 1 0 2 4, so my size of the K map table if it is a m by n if it is a m by n then this m into n equal to that should be 1 0 2 4. Now, see just now we have seen one 4 variable example, we have seen a 4 variable example and where for this 4 variable of example we have seen that there can be seven groups, I consider the all the quads and couples these are seven.

Now, if it is if it is a one larger variable function then the number of prime implicants will be more or if we consider all the quads couples octuplet or for larger variables that, hex box or the box containing 32, 164 1's then it will be the number of possibilities of grouping will be more. And then it will be almost impossible or even it is a possible it will it will be very slow to do that reduction by graphically.

And then we have to reduce, so just now the way we have told that redundant implicants prime implicants, so now another there are several other methods exist that again all are the basic concept is the K map, but improvement of that thing. This class we will discuss another one such improvements of the K maps, but then their exist many others and actually all that K map is a basics and that is why we will not discuss that all.

Now, one very important and very popular technique is the Quine-McCluskey. So, the rule is the eliminate as many terms as possible using logical adjacencies, the resulting terms will be prime implicants. So, mainly the K map the reduction principle is to notice the adjacencies because grouping is done by the adjacencies. So, here we are eliminating this term by using or by noticing only the minterms we will see later the example, and the say next what we have to do select the appropriate prime implicants to minimize the final expressions.

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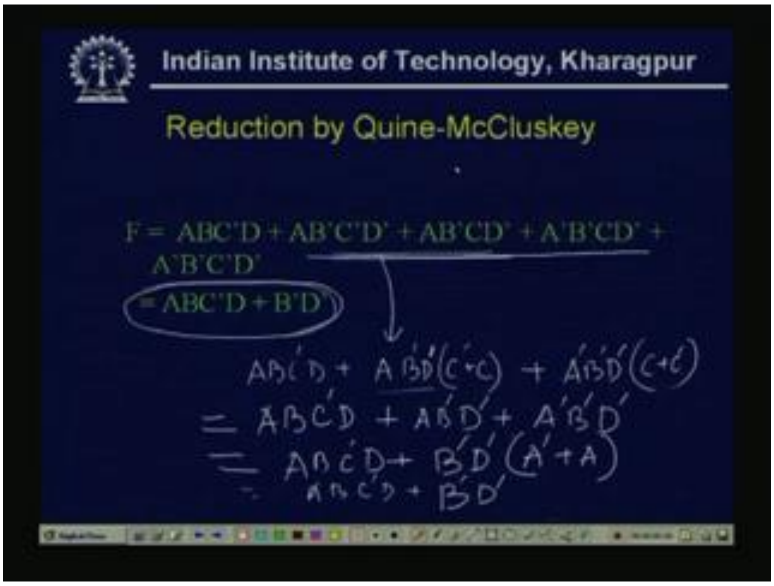

Now, see how we are doing that thing, see I have one function that the function have that five minterms, these are the minterms that A B C dash D, A B C dash D, A B dash C dash D dash A B dash C D dash A dash B dash C D dash A dash B dash C dash D dash. Now, if we do first we are doing we forget about all the K map etcetera, just to our using our theorems and postulates, now what it will be the thing see the if we keep the first term as it is because there is no common thing.

Here what we are see if we apply our, so we come from here we just currently being we forget about we forget about the result, see this is that A B C dash D we keep as it is because there are many other where we have to we can take the common terms. Say for these two, if we consider these two see here B dash C dash D dash B dash C D dash. So, A B dash D dash is common, so I can take C dash plus C. Similarly, the last 2 minterms I can take A dash B dash D dash A dash B dash D dash again this will be C plus C dash.

Now, C plus C dash is 0, so this reduces to AB C dash D plus A B dash D plus A dash B D, this equal to A B C dash D again see here from here A and D is common, so plus see AD then B dash plus B. So, this can be A B A B C dash D plus AD, now from a here again we can reduce that AD BC dash plus 1 that means, 1 and that can be reduce to A D, so this type of reduction is possible.

Now, this is from that if we apply the theorems, now see we again if we check A B C dash D A B C dash D dash there is one mistake here actually, here we have taken D, D as the here these two terms we have taken A B dash D dash D dash is there and that is why A B dash D dash. And similarly, here also A dash B dash D dash just, these are actually complimented D dash is complimented in all the terms, so just again if we do just the same thing, so A B A dash B dash D dash A dash B dash D dash.

