**Digital Image Processing**

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**Lecture - 40**

**Object Recognition**

Hello, welcome to the video lecture series on digital image processing.Now, in today's lecture, we will discuss about the final phase of image processing that is image understanding which we have termed as object recognition.

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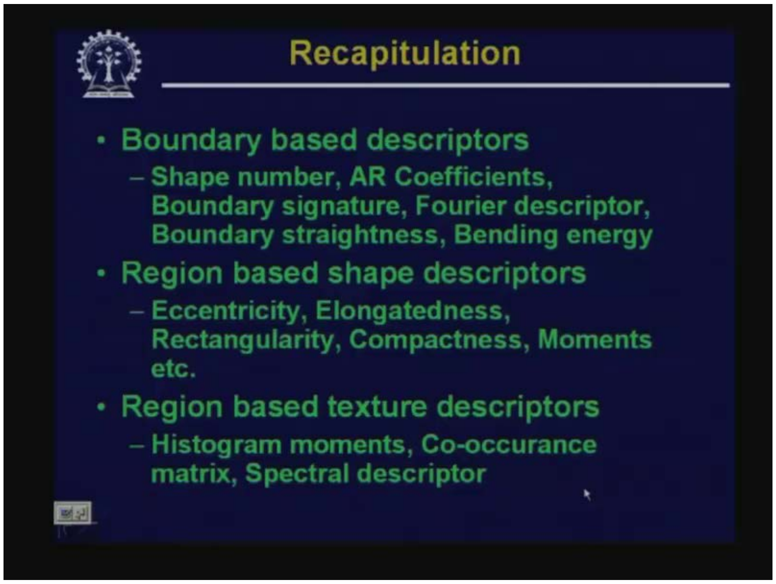


So,tillour last class, we have seen the different representation and description techniques and we had said that for understanding the images are to recognize the objects present in the images,wehave to have a proper representation mechanism so that the objects or the shapes present in the image can be represented properly.

Andthen, for such a representation scheme, we had to have a proper description; from the represented shapes,wehave to generate a proper description so that using these descriptions, the shapes can be matched against a state of similar such descriptions which are kept in the knowledge base and after this matching, we can identify that the object that we are getting from the image which particular object it is or we can roughly classify that which class to which class of the objects which are there in the knowledge base of the computer, this current object belongs. So, till our last class, we have seen a number of such representation and description techniques.

Sofirstly, we had obtained some boundary based descriptors and we have seen that the different boundary based descriptors or shape numbers, the shape number is something which is generated from differential chain code representation of the boundary.Wehave seen the auto regression coefficients where for getting these auto regression coefficients,we had to get a polygonal approximation, a polygonal representation of the boundary. Then the corners at vertices in this polygonal, they are represented by an auto regression model and by solving a number of linear equations, we can find out we can solve for those auto regression coefficientsand this auto regression coefficientsor state of regression coefficients that also act as a descriptor of the shape.

Then, we have seen a boundary signature and we have said that this boundary signature is nothing but some one - dimensional representation of a 2 dimensional boundary.So, there what we have to do is we have get the centroid of the shape and from the centroid of the shape, we have to get the distance of different boundary points and when you get the distance of different boundary points; then from the center, the boundary points have to be traced either in the clockwise direction or in the anticlockwise direction.

And, the direction of this particular distance, if I plot the distance is the direction from the centroid of that particular object; then what we get is 1 dimensional or 1D functional representation of the 2 dimensional boundaryand that is what we have called as a boundary signature and we have seen that if I have shapes of different types for different types of shapes, we get different types of boundary signatures and this signatures are obviously, they have to be normalized properly so that the values lies between0 and 1and these boundary signatures can also be used for recognition of the shape or shape description purpose.

Then, the other boundary based descriptor that we have obtained is Fourier descriptor.So, in that case what we have done is we have represented different boundary points, the points lying on the boundary as a complex numberand if you trace the boundary either in the clock wise direction or in the anticlockwise direction; then basically what I get is a sequence of complex numbersand if I take the descript Fourier transformation of this sequence of complex numbers, then what I get is a state of Fourier coefficients and in this case,in general,the state of Fourier coefficients are complex in nature.

So, this state of Fourier coefficients, they also act as descriptors which can be used for matching purpose or for recognition purpose.Thenwe have defined, we have obtained some more boundary based descriptors like boundary straightness, bending energy and so on.Then we have also talked about some region based shape descriptors.So, in region based shape descriptors, we have defined we have seen different descriptors like eccentricity whereeccentricity we have said that it is nothing but the ratio of the length of the major access and the length of the minor access of that particular shape.So, what you have to get is we have to obtain the length of the major access and we have to obtain the length of the minor access and the ratio of thesetwo is what is known as eccentricity.

Thenwe have seen another region based shape descriptor which we have said is elongatedness and for getting this elongatedness, what you have to obtain is a minimum bounding rectangle and the ratio of the sides of this minimum bounding rectangle is defined as this elongatedness.We have also talked about rectangularity, we have also talked about compactness, then we have seen some other moments, moment based descriptors and we have seen that there are 4 different moment invariants which can be used as descriptors of the shape.

Thenthe other kind of descriptor,we have said is here,wewant to find out the surface reflectance property of the object present in the image and we have said that this surface reflectance property, this may be a collar or the texture information feature present on the object surface.So, this region based descriptors, say for example, for textures, we have talked about the histogram best moments.Similarly, we have talked about the co-occurrence, the descriptors obtained from the co-occurrence matrix of the texture and we have also seen the different structural different spectral descriptors of a particular texture.So, all these different types of descriptors or a set of all these different types of descriptors, they can be used for recognition of an object or understanding the object present in the image.

Now,youfind that out of all these, it is the shape number which is slightly different because shape number basically gives a chain of different numerical values say ranging from 0 to 7 and this chain or cycle of numerical values actually describe the boundary of the objectwhereas the other kind of descriptors, we can assume that each of the other descriptors highlight a particular property of the shape.

So, if I take a set of such descriptors, say if I take n number of such descriptors and put those descriptors in an ordered manner; then whatI get is an n dimensional vectorand this n dimensional vector which is normally called as a feature vector; so each of this feature vectors represent a particular object or a particular object shape.So, we will see that for different objects or the different object shapes, this feature vector is going to be different.So, the recognition can be done based on this feature vectors because we assume that for different objects, they will be different.

