Digital Image Processing

Prof. P. K. Biswas

Department of Electronics & Electrical Communication Engineering

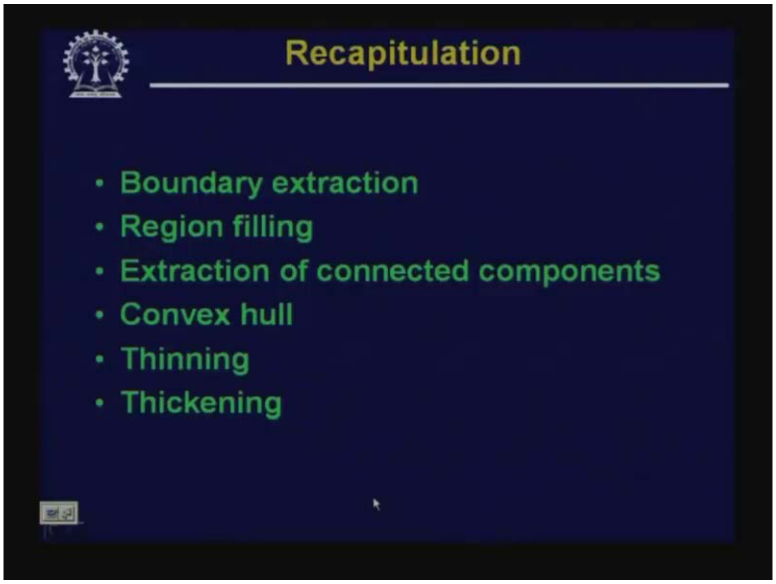
Indian Institute of Technology, Kharagpur

Lecture - 36

Mathematical Morphology - IV

Hello, welcome to the video lectures serieson digital image processing. For our last few classes, we are discussing on mathematical morphology and the application of mathematical morphology in digital image processing.

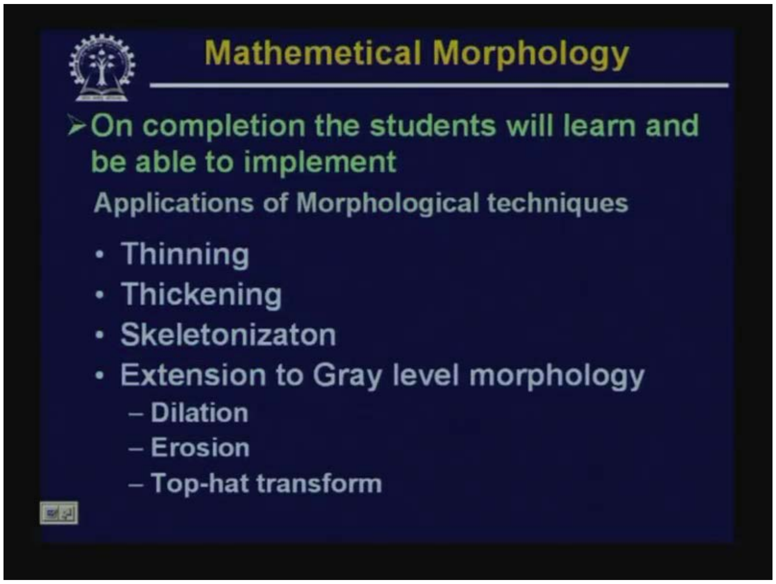
(Refer Slide Time: 1:15)



So, in your last class, we have discussed about the application of mathematicalmorphology in image processing techniques and we have talked about themorphological technique for boundary extraction, we have talked about the morphological technique for regionfilling, we have also discussed about the extraction of connected components using morphological operations we have talked about what is a convex hull and how to detect or how to form the convex hull for a given point set using the morphological operations.

Then we started our discussion on the thinning operation, thinning using the morphological operations which we will continue today and we will also discuss about the thickening operations along with some more applications.

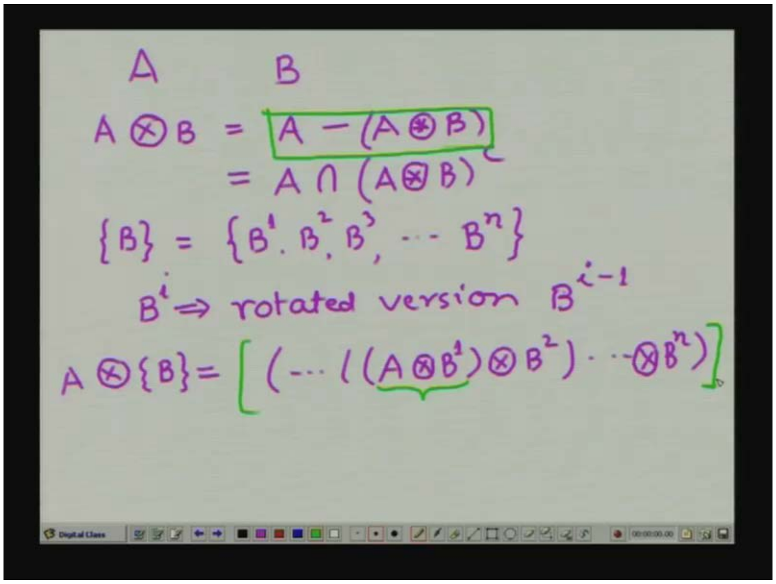
(Refer Slide Time: 2:12)



So, in today’s lecture, we will complete our discussion on thinning using the morphological operations.Wewill discuss aboutthickening and we will see that thickening is nothing but a dual of thinning operation. Wewill also discuss about the morphological techniques to obtain the skeleton of a given shapeand we have said earlier that the skeleton of a given shape is very very useful for describing an object shape and this description can be used for high level image understanding operations for image interpretation purpose.

Thenwe will extend; so for what till skeletonization,whatever we will discuss that is on the binary images; the application of morphological techniques on binary images.Thenwe will see that how to extend this morphological operations in gray level images,so we will call it as gray level morphology and we will see few of the operations morphological operations like dilation and erosion which is nothing but an extension of the binary morphology to gray level morphology and particular applications liketop hat transform, that we are going to discuss in today’s lecture.

(Refer Slide Time: 3:48)



So, to start with, as we were discussing about the thinning which we could not complete in our last lecture; so let us just quickly review what we have done in our last class.So, for thinning, suppose we are given a point set say A and this point set A has to be thinned by the structuring element B. So, a thinning operation using the morphological transformations can be obtained like this; A thinned with B, the structuring element B is defined as A minus A hit or miss transform with B and we know that from set operations, this set difference operation can be implemented using set intersection and set complementation.

