Digital Image Processing

Prof. P.K. Biswas

Department of Electronics & Electrical Communication Engineering

Indian Institute of Technology, Kharagpur

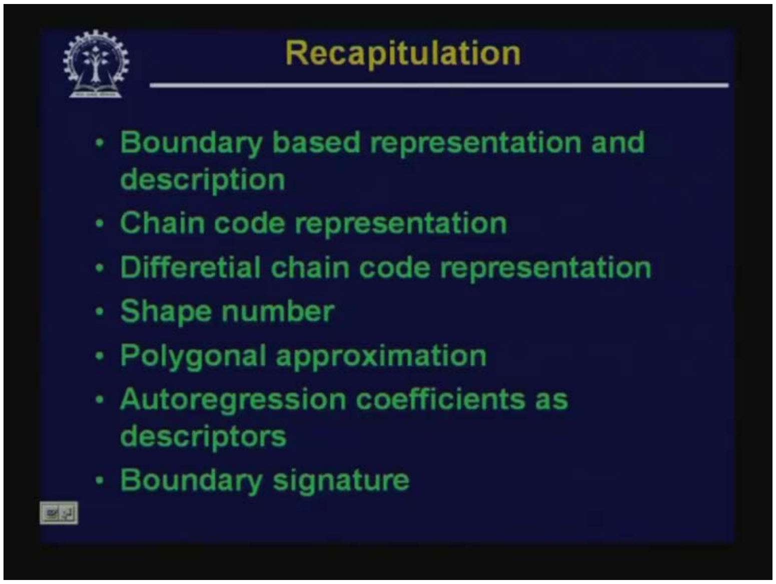
Lecture - 38

Object Representation and Description-II

Hello, welcome to the video lecture series on digital image processing.Sinceour last lecture, we have started our discussion on aspect, a particular aspect of image processing techniques that is we are moving towards understanding the object present in animage or we are going forinterpretation or image interpretation.Now, to start with, we have said that if we are going for imageinterpretation or image understanding, basically recognition of the objects presentin ascene; then the first operation that we have to do is we have to look for a proper representation mechanism of the object regions and once we have arepresentation mechanism, then from for that representation, we have to find out adescription of the object.

Now, once we get a description of the objet, then this descriptioncan be matched against similar such descriptions which arestood in the knowledge base in the computer and to whichever such modelsor description our current description matches, we can say that the object present in the scene is that kind of object,say it is an object x or object y and so on.

(Refer Slide Time: 2:22)



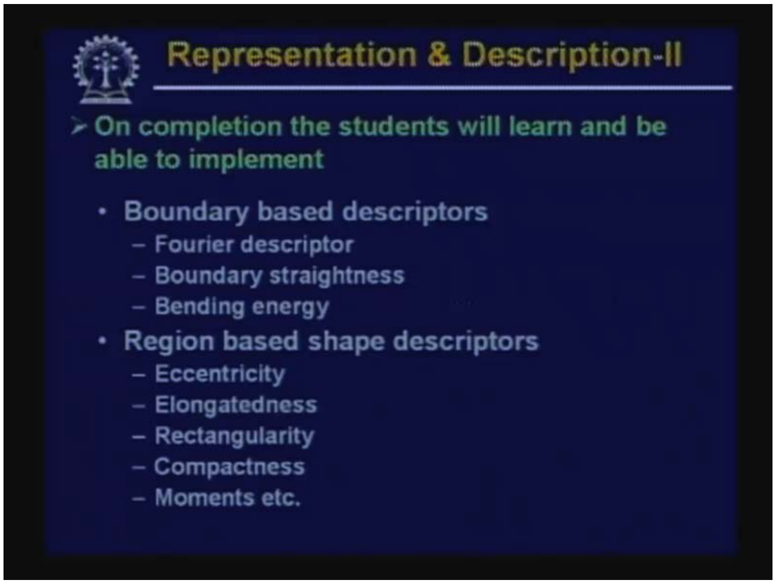
Soinitially, what we have done is in our last lecture,we have discussed about a representation technique, about some representation techniqueswhich are boundary-based representation and accordingly we have found out some descriptionfromthis boundary representation. So, the first kind of the boundary representation method that we have talked about is a chain code representation and we have also seen that when we have a chaincode representation,thechain code representationbyitself is not scale independentorrotationindependent though it istranslation independent.

So, to take care of its dependency on the scale as well as its dependency on rotation,what we have done is we have resampledthe boundary points by placing grids of differentsizes.So, the different scales of a region can be taken care by placing grids of proper spacing.Whereas, to obtain the translation invariance, what we have said is instead oftaking the chain code itself, we can go for differentialchain codeand we havesaid that we can obtain the differentialchaincode from the original chain code by considering that given two subsequent codes in the chain code,wehave to find out that how many rotationsonehas to give either in the clockwisedirection or the clockwise direction to move to the second code from the first code.

And, if I represent this number ofrotations that we have to give, the number ofsteps of rotation that we haveto give in the form of a chain thatbecomes a differentialchain code.And finally, to make it rotation invariant, what we have said is instead of considering that as a chain,we can consider this as a cycle and in that cycle, what we have to do is we have to redefine our starting point for chain code generation so that fromthat starting point if I open up that particularcycle, what I get is a numerical numberand we have said that these numerical number if I follow a convention, that I will start,I will select a starting point such that the numerical number that is generated is minimum; thenwhat I get is a chain code or differential chain code based description of the object shape or of the object boundary which is a descriptor that can be translation rotation and scale invariant and this particular minimum number that we get, we have said this is what is called shape number for that particular object shape.

Then we have discussed about anotherboundary-based representation technique which is the polygonal approximation of the boundary.So, wehavediscussedvarious techniquesofpolygonal approximation technique;how a given boundarycan berepresented bya polygon?Thenwe have talked abouta particular description mechanism that if wemodel the verticesorthe corners atdifferent vertices of the polygon in the form of anautoregressive model ofsayorder k;in that case, wecan form aset of linear equations.After solving those set of linear equations,we cansolve for thecoefficientsof thatauto regression modeland these set of coefficientscanalso be usedas a descriptorwhich representsthat particular polygonal shape and these coefficientscanalsobe used for matching purpose.