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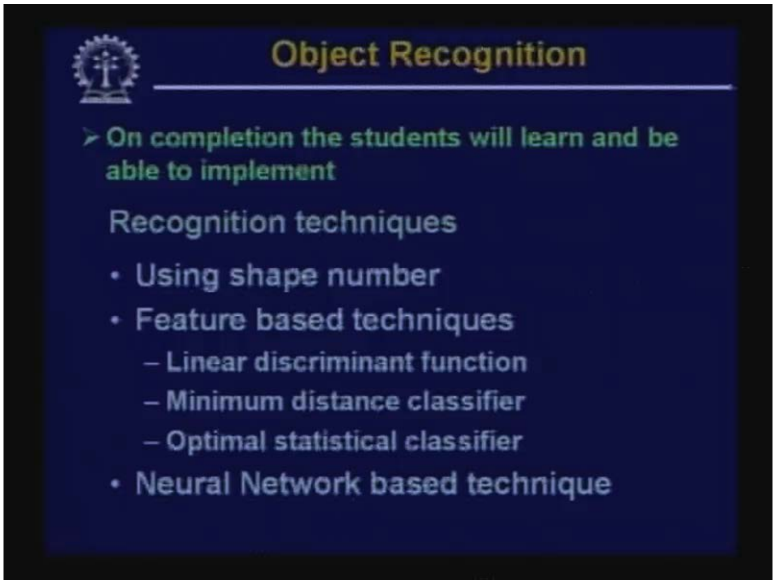

So,in today'slecture, we will talk about these object recognition problems.Sofirst, we will talk about the recognition technique using the shape number which is obtained from the boundary of the object,thenwe will talk about some feature based techniques for object recognition.So, under that we will talk about the linear discriminant function,we will talk about the minimum distance classifier and we will talk about optimal statistical classifier.

So, all these different the linear discriminator discriminant function, the minimum distance classifier or optimal statistical classifier; all these different types of techniques are using the feature vector as we have said that the feature vectors is nothing but an ordered state of the different features of descriptors that we have obtained earlier.Andlastly, we will talk about of another approach of recognition which is the neural network based technique and this neural network based technique also makes use of the feature vectors as the descriptors of different objects or different object shapes.So, the first one that we will talk about today is what we said is the shape number based approach.

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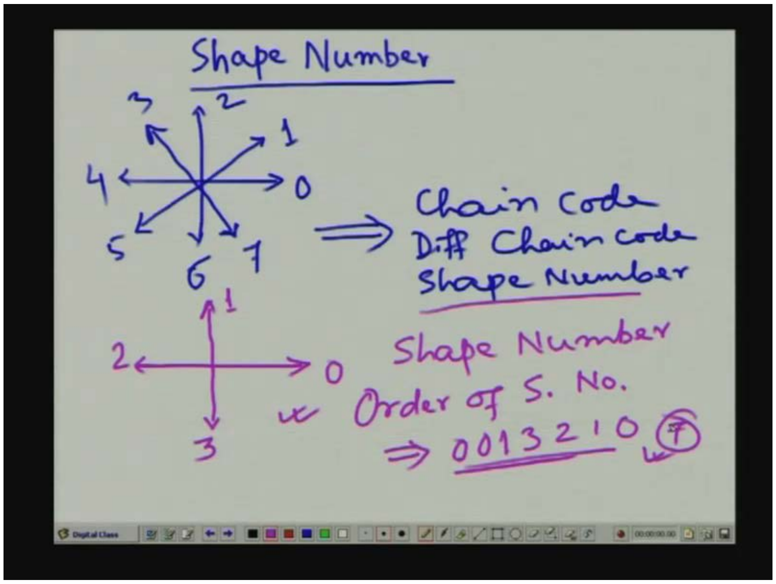


So,recognition technique using shape number:so earlier, we have talked about the shape number, generation of shape number using 8 directions.So, there we have said that in the 8 direction, we have identified the different directions of moves like this;so this is the direction which was given as 0, this is 1,this was 2, this was 3, this was 4, this was 5, this was 6 and this direction was 7 and using this 8 directions, we had obtained the chain code, we have obtained differential chain code and then we have obtained the shape number.

Now,for simplicity of the discussion, today we will talk about another kind of chain code generation which does not use 8 directions butit uses 4 different directions.So, today we will talk about the chain code using 4 directions and in these4 directions,our direction identifications are like this; so this direction will be mentioned as 0, this direction as 1, this direction as 2 and this direction as 3.

So, if I obtain a differential chain code using these 4 different directions; so first what we do is we obtain a chain code using the 4 different directions, then you obtain the differential chain code using 4 different directionsand we have seen that this differential chain code if I consider that to be a cyclic chain or cyclic set of numbers; then from here, from this differential chain code,I can redefine the starting point so that the resulting numerical value will be the minimumand we have said that that particular numerical value is what we have termed as a shape number.

Now,there is another term which is called order of the shape number order of shape number.Now, this order of shape number is nothing but the number of digits which are present in the shape number.So, if I have a shape number something like this, say 0 013210, something like this;so we find that in this particular shape number there are 1,2,3,4,5,6,7,7 different digits.So, the order of this particular shape number will be 7.

Now, the problem is if I want to use the shape number as a descriptor and this shape number is to be used for recognizing a particular object or to say that a particular object is similar to one of the state of objects which are present in the knowledge base of the computer. Thenwhat you have to do isthat this shape number must be independent of the starting point I mean whatever starting point we used to generate the shape number.So, this generated shape number must be independent of the starting pointand secondly the order of the shape number of the object of the object shape which we are trying to recognize and the order of the shape number which is therein our knowledge base which is stored in the computer, they must be same.

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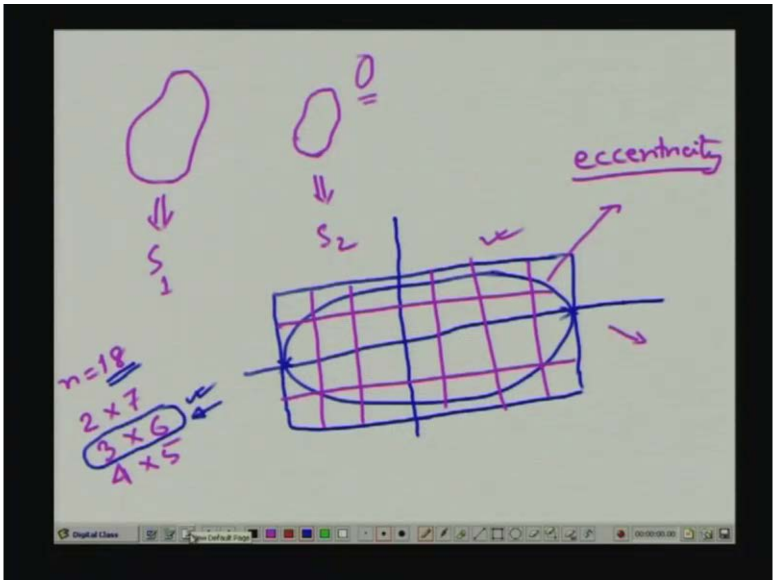


Now, the problem is something like this; suppose,I have an object of this form and I have a similar shape of object but the size is slightly different,so this will give me one particular shape number say $S_1$, this will give meanother particular shape number say $S_2$. So,our first job is when we want to generate the shape number of this particular object say O; in that case, the shape number that I have to generate must be starting point independent and at the same time, whatever is the order of the shape number of $S_1$, the order of the shape number $S_2$ must be same.

So,to obtain this, what we have to do is we have to align the grid in a particular way so that our generated shape number is independent of the starting point and not only that, we have decide that what should be that grid spacing to generate this particular shape number.So,one way to make it starting point independent is that you align the grids in such a way that the grid is aligned with the principle axisof the particular shape.So, if I have an object say something like this,then our principle axisis this one.