So, this is equivalent to,the same definition is equivalent to Aintersection with A hit or miss transform of B and complement of this.Andwe have said in our last class that for thinning operation, instead of using a single structuring element, what we use is a set of structuring elements and the thinning is performed with the help of that set of the structuring elements.

So, in our case, for the thinning operation,we will have a set of structuring elements the Band this set will contain a number of structuring elements,letus call them as B^1 B^2 B^3 and so on.Suppose there are n numbers of structuring elements,I will have upto $B^n$where all these structuring elements in this set will follow a particular property that if I consider a structuring element say B^i in this set then this B^i is nothing but a rotated version of set B^i minus 1.

So, if you rotate the set, the structuring element B^1, what I get is the structuring element B^2.Similarly,if I rotate the structuring element B^2, what I get is the structuring element $B^3$and so on. So, every structuring element B^i in this particular set of structuring elements is a rotated version of the previous structuring element that is B^i minus 1.

Now, using this set of structuring elements,now the thinning operation has to be performed by applying each of the structuring elements in sequence.So, the thinning of the point set A now is to be implemented in this form.So, thinning of the structuring element with thinning of the point set A with the set of structuring elements B has to be performed in this way.

Sofirst, A has to be thinned with the structuring element B^1, this has to be thinned with structuring element B^2 and we have to continue like this.Then finally, we have to thin with the structuring element B^n.So, you look the operation that we have performing.Weare taking the original set original point set A, take the structuring element B^1 from the set of structuring elements B,thin A with B^1; whatever thinned output you get, you thin that output with the structuring element B^2, thin this output with the structuring element B^3 and so on and you continue until you thin all the intermediate results with the last structuring element that is $B^n$and in this case, each of these thinning operations that is when we say A is thinned with structuring element B^1, this follows this particular definition that is A minus A hit or miss transform with the structuring element B^1.

Now, this entire operation forms onepass of an interactive algorithm.So, the way we have to implement is that this entire operation that is the thinning starting with the structuring element B^1 to the structuring element B^n, this entire operation has to be donerepetitively,say a number of times until and unlesswe find that in 2 subsequent operations, the output does not change.So, that is the stage when thealgorithm converges and the output at that particular instant of time at that particular time when the algorithm converges, gives the thinned version of the point set A with respect to the structuring element or the set of structuring elements B.

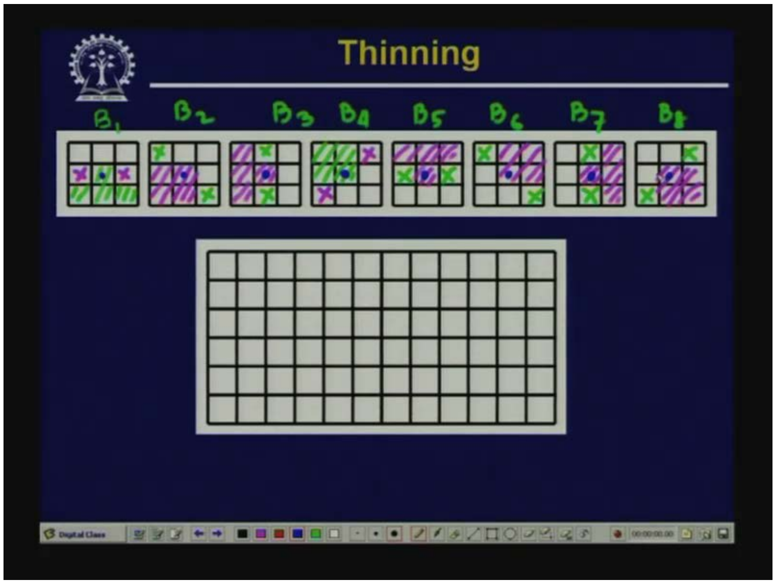
Now, let us take a particular example.Sayhere,as we said that the structuring element in this case is a set of structuring elements; so in this particular example, we take say 8 structuring elements starting from B_1, so this is structuring element B_1, structuring element B_2, structuring element B_3, structuring element B_4, structuring element B_5, B_6, B_7 and B_8.So, we consider 8 different structuring elements.

Now, thestructuring elements are something like this; so the first structuring element B_1 consists of these points and here we represent by cross, so these are the points which are don't care points.So, when we try to find out the hit or miss transform of the point set A with structuring element B_1, what we will look for is that a translated version of this structuring element B_1 at that translated location,I should get a match at all these different point locationswherever the points are 1 and all these3 points, the corresponding locations in the point set should also be equal to 0or the background pixels and we don't care about the condition of these 2 locations where we have put a cross.So, this is the first structuring element B^1 in our setof structuring elements.

The next structuring element B_2as we said that if we rotate B_1, what we get is B_2. So, the structuring element B^2 will be like this; this is our structuring element B_2 and these are the don't care locations.Wedon't care about the corresponding locations in our point set A.

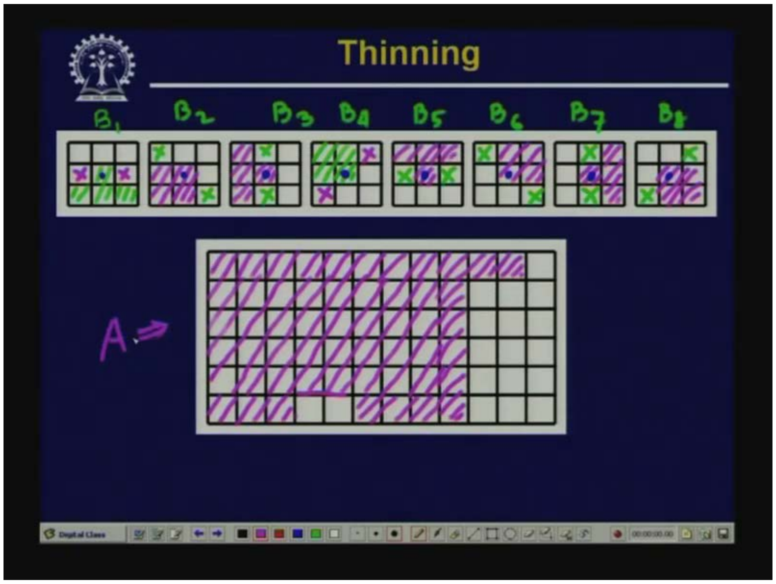
Similarly, the structuring element B_3will be like this; these 2 are the don't care locations, structuring element B_4will be like this with these 2 locations as don't care locations, structuring element B_5will be like this where these 2 locations will be don't care locations, structuringelement $B_6$is this - these 2 being the don't care locations, structuring element B_7 is this where these 2 are the don't care locations and finally the structuring element B_8 will take this form with these 2 locations as the don't care locations and for each of these structuring elements, we consider the origin of the structuring element to be the center location.That is in each of thesecases, the origin of the structuring element is the center location; these are the origins of different structuring elements and so on.

