

Digital Image Processing

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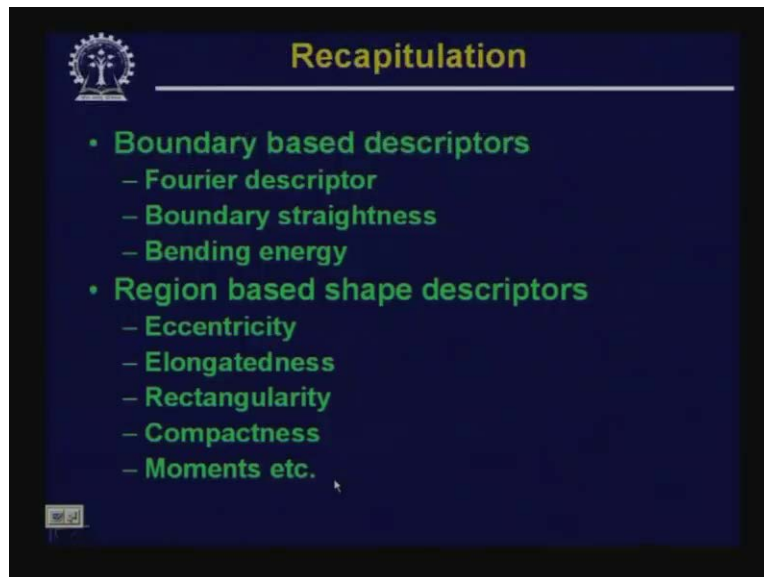
Indian Institute of Technology, Kharagpur

Lecture - 39

Object Representation and Description -III

Hello, welcome to the video lecture series on digital image processing. For the last 2 lectures, we have started discussion on the last phase of image understanding process that is representation and description of the regions present in an image.

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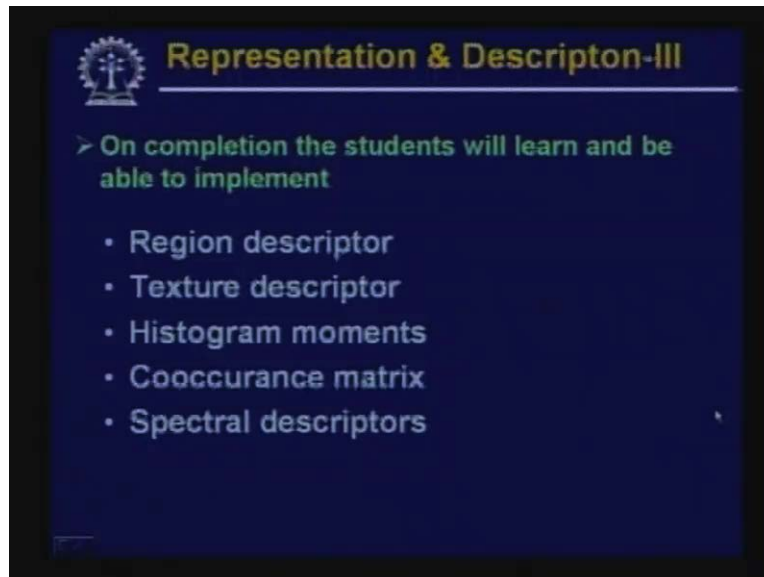


Till our last class, for the last 2 lectures, what we have done is we have found out some descriptors which we have said are shape descriptors and we have said that these shape descriptors can be obtained either from the boundary or from the region itself. So, in our last class, we have seen some of the boundary based descriptors like Fourier descriptor, then boundary straightness and bending energy. Similarly, some other shape descriptors which we have obtained from the region itself; some of them are eccentricity, elongatedness, rectangularity, compactness, moments etcetera.

In today's lecture, what we are going to discuss about is some other descriptors from the region itself. These are the descriptors that we will discuss; they take care of the surface nature, nature of the surface. You find that all other descriptors that we have discussed earlier, those are the shape descriptors which are obtained either from the boundary of the region or from the interior region but they do not give you any idea about what is the surface nature. It simply tells that what is the shape of

the object present in the scene. In today's lectures, we are going to get some descriptors, derive some descriptors which will tell us about what is the shape or what is the nature of the object surface.

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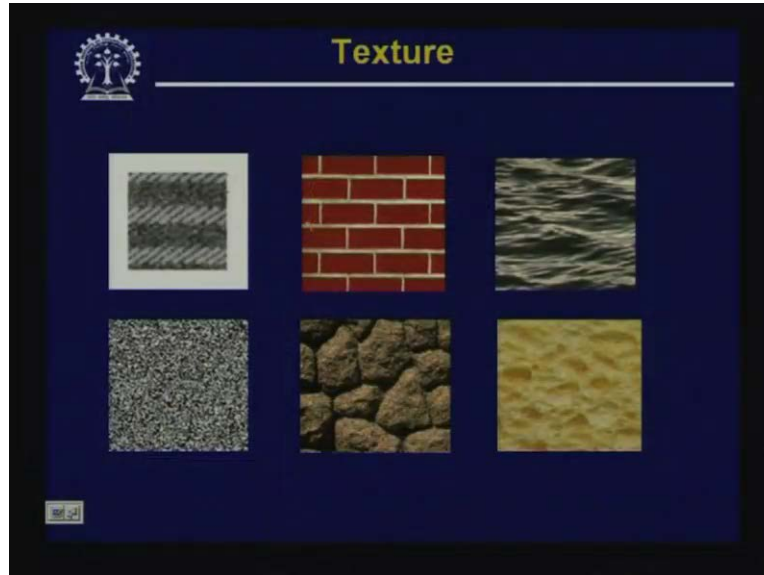


So typically, what we will be talking about the region descriptors and we will discuss about the region descriptors which are the texture descriptors. Now, as we have said earlier that such region descriptors which give you an idea about the nature of the surface, they may be derived from the colour of the surface or also may be derived from the texture of the surface. The colour information that can be obtained, that we have discussed earlier when we discussed about the colour image processing.

Today, what we will discuss is mostly the texture descriptor that is it will capture the texture information of the surface. Now, when we go about this texture descriptor, the texture descriptors can be obtained in various ways. So, one of the techniques that we will discuss is histogram moment based texture descriptor. We will also talk about the co-occurrence matrix based texture descriptor and we will also discuss about the spectral method or the spectral descriptors which are useful for describing textures.