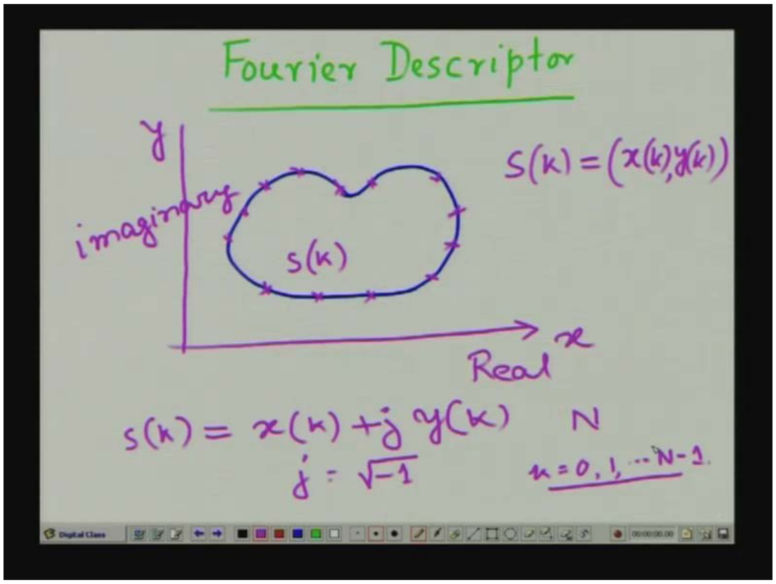
(Refer Slide Time: 6:32)



In today’s lecture, what we will talk about is wewill discuss aboutsome more boundary-based descriptors.Oneof the boundary-based descriptorwe will talkabout iswhat is called a Fourier descriptor,we will also talk about another descriptor whichis called boundarystraightness and theother kind of boundary descriptor thatwe will talkabout is called a bending energy. Then as we have saidthat all these boundary-based descriptors, we have said that this essentiallycaptures the shape information of the object region.

Now, to capturetheshape information of theobject region,similar such information we canalso get from the region not only from the boundary; from the entireregion,we can also capture theshape information ofthatparticular region.So, we will talk about someof the region-based shape descriptors likeeccentricity, elongatedness, rectangularity, compactness,momentsand some otherdescriptors which are the shape descriptorsthat can be extracted from the entire region not only from the boundary.So, let us first talk about thisboundary-based descriptorwhich we said is the Fourier descriptor.So, let us see what is the Fourier descriptor.

(Refer Slide Time: 7:58)

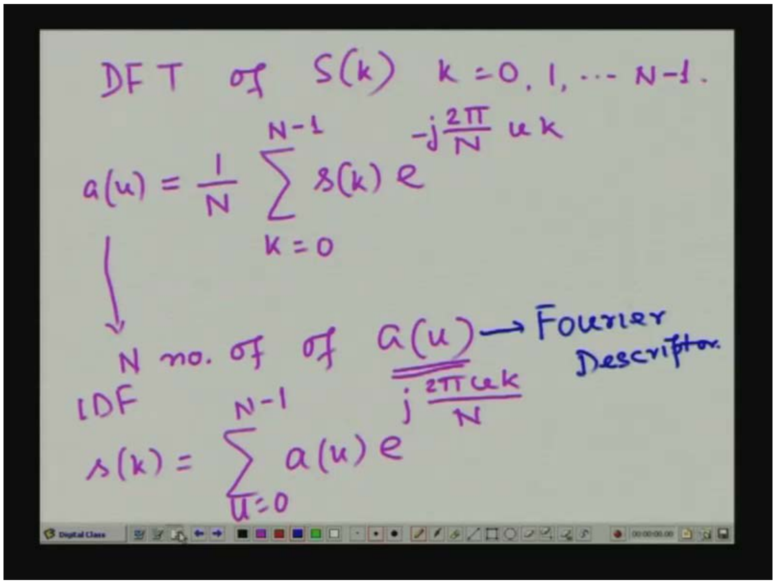


So, suppose you have an object boundarysay something like this;now obviously,I will have animage axis, seeimage axes are given byx and yand this boundary isnothing but a set of discrete points in this two-dimensional xy space.So, what is this boundary?This boundary is actually a set of points and let us represents the point a particular point byS (k).So, this boundary is nothing but a sequence of such pointsS (k)in our discretetwo-dimensional space xy.

So, this S (k)in the two dimensionwillhave a coordinatewhich is given byx (k)and y (k).Now, if I assumethis xaxis to be areal axis and y axisto be the imaginary axis; so what I am doing is I am interpretingthe xaxisas the real axisand y axisas the imaginary axis.Then every boundary point S (k)in thissequence ofboundary pointscan berepresented by a complex number which is given byx (k) plus j y (k)where j is equal to squareroot of minus 1.

So, based on this interpretation that we are interpreting the xaxisas the real axisand y axis asthe imaginary axis,every point every discrete pointon the boundarycan berepresented by acomplex number.So, ifI takethe sequence of suchboundarypoints when we tracethe boundaryeither in the clockwise direction or in theclockwise direction,what we getis asequence of complexnumberS (k)and suppose we havetotal of capitalN numberofpoints on this boundary;so in this particularcase, ourk will vary from0 to capital N minus 1.So,I have N number of complex numbers taken in a particular sequence which representsaboundary.

(Refer Slide Time: 11:09)



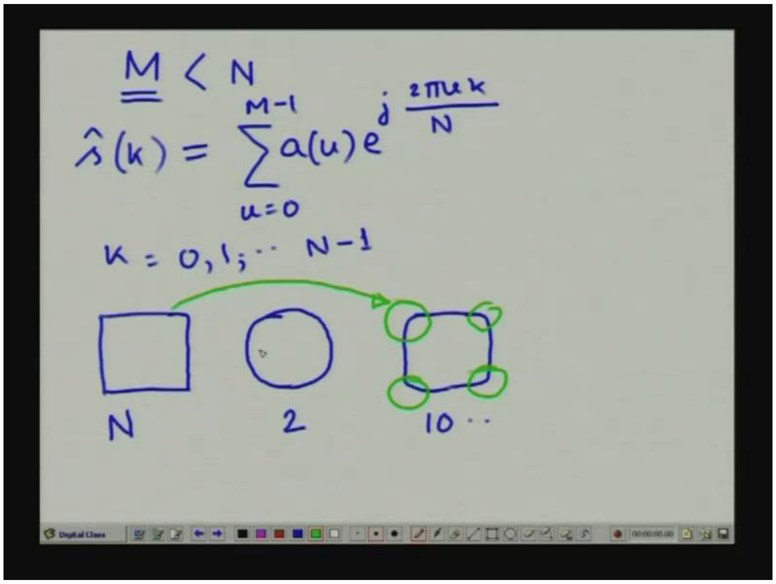
Now, what wewant todo is we want to takethediscreteFourier transform- DFT ofthis sequence ofcomplex numbers S (k)where kvaries from 0 tocapital N minus 1. So, as we have noted earlier when we had talked about thediscreteFourier transform, you find that ifI take the discrete Fourier transformof this N number ofsample points;in this particular case, each of the sample points is a complex number, what I get is capital N number of coefficientswhich are the Fouriercoefficients.