(Refer Slide Time: 10:30)



Now, let us consider a typical image and if we try to thin that particular image with this set of structuring elements; what kind of thinned output that we are going to have?So, I take an image like this; say,consider this particular binary image which is to be thinned with this set of structuring elements.

(Refer Slide Time: 14:20)

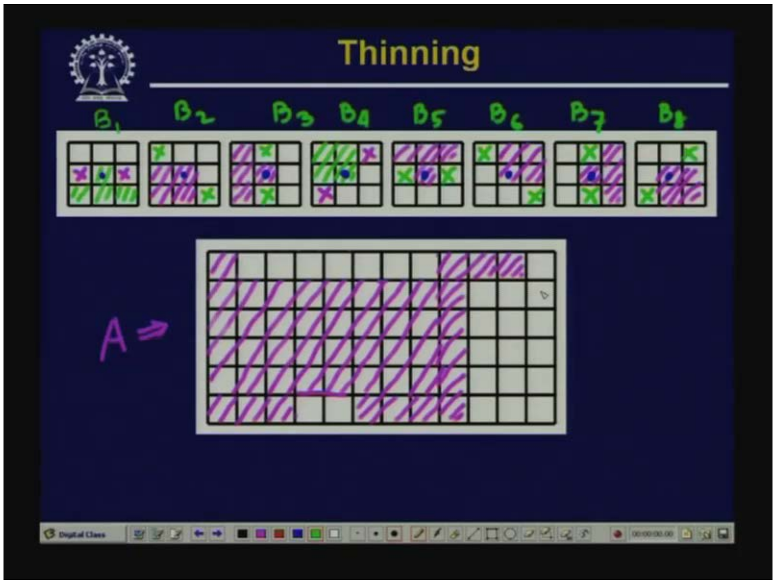


Obviously, in this case, this binary image is our given set A. Now, the way we have defined the thinning operation is that first we try to thin this point set A with the structuring element B_1, whatever output we get that we thin with the structuring element B_2, that output has to be thinned with structuring element B_1; so like this,we will continue upto the structuring element B_8 and this completesone pass.The inter operation has to be done second time, it has to be done third time, it has to be donefourth time and so on until and unless we get a convergence that is we get a situation that no other change,no change in the thinned output is possible.

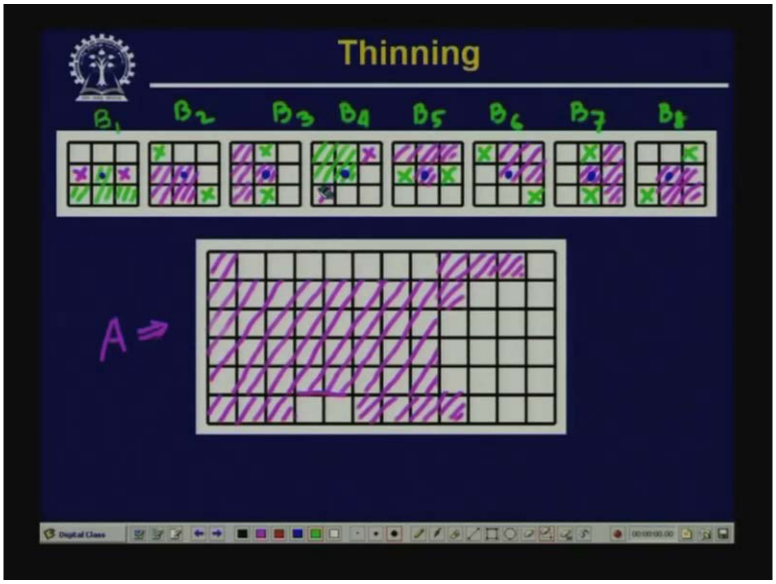
So, let us see, first let us consider the structuring element $B_1$and try to thin this point set A with the structuring element B_1. Now, if you look here, you will find that the points at which this particular structuring element B_1 will give a positive result for miss and hit and miss transform, at these particular locations;so this is 1 location,this is 1 location, this is 1 location, this is 1 location, this is 1 location, this is 1 location and this is another location.So, only at these locations,this particular structuring element B_1isgoing to give positive results.Thestructuring element B_1 will not give positive results or the match anywhere else within this particular image.

So, what we will do is because our thinning operations is defined as A minus A hit or miss transformwith B, so you will find that if I remove these points from my original point set A, then after performing the thinning operation with the set B_1, this is the intermediate result that I get.

(Refer Slide Time: 17:27)



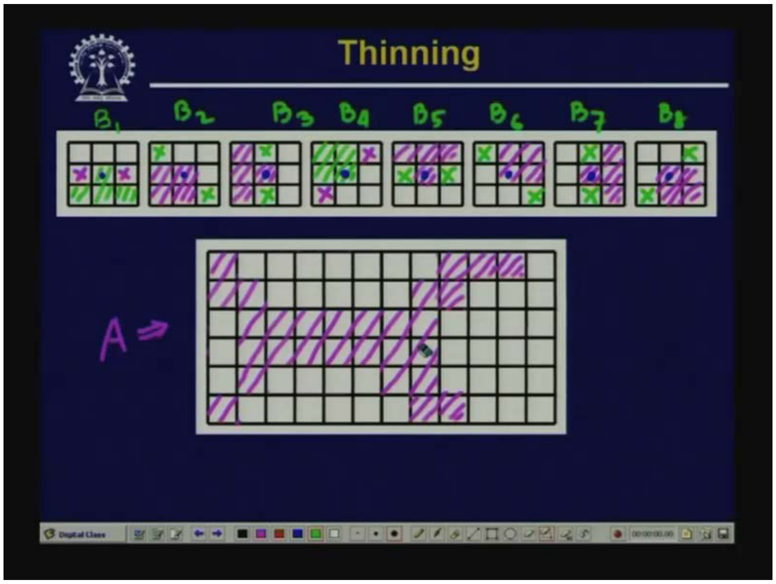
Now, if I try to thin this with the structuring element B_2, you will find that B_2 does not fit anywhere within this particular set. Try to thin with structuring element B_3 and you will find that B_3 with the structuring element B_3, the points that we can remove are only these points.So, this is 1 point, this is 1 point and this is the other point;so, these are the 3 points which can be where this structuring element B_3 gives a match.So, what we do is we remove these points from the original point set A.
(Refer Slide Time: 18:02)



So, at the end of the operation, this is what is going to be our thinned output, intermediate thinned output.Then you try to do the hit or miss transformwith B_4, you will find that are hit or the miss transformwith B_4in this particular image cannot remove any of the points present within this particular image.Try with B_5, again with B_5, the points that can be removed are these points.So, if I do this hit or miss transform with B_5; I can remove this particular point,I can remove this particular point, similarly I can remove this particular point,I can also remove this particular point.So, these are the points which can be removed after hit or miss transform after performing the thinning operation with our structuring element B_5.

