

# Digital Image Processing

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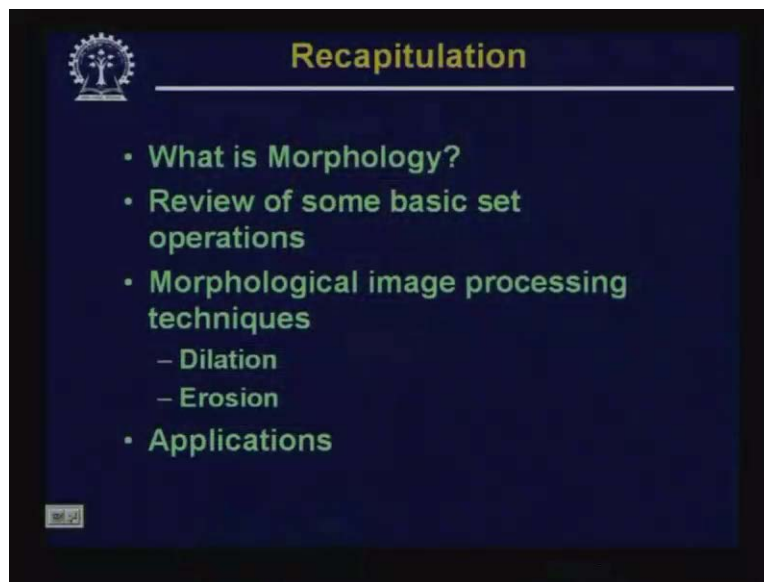
Indian Institute of Technology, Kharagpur

Lecture - 34

## Mathematical Morphology – II

Hello, welcome to the video lecture series on digital image processing. Since our last lecture, we have started discussion on mathematical morphology and the application of mathematical morphology in digital image processing.

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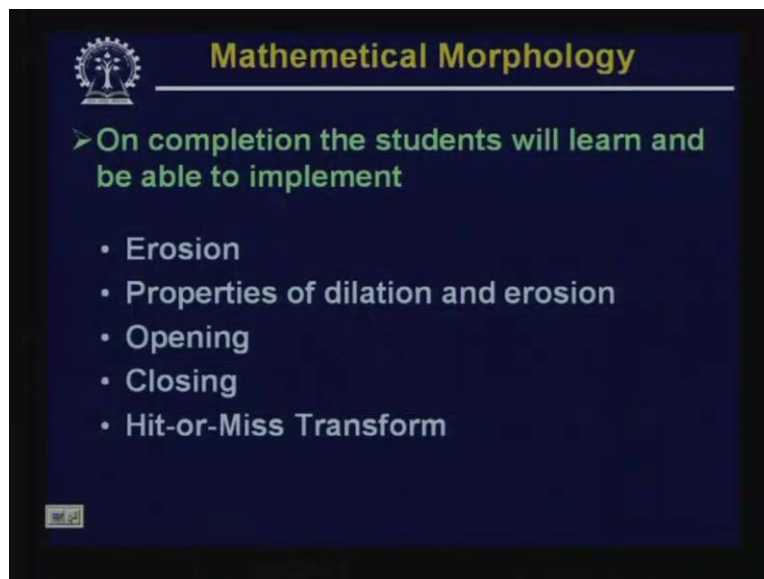
So in the last class, what we have seen is we have said what is morphology and we have said that morphology is basically a topic which was used extensively in the field of biology to talk about the structure or shape of animals and plants and in our image processing, we use mathematical morphology as a tool for doing various structure or shape related operations and we have seen that to use this morphological operations or image morphology, we must represent an image in the form of a point set and all the morphological operations are nothing but different set operations and the kind of set operations that we have to use depend upon what kind of morphological transformation we want to do and a morphological transformation of a given image  $x$  describes a relation of the image  $x$  with another small point set say  $b$  but this small point set is what we call as structuring element. So, any mathematical morphological operations or image morphological operation is always defined with respect to a given structuring element.

Then, in our last class, we have discussed about what is dilation. We have seen how the dilation can be implemented and we have seen how this dilation operation can be used to remove some noisy points within the object region within an image. Then, we had defined another operation which is called erosion and in today's lecture, we are going to discuss elaborately what is erosion.