So,I canwrite thatexpression in this formsay a (u) is equal to1 upon capital N, take the summationS (k)e to the power minus j2piby capital N ukwhere kvaries from0 tocapital N minus 1.So, by doing this, you find thatI getcapitalN number ofcomplexcoefficientsa (u). So, each of the coefficientsa (u) iscomplex. Now, these set of complex coefficientscan be used as a descriptor which describes this particular given shape or this particulargivenboundary.

Nowobviously,if I take the inverse discreteFourier transform,the IDFT ofthe set ofcomplex coefficientsa (u); thenI should get back S (k)or original setof boundarypoints. So, thisinverse discreteFourier transformis given by a (u)e to the power j2 pi ukby capital N. Now, u has to varyfrom 0 to capitalN minus 1.So, thisset of u, thiscapital N number of complexcoefficients,this is what is known as Fourier descriptor of the boundary points.

So, givena set of boundary points, we consider each of the boundary points as a complex number by tracing the boundaryin a particular order eitherin the clockwise direction or in the anticlockwise direction,I get a sequence of suchcomplex numbers;I take the discreteFourier transformation of that sequence and suchDFT coefficients,the set of DFT coefficients, that become the Fourier descriptor which describes the given boundary.Andnaturally,if I take the inversediscreteFourier transformationof theseFourier descriptors,then I get back ouroriginal boundary points.

(Refer Slide Time: 14:26)



Now, in most of the cases, for representing in a shape,we normally donot consider all the capital N number of Fourier descriptorsthat is all the coefficientsin thisFourier descriptor.Rather, we consider, sayfirst capitalM number of descriptors for describing theshapewhere obviously thiscapital M is less then capital N.

Now, using this capital M numberof Fourier descriptors, if I try toreconstructour original boundaryby taking inverse discreteFouriertransformation;then because we are cutting out some of the Fourier descriptors,naturally ourreconstruction will not be exactbutrather what we will get is an approximate reconstruction of our boundary points.So, that approximate reconstruction or approximate boundary points are given by S (k)is equal to a (u)e to the power j 2 pi uk by capital Nwherenow this summation will be for u equal to0 tocapital M minus 1.Sohere,you find that and k will vary from0 to capital N minus 1.

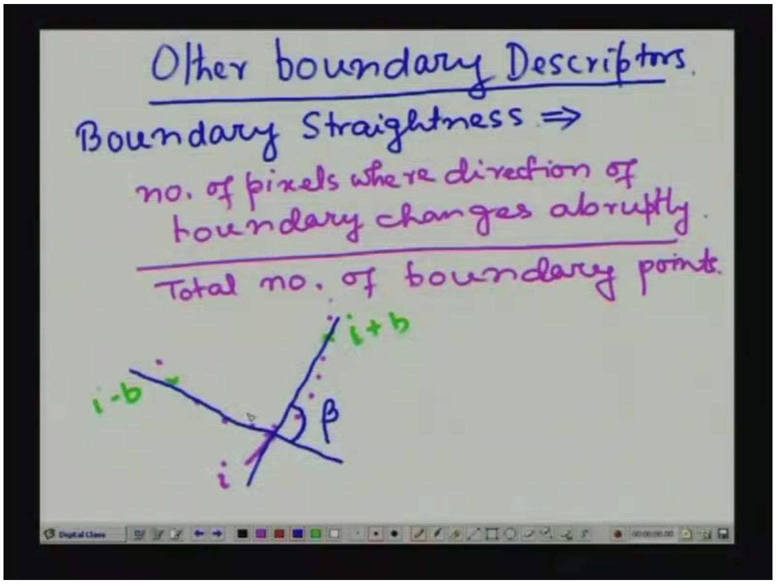
So, find that thoughwe are taking less numberof DFT coefficients orless numberofFourierdescriptors for reconstruction of our original boundary points, the number of boundary pointswhich arebeing reconstructed will remain thesame.Butwhat we will lose iswe will lose some details of the boundary.Sayfor example, if I have aset of boundary points whichcorresponds to a square; so if I consider very few number of coefficients,may be say one or twodiscreteFourier transformationscoefficient, in that case our capital Mwill become one or two,in that case after taking this inverse FourierdiscreteFourier transformation,I will get capital N number of boundary points which is same as our original number of boundary points.Butnow the points will lie on a circular shapelike this.

So,I will get back this original shape if I consider all the capital N number of Fouriercoefficients, the DFTcoefficientsforreconstructing the boundary by inverse discreteFouriertransformation.IfI consider say very few number of discreteFourier coefficientsay2, then I will get a circular shape. If I increase it further may be say 10 or so on,I will get a shape something like this.So, insuchcase, you find that thedetails of the cornerswill be lost but still Ican maintain the basic essence, the primary essencesofthe shape of ouroriginalboundary which was a square and it becomes almost a square.

So, by considering few number of DFT coefficients as Fourier descriptors to represent the boundary,I can approximatelyhavearepresentation of the boundaryand this gives an approximate description of our boundary.

So, thisFourier descriptor can bea very important descriptor which can be used forobject recognition purpose.Andobviously, if I have two different shapes which are widely different, then the Fourier descriptorsforthese two widelydifferent shapes aregoing to be widely different.So,I can easily discriminate between the descriptorsbetween the shapes between two or more shapes which are widely different.So, now let us take that what other boundary-based descriptors that we can have.

(Refer Slide Time: 18:59)



So, other boundary descriptors;so, another formof boundarydescriptor is what is called boundary straightness.This boundary straightnessis defined based on the concept of boundary curvature.So, as we know that in continuous case,the curvature at a point is given by rate of change of slope.So, if you take the derivative of the slope at a particular point, what you get is the curvature at that particular point.So, this boundary straightness even in discrete case, it is defined as number ofpixels, the ratioof the number of pixels where the boundary direction,the direction of boundary changes abruptlyto the total number of boundary points.

So,you find that in our case, we are not going for exactmeasurement or exactcalculation of the curvature at a particular point on the boundary.Butwhat we are doing iswe aretrying to find out, identify those points on the boundarywhere the direction of the boundarychanges abruptly because those are the points where the curvature is likely to be quite high and I consider the number of points on theboundarywhere the direction of the boundary changes abruptly and take the ratio of these number of points to the total number of boundary pointsand this ratio is what is called the boundary straightness.

So, you find that lesser the number, smaller the number is that is boundary straightness, the boundary is going to be more and more linear,itis moreand more straight whereas if this value is higher, if this ratiois higher that means there are more number of points on the boundary wherethe curvature is very high.So, we cannot say that the boundaryis straight in such cases.So, this gives a measureofthe straightnessof the boundary.