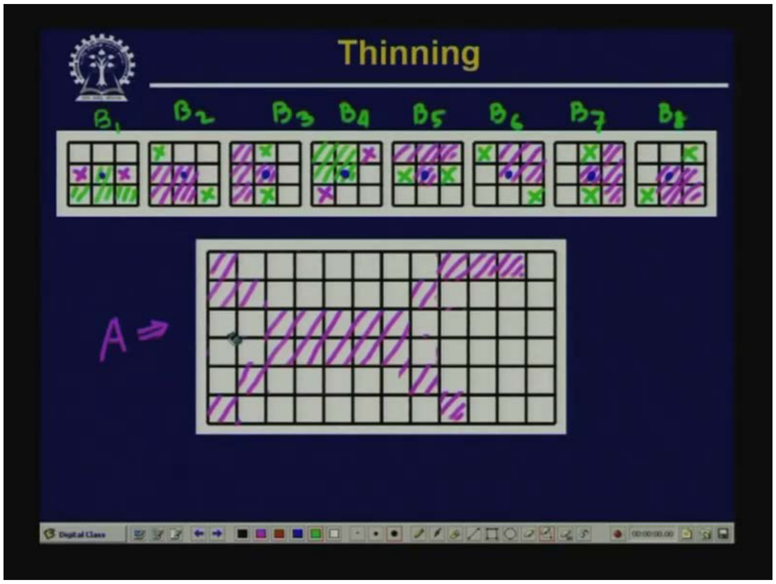
So, if I continue like this, you will find that at the end,the points which will remain within this image are these points.So, these are the points which will be remaining when the algorithm converges.Now, on this, if I impose the restriction that the connected component that this thinned output that I get that has to be m connected; in that case, some more points are to be removed from this particular thinned output.

(Refer Slide Time: 19:53)



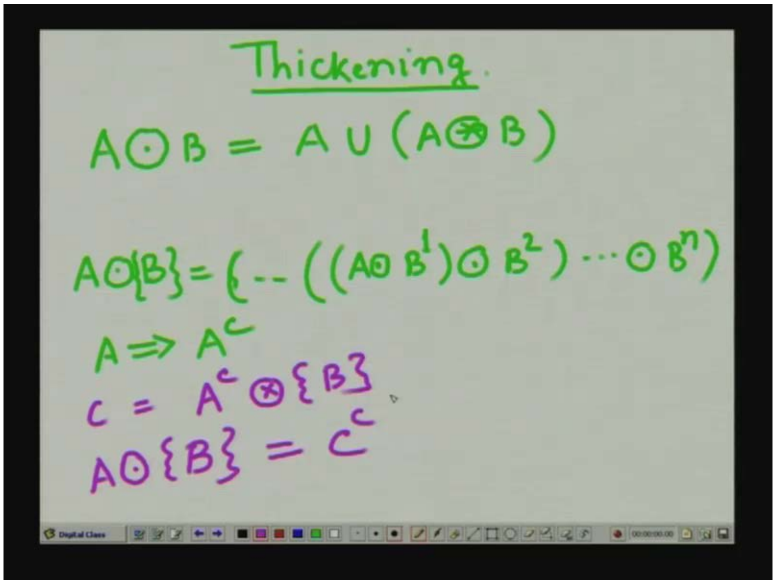
So, to make it m connected;I have to remove this point,I also have to remove this point,I also have to remove this particular point,I also have to remove this point,I also have to remove this point and this point also.

(Refer Slide Time: 20:48)



So, at the end, this particular output that I get, the point set that I get, this is the thinned version of my input original point set a which is thinned with the structuring elements or set of structuring elements B and you will find that because this is a skeleton kind of structure it can be used for obtaining the descriptions of a shape which is useful for high level image understanding operation.Now, as we said that the thinning is the thickening a dual operation of thinning; so just as in case of thinning, the thickening can also be represented can also be defined in this form.

(Refer Slide Time: 21:30)



So, thickening which is represented like this with the structuring element B, this is defined as A union with A hit or miss transform with B and in the same manner, here also, B is aset of structuring elements and because B is a set of structuring elements, if I consider the similar type of operation; then this thickening operation can be implemented in the same manner.

So first,what I have to do isthis B is a set of structuring elements; first I have to do the thickening operation of A with the structuring element B_1,this has to be thickened with this has to be thickened with structuring element B_2 and we can continue this way.Finally, thickening with structuring element B^n and this completes our thickening operation.

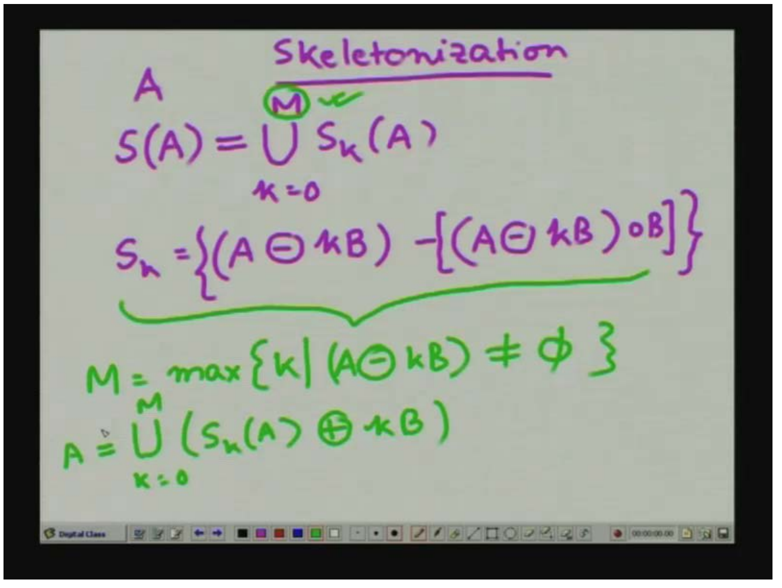
So,because thickening is adualof thinning,one way of implementing thickening operation isthat for the given set A, you first compute the A complement.Then what youcan do is you just do the thinning operation of A complement with the set of structuring elements.After performing this thinning operation, if you take the complement of the thinned a complement; what we get is the thickening version of the given point set A.