So, in today's lecture, we will have an elaborate discussion on the morphological operation called erosion. Then, we will see what are the properties of dilation and erosion operations and when we have discussed about dilation in our previous class, we have said that as the dilation operation removes noisy pixels **within** which are present within an object region; at the same time, the simple dilation operation also tries to expand the object region. That is after performing dilation operation; the object area gets expanded whereas we will see today that erosion operation is just an inverse operation where because of erosion operation, the object area gets reduced along its boundary.

So, in image processing operations, we use combination of this dilation and erosion operations. In one case, the dilation is done first followed by erosion and in the other case, the erosion is done first followed by the dilation operation and accordingly we have two different operations; one is called a morphological opening operation and the other one is called a morphological closing operations.

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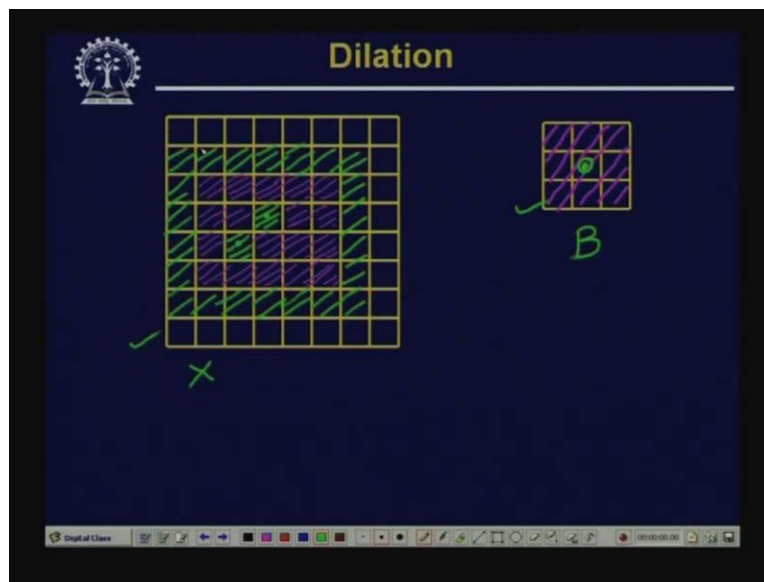


We will see with examples, what are the effects of this morphological opening and morphological closing operations on a binary image. Now, I talk about binary image because as we said that initially we will discuss about application of these morphological operations on binary images and later on we will try to expand the definition or the use of morphological operations to grey level images as well. But again in that case, our assumption will be that given a grey level image can be represented in the form of point set. Then, we will come to another application of this

morphological operation which is called as hit or miss transformation and this hit or miss transform is actually aimed to detect an object of a specific shape and size in a given image.

So, if we know what is the shape of the object and what is the size of the object that we are trying to locate within a given image, then we can make use of this morphological hit or miss transform and we will see that this hit or miss transform is nothing but basic morphological operations that is erosion and dilation operations in combination with other set operations which we have reviewed in our previous lecture.

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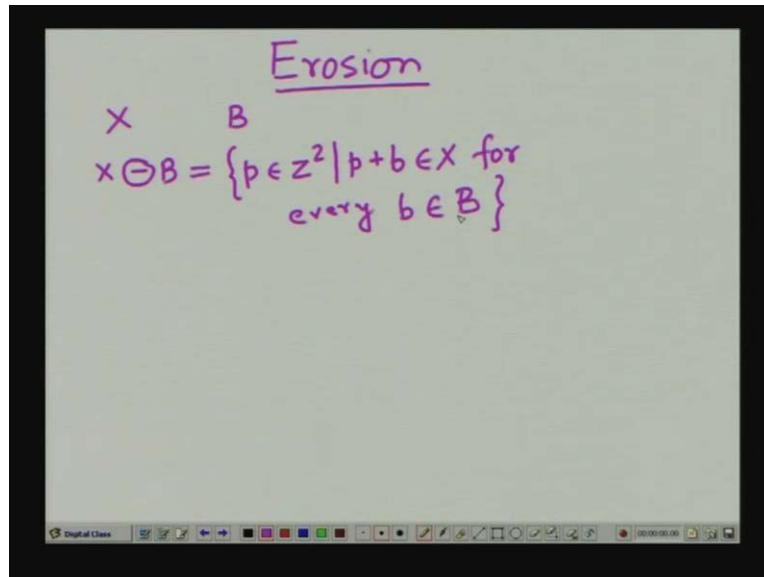
So, just to have a recapitulation, let us see what we have done in case of dilation operation. So, for dilation operation, suppose we have an object of this form; so suppose, this shaded region represents our object region and the black pixels in this figure represent the background region, so you find that here in this case, these pixels within the object region which appears to be black, these we assume that these pixels are because of the presence of noise. This should actually belong to object and along with this, we take a structuring element a 3 by 3 structuring element like this where the center of the structuring element, we take the origin at the center of the structuring element.

