Digital Image Processing

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Lecture - 7

Camera Model & Imaging Geometry

Hello, welcome to the video lecture series on digital image processing. In our last lecture, we had talked about a number of basic transformations and we have said that these transformations are very very useful to understand the image formation process.

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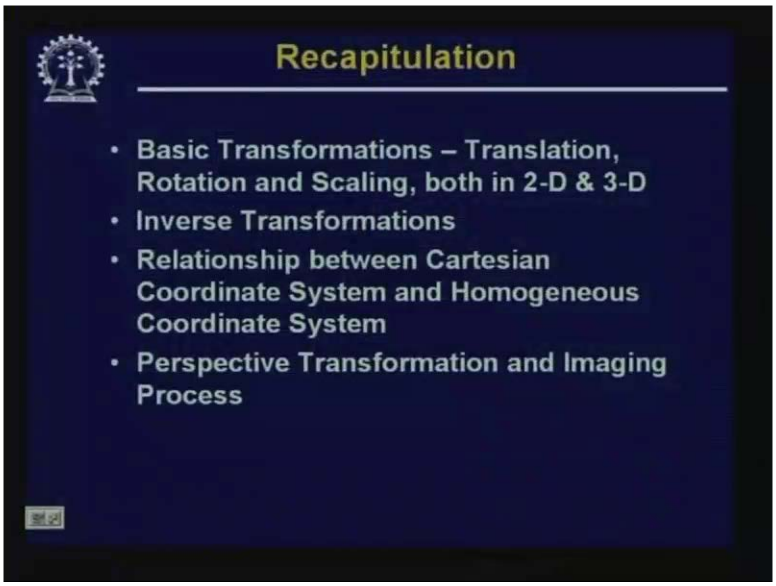

So, in the last class, what we had talked about is the basic transformations and we have talked about the transformations like translation, rotation and scaling and these transformations, we have said both in the 2 dimensions and 3 dimensional cases.

Then for all these transformations, we have also seen what is the corresponding inverse transformation. Then, after that we have gone for the conversion from the Cartesian coordinate system to homogeneous coordinate system and we have seen the use of homogenous coordinate system in perspective transformation where perspective transformation, we have said is an approximation of the imaging process so that when a camera takes the image of a point in a 3 dimensional world, then this imaging transformation can be approximated by the perspective transformation that we have discussed in the last class.

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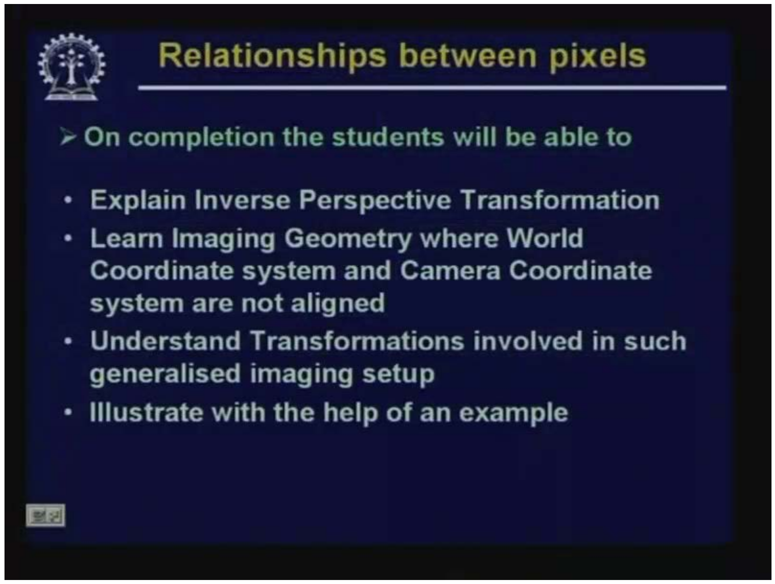

Today we will talk about the inverse perspective transformation. We have said that the perspective transformation takes an image of a point or a set of points in the 3 dimensional world and these points are mapped to the imaging plane which is a 2 dimensional plane. The inverse perspective transformation just does the reverse process that is given a point in the imaging plane; we will see that using this inverse perspective transformation, whether it is possible to find out that what is the point in the 3 dimensional coordinate system to which this particular image point corresponds.

Then we will also talk about the imaging geometry where the world coordinate system and the camera coordinate system are not aligned. You try to remember that in the last class, the imaging geometry that we had considered; there we have assumed that the 3 dimensional world coordinate system is aligned with I mean camera coordinate system. That is X axis of the camera is aligned with the X axis of the 3D world, Y axis of the camera is aligned with the Y axis of the 3D world and Z axis of the camera is also aligned with the Z axis of the 3D world.

In addition to that the origin of the camera coordinate system also coincides with the origin of the image coordinate system. In today's lecture we will take a generalized imaging model where the camera coordinates system and the 3D world coordinate system, they are not aligned and which is a general situation.

Then we will try to see that what are the transformations which are involved in a such in such a generalized imaging setup which will help us to understand the image formation process in a generalized setup. Then we will illustrate this concept with the help of an example. Now, let us briefly recapitulate what we had down in the last class.

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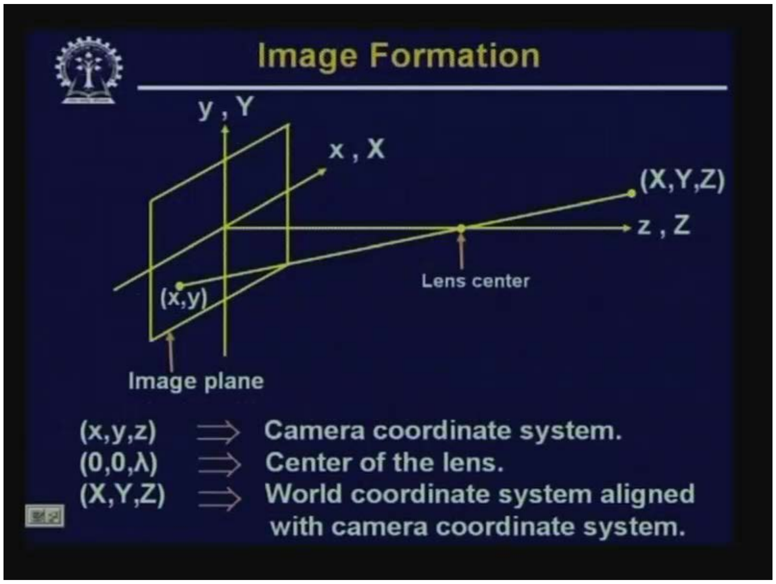

Now, this figure shows the image imaging geometry that we had considered where the 3D world coordinate system is aligned with the camera coordinate system. There we have taken a 3D point whose coordinates are given by X Y Z all in the capital and (x, y) lowercase coordinates are the corresponding image point in the imaging plane and we have assumed that the focal length of the camera is lambda that means the coordinate of the focal point of the lens center is (0, 0, lambda).

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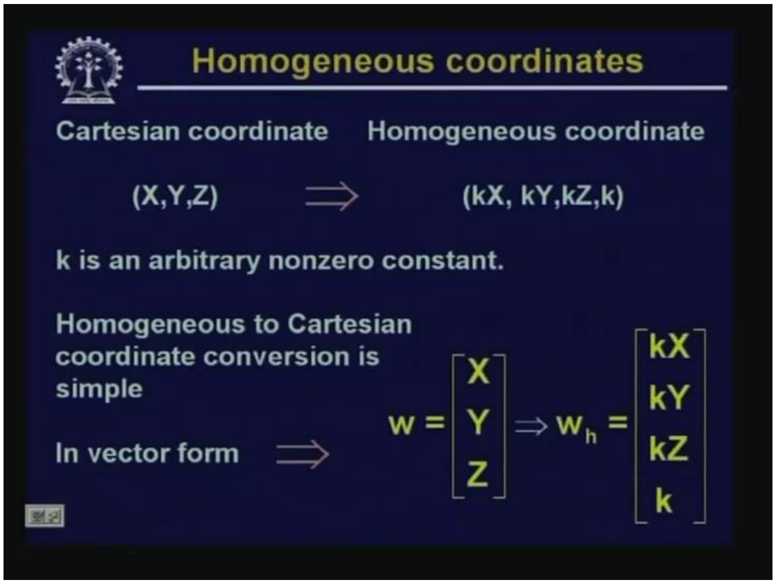

Now, using this particular figure, we have tried to find out a relation between the 3D world coordinate X Y Z and the corresponding image point which is (x, y). For that what we have down is we have taken a conversion from the Cartesian coordinate system to a homogenous coordinate system.

So, while doing this conversion, what we have down is every component of the coordinate that is X Y Z is multiplied by a non 0 arbitrary constant k and the same value of k is attended with the 3 components. So, for a Cartesian component (X, Y, Z) the corresponding homogenous coordinate is given by kX kY kZ and Z.

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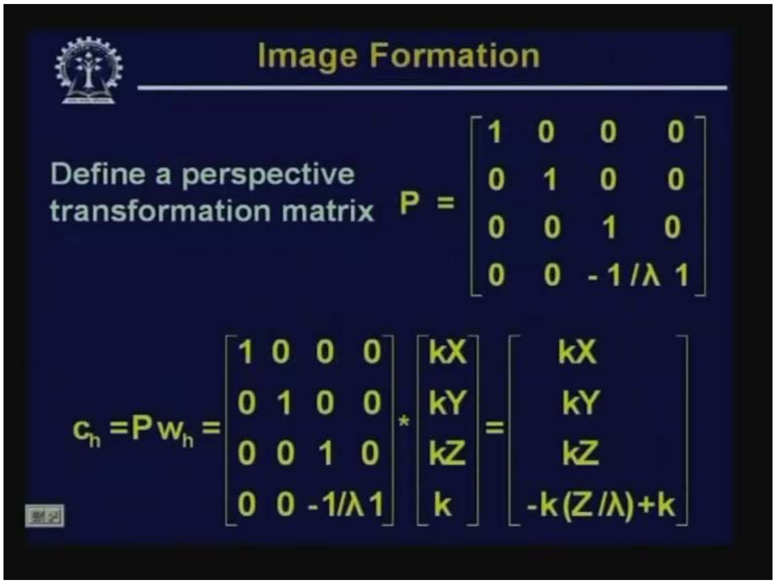

So, for a world coordinate point X Y Z, once we have the corresponding homogenous coordinate kX kY kZ and k, then we found that after this conversion if we define a perspective transformation; so this perspective transformation matrix P which in this case (1 0 0 0) (0 1 0 0) (0 0 1 0) and (0 0 minus 1 upon lambda 1) and the homogenous coordinate W_h is transformed with this perspective transformation matrix P, then what we get is the homogenous coordinate of the camera point to which this world point W will be mapped and the homogenous coordinate of the camera point of the image point after the perspective transformation is obtained as kX kY kZ minus k Z by lambda plus k.