So,I want to ensure that the grid that I face,the grid must be aligned with this principle axis and this is nothing but as we have said that major axisand minor axis is perpendicular to this major axis.So, our girds must be aligned with this major axis and the minor axisand then to decide about what should be the grid spacing because this is what we will decide about what will be the order of the shape number that we generate.

Now, since we are discussing about the chain code using 4 different directions,so you will find that if I specify the order of a particular chain code of a rectangle what we can do is we can enclose this particular shape using a rectangle and this rectangle has to be divided into a number of cells.So, you can divide this rectangle into a number of cells like this and this is what will be our grids.

So, I have to decide about that how many such cells I should have to generate the shape number of a particular order.So, for doing this, what we do is we consider that what is the eccentricity,wehave defined eccentricity earlier; so we define, we find out what is the eccentricity of this particular shapeand then we generate a rectangle.Suppose, we are interested in shape number of order say 18, so you want to make the order n equal to 18.So, we want to obtain a rectangle of whose shape number will be of order 18 and you will find that I have limited number of such possibilities,I can have limited number of shapes or rectangles having shape number of order of the shape number equal to 18 and these possibilities are I can generate a rectangle of size say 2 by 7 or I can have a rectangle of size say 3 by 6 or I can have a rectangle of size say 4 by 5.

So, you will find that for all these different rectangles, the shape number the order of the shape number will be equal to 18.Then, suppose the eccentricity of the object shape best matches with the eccentricity of the rectangle 3 by 6;so what we have to do is we have to put a rectangle of size 3 by 6centered at the middle point of the major axis of the shape and these grids must be aligned with the major axis and minor axis of that particular shape.Andonce I have a rectangle of this particular size 3 by 4 fitted into that particular rectangle of specific eccentricity, then what I can have is using this form of grid spacing if I generate the shape number, then that particular shape number will be of order 18.

So, depending upon a shape number of which particular order we want, we have to generate rectangles of similar size and using the corresponding rectangles of the grid spacings, we can generate the shape number.Now, our purpose is recognition.

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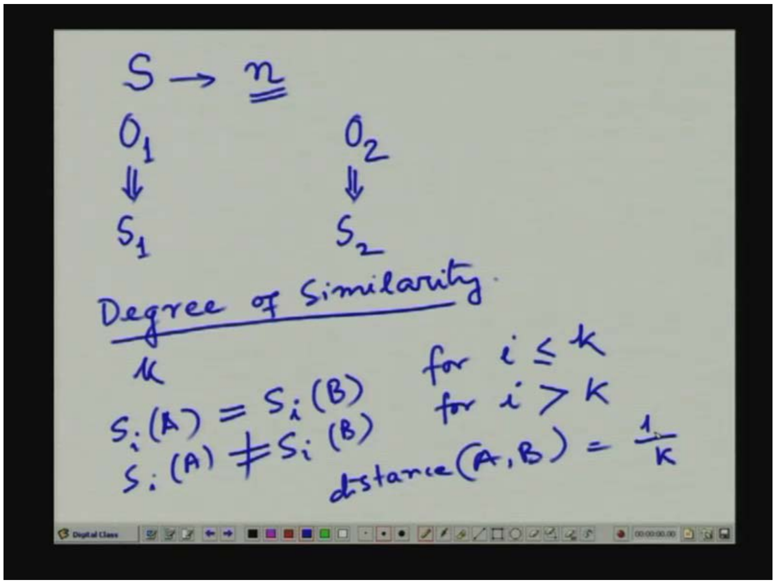


So, what we have got is we have got a shape number S and this shape number is so obtained that it is starting point independent and this shape number is having some order say nthat is the number of digits present in the shape number is equal to n.Now, given 2 objects; so I have one object $O_1$ and another object $O_2$. For$O_1$, I have the corresponding shape number $S_1$ and for $O_2$, I have the corresponding shape number $S_2$. Now, when I generate this $S_1S_2$, I make sure that they are of same order n.

Now, this order n can be varied depending upon spacing of the grids or on which the shape number is to be calculatedand we have said, discussed just now that how I can decide about what should be the girds spacing depending upon the shape number of which particular order, we want to generate.

Now, then comes a concept of degree of similarity.Now, what is this degree of similarity? When we are given these 2 particular shape numberssay $S_1$ and $S_2$ of the given order,then the degree of similarity say k is defined as the maximum order for which the shape numbers $S_1$ and $S_2$still matches.So, if I generate $S_1$ of say order 4 and $S_2$say of order 4, then $S_1$ of order 4 and $S_2$ of order 4 will be the same.Say $S_1$ of order 6 and $S_2$of order 6 will be same;$S_1$ of order 8, $S_2$ of order 8 will also be the same but I find that $S_1$ of order 10 is not same as $S_2$ of order 10.

In that case,I will say that the degree of similarity between these 2 shape numbers -$S_1$ and $S_2$ are between 2 objects shapes $O_1$ and $O_2$ is equal to 8.So,I define this degree of similarity as say if I say that $S_i$(A) is a shape number of object shape A of order is equal to $S_i$ (B), this is also the shape number of object shape B of the same order i; then $S_i$(A) and $S_i$(B) should be same for all iless than or equal to kwhereas $S_i$(A) and $S_i$(B) for all i greater than k, they should not be same.

So, in such case, this is the highest order for which the shape numbers still matches that is called the degree of similarity of these 2 shapes - A and Band using this degree of similarity, we can find out whether given2 different shapes are similar or notand you will find that from here, of course,I can define a distance function say distance between shapes A and B, this can also be defined as 1 upon the degree of similarity k because more similar the objects are the distance between those 2 objects shapes should be less.

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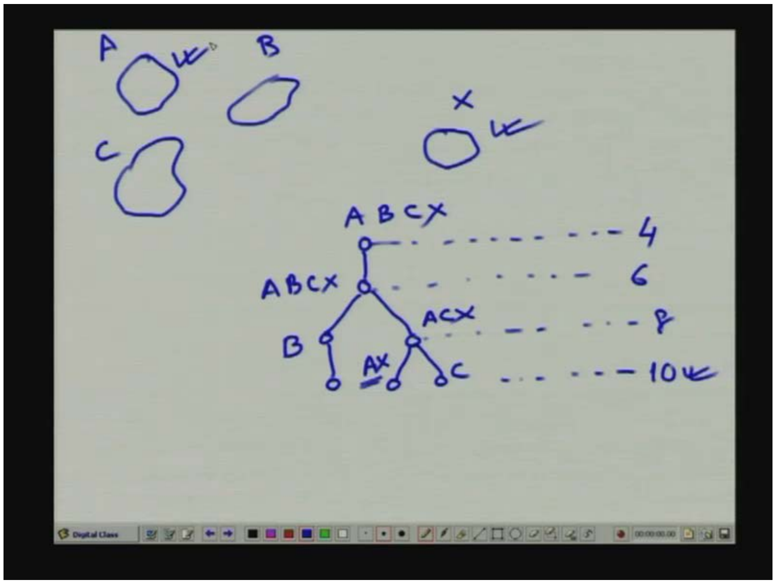


Sosuppose, we are given a number of shapes like this; in the data base, say something like this and suppose this is a shape which I want to recognize.So, for recognition purpose, what I have to do is or for matching purpose what I have to do is first I have to get all the shape numbers say this is object A, this is object B, this is object C and this is object X which I want to recognize.So, first I have to start with the lowest order and for 4 connected chain code, the lowest order of the shape number is equal to 4.