So, from here B dash D dash will be common this will be A dash plus A, A dash plus A is 0 this is B dash D dash actually, here D dash is complimented all the terms D dash is complimented. So, this is A A dash B C dash D B dash D dash this is from my reduction using the postulates and the theorem, now we see if I apply the Quine-McCluskey rule.

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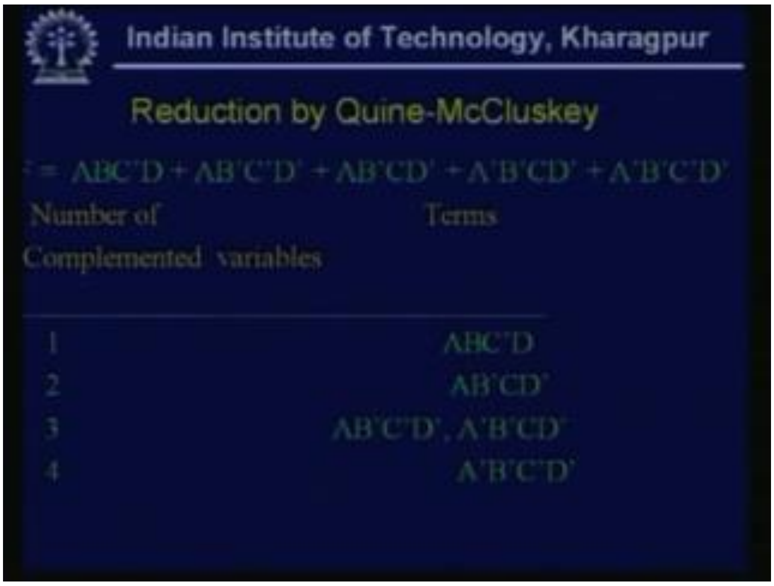

Now, see first what we are doing we are applying the rule that, so number of we are computing number of complimented variables and the terms. See here there are five terms now in the terms which are the complimented variables 1, see there are there is only one term only 1 minterm A B C dash D. See here the C dash is this C dash is complimented C dash is complimented. So, the it is one, if the number of complimented variables are 2 I have only 1 minterms, where this B dash and day dash the means two complimented variables are there.

If the number of complimented variables are three then there are two terms A B dash C dash D dash A dash B dash C D dash 3. Number of complimented variables are 4 I have only one term A dash B dash C dash D dash, so these are these are the minterms, now so, these are the table.

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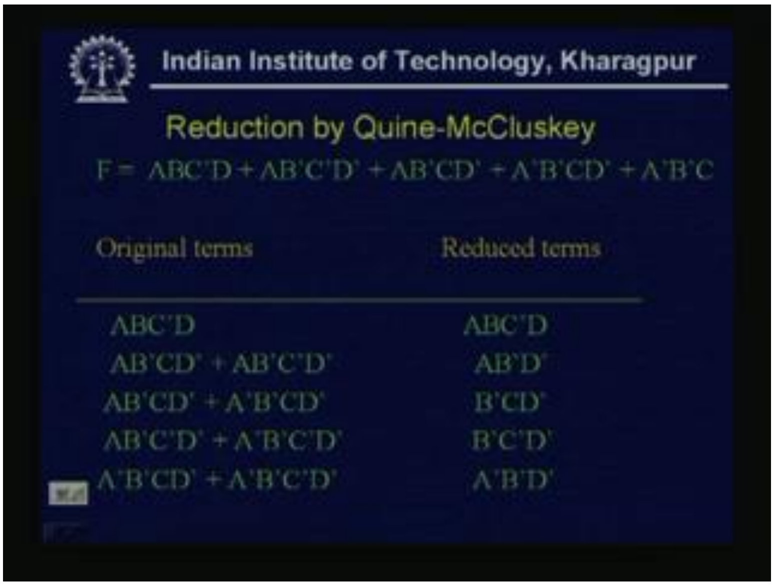

Now, I see that original terms and the reduced terms, so number of complimented variables is only 1, so I keep as it is, because it is not reducing anything. Now, here the concept we have applied is that mainly we are seeing the logical adjacencies, see that 2 columns are adjacent means, only one variable in one column it will be complimented the next column it will be uncomplimented.

So, if it is a 4 variable function then the 3 variable remain same and only one variable will vary complimented or uncomplimented for the 2 adjacent columns. And the same thing is true for the rows also, so this concept we are applying here, now see the because the we have taken the number of complimented variables are same. So, here we have A B dash C D dash plus A B dash C dash D dash, see here the A B dash and D dash this three variables are same whereas, only C varies.