So, given a point set A, the first operation that we have to perform is you take A complement.Then what you do is I make a set say C which is nothing but A complement thinned with the set of structuring elements B and finally if I take the complement of this set C, then what I get is the thinned the thickening version of this set A with the set of structuring elements B which is nothing but the complement of the point set C.So, you find that this thickening operation can be implemented by thinning A complement.

Now,there may be one problem that while doing this operation, there may be some spurious points which will arrive in the thickened version of the point set A which can be removed by some post processing operation like opening closing and so on and finally, what we get is the thickened version of the point set A.

So, as we have done thinning, we have said that this thinning operation gives us some sort of skeleton of a 2 dimension shape.Now, there are morphological operations, the morphological technique which can also be used to find out the skeleton of our given shape.

(Refer Slide Time: 25:03)



Now, let us see how we can implement the skeleton of a given point set A by using the morphological operations.So, given a point set A,sonow what we will discussing about the skeletonization;so given a point set A, we can find out the skeleton of A which let us represent as S of A. Thiscan be obtained by this operation - S $_{k}$ (A),take the union for k equal to 0 to say capital M where this S $_{k}$ (A) defined as A dilation with k times k times dilation with B, the structuring element B,this minus again A dilation k times dilation with B and opening of this with set B.

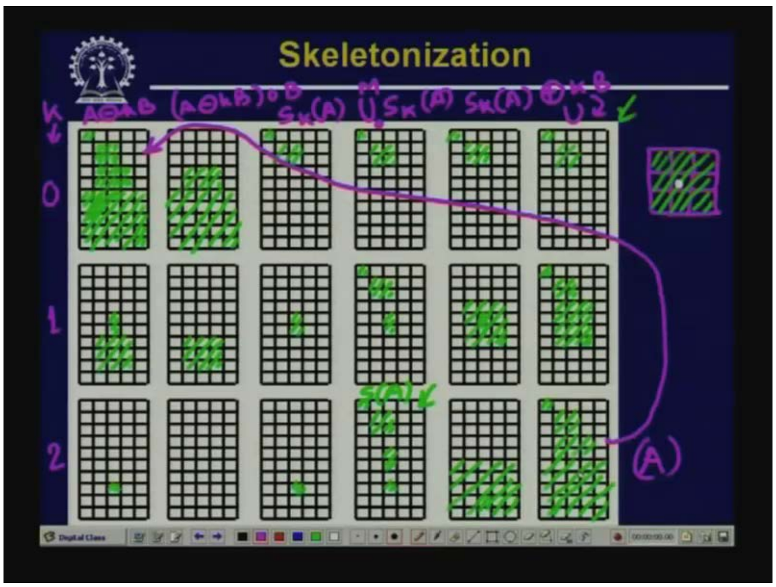
So, what this operation gives is this particular operation gives us a number of sub skeletons and the union of sub skeletons give us the skeleton of the final ofthe given set A which we represent by S of A. Now this capital M, in this particular case indicates that the last iterative step,as you find that here what we are doing is when I do this A dilation with kB; so this indicates that A is dilated with the structuring element B for successive k number of times and this capital M indicates the last iterative step before A erodes to an empty set.So, we can defined M as M is nothing but maximum of k such that A eroded with kB, this is not equal to a null set.

So, this M indicates the maximum number of iteration before our given set A erodes to a null set. Now, as we have found that the skeleton of a given set A, given point set A with respect to structuring element B, can be found out by repeated application by successive application of erosion with respect to the given structuring element B and an opening operation with the same structuring element B.

Similarly, given the skeleton, we can also re reconstruct our original point set A.So, this can be done in this fashion.So, given a point set A or the skeleton A, when we have all the sub skeletons S_k (A); what we can do is we can erode this S_k (A) with the structuring element successively k number of times where S_k (A) is a sub skeleton andtake the union of this for k equal to 0 to capital M and this is what is our original point set A.

So, let us illustrate this skeletonization and finally getting back our original point set A from the sub skeletons with the help of an example.So, the example is like this; for this skeletonization, the structuring element B we will consider is a 3 by 3 structuring element like this.So, this is our structuring element which we can use for skeletonization purpose and in this case, the origin of the structuring element is the center element.

(Refer Slide Time: 29:47)



Andnow, let us take an image like this;so this is the given image, now what we do is on this side, let us put the iteration number k,soon this side,we will put the iteration number k. So, this is k equal to 0, k is equal to 1and k is equal to 2 and on this side let us put the successive skeletonization operation.So,I put A erosion with A times B.This column, it represents A erosion with A times B, then opening with B.This column, suppose it represents S_k (A) that is the sub skeleton.This column, it represents the skeleton S_k (A) where we take the union of the sub skeletons S_k (A) where k varies from 0to capital N.

Thiscolumn let us assume that it represents S_k (A) dilated with kB and we will see what does this column represent is the union of this.So, if I do it like this, if I erode this given point set A with respect to our 3 by 3 structuring element; then after eroding it for the first time, the kind of output that I get is like this.So, this is the output that I will get after eroding.This given point set A with our 3 by 3 structuring element once.And, if I erode it for the second time, then the output that we will get is this.

Onthis column where if I perform the opening operation of this successively eroded images, then what I get over here is like this; in this particular case, what I will get is this and here this will be a null set.I will not get any point in this particular case that is after eroding for 2 subsequent operations, eroding it twice and then doing the opening with this structuring element B.

Now, from here, what I get is I get the intermediate or sub skeletons as we have defined the sub skeleton as a difference operations, the difference of A eroded by A eroded with k times B minus A eroded K times B open with B. So, if I take the difference of the first column and the second column, then what I get is our sub skeletons.So, in this case, the sub skeleton will be like this.So, if you find that this is nothing but the difference of the element in the first column and the element in the second column.

Similarly, in the second case, the sub skeleton will be like this and in the third case, the sub skeleton will be like this.Now, if you take the union of all these sub skeletons, as you have defined that the final skeleton is the union of all the sub skeletons; so the subfinal skeleton will have this particular shape.So, this our sorry here we will get it like this.

So, you find that this is the element which is actually our skeleton SA and here you find that when I get this skeleton, this skeleton is not really connected.So, what we get is a disconnected point set and that is not unnatural because when we have performed the morphological operations, nowhere we have guaranteed the connectivity.So, it is quite possible that given a set if I try to find out the skeleton of that point set by using morphological operations, then it may lead to a skeleton which is unconnected.