So, if I find that for the lowest order, all these shapes A B C and X, they are similar, then I put them as a single node in a chain.So, this is for fourth order.Then I go for sixth order, then still I may find that A B C and X, they are similar.Then, at the next order, next higher order, suppose at order 8,I find that B is different but A C and X, they are same.Thenif I go for eighth order, B will remain to be different and here I may find that A and C, they are similar whereas sorry A and X, they are similar whereas C is different.

So,this is of order tenand after this, A and X may be different if I go for say twelfth order,I may find that given A and X are becoming different.Butat least upto tenth order of the shape number,I find that A and X are similar and B and C are different.So, at this point,I infer that this shape X matches best with shape A which is or knowledge base.So, this is what is called a decision tree and using this decision tree, we can obtain or we can recognize a given shape against a set of shapes which are present in the knowledge base.

So, using the shape numbers and using the decision tree and following the concept of the degree of similarity, we can recognize a particular shape using the shape number itself.Now, the next topic that we will discuss is that is the featurebased recognition.

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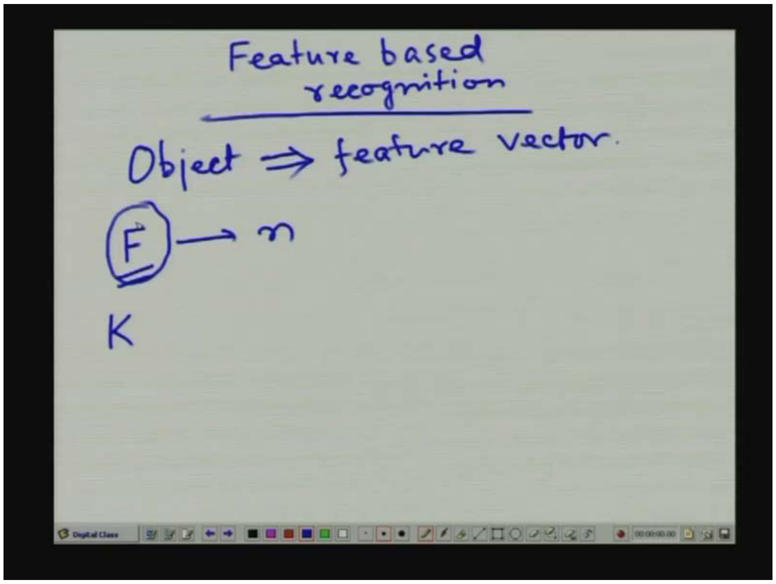


Andin this case, as I have said, what I assume is every object is represented by a set of features or ordered set of features which we call as a feature vectorand suppose, say a feature vectorsay F of a particular object contains n number of descriptors or n number of features.Now, each of these features maybe either eccentricity, elongatedness and so on.So, I saythat this feature vector F is of dimension n.

Now, if I consider an dimensional space or space of n different dimensions, then this feature vector F will be represented by a point in that n dimensional space.Now, findthat if I have a class of similar objects; in that case, the feature vectors of every object belonging to that particular class will be similar.So, if I plot all these feature vectors in that n dimensional space, then these different feature vectors say if I say that the class contains the n particular class of a objects, contains say K number of objects.So, I will get K number of feature vectors and the points, the K number of points corresponding to this K number of feature vectors n or n dimensional space will be placed close to one another. So, these points will try to form a cluster in the n dimensional space.

Now, using this concept that the feature vectors of the objects belonging to a particular class will be very close to each other and the feature vectors of the objects belonging to different classes will form 2 different clusters, they are located in 2 different locations in our n dimensional space; we can design a classifier which can classify an unknown object into one of the known classes.Now, let us say how that can be done.

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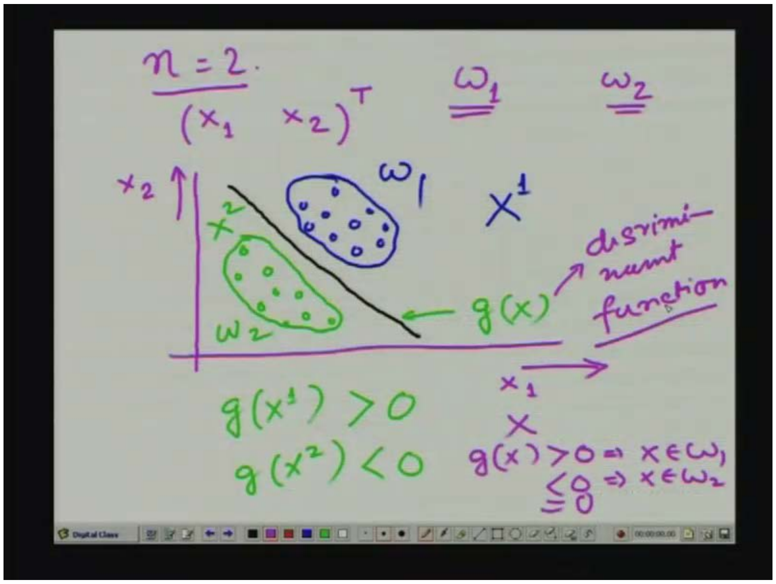


Forour discussion purpose and for simplicity,let us assume that dimension of the feature vectors that is n is equal to 2.So, if the dimension of the feature vector is equal to 2 that mean every feature vector is represented by a vector of this form.Itwill have 2 components $X_1$ and $X_2$ and every feature vector is nothing but a vector having these 2 components -$X_1$ and $X_2$. Now, what I do is let us plot these points in a 2 dimensional space.WhyI am doing for 2 dimension is it is easier for visualization.

So, suppose this horizontal direction is to represent the first component of the feature vector $X_1$ and I take this vertical direction to represent the second component of the feature vector that is $X_2$and suppose we have 2 different classes of objects and the classes of objects,I represent it by omega $_1$ and omega $_2$. So, omega $_1$ represents one class of objects and omega 2 represent another class of objects.

So, suppose the points belonging to or the feature vectors of the objects belonging to plus omega $_1$, I represent them by this blue line blue circles,so like this and the feature vectors of all the objects of the objects which belong to class omega $_2$, I represent them as green circles, like this.So, here you find that the feature vectors of the objects belonging to class omega $_2$, they form a cluster something like this and the feature vectors of the objects belonging to class omega $_1$, they form a cluster like this.

Now, given such a situation, it is possible that I can design or I can find out a line separating these 2 regions.Now, design of the classifier means I have to decide that what should be this line which demarcatesbetween the feature vectors of the objects belonging to class omega $_1$ and feature vectors of the objects belonging to class omega $_2$. So, this class on this side represents my class omega $_1$ and of this side represents my class omega $_2$.