So, C plus C bar is 1, so the reduce term will be A B bar D bar similarly, if I take the two this type of original terms, so all possible combinations of two then I will be getting this four such reduce terms. These are the four reduce terms I am getting and this is not reduced because only one is there.

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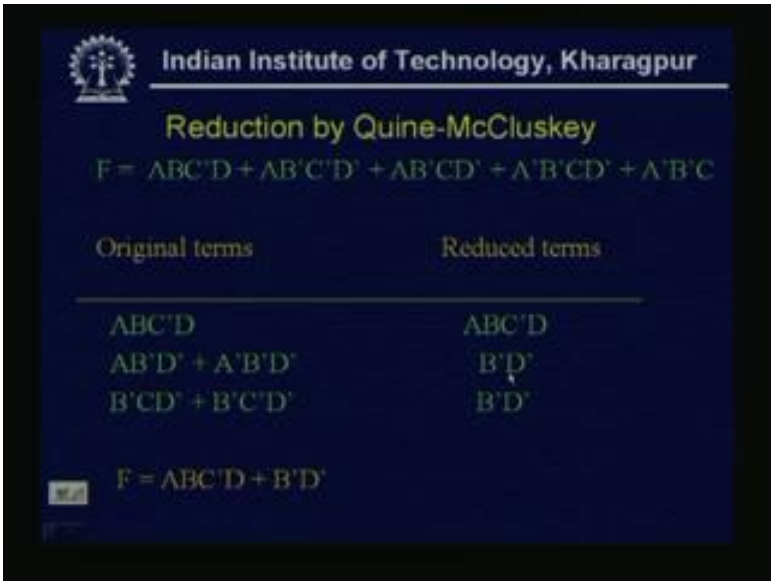

Now, again we continue, we continue that with this process with the reduced terms, so now the original terms becomes that which one was not reduced that is as it is. So, A B C dash D now from the previous table we get A B dash D dash and A dash B dash D dash. So that means, the from this 4 reduce terms this four reduce terms we are considering that 2 at a time and it is reduced to B dash D dash and this is reduced to B dash D dash.

Again this row reduce to B dash D dash, now again we apply we will be seeing, because here both the terms are B dash D dash so that means, X plus X equal to 1, so B dash D dash equal to 1. So, it becomes A B C A B C dash D plus B dash D dash what we have achieved by from the table laws, so the Quine-McCluskey we can apply for that automated reduction procedure.

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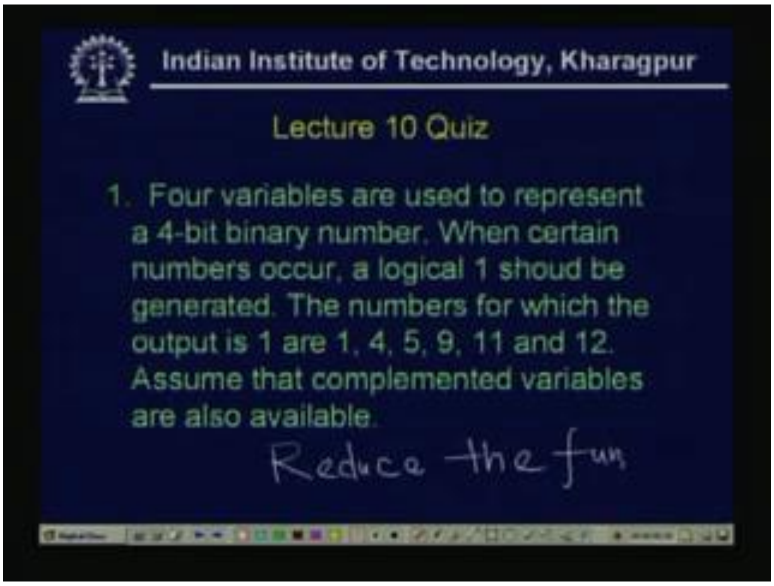

Now, today's quiz question that only one question is there that four variables are used to represent a found bit binary number when certain numbers occur a logical one should be generated. The numbers for which the output is 1 are 1, 4, 5, 9, 11 and 12, assume that complimented variables are also available, so you reduce this expression, so the thing is that reduce the find out the expression and reduce the function.