And, we will see that if I convert this homogenous camera point, the homogenous image point into the corresponding Cartesian coordinate, then this conversion gives us the Cartesian coordinates of the image point as (x y z) equal to (lambda X divided by lambda minus Z, lambda Y divided by lambda minus Z and lambda Z divided by lambda minus Z).

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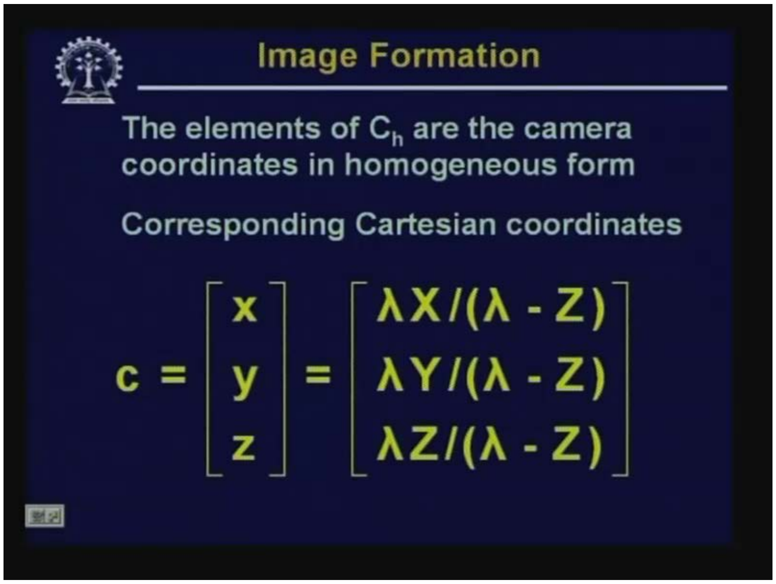

So, you just note that (x y z) in the lower case letters, these indicate the camera coordinate the image coordinate; whereas X Y Z in the capital form, this represents the coordinate in the 3D world of or the 3D coordinate of the world point W. Now, what we are interested in is the camera coordinate x and y at this moment, we are not interested in the image coordinate z.

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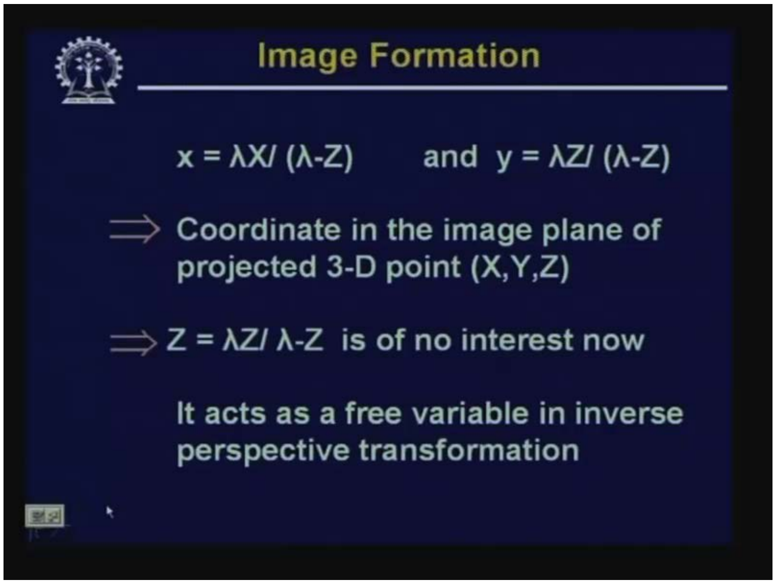

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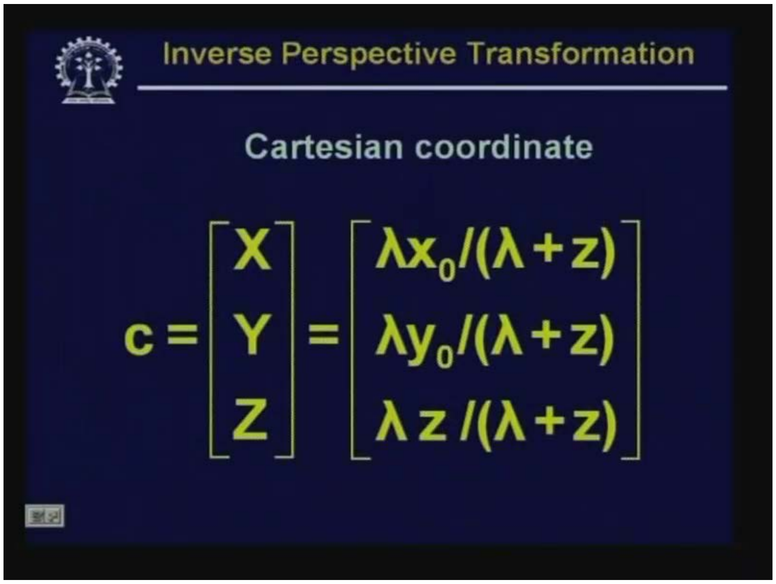

So, this can be obtained by simply simple conversion that if we find out the value of lower case z with respect to lambda and capital Z, then after solving the same equations here that is lower case x, lower case y and lower case z; we find that the image coordinate x and y in terms of the 3D coordinate capital X and capital Z is given by x equal to lambda X divided by lambda minus capital Z and image coordinate y equal to lambda times capital Z divided by lambda minus capital Z.

So, as we said that the other value that is the z coordinate in the image plane is of no important at this particular moment but we will see later that when we talk about the inverse perspective transformation, when we try to map an image point to the corresponding 3D point in the 3D world; then we will make use of this particular coordinate z in the image plane as a free variable.

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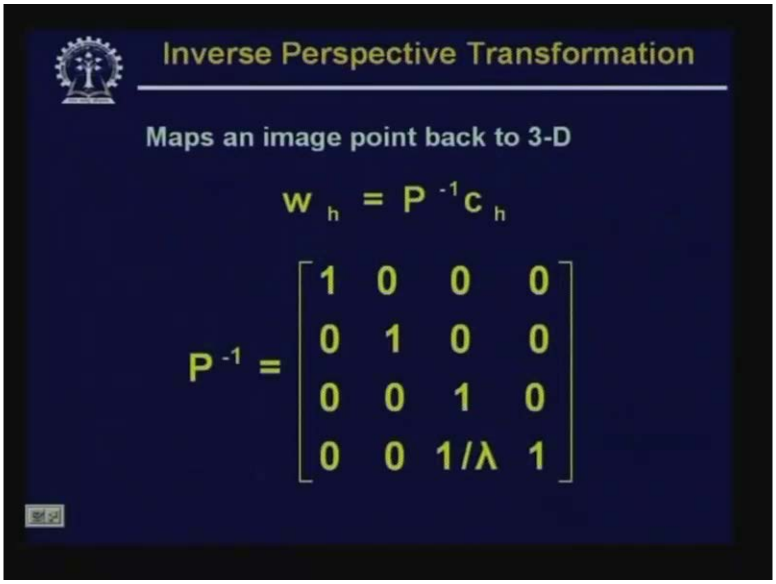

So, now let us see that what is the corresponding inverse perspective transformation that we can have. So, as we have said that a perspective transformation maps a 3D point onto a point in the image plane. The purpose of inverse perspective transformation is just the reverse. That is given a point in the image plane; the inverse perspective transformation or P inverse tries to find out the corresponding 3D point in the 3D world.

So, for doing that, again we make use of the homogenous coordinate system that is the camera coordinate C of the image coordinate point C will be replaced will be converted to the corresponding homogenous form which is given by C_h and the world coordinate, world point W will also be obtained in the form, in the homogenous coordinate from W_h.

And, we define a inverse perspective transformation P inverse which is given by (1 0 0 0) (0 1 0 0) (0 0 1 0) and (0 0 1 upon lambda 1) and you can usually verify that this matrix, this transformation matrix is really an inverse of the perspective transformation matrix P because if the if we multiply the perspective use transformation matrix by this matrix P inverse, what we get is really an unitary matrix.

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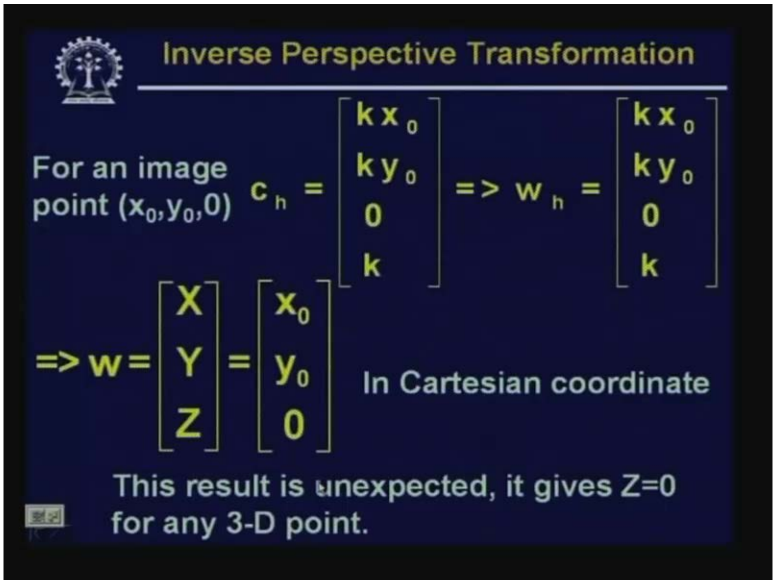

Now, given this inverse perspective transformation matrix as we said that if we assume an image point say x_0, y_0 and we want to find out what is the corresponding 3D world point W to which this x_0, y_0 image point corresponds.

So, the first step that we will do is to convert this image point x_0, y_0 to the corresponding homogenous coordinate which will be obtained as kx_0, ky_0 and 0 and then the forth component comes as k. Now, here you find that the third component or z coordinate we have taken a 0 because what we have is a point in 2 dimensions that is on the imaging plane. So, we have assumed the z coordinate to be 0.

Now, if we multiply or if we transform this homogenous coordinate kx, kx_0, ky_0, 0, k with the inverse perspective transformation P inverse, then what we get is the homogenous coordinate corresponding to the 3D world point which is obtained has W_h as given in this equation; W_h is equal to kx_0, ky_0, 0, k.

Now, from this homogenous coordinate system if I convert this to the Cartesian coordinate form, then the Cartesian coordinate corresponding to this homogenous coordinate is obtained as W equal to capital X capital Y capital Z which is nothing but x_0, y_0, 0.

So, you find that in this particular case, the 3D world coordinate is coming as x_0, y_0, 0 which is the same point from where we have started that is the image point from where we have started. Moreover, for all the 3D coordinate points, the z component always comes as 0. Obviously, this solution is not acceptable because for every coordinate or for every point in the 3 dimensional world, the z coordinate cannot be 0. So, what is the problem here?