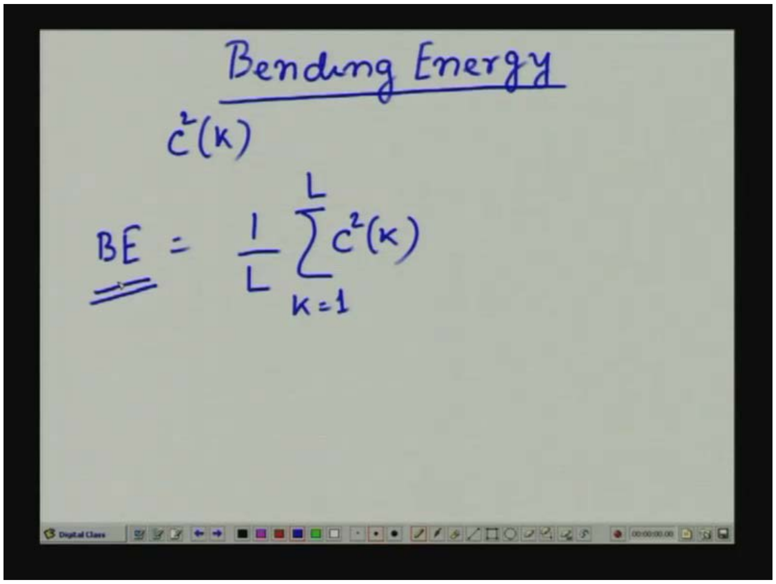
Now, the question is thathow to find out that what are the pixelswherethe boundary changesabruptly?So, if I consider say these are theset of points, so these are the set of points which are on aboundaryand I take this i'th pixelwhere I wantto find out whether theboundary changesabruptlyat thisi'thpixel or not.

So, what I do is I take a distance b and consider two more points; one at,say I consider one point - i plus b in one direction and I consider another point - i minusb in the opposite direction.Then whatI do is I draw a straight line passing through i and i plus b and I draw a straight line passing through i minusb and iand I try to find outthat what is the angle betweenthese two straight line, say anglebeta.

So, we find that if the boundary changesabruptly at thispixel location i,then thisvalue of this angle betawill be quitehigh whereas if the boundary does notchange abruptlyat this location i,then the value of this angle beta will be very low.So, depending upon the value of this angle beta between these two line segments; one passing between i minus b'th point and i'th point and the other onepassing betweeni'th pointandi pluspassing iplus b'th point,I can determine I can estimate whetherthere is a sharp change ofboundary direction at this i'th pixel or not.

So, this way I find out the number of pixels where the boundary changes abruptly and the ratioof that such number of points to the total number of points gives me a measure what is called a boundary straightness.The other form of boundary based descriptor can be what is called bending energy.

(Refer Slide Time: 24:17)



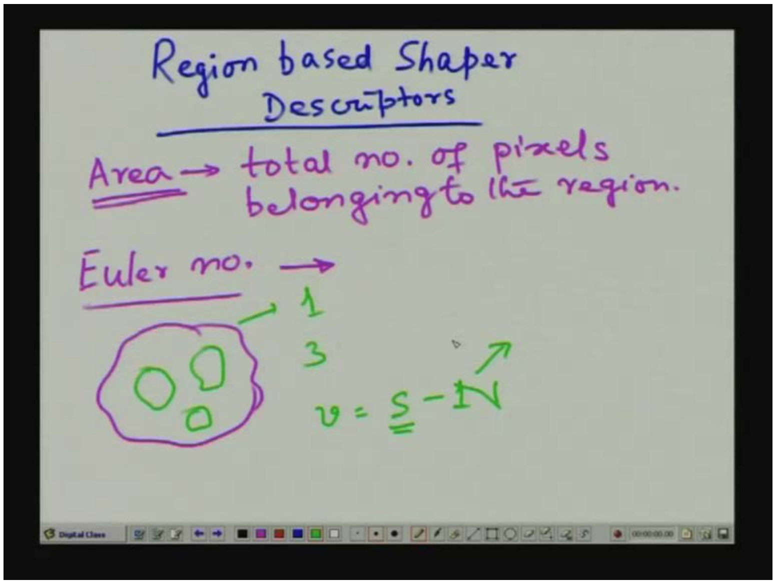
So, what is this concept of bending energy? Bending energy is supposed we have steel rod and we want to bend that steel rod to a given shape. Now, while doing so, the amount of energy that we have to spend to bend the steel rod to a given shape, that is what is called bending energy. So, considering this, you find that if I want to bend the steelrod suchthat thecurvature will be more,I have to spend more amount of energywhereas if the shape has a curvature which is less,thenI havetospend less amount of energy.

So, this bending energy,this can be computed fromthe concept of curvatureat different pointson the boundary itself.So, at the k'th point if I say that c (k) is the curvature atpointk;in that case,the bending energycorrespondingto that point is a square of this.So, the total bending energyof the boundary,this can be computed as say bending energyis equal tosummationof c(k) squarewhere c k is the curvature at point kand I have to take this sum for all the pointslying on the boundary.

So, this will be for k equal to 1 to say the capital Lif capitalL isthe total number of boundary points and I normalize this;soI have to take, divide this measure by 1by L.So, this becomes my normalized bending energyand this normalized bending energyor normally called bending energy, this is also a feature for a descriptorof the shape ofthe shape of the region.So,based on the boundary and asimilar in a similar manner, we can find out many other descriptors which represent theshape based on the boundary information itself.

So, as the shape information as we said earlier that the shape information can be described from the boundary of the region,similar shape information can also be obtained from the region itself or the region area.

(Refer Slide Time: 27:08)



So, letus seethat what aresuch descriptors that we can obtain from the shape informationthat wecan obtain fromthe region itself.Sonow,we will talk about region-based shape descriptors. Oneof the obvious descriptor isthe region area.Now, whatis region area?Region area is nothing butthe total number of pixels belonging to the area or belonging to the region.So, it is the total number of pixels belonging to that particular region.

Soobviously,we find that this regionarea can be a descriptorwhich is translation invariant or rotation invariant because if I translate the object or if I rotate the object, the total area or the total number of the pixel within thatobject will remain the same whereas the regionarea,this descriptoris obviously not scale independentbecause if I zoom the object, thetotal number of pixelsbelonging to the region willbemore whereas if I zoom itout,in that case, the total number of pixelsbelongingto that particular regionwill beless.