Now, what is a texture? The texture normally refers to properties of the nature of the object surface or the structure of the object surface. So, what I mean by texture, it is better explained with some examples. So, let us see some example textures.

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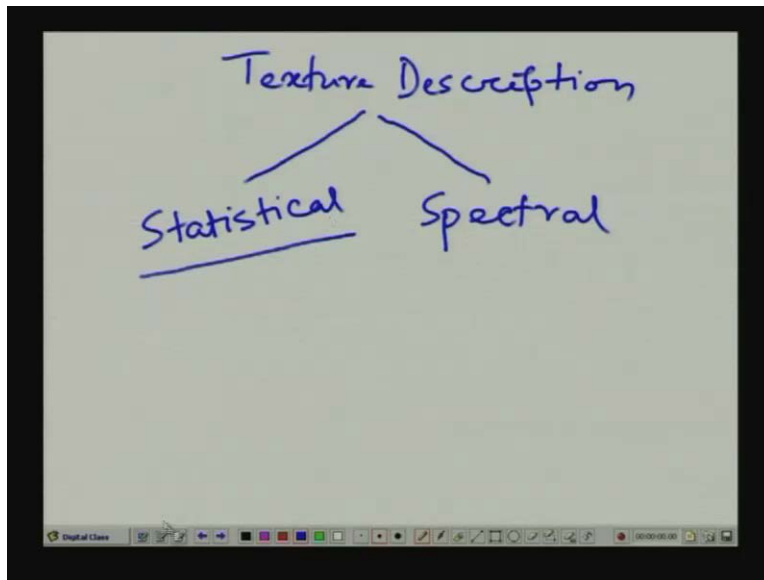
So, here you find that we have given **four different** 6 different images and each of these images has some textures. So, as you find that the texture can be of various form; for example, the pattern on a cloth, the way in which the threads are weaved that also forms a texture, if you take the image of plane surface of say wood that also forms a texture, the three lines, they also form a texture.

So, there are various varieties of texture which are available in nature and various varieties of texture which can also be generated synthetically. So, the notion of the texture, the concept of texture; this is quite obvious intuitively but because of this wide variation of textures, no precise definition of texture exists. So, what we can say is the texture is something which consists of mutually related elements.

So, when we say that it is something that consists of mutually related elements; so basically, what we are talking about is a group of elements or a group of pixels. So, in some cases, say for example, in this particular case, you find that we can say that these textures contain some primitive elements and by repetitive appearance of the primitive elements either it is completely periodic or semi-periodic; so, by repetitive appearance of these primitive elements, what I get is a texture image.

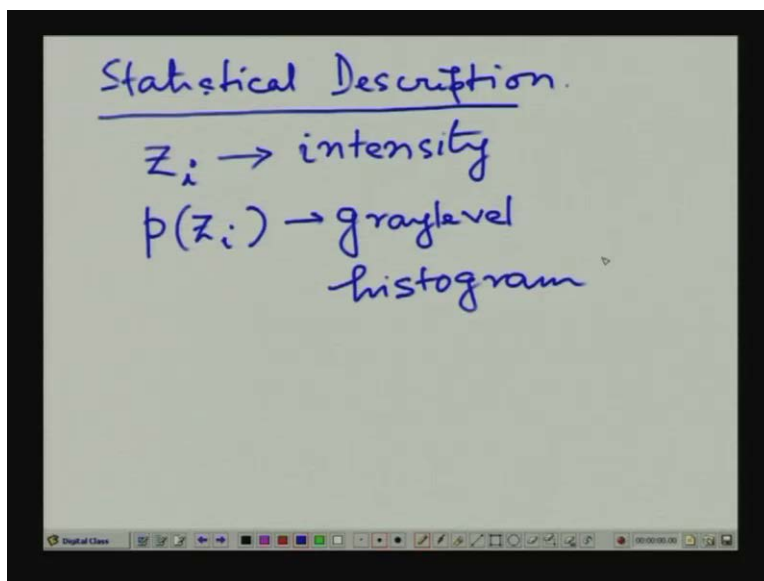
These primitive elements are the group of pixels which form a texture is called the texture element or **textures**. So, as we said that no precise definition of texture exists but the texture provides some very important information about the roughness or the smoothness of a surface. At the same time, it also gives us some information of the regularity of the surface. So, what are the ways in which we can obtain the descriptors which describe a particular texture? So, the texture descriptors can be obtained in various ways.

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We can have texture descriptions which are broadly categorized into 2 categories. So, we can have texture description. So, 1 of the ways in which the texture description can be obtained is statistical. So, we can have statistical means to obtain the descriptors of the textures and the other way of obtaining the texture descriptors is from the spectral domain. That is if you take the Fourier spectrum of the texture image and from the Fourier spectrum, we can obtain some textures, some descriptors which describe the nature of the texture. Let us first consider what are the statistical descriptors that we can obtain from a texture image.

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So, the simplest forms of statistical descriptors are obtained from the gray level histogram of a texture image and what we do is you find out the moments of the gray level histogram. So, let us

assumethat in a texture image; z is a variable, Z_i is a variable which represents the intensity of different pixels present in the texture image and suppose $p(Z_i)$, this represents the intensity histogram or gray level histogram. So, from this gray level histogram, we can find out different textures which are nothing but moments of different orders and the descriptors which are derived from these moments of the histogram.

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The image shows a whiteboard with handwritten mathematical formulas. At the top, it says "nth order moment". Below that is the formula for the n-th order moment:
$$\mu_n(z) = \sum_{i=0}^{L-1} (z_i - m)^n p(z_i)$$
 Then, the formula for the mean m is given as:
$$m = \sum_{i=0}^{L-1} z_i p(z_i)$$
 Below these, specific values are listed:
$$\mu_0 = 1, \quad \mu_1 = 0$$
 and
$$\mu_2 = \sigma^2(z)$$

So, given a histogram $p(Z_i)$, we can have an n 'th order moment. So, n 'th order moment of a histogram is defined like this; $\mu_n(z)$, this is n 'th order moment is given by Z_i minus m to the power n into $p(Z_i)$ and you take the summation for i is equal to 0 to L minus 1. So here, what we are assuming is that the intensity levels, the texture image has got L number of capital L number of intensity levels varying from z_0 to z_{L-1} .