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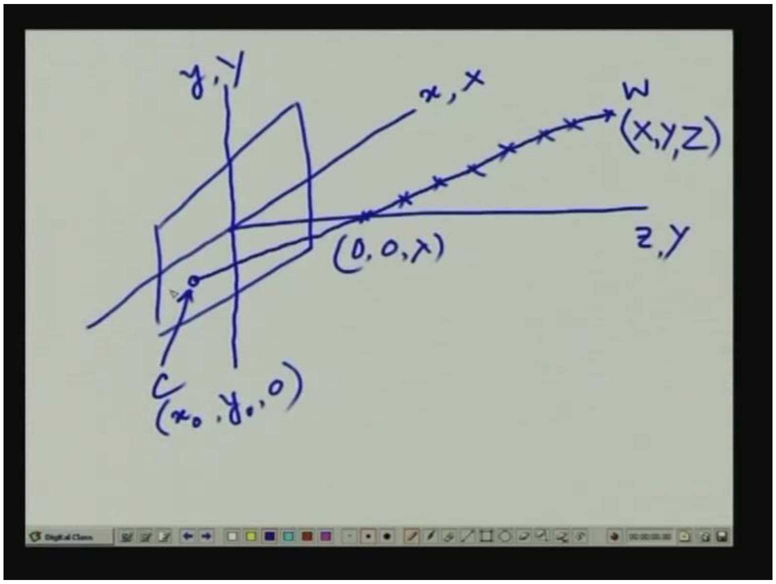

If you remember the figure of imaging system that we have used, let me just draw that particular figure. We had an imaging plane X Y plane like this on which the camera coordinate system and the image coordinate system camera coordinate system and the 3D world 3D coordinate system, they are perfectly aligned.

So, we had this x same as capital X, we had this y same as capital Y, we had this z same as capital Z and this is the origin of both the coordinate systems and we had somewhere here the optical center of the lens. Now, if I take some point here, some image point here and if I draw a line passing through this image point and the camera optical center and the world point W comes somewhere at this location.

So, we have seen in the previous figures that this point if I call this point as C, this point C is the image point corresponding to this 3D world point W whose coordinate is given by capital X capital Y capital Z and this C in our case has a coordinate of x_0, y_0 and 0 and when we have tried to map it back this point C to the 3 D world coordinate system; what we have got is for every point W, the value of z come out to be 0.

Now, the problem that comes here is because of the fact that if I analyze this particular mapping that is mapping of point W in the 3D world to point C in the image plane, this mapping is not a 1 to 1 mapping. Rather, it is a many to 1 mapping.

Say for example, if I take any point on this particular straight line passing through the point C and the point (0 0 lambda) which is nothing but the optical center of the camera lens; then all these points on this line will be mapped to the same point C in the image plane.

So naturally, these being a many to 1 mapping; when I do the inverse transformation using the inverse perspective transformation matrix from image point C to the corresponding 3D world, the solution that I will get cannot be acceptable solution. So, we have to have something more in this formulation and let us see what is that we can add over here.

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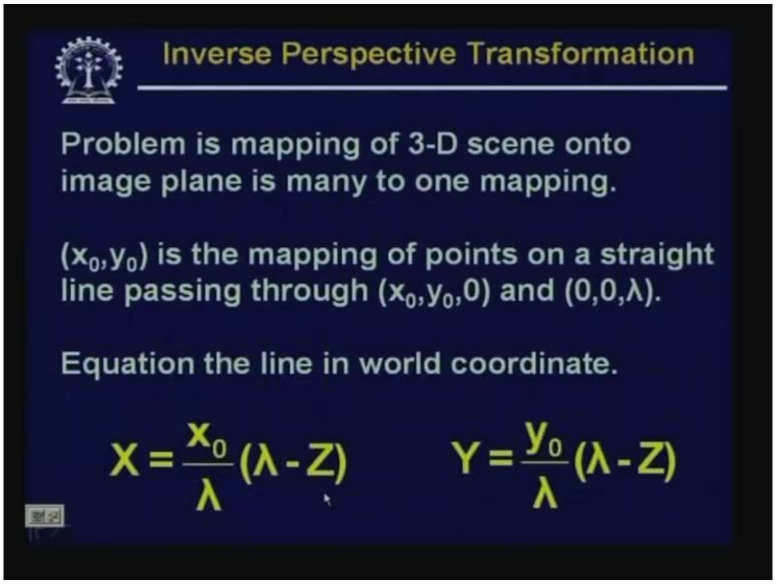

Now here, if I try to find out the equation of the straight line which passes through the point x_0, y_0 that is the image point and the point (0 0 lambda) that is the optical center of the camera lens; the equation of the straight line will come of this form. That is capital X equal to x_0 by lambda into lambda minus capital Z and Y equal to capital Y equal to y_0 by lambda into lambda minus capital Z.

So, this is the equation of the straight line so that the points every point in this straight line is mapped to the same point x_0, y_0 in the image plane. So, the inverse perspective transformation as we have said that it cannot give you a unique point in the 3D world because the mapping the perspective transformation was not a 1 to 1 mapping.

So, by using the inverse perspective transformation, even if we cannot get exactly the 3D point but at least the inverse transformation matrix should be able to tell me that the points belonging to which particular line maps to this point x_0, y_0 in the image plane. So, let us see whether we can have this information at least.

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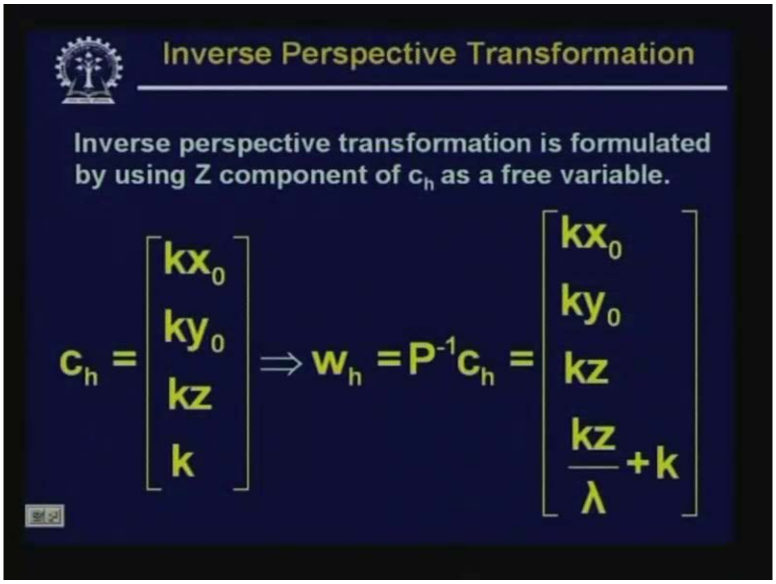

So for doing this, in earlier case, when we have converted the image point x_0, y_0 to the homogenous coordinate, then we have taken kx_0 ky_0 0 and k. Now here, the z coordinate, what we will do is instead of assuming the z coordinate to be 0, we will assume the z coordinate to be a free variable. So, in our homogenous coordinate, we will assume the homogenous coordinate to be kx_0, ky_0, kz and k.

Now this point, when it is inverse transformed, using the inverse transformation matrix, then what we get is the world coordinates, the world point in homogenous coordinate system as W_h is equal to P inverse C_h and this particular case, you will find that this W_h is obtained as kx_0, ky_0, kz, kz by lambda plus k. So, this W_h we have got in the homogenous coordinate system.

Now, what we have to do is this homogenous coordinate, we have to convert to the Cartesian coordinate system and as we have said earlier that for this conversion, we have to divide all the components with the last component. So in this case, kx_0, ky_0 and kz, z all of them will be divided by kz by lambda plus k.

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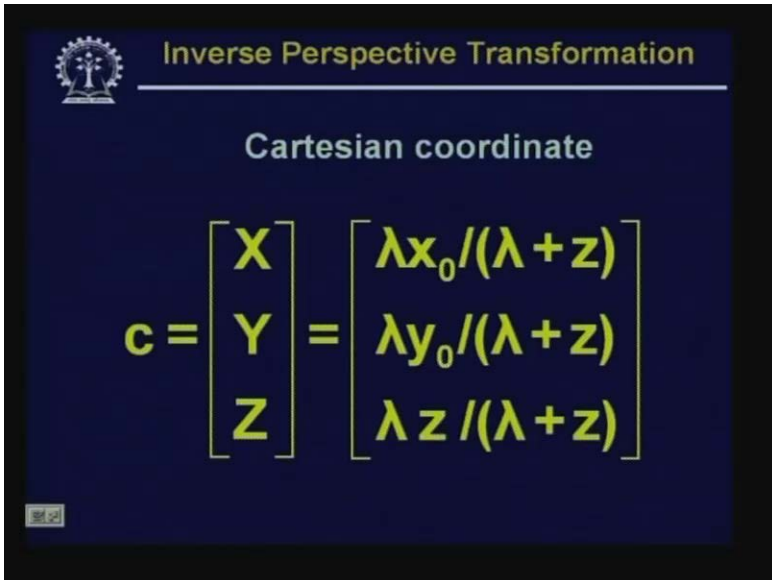

So, after doing this division operation, what I get in the Cartesian coordinate system is W equal to sorry here it is not C it should be W, so W equal to X Y Z which is equal to, so this point should be W; so, what we get is w equal to X Y Z which is equal to lambda x_0 divided by lambda plus Z, lambda y_0 divided by lambda plus Z and lambda Z divided by lambda plus Z.

So, on the right hand side all the z's are the lower case letters which is the free variable that we had assumed that we have used for the image coordinate and for the matrix, the column matrix on the left hand side, all X Y Z are in upper case letters which indicate that these X Y Z are the 3D coordinate.

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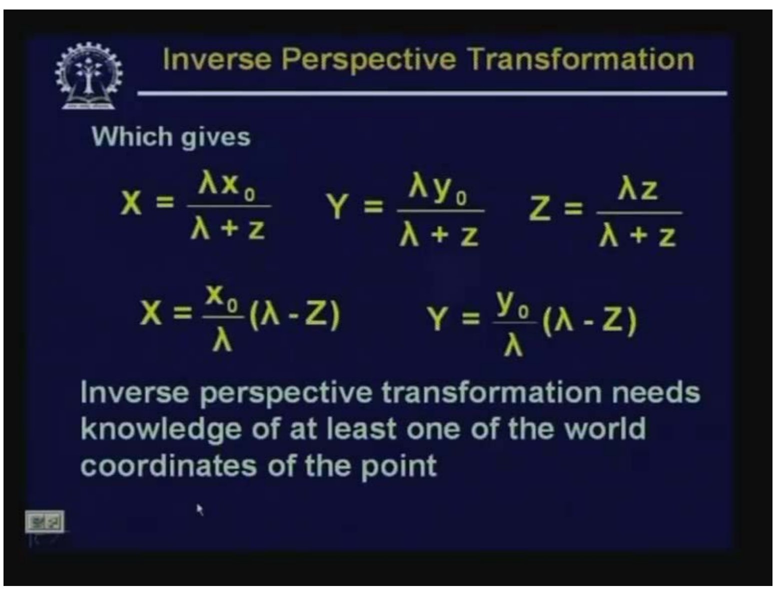

So, now what we do is we try to solve the values solve for the values of capital X and capital Y. So, just from this previous matrix, you find that the capital X is give by lambda x_0 divided by lambda plus lower case z, capital Y is given by lambda y_0 divided by lambda plus lower case z and capital Z is equal to lambda lower case z divided by lambda plus lower case z.