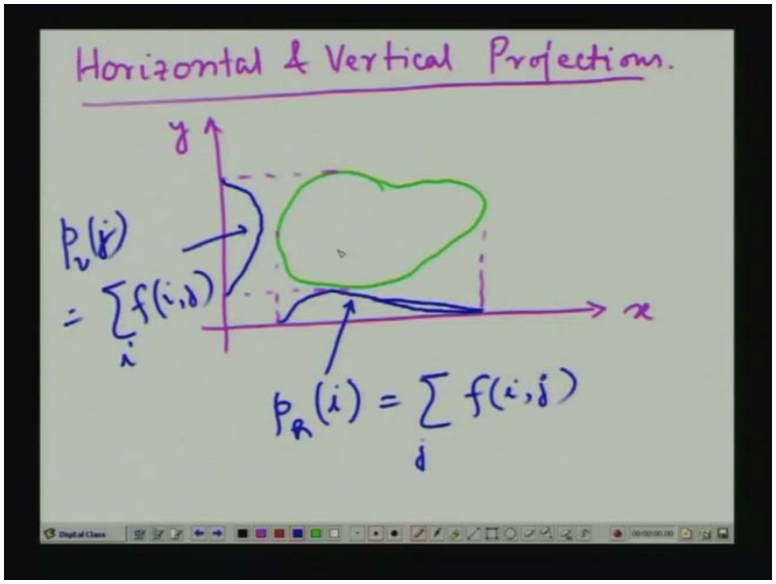
So,this region area, though it is a feature but thiscannot be used for object description or object recognition.The otherformof descriptor which is called Euler number;now Euler number, it isa simple topologicalinvariant. Now,what is Euler number?Suppose,I have a regionsomething like thisand within this regionI have a number of holes.So, youfind thatin this particularcase,I have oneconnected component of the regionand withinthis connected component,I have a number of holes. So, in this particular case,I have one connected componentand there are 3holes.

So, the Euler numberis defined like this;say Euler number is equal to s minus n where s is the total number of connected components of this particular region.So, s minus n and n is the total number of holes in this region.So, in this particular case, because it has got only one connected region and 3 holes; so Euler number in this particular case will be equal to minus 2whereas if I have an object of this form, say one connected region and one hole within this connected region, in this particular case, the Euler number will be equal to 0.

So,asI said that this Euler number is atopologicalinvariant descriptor.Whyit is invariant?because if I zoom this particular region or zoom out this particular region, if I translate the object or if I rotate the object,the Euler number will remain the same.Obviously, the restriction is if I zoom it, in that case, it should not be stretched so much that the single region breaks into two different regions or the viewing angle should not be such that one of the holes is not visible, one or more of the holes is not visible.So, within that constant, you find that this Euler number gives a descriptor,a topological descriptor which is invariant to rotation, translation and scaling.

Theother kind of descriptor which can be used which is again a region-based descriptor that is horizontal and vertical projections.

(Refer Slide Time: 31:29)

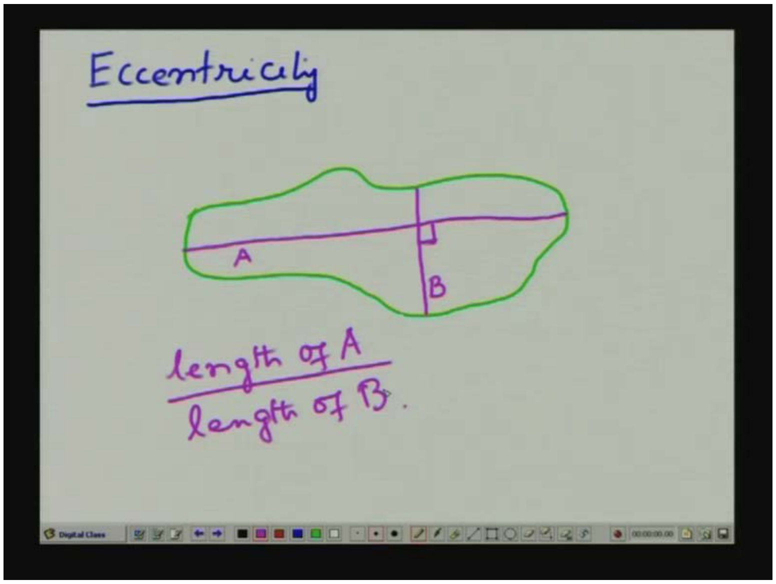


Now, what are these horizontal and vertical projections?Suppose,I have some object region say something like this and this is my x axis - horizontal axis and this is the vertical axis – y.So, what I do is I take the projection of this on my horizontal axis, this becomes the horizontal projection and if you take the projection of this on the horizontal axis,the projection will become something like this and I also take the projection of this area on the vertical axis.So, in this particular case, the projection on the vertical axis will become something like this.

So, this horizontal axis, the horizontal projection this i call $p_h(i)$ and this can be computed by simple expression f(i,j)where j varies from and this summation has to be done over this variable j that means I have to take the summation along a particular column.

Similarly, this projection I call vertical projection; so this is say $p_v(i)$ which can be simply computed in a similar manner.Itwill be the summation f (i, j)where now this summation has to be taken over i.So, this should be p_v (j).So,you find thatdepending upon the shape of this particular region;these two projections that is the vertical projection and horizontal projection, they are going to be different and normally these are used as descriptors in case of binary image.This is not normally used incase of gray level image.

(Refer Slide Time: 34:01)

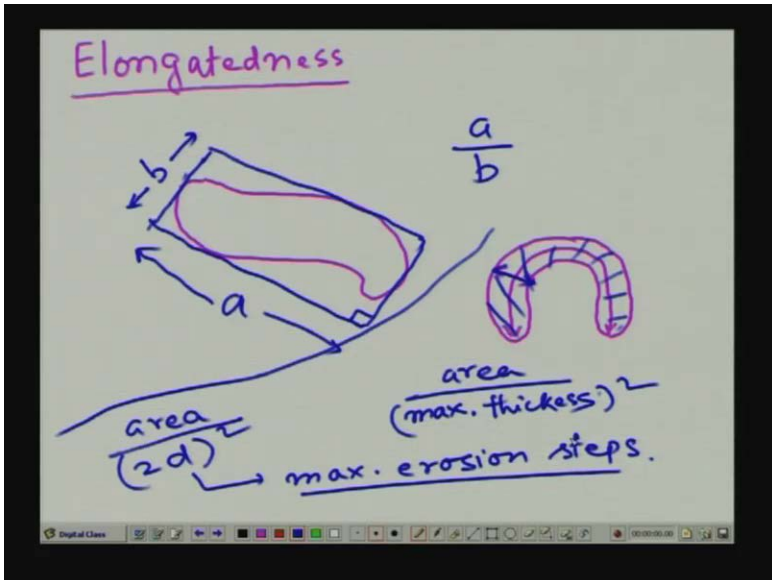


Theother kind of region-based descriptor that can be used what is called eccentricity.Now, what is eccentricity?Suppose,I have a region say something like this; what I do is I find out a code of this particular region of maximum length. So, suppose I have a code this which is of maximum length and I call this code as code A and then I find out another code of maximum length of the same region which is perpendicular to the code of maximum length.So,I call this code as code B and my restriction is that these two codes have to be perpendicular to each other.