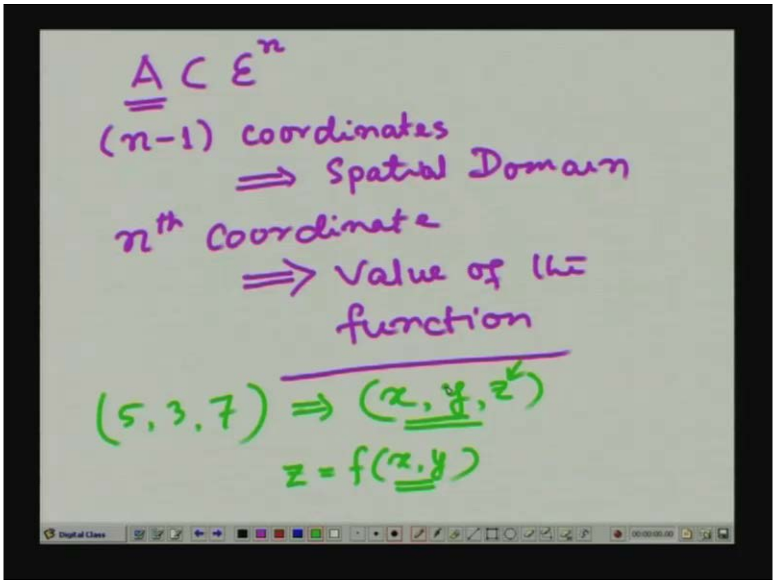
Now, let us say that given this sub skeletons whether it is possible to find out the original set A by using the reverse process that is while doing the skeletons, what we have used is the successive erosion.Now, in the reverse process, what we have used is the successive dilation.So, after defining those dilation operations as we have defined earlier, now we can try to the original point set A from these sub skeletons.

Sohere, you find that in the fifth column which is nothing but S_k (A) that is sub skeleton A eroded successively K times with the structuring element B.So, if I do this; in that case, this particular column will give me this output and finally in the last column which is nothing but union of all these operations, so if I take the union of all of them, what I get is this.So, this is what I get.So, if I compare now, this output with our original point set A, you find that this is nothing but the point set A which was given.

So, by this type of morphological operations, it is possible to obtain the skeleton of a given shape and once I have this sub skeletons while doing this skeletonization operation, then from this soft skeletons, it is also possible to obtain the original point set A by applying the inverse operation.So, so far what we have discussed, all the morphological operations that we have discussed, they are meant for the binary images.

Now, let us see whether we can extend this binary morphology,these morphological operations to gray level images as well.Obviously, our assumption will be that as we have started that every image, we should be able to represent as a point set if we want to apply the morphological operations on the image.So, here also the gray level image we should be able to represent as a point set or set of points.

(Refer Slide Time: 40:00)



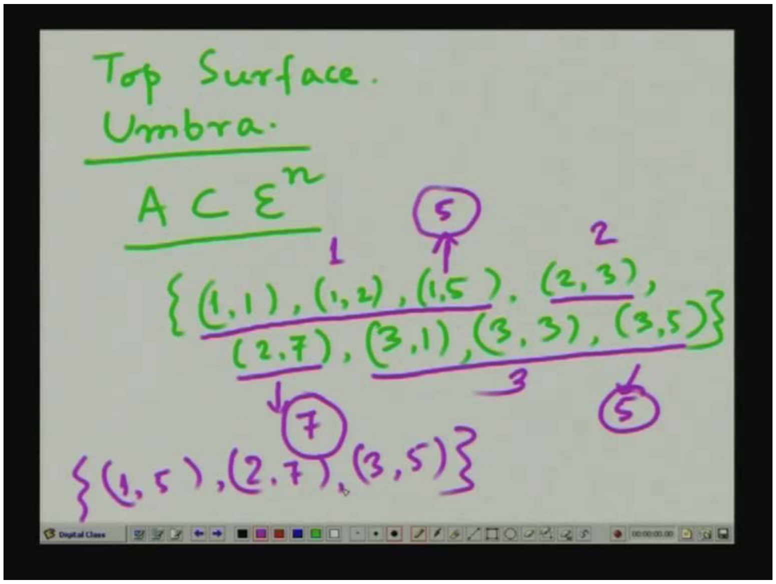
So, let us assume that we are given a point set A in say n dimensional equilibrium space. So, this A represents a point set A in n dimensional equilibrium space. Now,out of for this point set A; the first n minus 1 components, the first n minus 1 coordinates, this represents a spatial domain and the n'th coordinate, this represents the value of the function.So, this is our basic interpretation. So, what we are doing is we are taking a set A, a point set A in n dimensional equilibrium space.So, if I take a point belonging to that set A, then first n minus 1 coordinates of that particular point, this represents its location in the spatial domain and the n'th coordinate that is the last coordinate, this represents what is the value of the function at that particular location in space.

Say for example, if I take a 3 dimensional point say (5,3,7)or in general if I assume a coordinate systems say (x, y, z). Now, what does it mean?This means that I have a 2 dimensional phase which is given by (x, y) and z is the value at that location.So, I can represent this as z is equal to sum function f (x, y). So, find that this first 2 coordinates x and y, this represents a spatial domain and the last coordinate that is the third coordinate, this represents what is the value at that particular location.

Sosimilarly, for a gray level image; a gray level image, we will represent as a tripletlike this say (5,3,7) where this (5,3) this represents a location in a 2 dimensional phase and the last component 7 represents what is the value in that particular locations.So, when I have a gray level image, this value represents what is the intensity value or the gray level value at location (5,3) in the inverse.

Now, you find that if I have this sort of interpretation of a gray level image,I get some advantage.Now, what is the advantage?The advantage is that now the gray level image, typically represents a topological sketch and the intensity value I can assume that it represents the height within that topological sketch.