So, from this these 3 equations, I can obtain capital X equal to x_0 by lambda into lambda minus Z and Y equal to y_0 by lambda into lambda minus Z. So, if you recall the equation of the straight line that passes through x_0, y_0 and (0 0 lambda), you will find that the equation of the state line was exactly this; that is capital X equal to x_0 by lambda into lambda minus z capital Z and capital Y equal to y_0 by lambda into lambda minus capital Z.

So, by using this inverse perspective transformation, we have not been able to identify the 3D world point which is of course not possible but we have been able to identify the equation of the straight line so that the points on this straight line maps to the image point x_0, y_0 in the image plane. And now, if I want to exactly find out a particular 3D point to which this image point x_0, y_0 corresponds; then I need some more information.

Say for an example, I at least need to know what is the Z coordinate value of the particular 3D point W and once we know this, then using the perspective transformation along with this information of the Z coordinate value, we can exactly identify the point W which maps to point x_0, y_0 in the image plane.

Now, till now all the discussions that we have done, for all these discussions we have assumed that the image coordinate system and the camera coordinate system they are perfectly aligned. Now, let us discuss about a general situation where the image coordinate system and the camera coordinate system, they are not perfectly aligned. So, here we assume that the camera is mounted on a Gimbal.

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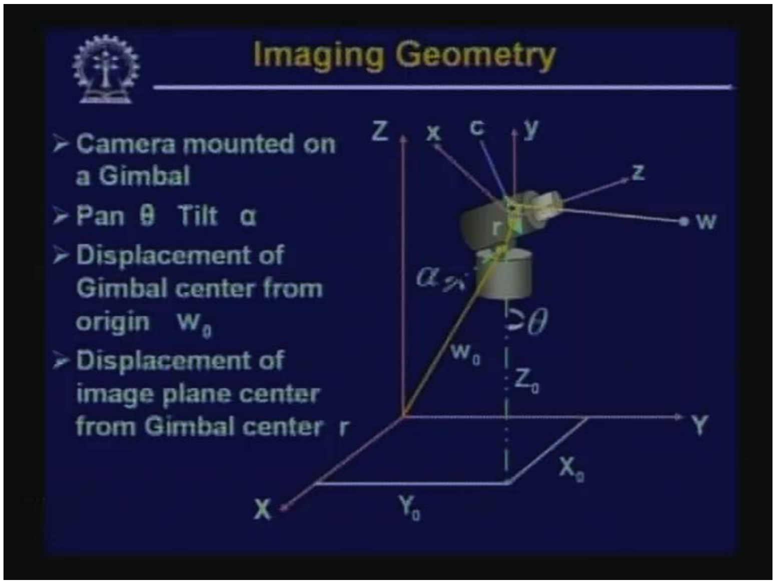

So, if you mount the camera on a Gimbal, then using the Gimbal; the camera can be a pan of angle theta, it can also be given a tilt by an angle alpha. So, you remember that pan is the rotation around Z axis and the tilt is rotation around X axis.

We also assume that the Gimbal center is displaced from the 3 world coordinate origin (0 0 0) by a vector W_0 which is equal to x_0, y_0, z_0 and finally, we also assume that the camera center or the center of the imaging plane is displaced from the Gimbal center by a vector r which will have component say r_1, r_2 and r_3 in the X, Y and Z direction of the 3D world coordinate system.

Now here, our interest is given such a type of imaging arrangement, now if we have a 3D world a point in the 3D world coordinate W; what will be the camera coordinate, what will be the image point C to which this world point W will be mapped. So, this is a general situation. And now, let us see that we can obtain the solution to this particular problem that for this generalized imaging setup for a world point W, what will be the corresponding image point C.

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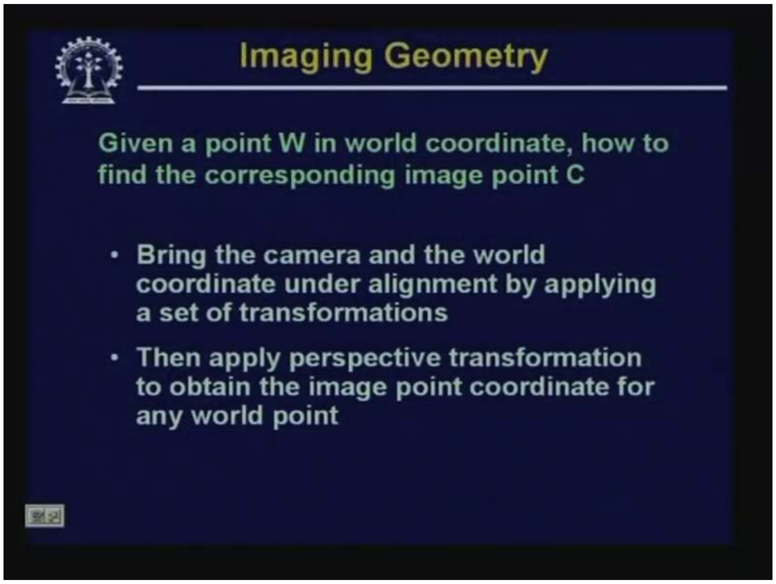

So, the steps will be like this. Since our earlier formulation were very simple in which case we have assumed that both the camera coordinates system and the 3D world coordinate system, they are perfectly aligned; in this generalized situation, we will also try to find out a set of transformations which if applied one after another will bring the camera coordinate system and the world coordinate system in perfectly aligned.

So once that alignment is made, then we can apply the perspective transformation to the transformed 3D world points and this perspective transformation to the transform 3D world point, give us the corresponding image coordinates of the transform point W. So, what are the transformation steps that we need in this particular case?

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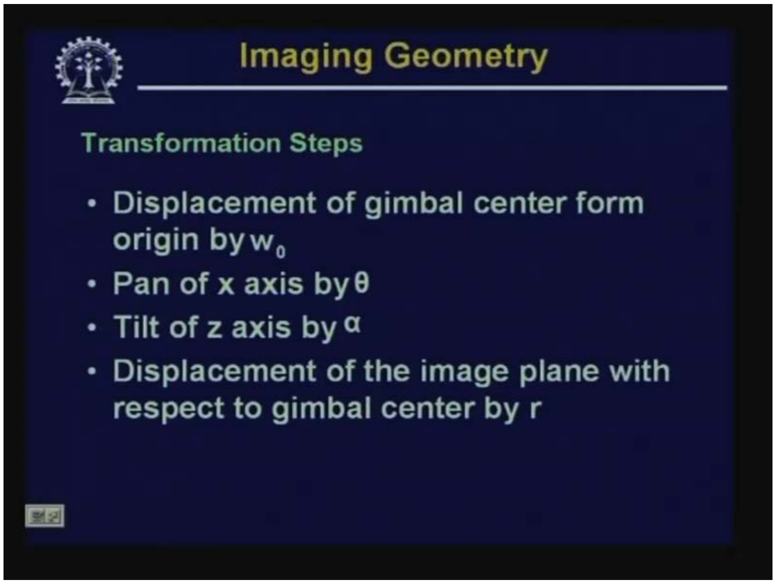

So, the first step is we assume that the image coordinate system and 3D world coordinate system, they are perfectly aligned. So from this, we displace the Gimbal center from the origin by the vector W_0 and after displacing the Gimbal center from the origin by W_0, we pan along X axis by an angle theta followed by tilt around Z tilt of Z axis by angle alpha which will be followed by the final displacement of the image plane with respect to Gimbal center by the vector r.

So, we have 4 different transformation steps which are to be applied one after another and these transformation steps will give you the transformed coordinate of the 3D world point W. So, let us see, how this transformation is to be applied one after another.

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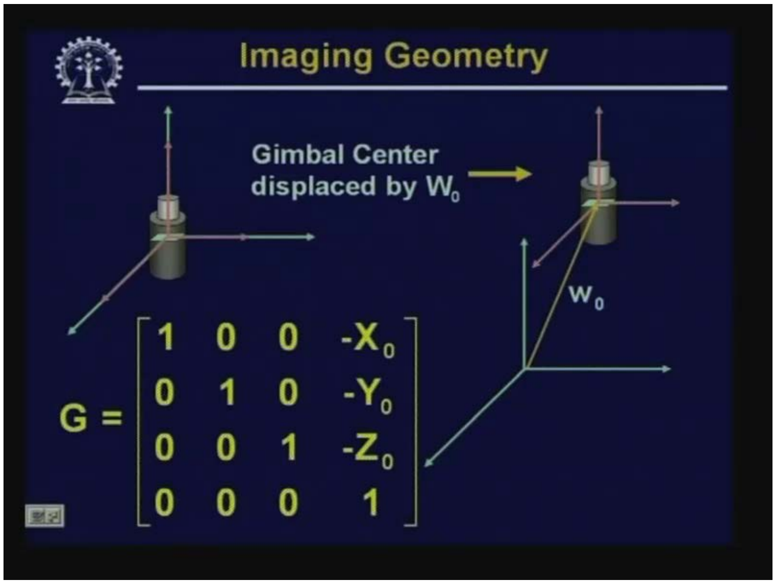

So here, on the left hand side, we have shown a figure where a camera coordinate system and the world coordinate system are perfectly aligned. Now, from this alignment, if we give a displacement by a vector W_0 to the Gimbal center, then the camera will be displaced as shown on the right hand side of the figure where you find that the center is displaced by vector W_0.

You remember that if I displace the camera center by vector W_0, then all the world coordinates all the world points will be displaced by a vector minus W_0 with respect to the camera. Now, you just recollect that when we try to find out the image point of a 3D world point; then the image point, the location of the image point is decided by the location of the 3D world point with respect to the camera coordinate, it is not with respect to the 3D world coordinate.

So, in this case also after a set of transformations, we have to find out what are the coordinates of the 3D world point with respect to the camera coordinate system where originally the coordinates of the 3D world point are specified with respect to the 3 world coordinate systems.

So here, as we displaced the camera center by vector W_0; so all the world coordinate points, all the world points will be displaced by the vector which is negative of W_0 that is by minus W_0 and if W_0 has components of x_0 along X direction, y_0 along Y direction and z_0 along Z direction, so the corresponding transformation to the 3D points will be minus x_0, minus y_0 and minus z_0.

And, we have seen earlier that if a 3D point is to be displaced by minus x_0, minus y_0, minus z_0, then in the unified representation, the corresponding transformation matrix for this translation is given by G equal to (1 0 0 minus x_0) (0 1 0 minus y_0) (0 0 1 minus z_0) and then (0 0 0 1). So, this is a transformation matrix which translates all the world coordinates, all the world points by vector minus x_0, minus y_0 minus z_0 and this transformation is now with respect to the camera coordinate system.