So, there are capital L number of distinct intensity levels and if $p(Z_i)$ is the histogram of such a texture image, then we can obtain an n 'th order moment of this texture image from the histogram as given by this definition. Now, in this particular case, m is nothing but the mean. So, you define m as $\sum Z_i p(Z_i)$ where you have to sum up from i equal to 0 to capital L minus 1. So, this is what is the mean intensity value within the texture image.

Now, we can infer a number of interesting properties from this n 'th order histogram moment. You find that if I take the 0th order moment that is μ_0 where value of n equal to 0; in that case, Z_i minus m to the power n that becomes equal to 1. So, what we are left with this summation of $p(Z_i)$ where i vary from 0 to capital L minus 1. So, if I add all these different probability terms, then the summation becomes equal to 1. So, you find that in this particular case, **the 0th order moment of** the 0th order moment of this histogram is simply equal to 1.

Similarly, if I take the first order moment that is μ_1 , you will find that μ_1 will be equal to 0 but very important is the second order moment μ_2 . So here, you find following this same

expression, μ_2 is nothing but Z_i minus m square $p(Z_i)$, take the summation for i equal to 0 to capital L minus 1.

So, from this particular expression, you will find that μ_2 is nothing but the variance which we normally write as $\sigma^2 z$. So, the second order moment is basically the variance of the intensity values present in the image and this variance is a very very important information because it tells us that what is the variability or what is the range of the intensity values which are present in a particular image in a given texture image and from this second order moment or the variance, we can derive very very important texture descriptor. So, we can define an important texture descriptor like this.

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$$R = 1 - \frac{1}{1 + \sigma^2(z)}$$

$\mu_2(z)$

$$\sigma^2(z) = 0 \rightarrow \text{Uniform intensity.}$$

$$\Rightarrow R = 0$$

So, R is equal to 1 minus 1 upon 1 plus $\sigma^2 z$ and as we said that the $\sigma^2 z$ is nothing but μ_2 that is the second order moment. So, what is the importance of this particular descriptor R ? You will find that if I have a completely plane surface, a completely smooth surface of uniform intensity; in that case, the value of $\sigma^2 z$ or the variance will be equal to 0 for uniform intensity region and in such case, because variance $\sigma^2 z$ is equal to 0, you will find that this complete expression R will become equal to 0 in such case.

So, if I have an image of uniform intensity, the same intensity value for that, value of R will be equal to 0. But if I have variation on the image, the intensity variation in the image; in that case, $\sigma^2 z$ will be non 0 values and more the variation is the value of the $\sigma^2 z$ will even be more. So, as the value of $\sigma^2 z$ increases, in this particular case, you will find that 1 upon this particular quantity, 1 upon 1 plus $\sigma^2 z$, this tends to be 0. As you increase $\sigma^2 z$ more and more, the value of this quantity 1 upon 1 plus $\sigma^2 z$, this tends to be 0.

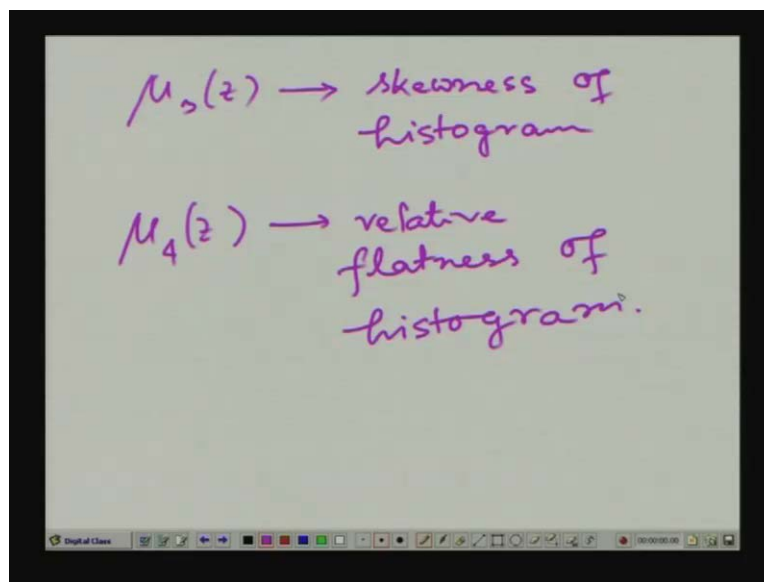
So, as the surface becomes more rough in the sense that is more variation of intensity values in the image, the value of R turns to be 1. So, for a completely uniform surface, we have value of R

equal to 0 whereas for rough surface, depending upon the degree of roughness or the degree of variation of intensity values, the value of R increases and it reaches a maximum value equal to 1.

So, it is the value of this particular descriptor R which is very very important which catches what is the variation of the intensity values in a given texture. So, we have said, we have seen the meaning of three different histogram moments; μ_0 - the 0th order histogram moment which is equal to 1, μ_1 - the first order histogram moment which is always equal to 0.

So, these 2 histogram moments does not give you any information about the nature of the texture whereas the second histogram moment μ_2 which is equivalent to variance of the intensity values in the image; from this μ_2 , we can generate an important texture descriptor which for a flat surface or for a uniform image will be equal to 1 and it will gradually increase and reaches a value 1 for a uniform surface, this value becomes equal to 0 and as the surface roughness increases, the value of R also increases and it reaches a maximum value of 1 depending upon the degree of roughness of the surface.

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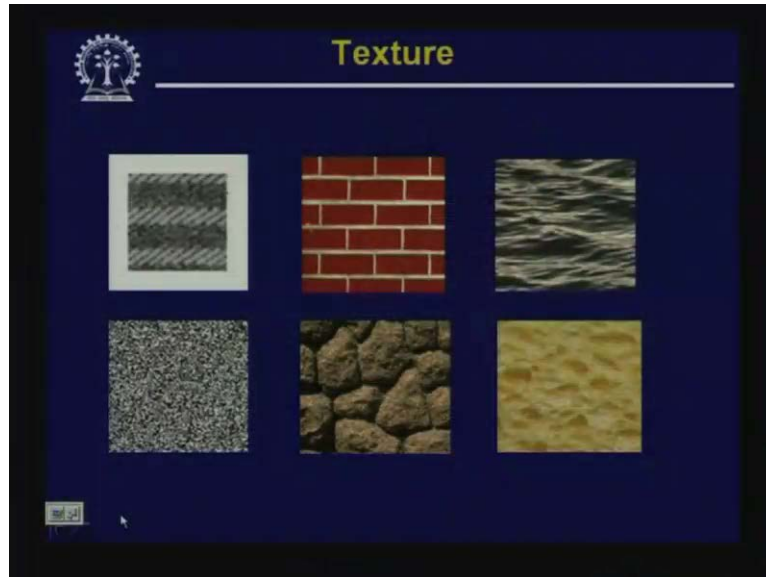
Now, there are other moments, say for example, μ_4 or μ_3 that is the third order moment; you will find that this μ_3 (Z) third order moment, this tells you that what is the skewness of the histogram. Similarly, the fourth order moment μ_4 , this tells us the relative flatness of the histogram.