(Refer Slide Time: 43:38)

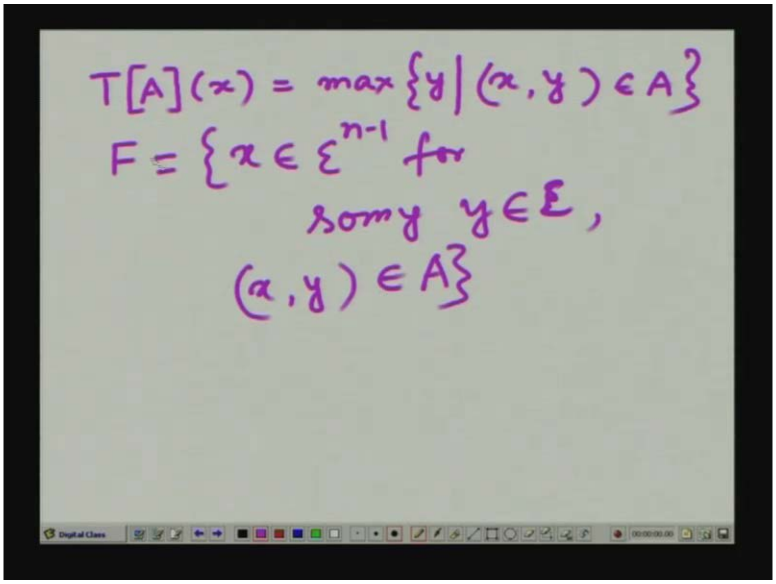


So, given this, when I have this sort of interpretation; now to extend our gray levelextend our morphological operations through gray level images, we try to define 2 different terms.One of the terms we will define is the top surface and the second term that we will define is what is called an umbra.Now, what is this top surface?Wehave said that we are assuming a point set A which is in n dimensional space, n dimensional equilibrium space and we have said that the first n minus first components that represents spatial domain and the last component that is the n'th component represents a value of the function at a point in the in space.

Now, over here, if I take like this that suppose I have,if we considerin 2 dimension,suppose I have a set of points something like this, say (1, 1)(1,2)(1,5),then say(2,3), then suppose (2,7) then say (3,1)(3,3) and say(3,5).Suppose these are the set of points which are given.Now, if we analyze this point set,I find that for these points, the first component is same which is equal to 1.For these points, the first components is again same which is equal to 2 and for these points, the first components is again same which is equal to 3.

So, as we have said that the first component represents a point in space.So, coming to our n dimensional space, the first n minus first components represent the space.So, what we do is for each n minus 1, we try to find out what is the maximum value of the n'th component in set A.As in this case, for all these components, the maximum value is 5.For this case, the maximum value is 7, whereas for this case, the maximum value is again 5.So, the top surface consists of these values 5,7 and 5.So, the top surface consists of the points (1,5)(2,7) and (3,5).So, these are the points which make the top surface.

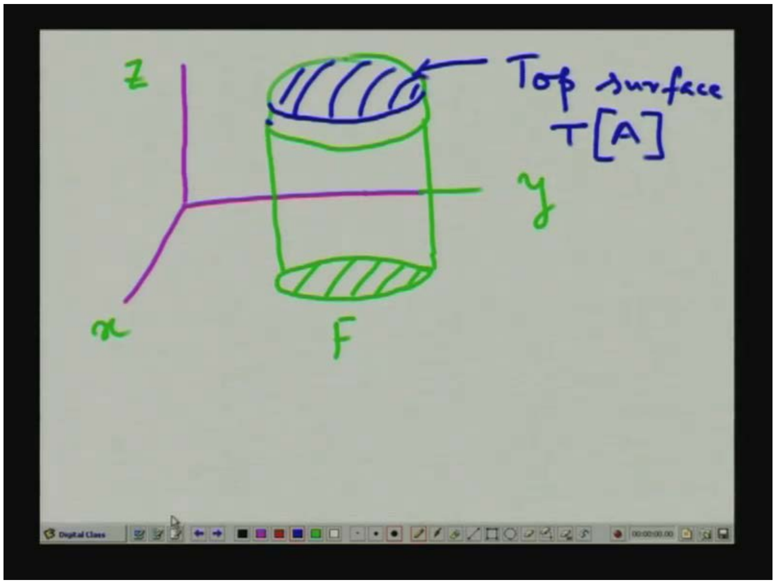
(Refer Slide Time: 47:09)



So, coming to our formal definition, what I have is for each of n minus for each n minus 1,I try to find out what is the maximum value of the n'th component in set S in the given set A and that maximum value forms the top surface.Soformally, we can define the top surface as like this that given a set A the top surface T [A] at location x where x is our n minus 1 dimensional ((dual)) this will be given by maximum of y where (x, y) belongs to set A.

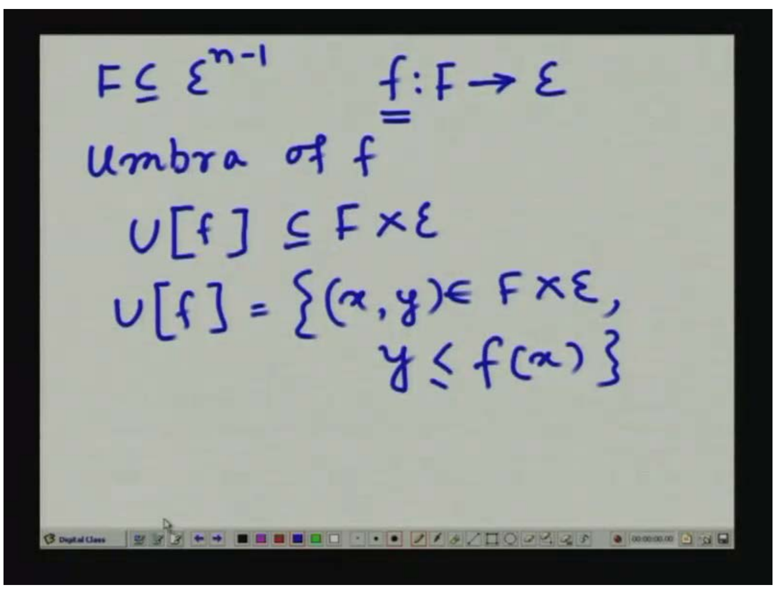
So, we find that this(x, y) is an n dimensional an n dual x is n minus 1 ((dual)) and we try to find out the maximum y for each n minus 1 ((dual)) which is the top surface and for these cases, we also define the region of support which we define as F.This region of support is defined like this; it is x belonging to n minus1 dimensional space for some y belonging to 1 dimensional equilibrium space such that (x, y)belongs to set A. So, we find that we have the region of support, we have the top surface T [A] anddiagramicwith the help of a diagram, we can represent like this.

(Refer Slide Time: 49:04)



Suppose, we have a set of points given like this;so suppose this is our x dimension, this is y dimension, this is z dimension,so the projection of these points on the (x, y)dimension, these gives us the region of support F and if I take the maximum of the values, the maximum of z values at each point in F, this is what give us the top surface, so T of A.