The next operation as we said that after this displacement, we pan the camera by angle theta and this panning is done along the Z axis. So, when we pan along Z axis, the coordinates which are going to change is the X coordinate and the Y coordinate; the Z coordinate value is not going to change at all and for this panning by an angle theta, again we have seen earlier that the corresponding transformation matrix for rotation theta is given by r theta equal to (cosine theta sin theta 0) (minus sin theta cosine theta 0 0) then (0 0 1 0) and then (0 0 0 1).

So, when we rotate the camera by an angle theta; all the world coordinate points, all the world points will be rotated by the same angle theta but in the opposite direction and that corresponding transformation matrix will be given by this matrix R theta.

So, we have completed 2 steps. That is first displacement of the camera center with respect to origin of the world coordinate system, then panning the camera by angle theta. The third step is now we have to till the camera by an angle alpha and again we have to find out what is the corresponding transformation matrix for this tilt operation which has to applied to all the 3D points.

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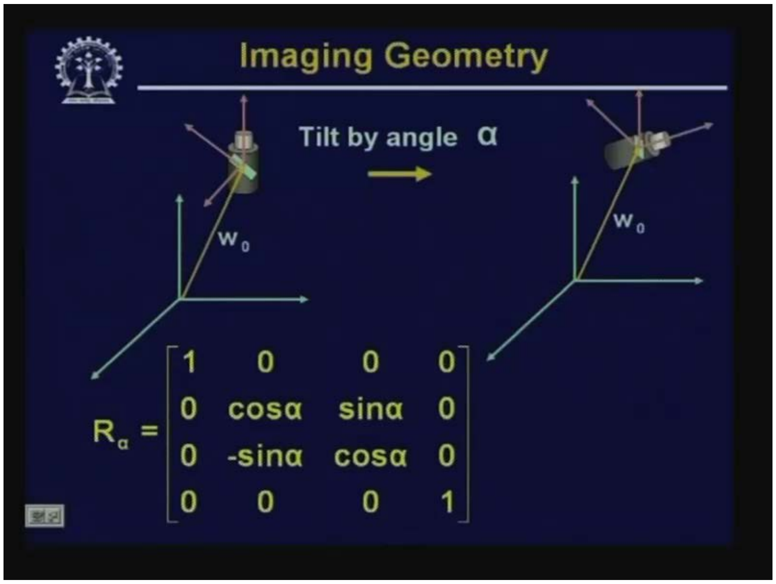

So, for this tilt operation by an angle alpha, the corresponding transformation matrix R alpha will be given by (1 0 0 0) (0 cosine alpha sin alpha 0) then (0 minus sin alpha cosine alpha 0) and (0 0 0 1). So, just you collect that these are the basic transformations which we have already discussed in the previous class and how these transformations are being used to understand the imaging process. So, so far we have applied one displacement and 2 rotation transformations along R theta and R alpha.

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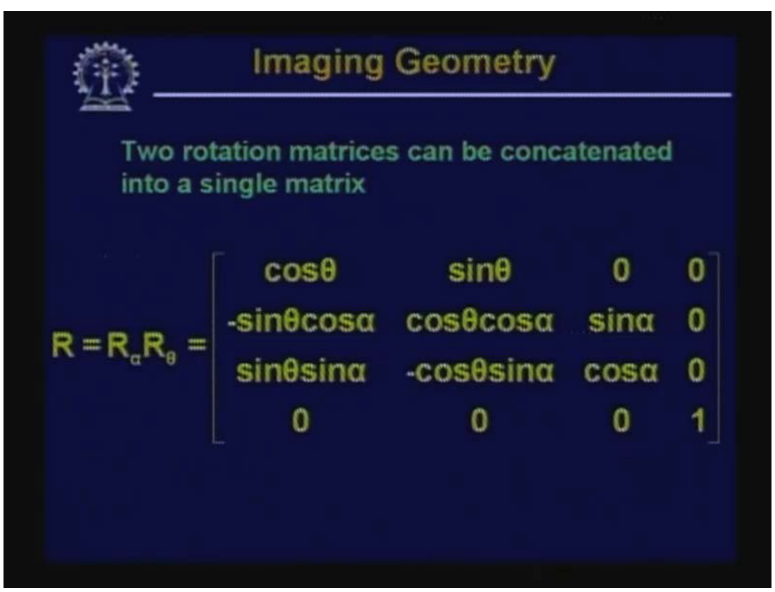

Now, find that this R theta and R alpha, they can be combined into a single rotation matrix R which is equal to R alpha concatenated with R theta and the corresponding transformation matrix R will be given by (cosine theta sin theta 0 0) then (minus sin theta cosine alpha cosine theta sin alpha sin alpha 0) then (sin theta sin alpha minus cosine theta sin alpha then cosine alpha 0) and then (0 0 0 1).

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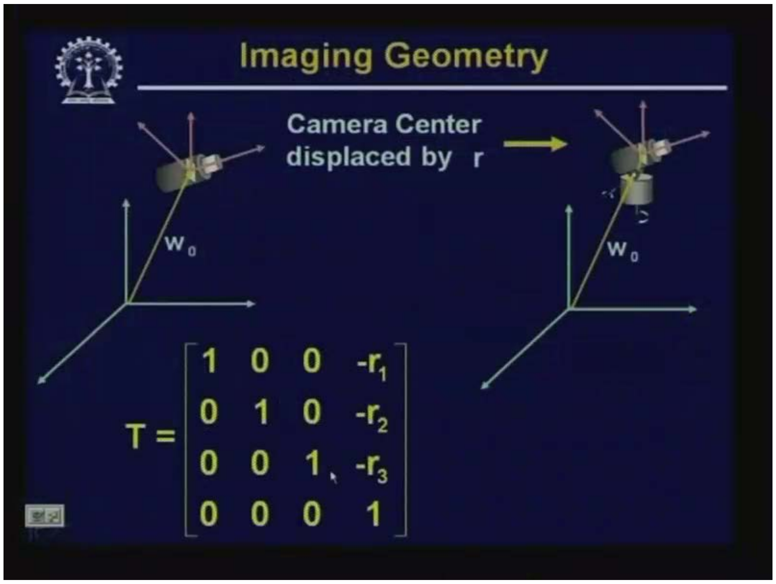

Then final transformation that we have to give to the camera center or the center of the imaging plane from the Gimbal center by a vector R and this vector R has the components r_1, r_2 and r_3 along X, Y and Z directions and by this transformation, now all the world points are to be transformed are to be translated by a vector minus r_1 minus r_2 minus r_3 and the corresponding translation matrix, now will be T equal to (1 0 0 minus r_1) (0 1 0 minus r_2) (0 0 1 minus r_3) and then (0 0 0 1).

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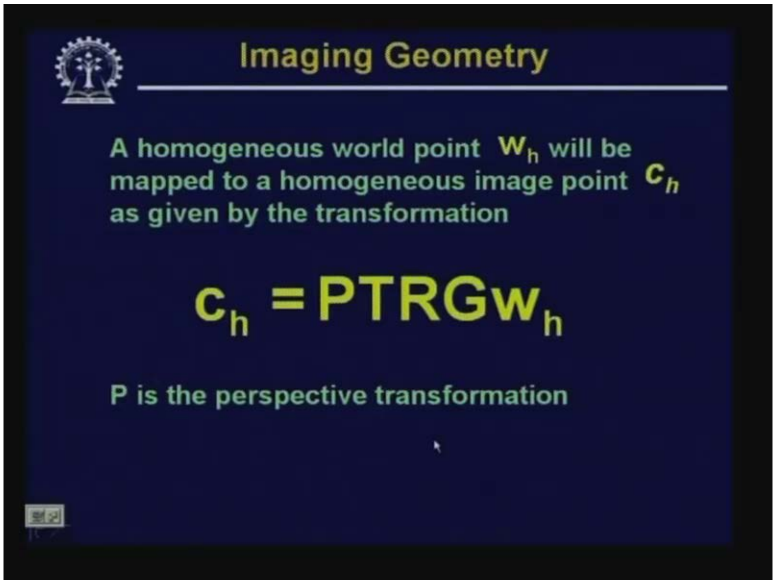

So, if I apply all these transformation one after another and I represent the 3D world point W by the corresponding homogeneous coordinates W_h; then your find that these transformations T, R and G taken together on this homogeneous coordinate W_h gives you the homogeneous transformed point W as seen by the camera. And once I have this transformed 3D point, then simply applying the perspective transformation on this transformed 3D points will give you the camera coordinate in the homogeneous point.

So, now the camera coordinate C_h is given by PTRG into W_h. So, you remember that this coordinate comes in the homogeneous form. Then final operation that we have to do is to convert this homogenous coordinate C_h into the corresponding Cartesian coordinate C.

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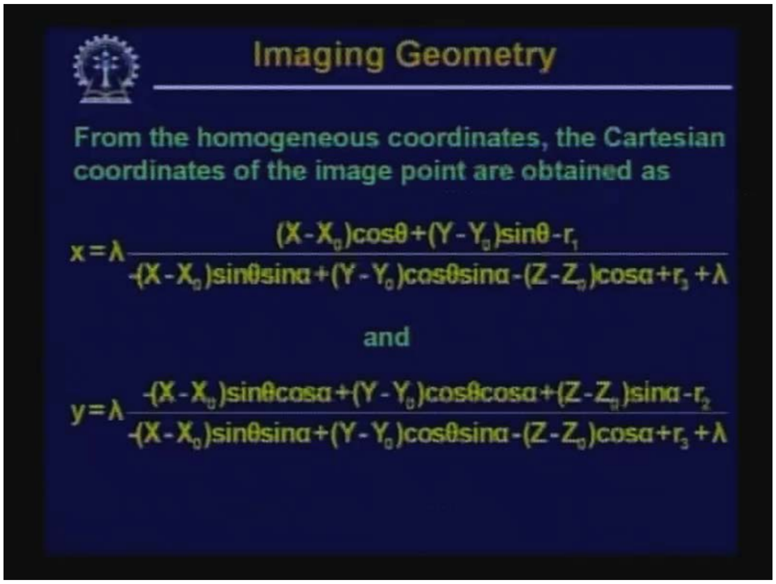

So, that Cartesian coordinate if I solve those equations will come in this form, you can try to derive this equations that X equal to lambda into X minus X_0 cosine theta plus Y minus Y_0 sin theta minus R_1 divided by minus X minus Y X_0 sin theta sin alpha plus Y minus Y_0 cosine theta sin alpha minus Z minus Z_0 cosine alpha plus R_3 plus lambda.

And, the image coordinate Y is given by lambda X minus X_0 sin theta cosine alpha plus Y minus Y_0 cosine theta cosine alpha plus Z minus Z_0 sin alpha minus R_2 divided by minus X minus X_0 sin theta sin alpha plus Y minus Y_0 cosine theta sin alpha minus Z minus Z_0 cosine alpha plus R_3 plus lambda.