So normally, as texture descriptor, the texture descriptors which are generated which are derived from the gray level histogram, the descriptors which are used up to fourth order histogram moments, the moments of higher order like fifth order or even higher order are normally not used but if you use them, they will give more and more final descriptors of the texture.

So, though these histogram moment based descriptors, they are very simple but they have one problem. The problem is these histogram descriptors, histogram moment based descriptors, they

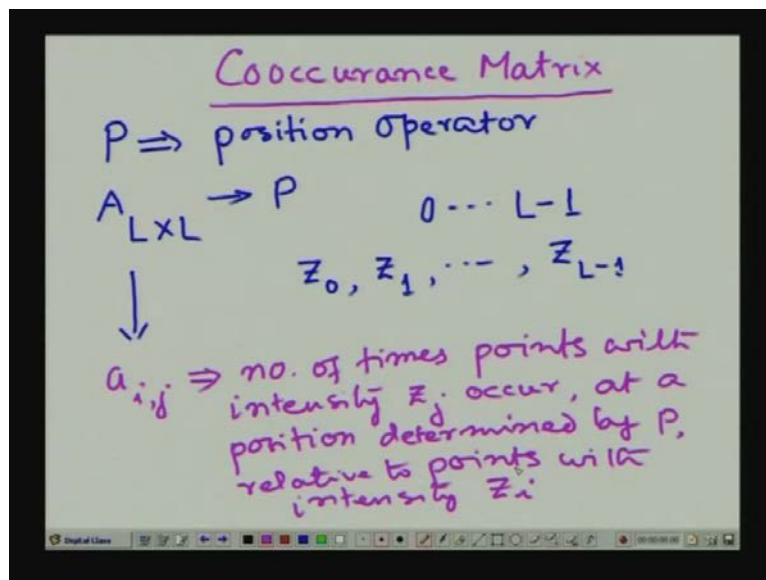
do not provide us any information about the relative position of the pixels. But as we have seen, you will find that in this particular texture, the intensity values at a position with respect to some other position that carries a lot of information.

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So, what is the way in which this relative position information can also be obtained for the texture images, that information that relative position information also gives you a lot of important descriptors which are useful for describing a texture.

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So, towards that we will talk about another technique for obtaining the texture descriptors which are obtained from what is called co-occurrence matrix. So, as we said that what we are interested in is along with the intensity values, the relative positions of the points with various intensity values within the texture; so, we have to define an operator.

So, we define an operator say P which we call as position operator and before deriving the co-occurrence matrix, we will generate a matrix say A which will be of dimension capital L by capital L and this matrix A will be generated using the constraint which is specified by the position operator. So, we are saying that we will generate a matrix of A of dimension capital L by capital L . So, here again our assumption is that the intensity values which are present in the texture image varies from intensity gray levels varies from 0 to L minus 1. So, we will have gray level of z_0 value, we will have gray level of z_1 value and like this the maximum gray level that we can have is Z_{L-1} and in this matrix, capital A which is of size capital L by capital L , a particular element say $a_{i,j}$ this will indicate the number of times points with intensity value Z_j occur at a position determined by P relative to points with intensity Z_i .

So, what we are doing is we take a point with certain intensity value within the texture image, then following our position operator capital P , we come to some other point. So, I find that what is the intensity value; suppose the intensity value at the location that we are considering is Z_i and following the position operator P , the other location where I come the intensity value of that location is say Z_j ; so this $a_{i,j}$ this indicates that how many times such a pair $Z_i Z_j$ as indicated by the location operator capital P appears within the given texture.

Obviously, in this particular case, i and j the indices i and j will have values within the range 0 to capital L minus 1 so that I have a matrix A of size capital A by capital P . So, what I mean by this, this will be better explained with help of an example.

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Ex.

$$I = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 2 & 2 & 1 & 0 \\ 1 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$P \Rightarrow$ one pixel to right

$$A = \begin{bmatrix} 4 & 3 & 1 \\ 3 & 3 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$z_0 = 0$
 $z_1 = 1$
 $z_2 = 2$

$A_{3 \times 3}$
 $z_i = 0$
 $z_j = 0$

So, let us take an example matrix assuming that that is the representative of a particular texture image that we are considering. So, let us take an example like this; so we have a given matrix, image matrix say i which is given by $\begin{bmatrix} 0 & 0 & 1 & 2 \\ 1 & 1 & 0 & 1 \\ 2 & 2 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$, then I can have say $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$.

So suppose, this our given image and let us assume that the position operator that we are saying say P , this position operator indicates 1 pixel to right. So, from this given image and this position operator; how we can generate our given matrix A ? So, here you notice that this particular image or the sample of the image that we have said, this contains three distinct intensity levels.

So, those intensity levels are say Z_0 is equal to 0. So, 0 is one intensity level within this image. Z_1 is equal to 1 and Z_2 is equal to 2. So, there are 3 distinct intensity levels within this image. So, the matrix A that we calculate will be of dimension 3 by 3. Now, we have find that following this position operator, what I have to do is following this position operator, I have to get the pair of pixels and we have to compare and we have to count that for those pair of pixels, whatever is the pair of intensity values, how many times that pair appear within our given image.