So, given such a situation; then the ratioof the length of the maximum code, so length of A to the length of B, so the ratio of these two codes - one is a code of maximum length and the second one is a second code which is again of maximum length but perpendicular to the first code and ratio of these two, this is what is called eccentricity of this particular shape.

So, this eccentricity can also be used as one of the descriptors, one of the shape descriptors and as you see that it is also a region-based descriptor.Ofcourse, a similar such descriptor,I can also obtain from the boundary of the region.

(Refer Slide Time: 36:06)



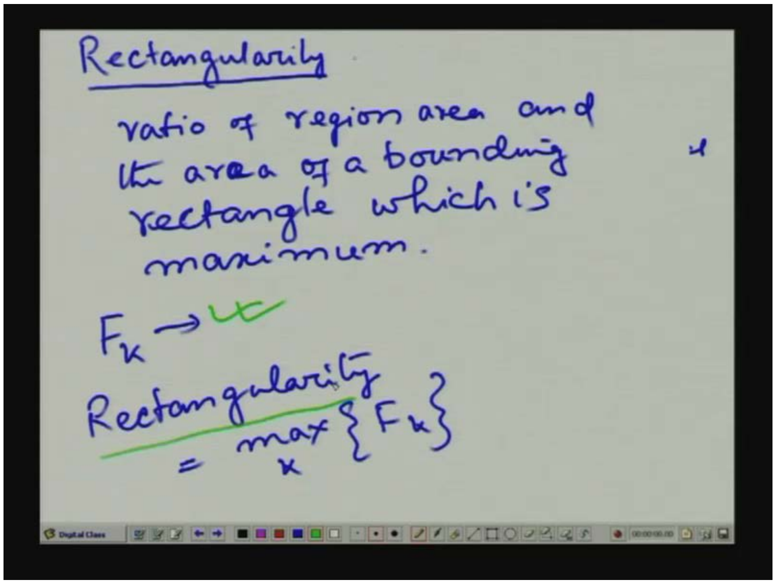
Anotherkind of descriptor that can be defined shape-based descriptor which is called elongatedness. Now, to defineelongatedness, we canstart something called a region minimum bounding rectangle.Now, what is this?Suppose,I have a shape, a region shape of this form, saysomething like this and what I do is I find out a rectangle,a bounding rectangle of thisregion which is of minimum size say may be such a bounding rectangle can be of this form, say I have a bounding rectangle like this.

Now, in this particular case, this shape has not become a proper rectangle because the angles are to be 90 degree.So,I have a bounding rectangle like this and this bounding rectangle have to be of minimum shape, minimum size.So, this is what is called minimum bounding rectangle.Now, once I have such a minimum bounding rectangle, suppose a is the length of the larger side of this bounding rectangle and b is the length of the smaller side of this bounding rectangle and the elongatedness in such case is defined as the ratio of a to b that is the length of the larger side of the minimum bounding rectangle to the length of the smaller side of the minimum bounding rectangle.This is what is called the elongatedness of this particular shape.

Now,you find that such a kind of elongatedness,I mean this simple definition of elongatedness is not valid if my region is acurved region.Forexample, if Ihave a region something like this; for this kind of region, the elongatednessmeasure cannot be simply defined by this ratio of a and b.So, in such cases to define the elongatedness of this kind of region, what I have to take into consideration is what is the thickness of this elongated region along with the length of the elongated region.

So, one of the measure of elongatedness,I mean one way of finding out the elongatednessfor such curved region is like this; it is defined as for such curved regions as area by maximum thickness square.So,I find out what is the area of this curved region,I find out what is the maximum thickness and area divided by maximum thickness square, these gives me a measure of the elongatedness for such a curved region and one of the way to find it out is you find out area divided by say2d square where d is the number of erosion steps that is needed before this region erodes to a null set.So, this is the maximum erosion steps. So,I can defineelongatedness for a curved region in this way and this elongatedness is one of the descriptors which captures what is the region shape.

(Refer Slide Time: 40:40)

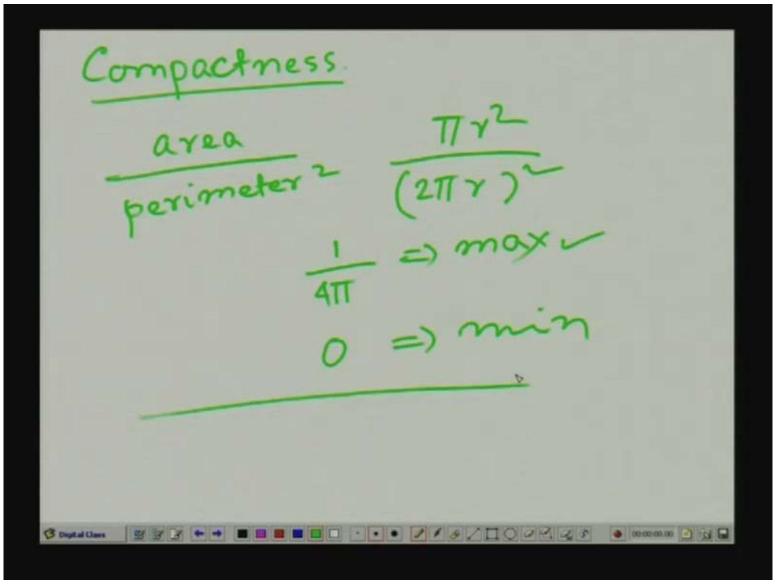


Someother descriptors,some other region-based descriptors can be one of them, can be say rectangularity.Rectangularityis the ratio of a region area and the area of a bounding rectangle having a particular direction say k which is maximum.So,I can define this rectangularity as the ratio of region area and the area of a bounding rectangle which is maximum.So, if I say that $F_k$is the ratio of the region, ratio of the region area to the area of a bounding rectangle having direction k; then the rectangularity will be defined as will be simply maximum k over F_k.

So, what we have said that this F_k is the ratio of the region area to the area of a bounding rectangle having direction k and the maximum of these ratios is what is called the rectangularity of that particular given shape, regionshape and here obviously you find that when I say that what is the direction or a bounding rectangle having a direction k, we have to define what is the direction of a bounding rectangle.The direction of a bounding rectangle is defined as the direction of the larger size side of the bounding rectangle.

So, what we have do is given a particular region,I find out bounding rectangles of various orientations and for each of these orientations, for each of the directions,I find out the ratio of the region area to the area of the bounding rectangle and for different directions whichever gives me the maximum ratio that is what is defined asthe rectangularity of that particular region.