So, these are the various transformation steps that we have to apply if I have a generalized imaging setup in which case the 3D coordinate axis and the camera coordinate axis, they are not aligned. So, the step that we have to follow is first we assume that the camera coordinate axis and the 3D coordinate axis, they are perfectly aligned. Then, give a set of transformations to the camera to bring to its given setup and apply the corresponding transformations but in the reverse direction to the 3D world coordinate points.

So, by applying these transformations to the 3D world coordinate points, the image points the 3D world coordinate points as seen by the camera will be obtained in the transformed form and after that if I apply the simple perspective transformation to this transformed 3D points, what I get is the image point corresponding to those transformed 3D world points.

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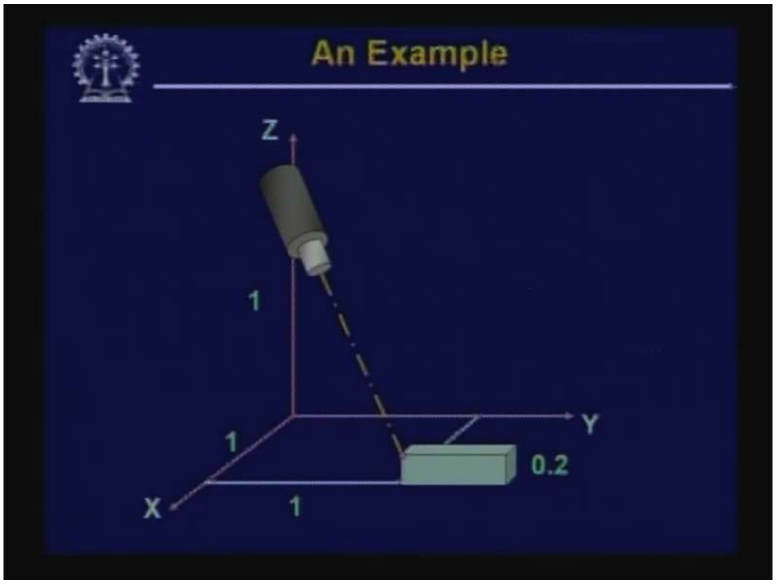

Now, let us try to see an example to illustrate this operation. So, let us take a figure where we assume that the camera or the center of the image plane imaging plane of the camera is located at location (0 0 1) with respect to the 3D world coordinate system X, Y, Z. And, we have an object placed in the X Y plane where one of the corners of the object is at location 1, 1, 02 0.2 and we want to find out that what will be the image coordinate for this particular 3D world point which is now a corner of this object as placed in this figure.

So, what we will try do is we will try to apply the set of transformations to the camera plane one after another and try to find out that what are the different transformations that we have to apply or what are the different corresponding transformations to the 3D world point that will bring that will give us the world coordinate points, world points as seen by the camera. So, initially I assume again that all the points are the image coordinate system and the camera coordinate system, they are perfectly aligned.

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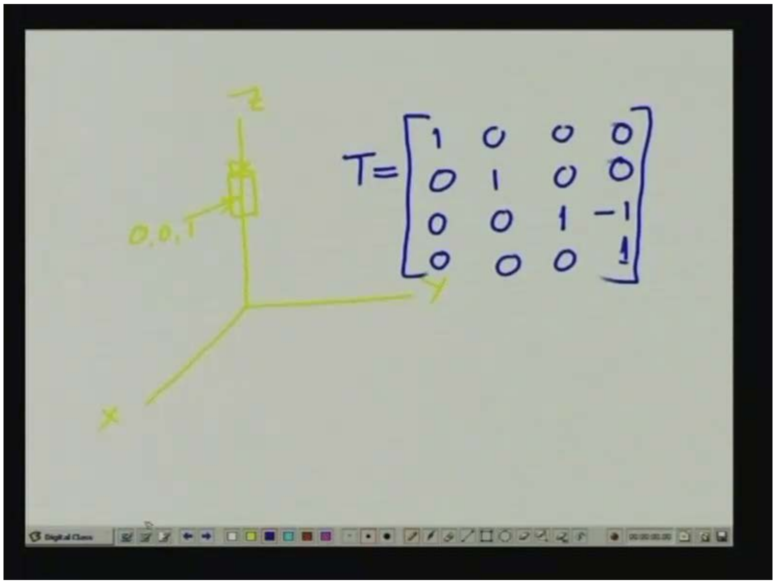

Now, after this assumption, what I have to do is I have to give a displacement to the camera by the vector (0 0 1). So, what I will do is I will bring the camera to a location here, so this is my camera where the image plane center is at location (0 0 1); so, this is my X axis, this is my Y axis this is the Z axis. Now, if I do this transformation, then you find that all the 3D points will be transformed by the vector (0 0 minus 1) with respect to the camera coordinate system.

So, the first transformation matrix which has to be applied to all the points in the 3D coordinate system is given by (1 0 0 0) (0 1 0 0) (0 0 1 minus 1) and then (0 0 0 1). So, this is the first transformation that has to be applied to all the 3D points. Now, after the camera is displaced by the vector (0 0 1), the next operation that we had to apply is to pan the camera by an angle 35 degree.

I just forgot to mention that in that arrangement, the pan was 135 degree; the tilt was also 135 degree. So, after this initial transformation, displacement of the camera by vector (0 0 1); we have to apply a pan of 135 degree to this camera.

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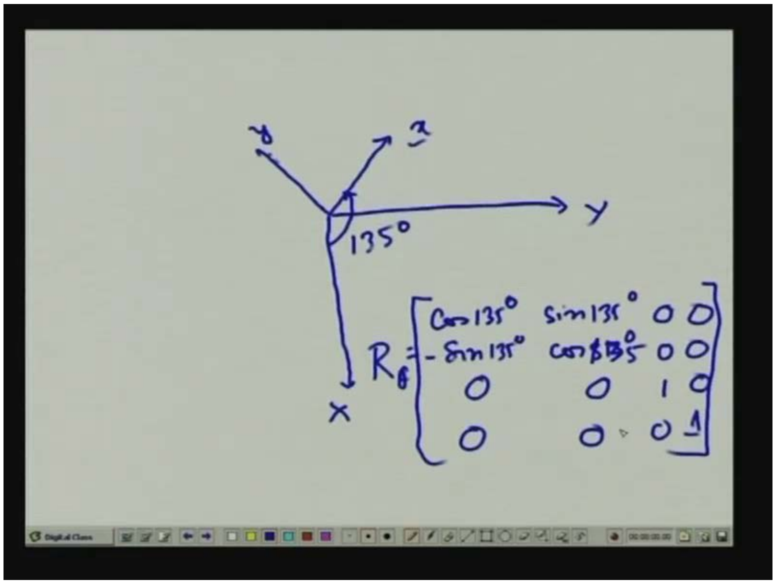

So, if I represent that, let us take a 1 2 dimensional view. As we said, the panning is nothing but rotation around Z axis; so if I say that this is the X axis, this is the Y axis, then by panning, we have to make an angle of 135 degree between the X axis of the camera coordinate system. So, the situation will be something like this.

So, this is the Y axis of the camera coordinate system, this is the X axis of the camera coordinate system and by pan of 135degree, we have to rotate the camera imaging plane in such a way that the angle between the X axis of the camera coordinate and the X axis of the 3D world coordinate is 135 degree and once we do this, here you find that this rotation of the camera is in the anticlockwise direction.

So, the corresponding transformation on the 3D world points will be in the clockwise direction but by the same angle 135 degree and the corresponding rotation matrix which is given now given by R theta will be equal to (cosine 135 degrees sin 135 degree 0 0) then (minus sign 135 degrees cosine 135 degree 0 0) then (0 0 1 0) then (0 0 0 1).

So, this is the rotation transformation that has to be applied to all the world coordinate points. So, after we apply this R theta, the next operation that we have to perform is to tilt the camera by an angle 135 degree.

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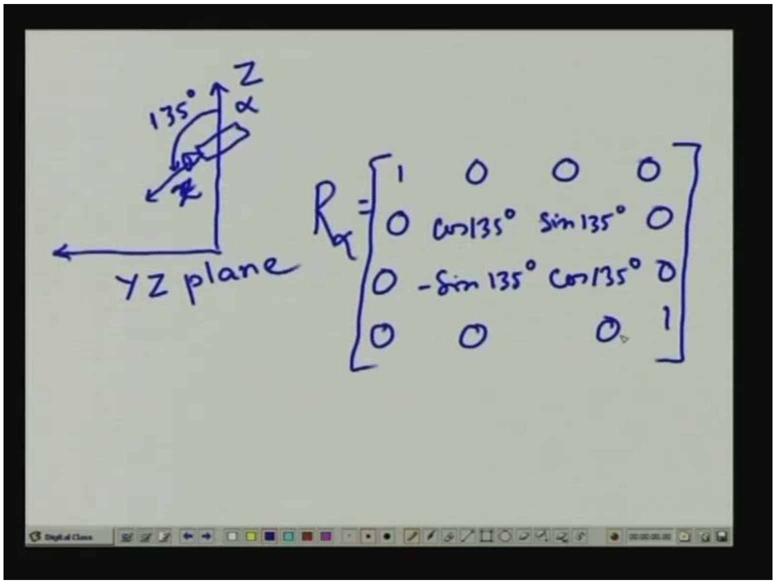

So again, to have a look at this tilt operation, we take again a 2 dimensional view. So, the view will be something like this. So, we take this as the Z axis of the 3 D world coordinate system and in this case, it will be the Y Z plane of the 3D world coordinate system and by tilt what we mean is something like this.

This is the Z axis of the camera coordinate system and the angle between the Z axis of the 3D world coordinate system and the camera coordinate system is again 135 degree. So, this is the angle tilt angle alpha. So here, again you find that the tilt is in the anticlockwise direction, so the corresponding transformation in the 3D world point will be rotating the 3D world points by 135 degree in the clockwise direction along the around the X axis and the corresponding transformation matrix in this case will be R alpha is equal to (1 0 0 0) (0 cosine 135 degrees sin 135 degree 0) (0 minus sin 135 degree cosine 135 degree 0) (0 0 0 1).

So, this is the transformation matrix that has to be applied for the tilt operation. So, after doing this, you will find that the 3D world coordinate, the 3D world point for which we want to find out the corresponding image point is given by (1, 1, 0.2).