So firstly, let us assume that Z_i is equal to 0, Z_j is also equal to 0. So, here you find that since it says that P - the position operator is 1 pixel to the right; so I come to a pixel Z_i which is having a value Z_i equal to 0 and I have to go to the next pixel to the right where the value will also be equal to 0. So, how many times, I have to count how many times this 00 pair; one in the horizontal direction at a distance of 1 pixel appears within the given image. So, here we find that I have one such pair; this is another pair, this is another pair and this is one more pair. So, number of times this 00 pair appear within in our given image is 4. So, when I compute this matrix A , the 00 location A_{00} will be equal to 4.

Similarly, how many times A_{01} will appear within this image? So, A_{01} , this is one occurrence; let me use some other colour, so this is one occurrence of A_{01} , this is another occurrence of 01, this is another operands of 01. So, this is 01 pair appear thrice within this given image. So, A_{01} location will contain a value equal to sorry this is one more location pair A_{01} occurs. So, this A_{01} element will have a value is equal to 3. So, this will have the value equal to 3.

Similarly, let us say how many times A_{02} appear in this image. So A_{02} , we can find that this is the only occurrence of A_{02} in this particular image. So, this location A_{02} will have a value equal to 1. Similarly, I have to check how many times 10 appear in this particular image. So, you will find that the number of times 10 appear in this image is here I have a 10 transition, this one, here also I have 10 transitions, here also I have 10 transitions, 10 pair. So, A_{10} will have a value equal to 3.

Similarly A_{11} , this is one pair, A_{11} this is one more pair, A_{11} this is another pair. So, A_{11} will also have a value equal to 3. A_{12} this is 1 pair, A_{12} and possibly this is the only pair A_{12} within this image. So, A_{12} will also have a value equal to 1. Similarly A_{20} , you come to this particular image, you will find that this is one pair A_{20} and there is no other occurrence of A_{20} pair within this given image. So, A_{20} will also be equal to 1.

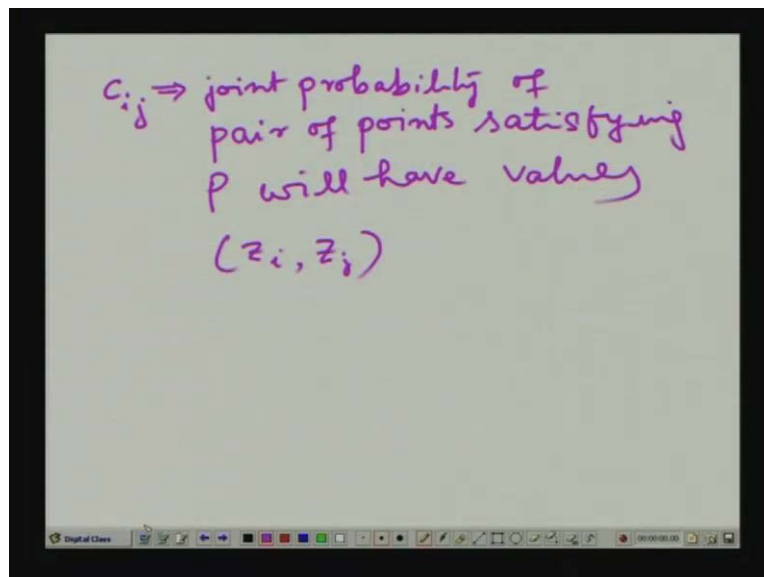
Similarly A_{22} , this is the only pair within this image. So, A_{22} sorry this is A_{20} then A_{21} , A_{21} , this is the only occurrence of A_{21} within this image. So, this A_{21} will also be equal to 1. Similarly A_{22} , this

is the only occurrence of A_{22} . So, this element will also be equal to 1. So, as a result, I get my A matrix which I just computed in this particular form.

Now, from this matrix A, we can compute the co-occurrence matrix for this particular image. How do we get the co-occurrence matrix? If I take the total number of occurrences of all these pairs within this image which follows which satisfies our positional operation, the positional restriction as given by the positional operator capital B; then you find that the total number of or the total such occurrences within this matrix is say n is equal to in this particular case if you take the summation of all these elements, it will be equal to 19. So, if I divide all the elements of matrix capital A by this total number of occurrences, then what I get is what is called co-occurrence matrix.

So, in this particular case, our co-occurrence matrix C will be simply given by $1/19$ times capital A. So, what are to do is I have to divide each and every element in the matrix capital A by the total number of occurrences of pairs of intensity values as dictated by our positional operator capital P and this resultant matrix that I get is our co-occurrence matrix. So, what does this co-occurrence matrix tell us?

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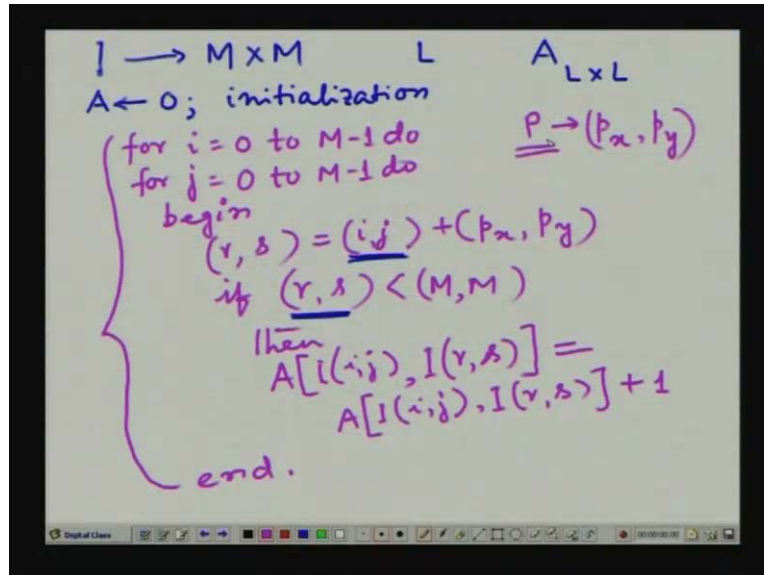


Every element say C_{ij} within this co-occurrence matrix, it indicates the joint probability of pair of points that satisfies; so pair of points satisfying P will have values Z_i and Z_j . So, when I have this co-occurrence matrix, every element C_{ij} within this matrix that indicates the joint probability of pair of points which satisfies our positional operator capital P that will have values Z_i and Z_j .

So, you find that in histogram based moments or available histogram based descriptors that we have generated, we did not have any positional information. But when we have a co-occurrence matrix, in the co-occurrence matrix, we have the positional information as well as the intensity relation of the points following certain positional information.