Now, that can be obtained very easily because from our school day mathematics we know that if we have a straight line,then for the points belonging to one side of the straight line,I will get avalue which is greater than 0 and for other sides on the straight line,I get a value equal to 1.Sobasically, what I have to do is I have to get an equation of this straight line say g (X),I call it g (X) such that for any point or any vector say $X_1$,let me use some of the notation because otherwise it will be confusing.So, let me represent them by super script.So, I say that X super script 1 is the feature vector of an object belonging to class omega $_1$ and X super script 2 is a feature vector for an object belonging to class omega $_2$.

So, I can assume that for all these points $X_1$ my g $(X^1)$ that should be greater than 0and g $(X^2)$ super script 2 for all the objects belonging to class omega $_2$, this should be less than 0.So, once I decide once I design this g (X) having this sort of property; then for any unknown feature vector X if I find that g (X) is greater than 0,I immediately infer that X belongs to class omega $_1$ whereas if g (X) is less than 0,I immediately infer that X belongs to class omega $_2$whereas of course, g (X) equal to 0 that is the boundary case that means these are the feature vectors which fall on this particular lineand such a function g (X), this is called a discriminant function.

So,I can design a discriminant function which basically divides the entire 2 dimensional space into different halves, one half for all objects belonging to class omega $_1$ and the other half corresponds to the objects belonging to class omega $_2$ and for any unknown feature vector X, we just find outwhat is the value of g (X).Ifg (X) becomes greater than 0, wedecide that this particular X belong to class omega $_1$; if g (X) is less than 0, we decide that the particular X belongs to class omega $_2$.

So, the design consideration is we have to find out what is this g (X) and this is what we have to do for a 2 class problem thatmeans if we have only 2 classes of objects. Now, what happens if we have multiple classes of objects?

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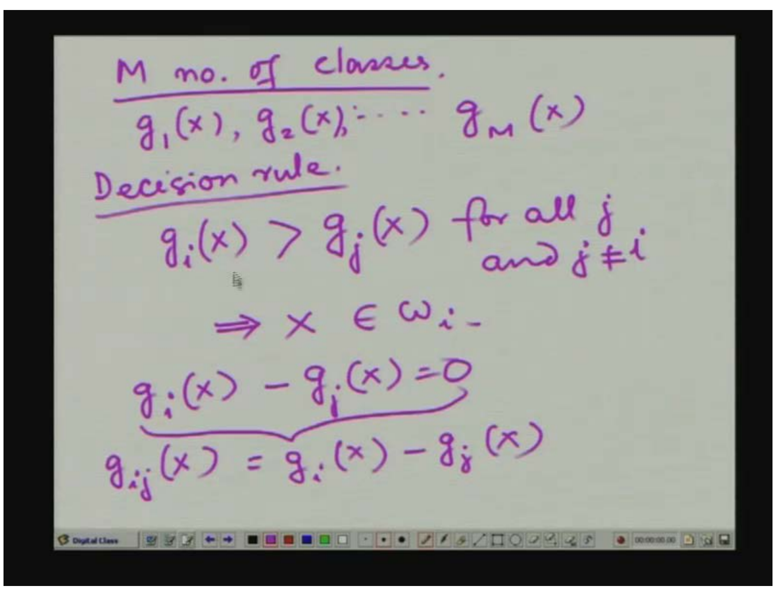


So, suppose we have M number of classes of different objects; so in such case, the usual practice is for every class, you design a discriminant function.That means I have a set of discriminant functions $g_1(x)$, $g_2(x)$ like this,I have M number of discriminant functions $g_M(x)$ and in such case,our decision rule will be something like this.

Thedecision rule is if I find that for a feature vector X, $g_i(X)$, so $g_i(X)$ is the discriminant function for class omega $_i$, $g_i(X)$ is the discriminant function for class omega $_I$;so, if I find that $g_i(X)$ is greater than $g_j(X)$ for all j and j is not equal to iobviously.Inthat case,I decide that this feature vector or the object having the feature vector X belongs to class omega $_i$and here you find that that the decision boundary between these 2 classes - omega $_i$ and omega $_j$ is given by $g_i(X)$ minus $g_j(X)$ that is equal to 0because obviously for all the feature vectors all the points lying on the decision boundary which is the boundary between the class omega $_i$ and omega $_j$, I have to have the corresponding functional values to be same that is $g_i(X)$ should be same as $g_j(X)$and the usual practice is that you design a discriminator or discriminant function for every pair of classes.That means we design $g_{ij}(X)$which is nothing but $g_i(X)$minus $g_j(X)$ and here you find that for a feature vector X if I find thatg $_{ij}$ (X) will be greater than 0.

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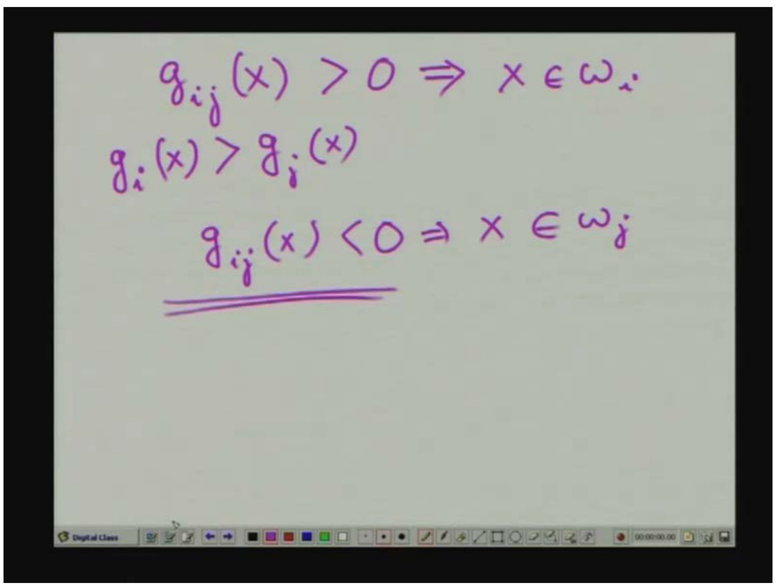


Thisclearly says that I should have I must have $g_i(X)$ which is greater than $g_j(X)$.So, if I find that $g_{ij}(X)$ is greater than 0,I immediately infer that X belongs to class omega iwhereas if $g_{ij}(X)$ is less than 0,I immediately infer that X belongs to class omega $_j$. So, what we have to do in this case is for every pair of classes, we have to generate a discriminant function of the form $g_{ij}(X)$.Now, this particular functional form, this gives ofa basis of designing a number of classifiers.