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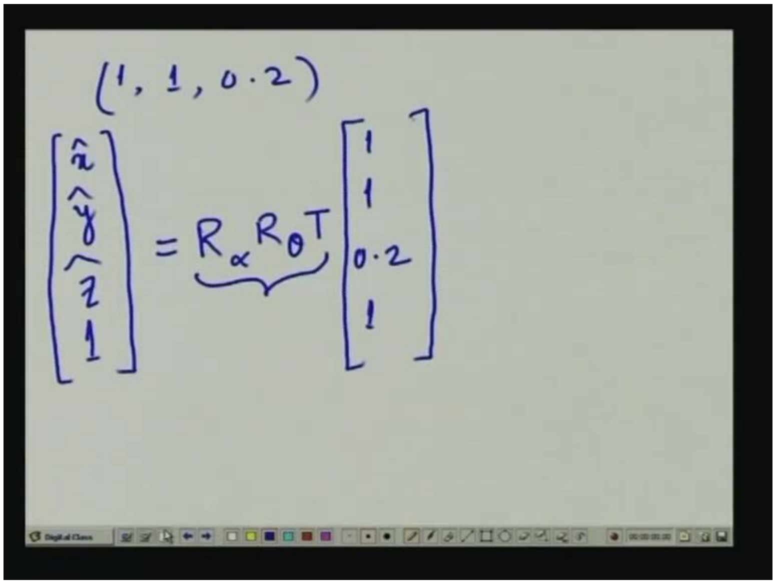

This is the 3D world coordinate point and after application of all these transformations, the transformed coordinate of this 3D world point if we write it as x hat, y hat, z hat and this has to be represented in unified form; so, this will be like this.

It has to be R alpha R theta then T and the original world coordinate (1, 1, 0.2 and 1) in the unified form. Now, if I compute this R alpha R theta and T using the transformation matrix that we have just computed that we have just derived, you will find that this transformation matrix can be computed as (minus 0.707, 0.707, 0, 0) then (0.5, 0.5, 0.707 minus 0.707) then again (0.5, 0.5, minus 0.707, 0.707) then (0, 0, 0, 1).

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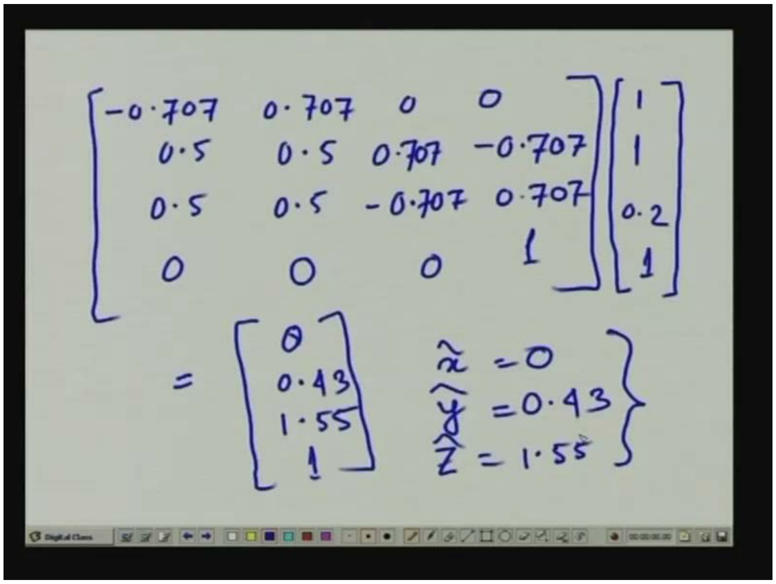

So, this is the overall transformation matrix which takes care of the translation of the image plane, then pans by angle theta and also tilt by angle alpha. So, if I apply this transformation to my original 3 D world coordinates which was (1 1 0.2 then 1); then what I get is the coordinates of the point as observed by the camera. So, if you compute this, you will find that this will come in the form (0, 0.43, 1.55 and then 1); again, this is in the unified form.

So, the corresponding world and the corresponding Cartesian coordinates will be given by x hat equal to 0, y hat equal to 0.43 and z hat is equal to 1.55. So, these are the coordinate for the same 3D point as seen by the camera. So now, what we have to do is we have to apply the perspective transformation to these particular points.

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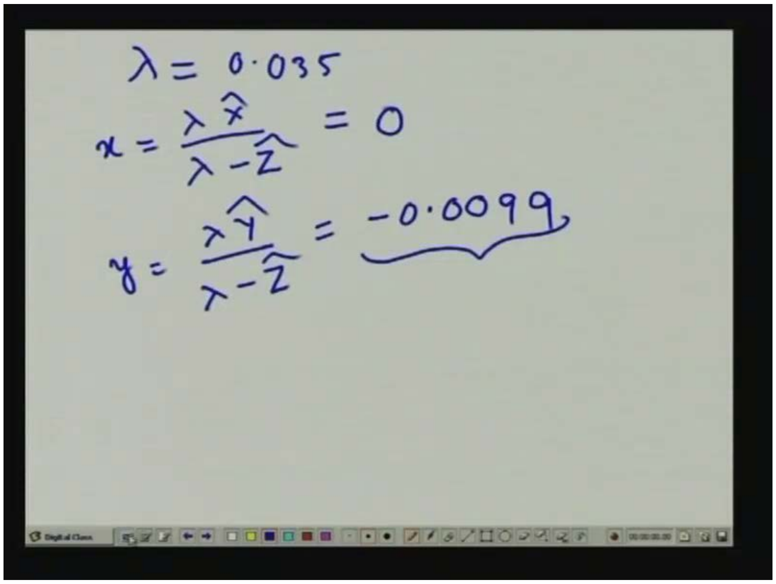

So, if I apply the perspective transformation and if I assume that the focal length of the camera is 0.035, then we obtain the image coordinates as x equal to lambda x hat divided by lambda minus z hat which will be in this case of course 0 and y equal to lambda y hat divided by lambda minus z hat which if you compute this will come as minus 0.0099. So, these are the image coordinates of the world coordinate point that we are considered.

Now, note that the Y coordinate in the image plane has come out be negative. This is obvious because the original 3D world coordinate has obtained by after applying the transformations came out to be positive. So obviously, in case of image plane, there will be an inversion. So, this value of Y coordinate will come out to be negative.

So, this particular example illustrates the set of transformations that we have to apply followed by the perspective transformation so that we can get the image point for any arbitrary point in the 3D world. So, with this we complete our discussion on the different transformations and the definite imaging models that we have taken.

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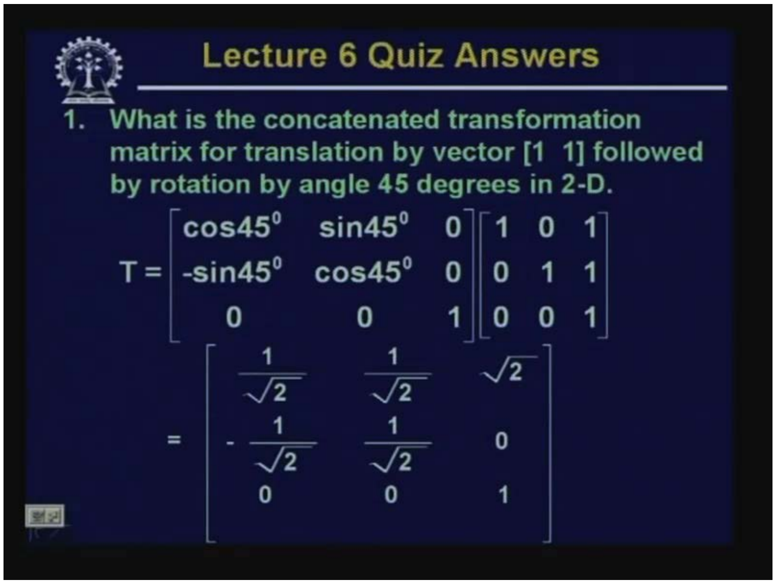

Now, let us try to see the answers to the quiz questions, the questions that we have given in the last class. So, the first question was what is the concatenated transformation matrix for translation by vector [1 1] followed by rotation by angle 45 degree in 2 dimension? So, here you find that first you have to apply the transformation followed by the rotation.

So, the concatenated transformation will be (cosine 45 degrees sin 45 degree 0) (minus sin 45 degree cosine 45 degree 0) (0 0 1) then (1 0 1) (0 1 1) (0 0 1). So, if you multiply these 2 matrixes, the concatenated transformation matrix will come out to be (1 over root 2, 1 over root 2, root 2) then (minus 1 over root 2, 1 over root 2, 0) then (0 0 1).

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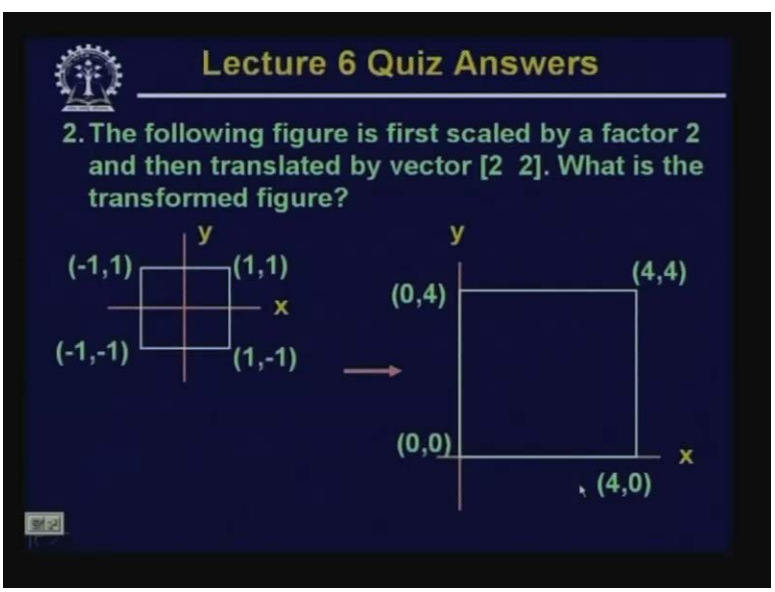

The second question that we are asked that for this square figure whose are at locations (1, 1) (minus 1, 1) (minus 1, minus 1) and (1 minus, 0); if this figure is first scaled by a vector 2 and then translated by vector [2 2]; then what is the transformed figure? In this case, the transformed figure will again be a matrix will again be a square whose coordinates after both these transformations will lie at (0, 0) (4, 0) (4, 4) and (0, 4).

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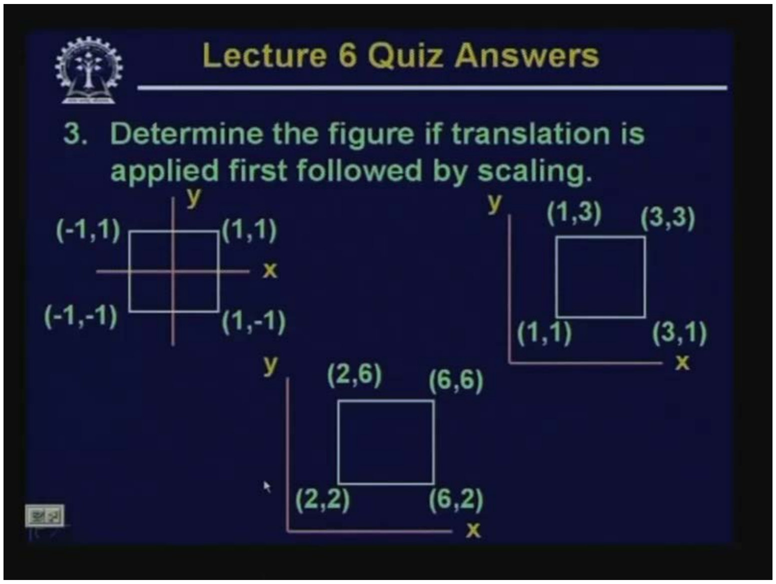

The third question we had asked is what will be the figure if the translation if the transformations are applied in the reverse manner? So, in this case, first you have to apply the translation followed by the scaling and then after final transformation, your coordinates will be (2, 2) (6, 2) (6, 6) and (2, 6).