So, the question is once we have defined such C_{ij} or once we have obtained a co-occurrence matrix; what is the algorithm following which we can obtain a co-occurrence matrix? So now, let us see that what kind of algorithm can be used to obtain this co-occurrence matrix.

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So for this, let us assume that we have our image I given image I which is of size say capital M by capital M . So, this capital M by capital M is our original image and let us assume that the intensity levels, the discrete intensity levels, the number of discrete intensity levels which are present in the image is say L . So, first of all, what we have to do is we have to generate a matrix capital A whose size will be capital L by capital L . So, an algorithm can be written like this; this matrix capital A of size L by L can be initialized to 0. So, this is my initialization. What I mean by this is we make every element in the matrix capital A equal to 0.

Then our algorithm can walk in this fashion; say for i equal to 0 to capital M minus 1 do for j equal to 0 to capital M minus 1 do begin, so here comes the operation for computation of our matrix capital A . So, for this computation what I assume is suppose the position operator P is indicated by a vector because as we said that the position operator says that given a particular pixel location, what is the location of the other pixel that we are interacted in. So, that can be obtained simply by some vector addition operation.

So, I assume that P is indicated by a vector, by a position vector whose components are say P_x in the x direction on P_y in the Y direction. So, what I have is ij maybe index of one particular pixel location and from this, I generate the index of another pixel location (r, s) which is nothing but the vector addition ij plus (P_x, P_y) . So, this is a vector addition operation.

Then what I have to do is I have to compute whether this image index, image point index (r, s) that you have generated by this vector addition operation, whether that is within our image or not. So, I have to check if (r, s) , this vector is less than M, M . Then only this (r, s) position is within our image because our image is of dimension capital M by capital M which are indexed from 0 to

capital M minus 1 and 0 to capital M minus 1. So, if this point is satisfied, this condition is satisfied; that means (r,s) point remains in our image.

So, if this is satisfied, then what I see is what is the intensity value at location ij and what is the intensity value at location (r, s)? So, using those intensity values as indices in our matrix capital A, I can increment the corresponding location in matrix capital A by 1. So, our operation will be that if this is true, then A [I (i,j), I (r,s)], these has to be incremented by 1. So, this becomes A [I (i,j), I (r,s) plus 1 and this is where our iteration ends.

So, you find that at the end of execution of this, the matrix A will contain the number of times a pair of pixels having a pair of intensity values occur within the image I where the these pair of pixels follow the position relation as indicated by this position operator or position vector capital P. So once I get this A matrix, the matrix capital A; then from this matrix capital A, if I add all the elements in the matrix A and divide every element of the matrix capital A by that summation, what I essentially get is our co-occurrence matrix capital C.

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The image shows three handwritten mathematical expressions on a whiteboard:

1. Max. prob. $\max_{i,j} \{C_{i,j}\}$
2. Element difference moment of order k $\sum_i \sum_j (i-j)^k C_{i,j}$
3. $\sum_i \sum_j C_{i,j} / (i-j)^k$

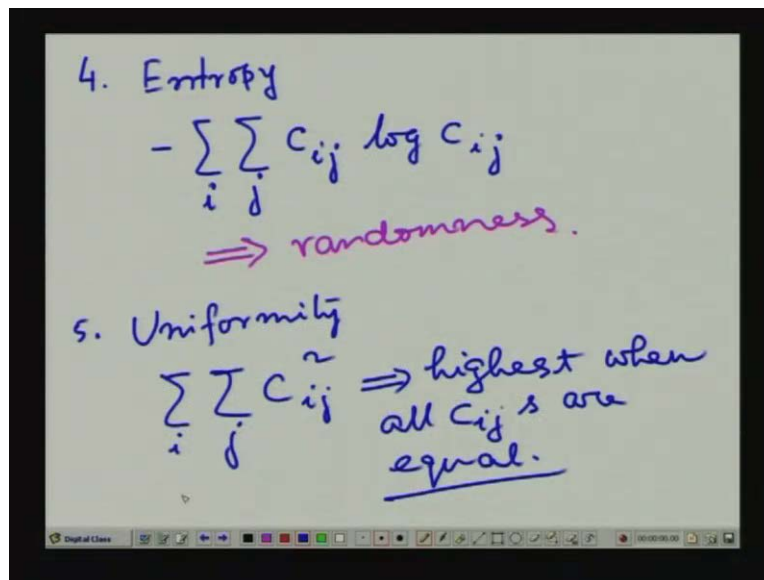
The next operation to obtain the feature descriptors, the texture descriptors from this co-occurrence matrix is you analyze this co-occurrence matrix to generate various kinds of descriptors. So, what are the different types of descriptors that we can obtain from the co-occurrence matrix? The first type of descriptors that we can obtain from the co-occurrence matrix is the maximum probability this is nothing but maximum of C_{ij} over all ij .

So, what is this maximum probability indicates? This maximum probability indicates the strongest response to the position vector P of the given image I. So, this will indicate that whether given a position operator say capital P if the capital P indicates a position in the horizontal direction, then how the texture is responding to that horizontal variation.

The other kind of descriptor that we can obtain from this co-variance matrix is what is called element difference moment of order k and this element difference moment is defined like this; i minus j to the power k $\sum_i \sum_j C_{ij} (i-j)^k$ where we have to take the summation over i and j . So, this is what is called the element difference operator and this element difference operator, you will find that it will be low if higher values appear along the main diagonal because along the main diagonal, we will have i is equal to j , so if higher values of C_{ij} they appear along the main diagonal of the co-occurrence matrix; then this element difference moment, k 'th order element difference moment will assume a low value.

Similarly, we can have the other operator which is just inverse of this that is C_{ij} divided by i minus j to the power k and you have to take the summation of these over i and j and this gives you just the inverse effect. That means if the higher element values if larger will the value of C_{ij} appear along the main diagonal; in that case, **this particular value will have** this particular quantity will have a higher value and if the elements along the main diagonal, they have low values, then this will also give a lower value.