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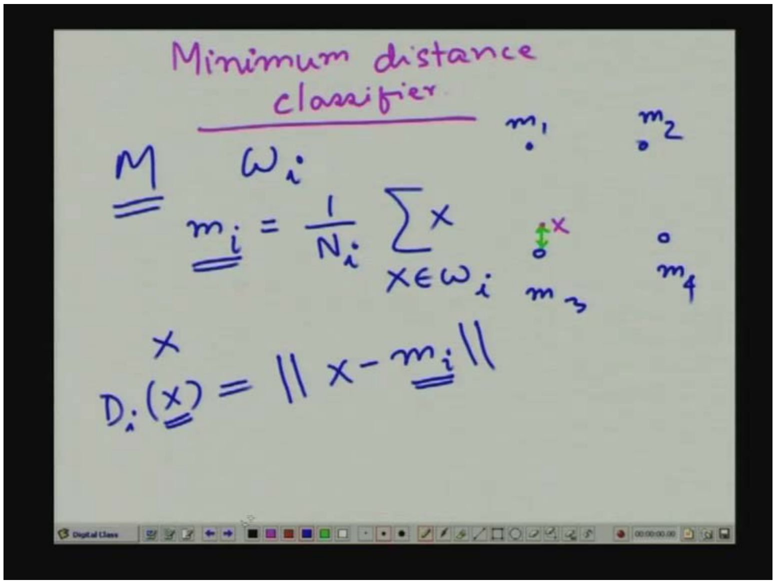


So,one such basic classifier let us discuss which is called a minimum distance classifierminimum distance classifier.So, what is this minimum distance classifier?Suppose,I have capital M number of object classes and for the objects belongingto a particular class,I have a set of feature vectors.So, what I say is every class, say every i'th class is represented by a mean feature vectorbut the mean feature vector of object class say omega $_i$ is designed as is obtained as $m_i$ equal to 1 upon $N_i$ summation of all the feature vectors X for all X belonging to class omega $_i$. So, what it say is I have say $N_i$ number of feature vectors of the objects belonging to plus omega $_i$.

So, if I take the mean of all those feature vectors, then the mean vector that is $m_i$, this is a representative of class omega i.So, this $i_2$is for each and every class.So, when I have capital M number of classes,I will have capital M number of such representative mean factorsand these are the representatives of different classes.Then, what I do is given any unknown vector X; what i do is i find out the Euclidian norm of this vector X from these mean representative classes.

So,I just find out $D_i(X)$ which is X minus $m_i$. So, this gives me the Euclidian norm or the Euclidianthe distance of this feature vector X from the representative of m'th class.So,I will say that this particular feature vector X will belong to that particular class whose distance or Euclidian norm, the distance of whose representative, the mean vector from this particular vector X is the minimum.

So, when I have representatives like this; say this is $m_1$, this may be say $m_2$, this may be say $m_3$, this may be say $m_4$. So, these are the representatives of 4 different classes and if I have a feature vector $X_1$ in somewhere here,then if I compute the Euclidian distance of X from each of these different representatives,I find that the Euclidian distance between X and $m_3$, this is minimum.So, what I say is that this X belongs to class $m_3$. Now,we find that what does this Euclidian norm mean.

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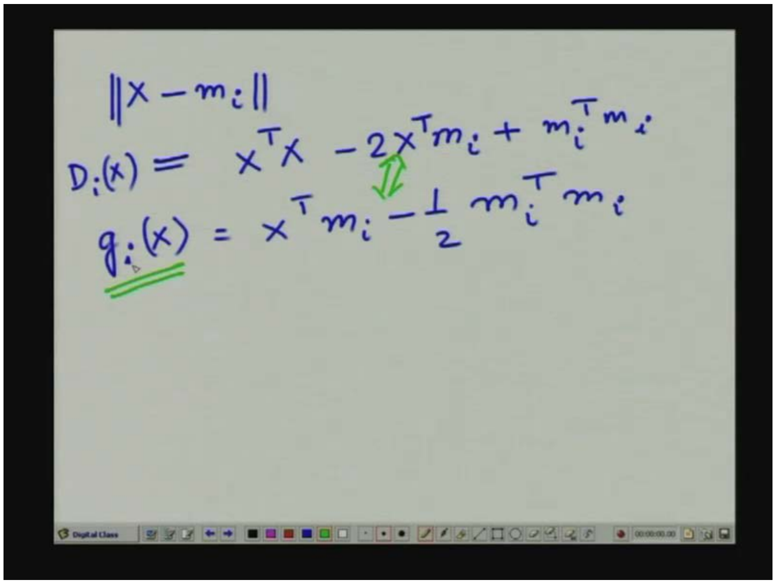


ThisEuclidian norm is nothing butX minus $m_i$. ThisEuclidian norm if I expand this, it simply becomes X transpose X minus 2 X transpose $m_i$ plus $m_i$ transpose $m_i$. So, what I am saying is this becomes my $D_i(X)$.So, what I am saying is if these $D_i(X)$ is minimum for a particular value i, then X is assigned to that particular classand that is equivalentto having a discriminant function of the form $g_i(X)$ which is nothing but X transpose $m_i$ minus half of $m_i$ transpose $m_i$. So, that can be easily obtained or easilygotfrom this particular expression.

So, I will say that this X will belong to that particular class for which this discriminant function $g_i(X)$ is the maximum.So, X will be assigned to that particular class for which the $g_i(X)$ gives the maximum numerical value.So, this is a particular class of classifier which is known as minimum distance classifier because we are assigning the vector X to a particular class which gives the minimum distance.

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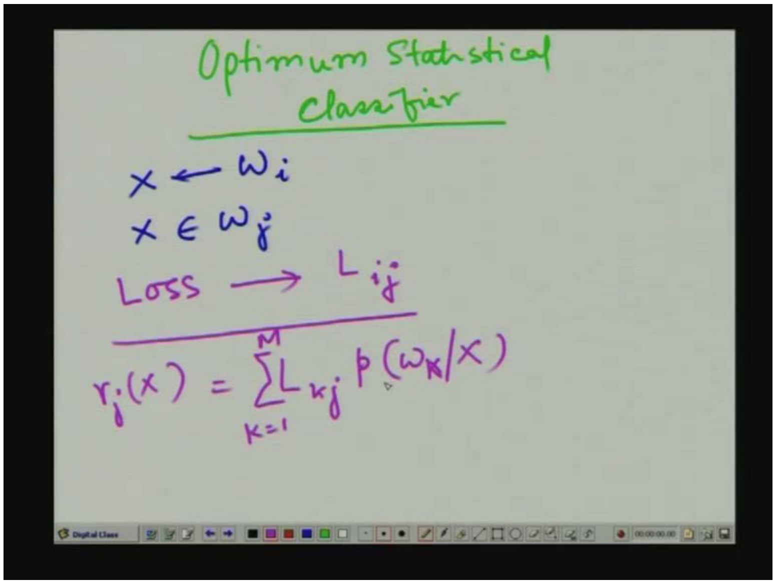


Now,following the same approach, we can have a classifier which is called optimum classifier or optimum statistical classifier.So, what is this optimal statistical classifier?Here, you find that every feature vector X that is coming from objects belonging to a particular class omega i. Then the job of the classifier is the classifier does not know that from which class this feature vector X have been generated.So, the classifier based on it is decision rule has to decide that to which particular class, the feature vector X has to be assigned.

Nowsuppose, the classifier decides that X should belong to omega $_j$. So, find that X has been generated form class omega i but the classifier has decided wrongly that it belongs to class omega $_j$. So, once the there is such a wrong decision, then the classifier incurs a loss.So, we represent this loss by say L $_{ij}$. That means the loss incurred for taking a decision in favor of class omega $_j$ when the actual class is omega $_i$. Then this optimal classifier is designed based on the concept that average loss of taking a decision will be minimized.So, how we can represent this average loss?