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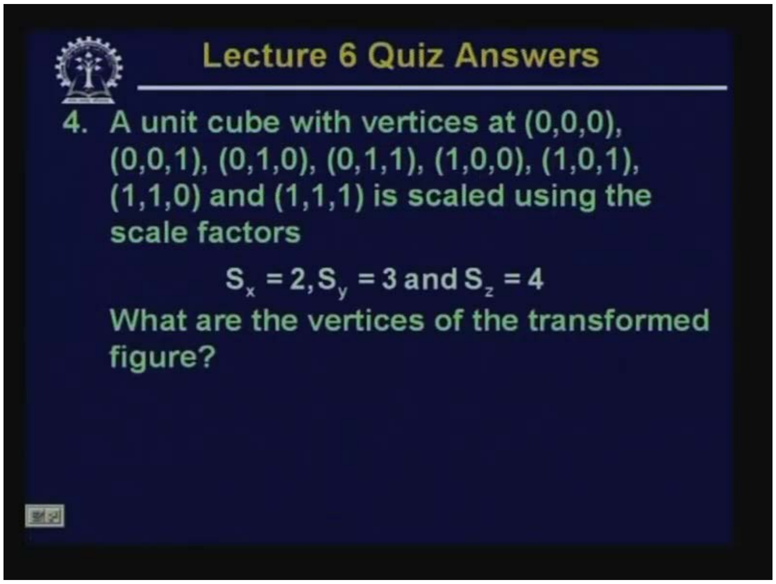

Then we had given another problem with vertices of a queue where the scale factors were 2 along X axis, 3 along Y axis and Z along and 4 along the Z axis.

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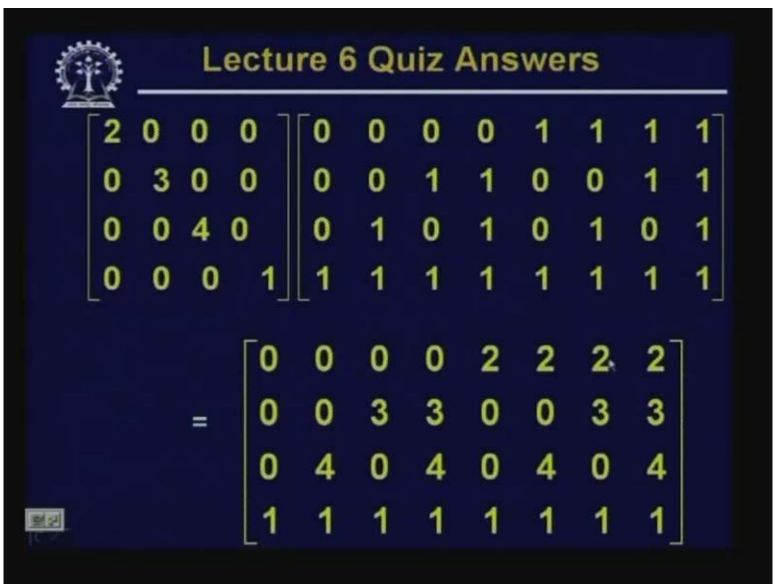

So, if we transform this queue with the help of these scale factors, then finally I will get the transformed coordinates like this – (0 0 0), (0 0 4), (0 3 0), (0 3 4), (2 0 0), (2 0 4), (2 3 0) and (2 3 4).

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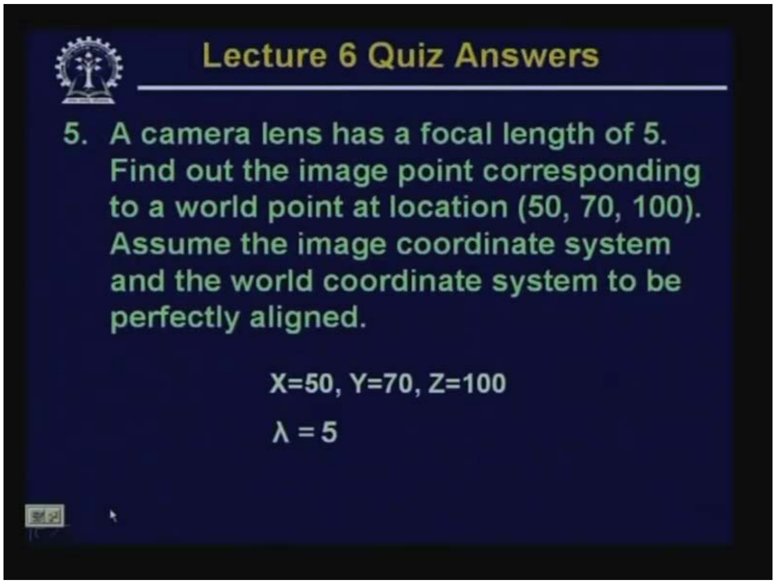

Then we had given the fifth camera fifth problem where you had given a 3D world point and the coordinate of the camera. We had to find out we have to find out what is the corresponding image point?

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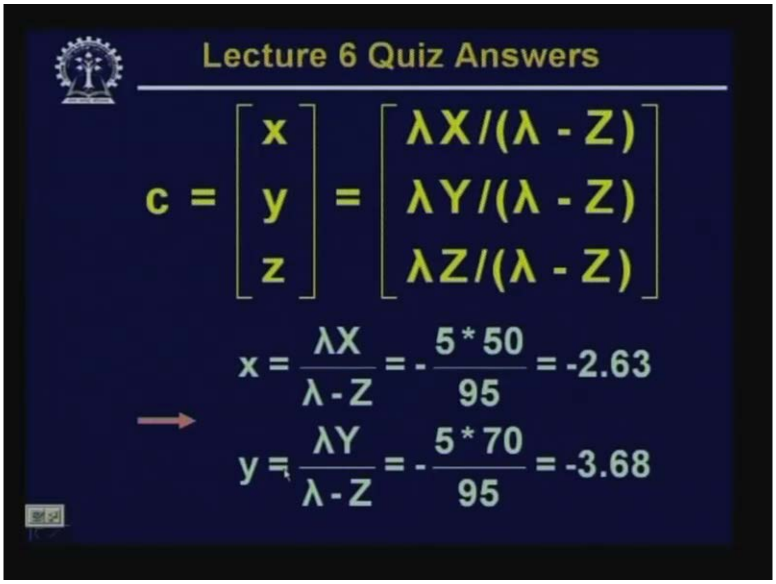

So here, the image point is obtained as x equal to minus 2.63 and y equal to minus 3.68 after applying the transformations that we have discussed.

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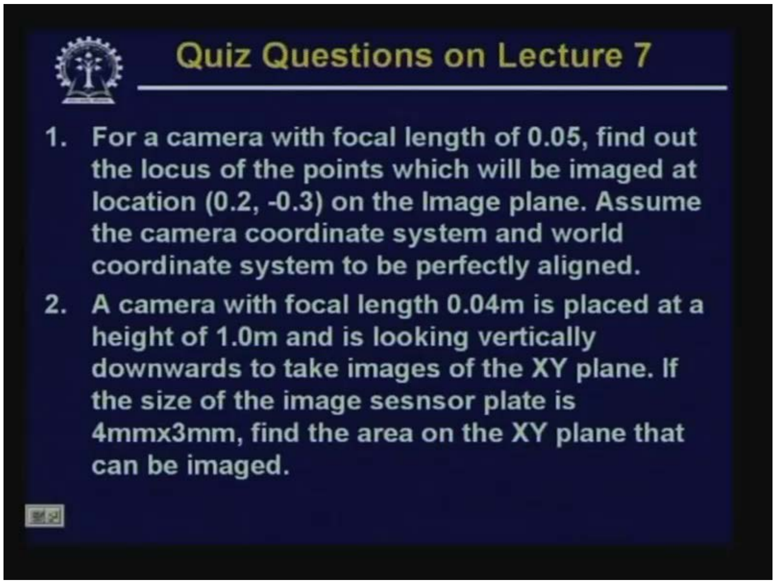

Now, coming to today's quiz question. The first question is for a camera with focal length of 0.05, find out the locus of the points which will be imaged at location (0.2, minus 0.3) on the image plane. You assume that the camera and coordinate system and the world coordinate system are perfectly aligned.

The second question; a camera with focal length of 0.04 meter is placed at a height of 1 meter and is looking vertically downwards to take image of the XY plane. If the size of the image sensor plate is 4 millimeter by 3 millimeter, find the area on the XY plane that can be imaged.

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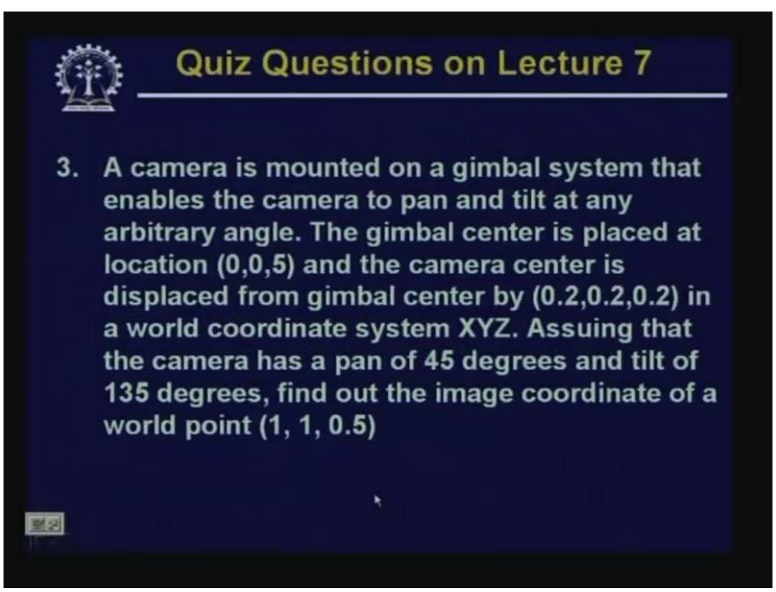

Then the third question: a camera is mounted on a Gimbal system that enables the camera to pan and tilt at any arbitrary angle. The Gimbal center is placed at location (0, 0, 5) and the camera center is displaced from the Gimbal center by (0.2, 0.2, 0.2) in a world coordinate system XYZ. Assuring that the camera has a pan of 45 degrees and tilt of 135 degree, find out the image coordinate of a world point (1, 1, 0.5).

Thank you.