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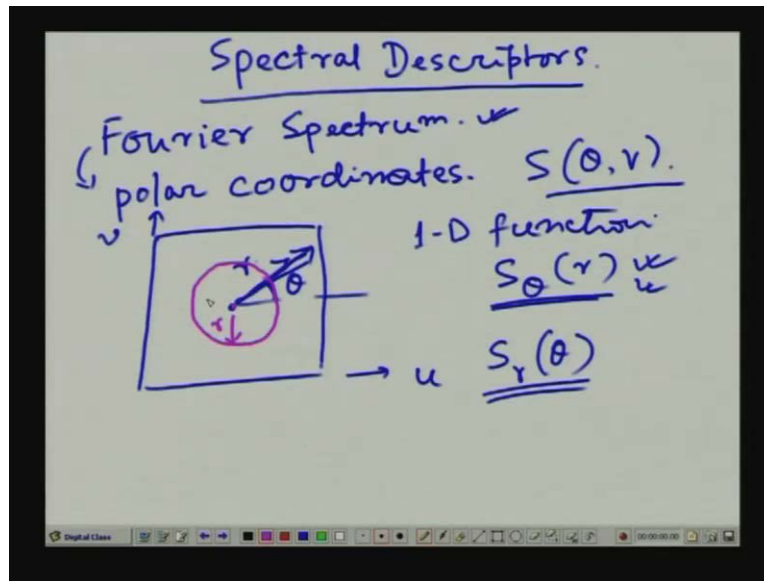


The other descriptors that we can obtain, fourth descriptor from this co-occurrence matrix C is what is called entropy and we know that an entropy is defined in this manner; we have C_{ij} , then \log of C_{ij} and we have to take the summation over i and j and as we know that this entropy is nothing but a measure of randomness, so it tells us that how random the given texture measures.

Similarly, the other texture measure that we can obtain from the co-occurrence matrix say fifth measure is what is called uniformity and which is defined as summation of C_{ij} square and here again, you have to take the summation over i and j . So, we will find that this particular value uniformity, it will have highest value when all C_{ij} 's are equal. That means our texture is the co-occurrence matrix for that particular texture is a uniform co-occurrence matrix. So, these are the various texture descriptors that can be obtained from the co-occurrence matrix.

And, as we have said, in our algorithm that given a texture image, how we can compute the co-occurrence matrix of the given texture image. So, these are the difference descriptors, the texture descriptors whether they are used, whether they are derived from histogram moments or they are derived from the co-occurrence matrix, these are the statistical descriptors that we can have of a given texture.

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And, as we said that along with these statistical descriptors, we can also generate some spectral descriptor. That means we can generate descriptors from the spectral domain. So, to obtain the spectral descriptors, what we have to do is we have to find out what is the Fourier spectrum of the given image. So firstly, we have to find out the Fourier spectrum.

So, what we get from this Fourier spectrum? The prominent peaks in the Fourier spectrum will tell us what is the principal direction of the texture patterns. That is whether the principal direction of the texture pattern is horizontal or principal direction of the texture pattern is vertical or they are diagonal and so on.

So, this principal direction of the texture patterns is obtained from the Fourier spectrum or the prominent peaks in the Fourier spectrum and another information that we get is from the location of the peaks. So, the location of the peaks within the Fourier spectrum tells you what are the fundamental special periods of the patterns. As we had seen earlier that whenever you have a texture, the texture has some special variation and this special variation can have some periodicity, the periodicity may be strictly defined or it may not be strictly defined.

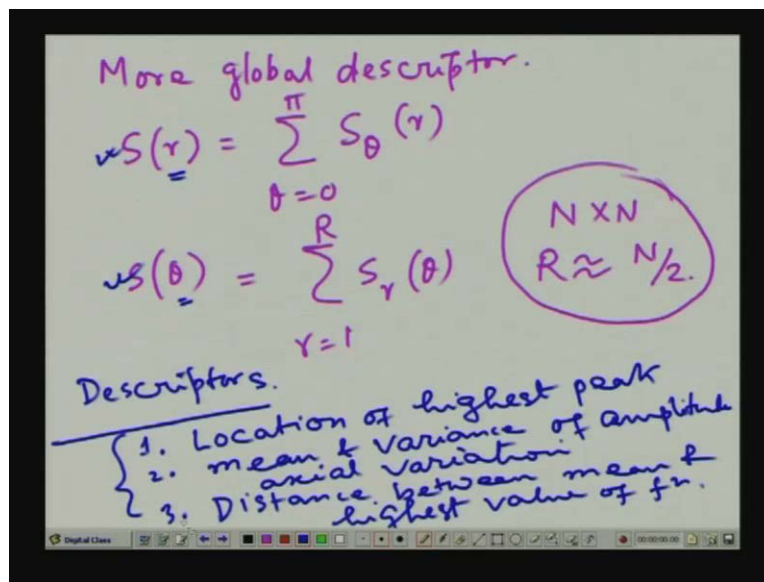
So, the location of the peaks, location of the prominent peaks in the Fourier spectrum, they give you information about what are the fundamental periods of the texture pattern. Now, for convenience of computation, what we do is this Fourier spectrum is converted into polar coordinates. So, polar coordinate means I have the Fourier spectrum which is originally Cartesian coordinate. So, I have directions u and v and these I would like to convert in the polar coordinate.

So, in polar coordinate, a particular direction theta, if I find out the variation in a particular direction of theta; so each value of theta will give me a 1 dimensional function which I call as $S_{\theta}(r)$ because r is the radial distance. So, if I convert this into polar coordinates, the Fourier spectrum into polar coordinates; then for each value of theta, so what I have is now the Fourier spectrum which is represented in the form of $S_{\theta}(r)$ as we are going for polar coordinate representation and each value of theta gives us a function a 1 dimensional function which I can represent as $S_{\theta}(r)$.

Similarly, for each value of r that is the radial direction, I can have a 1 dimensional function which I can represent as $S_r(\theta)$. So, what do this $S_{\theta}(r)$ and $S_r(\theta)$ indicates? $S_{\theta}(r)$ as we are saying that we are moving in a particular direction along the radial axis; so this tells us what is the behavior of the spectrum along a radial direction from the origin. So, as we are moving in this particular direction and for this particular theta, we are computing this $S_{\theta}(r)$, it tells us that what is the behavior of the spectrum along a radial direction as you move from the origin within the spectrum.

Similarly $S_r(\theta)$, because this $S_r(\theta)$ is for a particular value of r; this indicates that what is the behavior of the spectrum if I move along a circle of radius r centered at the origin. So, these are the 2 behaviors of the spectrum that can be obtained if I represent the Fourier spectrum in the polar coordinates.