(Refer Slide Time: 43:57)

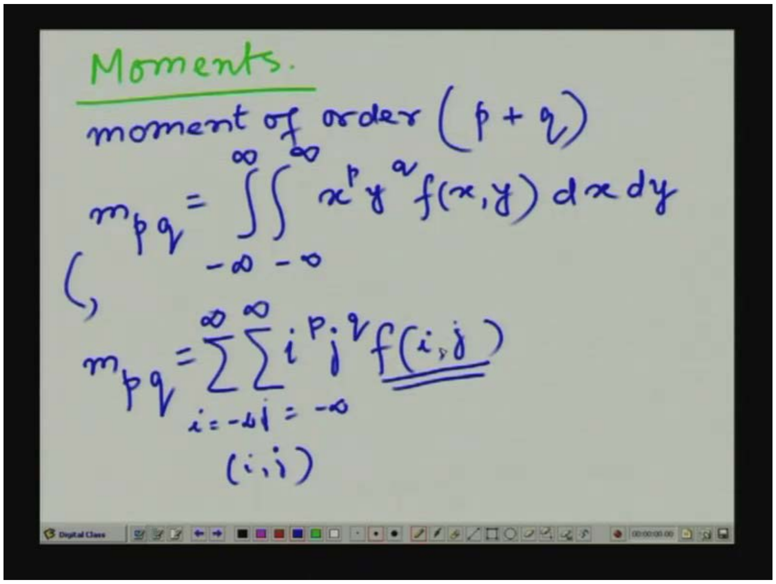


Theother kind of descriptor that can be used is what is called compactness and this compactness is defined as the ratio of area by perimeter square.So, what does it give?Weknow that a circle is maximally compact.So, this compactness for a circle, for a circle we know that area is given by pi r square and the perimeter is given by 2 pi r and if I take the square of this and then take the ratio, this simply becomes 1 upon 4 pi.So, the maximum value of the compactness is 1upon 4 pi and the minimum value of the compactness can be 0.

Suppose,I have a long, very long region whose thickness is very very small; so in such case,in the limiting case, what we are going to have is an infinite perimeter but zero area.So, if I take the ratio in such case that area by perimeter square these gives me a value equal to 0.So, these are the two limiting values of this compactness; one is 1 upon4 pi which is the maximum and this we get for a circular region and as the region shape deviates from circularity, the compactness will be less and less and in the limiting case the minimum value of the compactness will be 0.

So, these are some simple descriptors, the shape descriptors which can be obtained from the region itself not only from the boundary.Now, there are some more shape descriptors which are also obtained from the region and those are called moments.

(Refer Slide Time: 46:01)



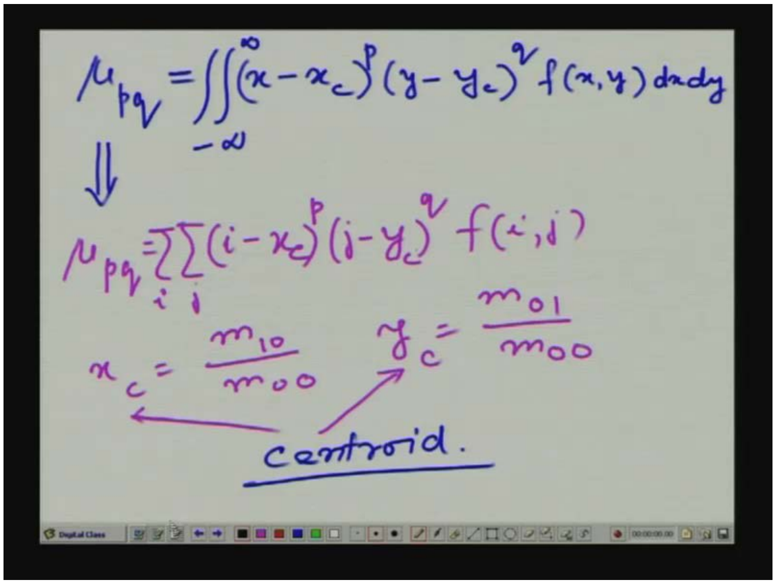
Now, to compute this moment of a region or interpretation is the gray level image that you get is a probability density that represents a normalized gray level image represents the probability density of a two-dimensional variable.Now, with this interpretation that a normalized gray level image tells us the distribution of a two-dimensional variable, we can find out some statisticalproperties of the graylevel itself and this statistical properties are what are nothing but moments.

So, a moment of ordersay p plus q, moment of order p plus q is defined like this; we have m_{pq} which is given by x to the power p, y to the power q, f (x, y) where f (x, y) is the normalized gray level image,take the integral dx dy and this integral has to be taken in the limit minus infinity to infinity over y and minus infinity to infinity over x.So, this is what is the definition of a moment of order p plus q for a normalizedgray level image f (x, y).

Indiscrete case, this expression is converted into m_{pq} is equal to i to the power p, j to the power q, f (i, j), take the double summation where j varies from minus infinity to infinity and i also varies from minus infinity to infinity.Obviously,for a finite image, this (j, i) and j limits will be changing accordingly and in this case (i, j)(i, j) is the location of a pixel in the image and f (i, j) is the normalized gray level of that particular image.

Now, you find that the moment as it is defined in this particular case, this moment is not translation invariant.So, to make the moment translation invariant, what we take is what is called as central moment.

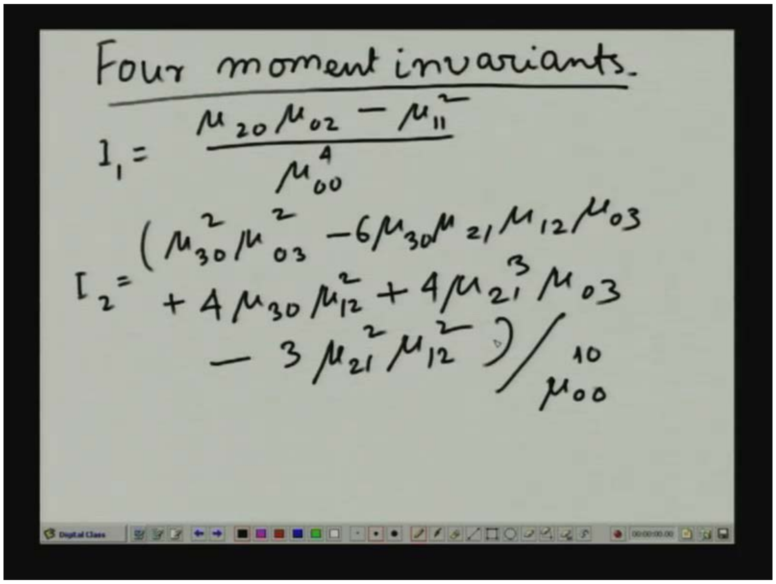
(Refer Slide Time: 48:52)



The central moment of order p plus q is defined like this; thecentral moment mpq mu $_{pq}$is defined as (x minus x_c) to the power p (y minus y_c) to the power q f (x, y) dxdy and again you take the double integral in the limit minus infinity to infinity.So, this is what is called the normalize moment and again in the discrete case,this normalized moment will be expressed as mu $_{pq}$ is equal to (i minus i_c) to the power p (j minus j_c) to the power q f (i, j) take the double summation over i and j.Sorry, this should be (i minus x_c) and this should be (j minus y_c)where this x_c is nothing but m_{10} by m_{00} and y_c is nothing but m_{01} by m_{00}.