(Refer Slide Time: 49:58)

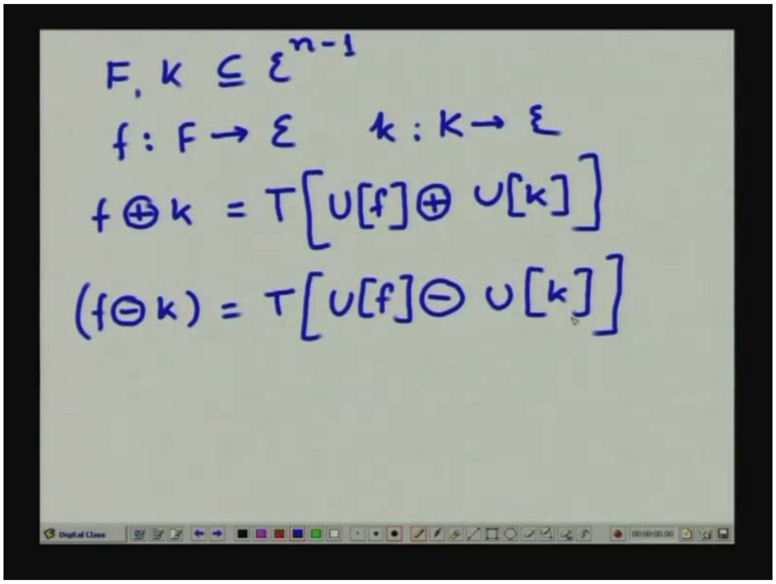


Now, we define the umbra.So, given a region of support say F which is a subset of n minus 1 dimensional equilibrium space;soone more thing here we find that this top surface, it is something like a mapping function.Wecan represent these as amapping function which maps this region of support F to a 1 dimensional equilibrium space.

Now, given this top surface which as we have said that it is the mapping function, we can define umbra of f asU [f] which is obviously a sub set of an n dimensional equilibrium space.So, F is an n minus 1 dimensional equilibrium space if I take the Cartesian product with 1 dimensional equilibrium space, what I get is n dimensional equilibrium space.

Sonow, this U [f],theumbra of the top surface, this is defined as the set (x, y) belonging to obviously F Cartesian product with the 1 dimensional equilibrium space where this y is less than or equal to f (x). So, what does it mean?This expression means that umbra consists of all the points including the top surface, all the points below the top surface, including the top surface itself. So, the top surface gives you the maximum value at a particular location in the region of support in the special domain and umbra is everything below the top surface including the top surface.

(Refer Slide Time: 52:06)



So, after giving these definitions, let us see that how the dilation and erosion operation can be defined in case of gray level images.So, as we have said that a gray level image is a subset of a 3 dimensional space, so given a gray level image; so we define it like this, say suppose we have given 2 region of support,one is F and other one is K. Theseregions of supports are obviously in dimension n minus 1dimension equilibrium space.Incase of gray level image, this will be a 2 dimensional equilibrium space.Wehave 2 top surfaces F which maps the region of supports into 1 dimensional equilibrium space and k which maps the region of support K again to 1 dimensional equilibrium space; then we can define the dilation of f and k as the top surface of the dilation of umbra of f with the umbra of k.

So, what we have to do is given a gray level image, we have to find out the umbra of it. Similarly, given a structuring element, we have to find out the umbra of the structuring element.Then we have to dilate the umbra of the image with the umbra of the structuring element.Then we have to find out the top surface of this dilated output.

So, you take the umbra of the gray level image, you take the umbra of the structuring element; dilate these 2 umbras, then the top surface of this dilations tells you what is the dilation of the given gray level image with respect to a given structuring element and in the same manner, the erosion in case of gray levelimage, this can also be defined in terms of the top surface of erosion of umbra of f with the umbra of k.

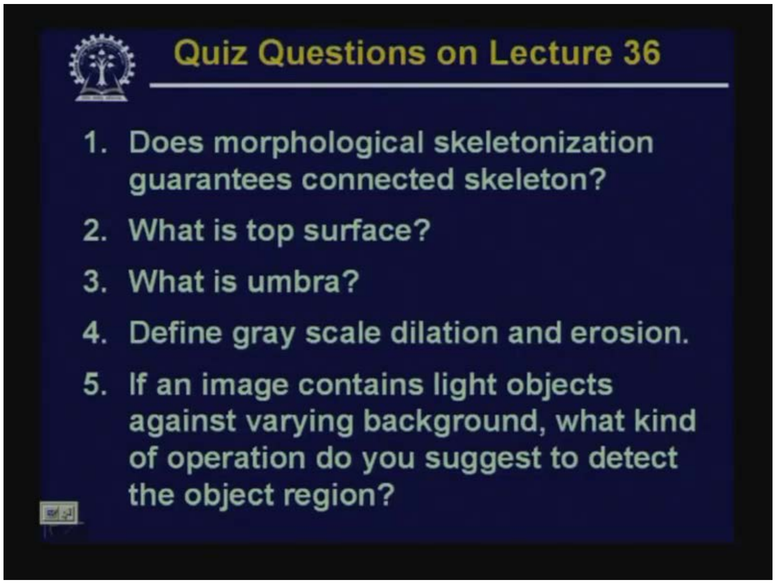
So, you find that extension of the morphological operations from binary image to gray level images is quite simple.What we do is we convert the images into their umbras.Similarly, you convert the structuring element into its umbra, then take the erosion or dilation of the corresponding umbras and the top surfaces of this erosion or dilation gives the corresponding erosion or dilation of the gray level image with respect to the corresponding with respect to the given structuring element.

So with this, we stop our discussion on erosion and dilation operations,discussion on morphological operations.So, in this discussion, what we have done is first we have defined the basic morphological operations like erosion and dilation and we have seen that these basic morphological operations are implemented by different set operation.Ourbasic assumption is any form of image on which this morphological operations are to be applied, they are to be represented as point sets and then all subsequent applications of this morphological operations in case of image whether it is for a binary image or for gray level image, we have seen that they are nothing but applying the basic erosion and dilation operations along with few set operations in different order.

Andthe net result of applying this morphological operations is that given any point set or given an image, we can regularizes an object present in the image.Byregularization,what I mean is in the object region if there are some noises, we can remove those noises from by morphological operations; in the background region if there are some noises, we can remove those noises by morphological operations, if 2 object regions are connected to each other because of some noises,we can remove that.

So, these are various regularization operations that we can perform through morphological operations.Thebasic aim is of course mathematical morphology is very wide but we are concentratingon a very very narrow application of it inour image processing applications and the basic purpose of doing this regularization is we can have a better description of object regions or the different foregroundregions.Now, let us see some questions on today’s lectures.

(Refer Slide Time: 57:31)



So, the first question is does morphological skeletonization guarantees connected skeleton?Secondquestion, what is top surface?Thethird question, what is umbra?The fourth question, define gray scale dilation and erosion operations.Thefifth, if an image contains light objects against varying background; what kind of operation do you suggest to detect the object region?

Thankyou.