So, you find that this average loss can be written as r (X) which is nothing but L $_{kj}$ into P of omega $_k$ given feature vector X.So, what is this L $_{kj}$? L $_{kj}$ is the loss incurred for taking a decision that the feature vector belongs to class omega $_j$ no omega $_k$and we have to take the summation for k equal to 1 to M.That is for all the possible classesand what is this P (omega $_k$X)?P (omega $_k$ X) is the probability of class omega k given a vector X.

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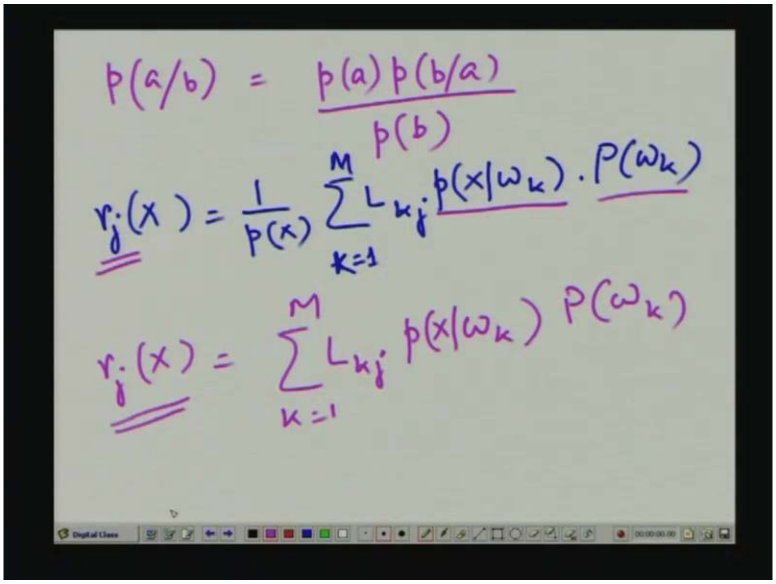


Now, from our basic probability theory,we know that P (a given b) can be written as P (a), probability a into probability of b given a divided by probability of b.Now, using this, you will find now this $r_j(X)$, the average loss, this can be written as 1 upon probability of X into summation k equal to 1 to capital M where capital M is the total number of classes into L $_{kj}$P (X given omega $_k$) multiplied by P of omega $_k$.

Now, here you find that what is this P (X given omega $_k$)?P (X given omega $_k$) is nothing but the probability density functions of the feature vectors belonging to class omega $_k$and capital P (omega $_k$) this is the probability of occurrence of class omega $_k$. Now, here you find because this P (X) will be common to all the functions to all this loss functions $r_j(X)$; so this P (X) can be removed from the particular expression.So, once I remove this P (X), the form of $r_j(X)$ is something like this.$r_j(X)$ k can now become L $_{kj}$ into P (X given omega $_k$) into P (omega $_k$) where k is equal to 1 to capital M.So, as we have said that the job of the classifier is to take that particular decision for which the average loss is minimum; so here the classifier will assign this vector X to a particular class i for which this loss function $r_i(X)$ will be minimum.

Now, usually for a practical purpose, we assume that this loss function L $_{kj}$ that is the loss incurred for taking a decision in favor of k when the actual class is say j is equal 0 for taking a correct decision and equal to 1 for taking a wrong decision.

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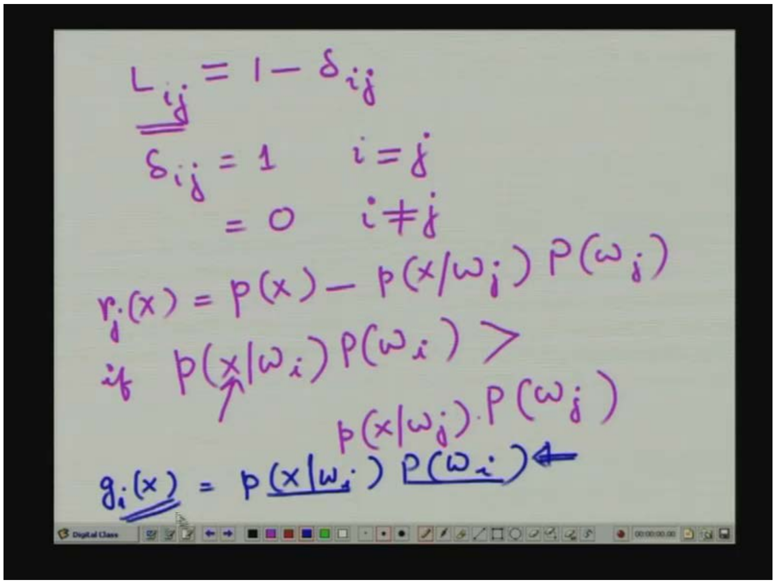


So, this L $_{kj}$ or I can write it as L $_{ij}$ is actually written as 1 minus delta $_{ij}$. That means I am taking decision that the object belong to class j whereas the object actually belongs to class omega $_i$. So, in this particular case, this delta $_{ij}$, this will be equal to 1 whenever i is equal to jso that the loss function becomes equal to 0and this will be equal to 0 whenever i is not equal to j so that the loss functions become equal to 1and by taking this particular modification, now you will find that we can write this loss function $r_j$(X), the average loss r $_j$(X) which can be written as p (X) minus p (X given omega $_j$) into p (omega $_j$).

So, you will find that in this particular case, the feature vector X will be assigned to a class i for which this $r_i$(X) will be minimum or we can say that it will be assigned to class i if we find that P (X given omega $_i$) into p (omega $_i$),this is greater than P of X probability density function omega $_j$ into the probability of occurrence of class omega j.So, whenever such a situation occurs that P of X given omega $_i$ into P omega $_i$ is greater than P of X given omega $_j$ into P omega $_j$; in that case, the feature vector X will be assigned to class omega $_i$and here you find that this is nothing but this is equivalent to having a discriminant function of the form $g_i$(X) which is equal to P of X given omega $_i$ into p of omega $_i$.

So, here you find that to obtain such a discriminate function,I need to have 2 different probability terms.One ofthe probability terms is the probability distribution of the feature vectors belonging to class omega $_i$ and the second term that I must have is the probability of occurrence of class omega $_i$. So, only after having these 2,I can have I can design our requireddiscriminant function $g_i$(X).

So, to obtain these 2 probability terms, you have to do a lot of experiments.Tosimplify the matter, what is done is in most of the cases, you assume a particular probability distribution function or probability density functionand as we have seen that for most of the applications, we normally use Gaussian probability density function.