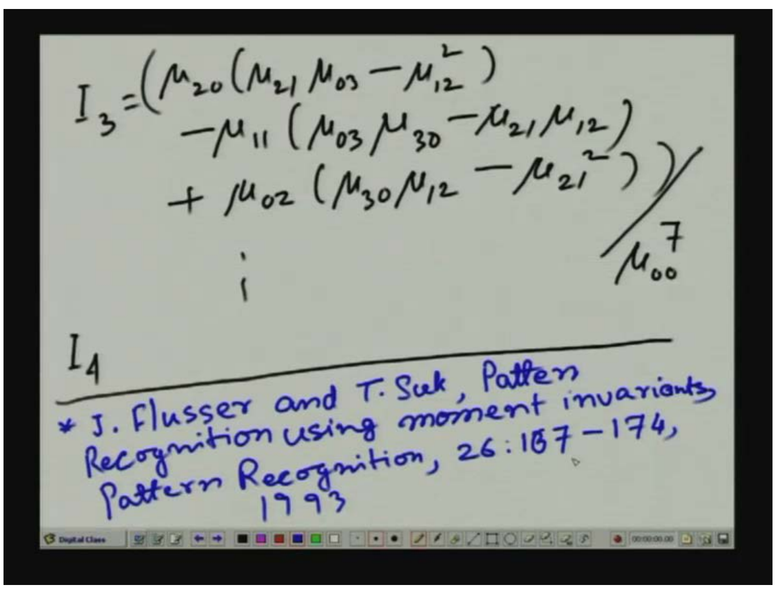
So,you find that this x_c and y_c, these two coordinates, what it specifies is nothing but the centroid of the region.So, that is why it is called central moment and once we have the central moment, the central moment becomes translation invariant.Butyou find that these moments and not rotations and scale invariant.So, people have tried to find out the rotation and scale invariants from these moments.

(Refer Slide Time: 51:11)



So, there are 4 different moment invariants which have been suggested, so 4 and those 4moment invariants are defined like this; sayI_1equal to mu$_{20}$ mu$_{02}$ minus mu$_{11}$square upon mu$_{00}$to the power 4,I_2 is given by mu$_{30}$ square mu$_{03}$ square minus 6 mu$_{30}$mu$_{20}$ mu$_{12}$ mu$_{03}$ plus 4 mu$_{30}$ mu$_{12}$ square plus 4 mu$_{21}$ cube mu$_{03}$ minus 3 mu$_{21}$ square mu$_{12}$ square this divided by mu$_{00}$ to the power 10.

(Refer Slide Time: 52:51)



So, the third invariant is given by I_3 is equal to mu$_{20}$ to mu$_{21}$ mu$_{03}$ minus mu$_{12}$ square minus mu$_{11}$ into mu$_{03}$ into mu$_{30}$ minus mu$_{21}$ into mu$_{12}$ plus mu$_{02}$ into mu$_{30}$ into mu$_{12}$ minus mu$_{21}$ square; this whole thing divided by mu$_{00}$ to the power 7 and similarly there is another moment invariant which is $I_4$and that has a very big expression like this.

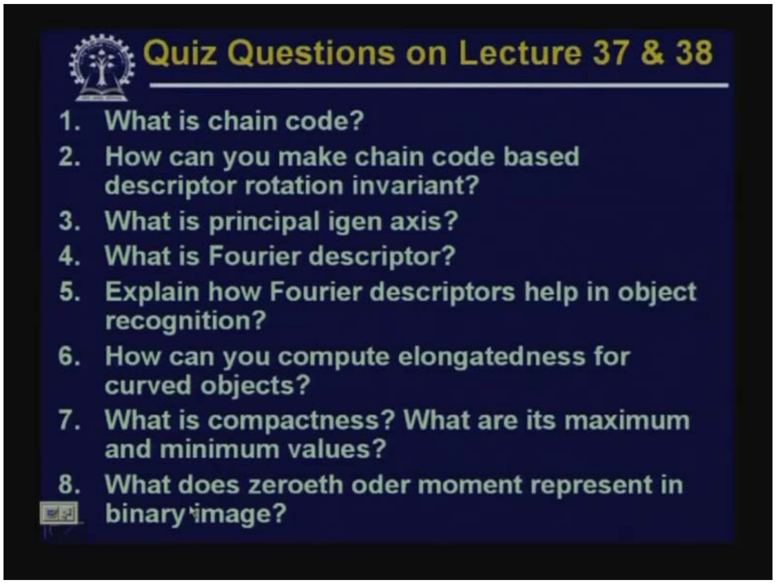
So, we are not going into the details of how these moment invariants arederived but the details of this information, if any of you is interested can be obtained from this particular reference.So, this reference gives the details of how these moment invariants are generated, how these four different moment invariantsare generated from the moments and these are rotation, translation and scale invariant.

So, today what we have discussed is we have discussed some of the boundary descriptors, some of the descriptors using the boundary as well as some of the shape descriptors using the region itself and these descriptors can be used for high level recognition purpose when we try to match these descriptors with similar such descriptors which are stored in the knowledge base.

So, we stop our discussion here today.Inour next class,we will discuss about some more region-based descriptors.Now,today what we have discussed is the region-based descriptor but this these region-based descriptors does not take into account the reflectance properties such as colour or texture of the surface. Wewill talk about other region-based descriptorsay for example, texture descriptors in our next lecture.

So, let us see some of the quiz questions based on today's lectures and as well as the lecture that we have a given in our earlier class.

(Refer Slide Time: 56:32)



So, the first question is what is chain code?Second question, how can you make chain code-based descriptor rotation invariant? Third question,what is principal igen axis?Fourth question, what is Fourier descriptor? Fifth question,explain how Fourier descriptors help in object recognition? Sixth question, how can you compute elongatedness for curved objects? Seventh question, what is compactness?What are its maximum and minimum values? And, the last question, what does zero'th order moment represent in binary images?

Thank you.