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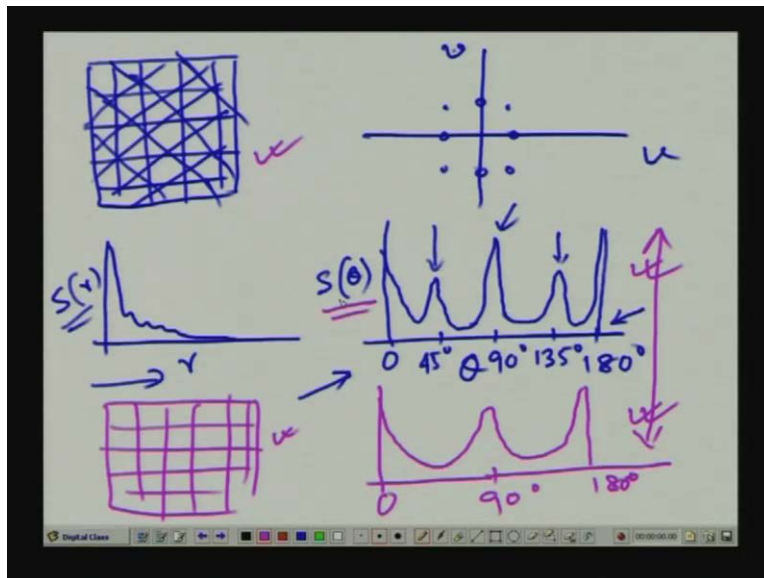
And, a more global descriptor can be obtained from these 2 functions. So, you can have more global descriptors which are given as S_r is equal to summation $S_{\theta}(r)$ where you take the summation from theta equal to 0 to pi and the other descriptor that we can obtain is S_{θ} which we have to take from $S_r(\theta)$ where this r will vary from say 1 to capital R where this capital R is related to dimension of the image or similarly it is related to dimension of the Fourier spectral coefficients that we get.

Normally, if I have an image of size capital N by capital N, then value of R is typically taken as capital N by 2. So here, you find that by varying the values of R and theta, R is the radial variable and theta is the angular variable. So, by varying the values of these R and theta, we can generate two 1 dimensional functions which describe the texture energy content. So, if I vary the value R, I get one 1 dimensional function which gives us an indication of the variation of the texture energy content as we move along the radial direction globally and if I vary the value of theta, this gives us the variation of the textural energy content as we move along the concentric circles within the spectral.

So, from these 2, the descriptors which are normally generated is one kind of descriptor is the location of highest peak, the other kind of descriptor can be mean and variance of amplitude as well as axial variation and which can also have the distance between mean and highest value of the function.

So, these are the various descriptors that can be obtained from these 2 functions; S_r and S_{θ} . Now, what we mean by this? Let us take look at an example.

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Suppose, I have a texture image which is given like this; so this shows atypical texture image. So, if I have a texture image of this form, you will find that in the frequency domain if in the Cartesian coordinated domain if I put it as uv, there are few horizontal lines and few vertical lines and for that I will have some spectral components in these difference locations. Similarly, there are diagonal lines which are oriented at 45 degree and 135 degrees; so these are the points which correspond to those diagonal patterns.

If I plot the S_r with r, a typical pattern of this S_r 1 dimensional function of this is you can obtain something like this. So, it says that as we move radially from the center of the spectral plan, what we are doing is we are moving into more and more frequency components as it is quite obvious

that in most of the images, we have higher energy content of higher low frequency energy content whereas lower, high frequency energy content and this is the typical pattern that we will get when we plot S_r versus r . But we get very very important information if I plot $S(\theta)$, $S(\theta)$ versus θ .

So, if I plot it from 0 to 180 degrees; so somewhere here we have 90 degrees, somewhere here we have 45 degrees, somewhere here we have 135 degrees and we will find that we will get a pattern something like this. So, it clearly shows that we have the periodic nature of the pattern in the horizontal direction with θ equal to 0, we have the strong periodic nature in the vertical direction that is θ is equal to 180 and we also have sorry θ equal to 90 degree and we also have stranger periodic patterns in the 2 diagonal directions that is θ equal to 45 degree and θ equal to 135 degree.

Now, opposed to this if I have a texture pattern simply of this form, I have horizontal pattern and I have vertical pattern; so if I plot $S(\theta)$ versus θ of this particular texture pattern, that kind of plot that we will get is something like this. So, this is 180 degree, this is 90 degree and this is 0 degree.


So, here you will find that if I consider this $S(\theta)$ plot, from this $S(\theta)$ plot, I can clearly demarcate between this texture and this kind of texture. So, this $S(\theta)$, this 1 dimensional function provides very very important descriptor which can be used to describe texture as well as to discriminate among the textures and the various kinds of descriptors as we have said that the location of the peaks, similarly mean and variance and also the distance of the highest value from the mean value; these are the different type of descriptors that can be obtained from this spectral domain.

So, till now what we have done is we have discussed about different descriptors, the shape descriptors, we have discussed about the shape descriptors which are obtained from the boundary, the shape descriptors which are obtained from the region as well as the texture descriptors which are also region descriptors.

Now, for object classification or object identification, what we have to do is we have to make use of these descriptors. So, if I can assume that each of the descriptor to be a scalar quantity and each descriptor is a component of a particular vector; so if I put if I arrange different few of these descriptors in an ordered manner, what I can have is a vector representation of difference descriptors and these vector as we will see in our next class, we will call as feature vectors and based on the feature vectors, we can design some classifier or some recognizer using which we can recognize the object present in the image.

So, we will discuss about the object recognition or object understanding problems in our next class. Now, let us see some of the questions quiz questions from our today's lecture.

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Quiz Questions on Lecture 39

1. What is the significance of second order histogram moment?
2. What is co-occurrence matrix?
3. How do you measure texture entropy from co-occurrence matrices?
4. A texture has prominent periodic intensity variation in vertical direction. What will be the nature of $S(\theta)$?

The first question is what is the significance of second order histogram moment? The second question, what is co-occurrence matrix? Third question, how do you measure texture entropy from co-occurrence matrices? The fourth question, a texture has prominent periodic intensity variation in vertical direction; what will be the nature of $S(\theta)$?

Thank you.