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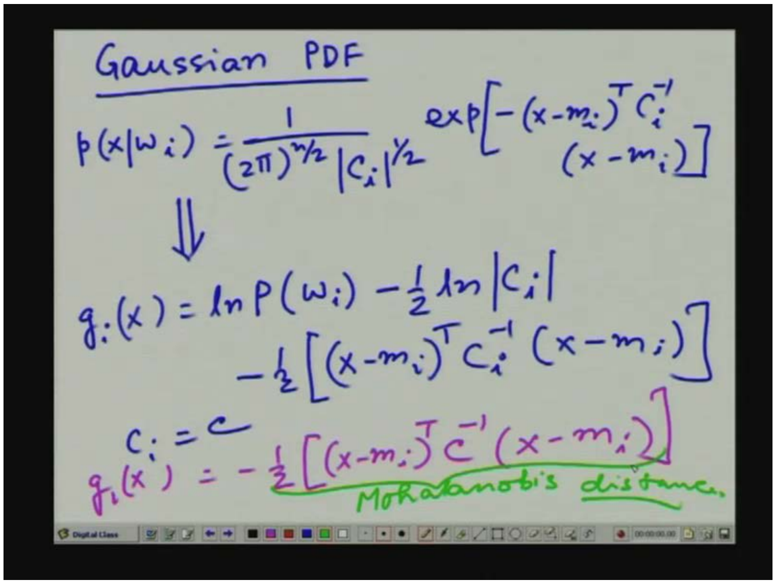


So, if I assume Gaussian probability density function that is Gaussian PDF which is of the form P of X given omega $_i$ is equal to 1 upon 2pi to the power n by 2 into $C_i$ to the power half exponential minus X minus $m_i$ transpose $C_i$ inverse into X minus $m_i$where $m_i$ is the mean of the vectors X belonging to plus omega $_i$ and $C_i$ is the covariance matrix of all the vectors belonging to class omega $_i$.

So, from here, it can be deduced that $g_i(X)$ can be obtained which is of this form, a discriminantfunction $g_i(X)$ can be written like this; ln P of omega $_i$ minus half ln $C_i$ minus half X minus $m_i$ transpose into $C_i$ inverse into X minus $m_i$. So, if we make few further simplificationsthat if we say that the covariance matrix for all the classes is same that is $C_i$ equal to C and all the classes are equally probable; in that case, $g_i(X)$ can be further simplified as it simply becomes minus half into X minus $m_i$ transpose C inverse into X minus $m_i$.

So, here you find that such a discriminant function again leads to a particular type of minimum distance classifier and for that minimum distance classifier, we have to take this particular function as the distance function and this is what is known as Mohalanobis distance.So, just byextending the same concept, we can have a probabilistic.Thisis also called optimal statistical classifier because it tries to minimize the loss incurred, average loss incurred for taking a particular decision.Now, this type of features can also be used to train a neural network and we can use a neural network for the recognition purpose and the type of neural network which is most common for recognition is what is called a multi-layer feed forward network.

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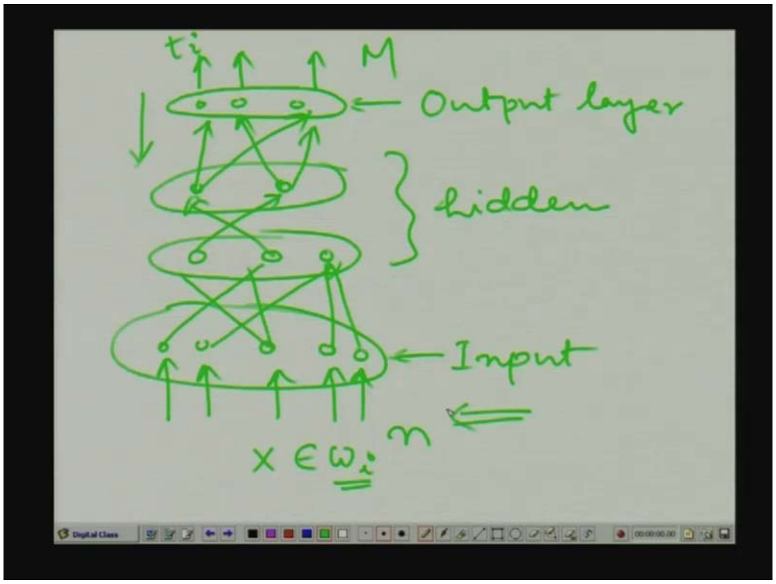


So, here it is something like this; in each of these layers, we have a number of neurons.Thisis called input layer and this is the output layer.Thenumber of neurons in the output layer is same as the number of classes that we have and the number of neurons in the input layer is same as thedimensionality of the feature vectorsand there are one or more hidden layers.So, these are the hidden layers and from every layer, the neurons are connected to the neurons of the upper layer through some connection vets,so something like this.

So, what you can do is you can train this particular neuralnetwork using some feature vectors whose class belongingness are known so that these vets are adjusted properly and once the neuron this neural networkis trained, then for any given unknown feature vector, the neural network will be able to classify that feature vector into one of the m different classes.So, while training, what you have to do is suppose we feed a feature vector X which belongs to class say omega $_i$and we know that if the input is from a class omega $_i$, I should get an output which is say $t_i$ but actually I get something other than $t_i$.

So, in that case,what I have an error and based on the error, the error information is propagated backwards to adjust all these connection vets and that is why this is also known as backpropagation learning.Thatis when you train the neural network, what you give is the error back propagation concept for training the neural network.So, we will not go into the details of this neural network approach.

Sotypically, these are the different approaches which can be used for object recognition purpose.Obviously, of course, there are other schemes like we can represent an object in the form of a graph and we can go for graph matching techniques for recognition of the object.So, with this we come to the end of today’s lecture.

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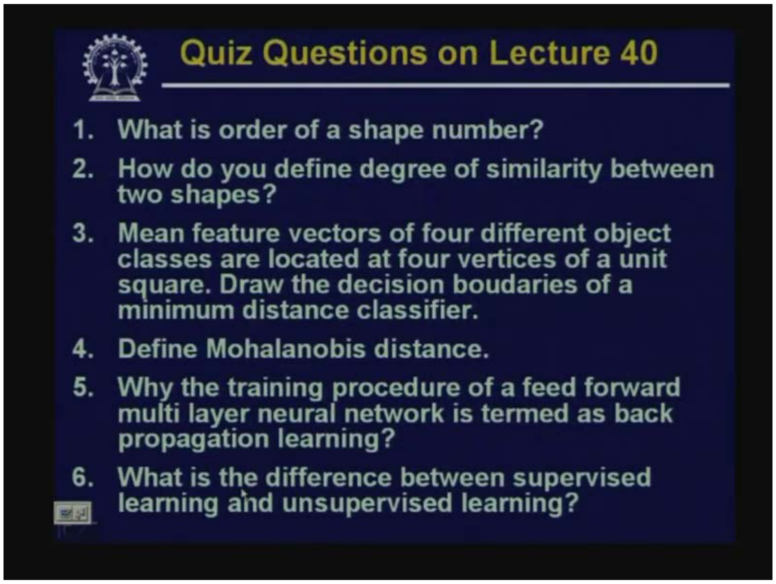


Now, let us see some of the questions based on today’s lecture.Thefirst question is what is order of shape number?Howdo you define degree of similarity between 2shapes?Themean feature vectors of 4 different object classes are located at 4 vertices of a unit square.Drawthe decision boundaries of a minimum distance classifier?Then, define Mohalanobis distance.Why the training procedure of a feed forward multi-layer neural network is termed as back propagation learning?And, what is the difference between supervised learning and unsupervised learning?

So, with this, we come to the end of our video lecture on digital image processing and I hope that you will find this material quite useful.

Thankyou.