So, this is our given image  $x$  and this is our structuring element  $B$ . So, if we dilate this given image  $x$  with this structuring element  $B$ , then as we have said in the last class that after dilation, these black pixels inside the object region will turn out to be object pixels and at the same time, the pixels along the boundary of the object region, they will also turn out to be white indicating that these are also converted to object pixels. So, these are the pixels which are actually converted to the object pixels.

So, as we have said that when we do this dilation operation, the dilation operation removes the noisy pixels inside the foreground region or inside the object region but at the same time, this dilation operation expands the area of the object region along the boundary. So, this is aside

effect of the dilation operation. Then we have defined another kind of operation which we have said as erosion.

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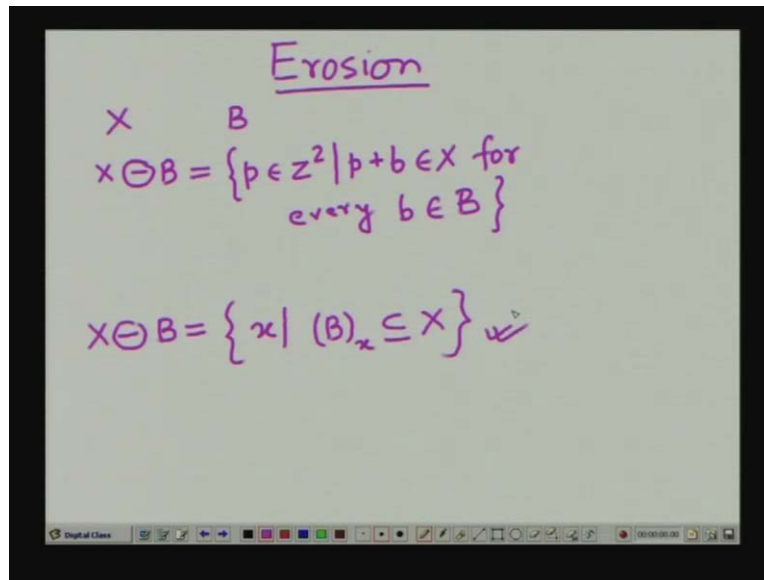
The image shows a whiteboard with the word "Erosion" written in purple at the top. Below it, the variables "X" and "B" are written. The main equation is 
$$X \ominus B = \{p \in Z^2 \mid p + b \in X \text{ for every } b \in B\}$$
 The whiteboard also features a toolbar at the bottom with various drawing tools and a timestamp of 00:00:00:00.

So, let us see how we have defined our erosion operation. We have defined erosion as, so now we are going to discuss about erosion. So again, we assume that we have given an image  $x$ , obviously  $x$  is in the form of point set and along with image  $x$  say we are also given with a structuring element say capital  $B$  and in the last class, we have just mentioned that erosion operation image,  $x$  eroded with structuring element is represented by this symbol, it is a circle and within the circle you have a negative sign.

So,  $x$  eroded with  $B$ , this can be defined as the set of points of  $p$  belonging to the two dimensional space say  $Z$  square such that  $p$  plus  $b$  belongs to the set  $x$  for every  $b$  belonging to the structuring element capital  $B$ . So, this is the first definition of the erosion operation that we have said. This is the set of points  $p$  in the two dimensional space  $Z$  square such that  $p$  plus  $b$  where  $b$  is an element in the structuring element capital  $B$ . So, this  $p$  plus  $B$  must belong to our image capital  $X$ .

We have also said that there is an alternative definition of the erosion operation. As we have seen that for dilation, we have two definitions but both the definitions lead to identical results meaning that the definitions are equivalent; same way, in case of erosion, we can have two definition. So, the alternate definition for the erosion operation is given like this.

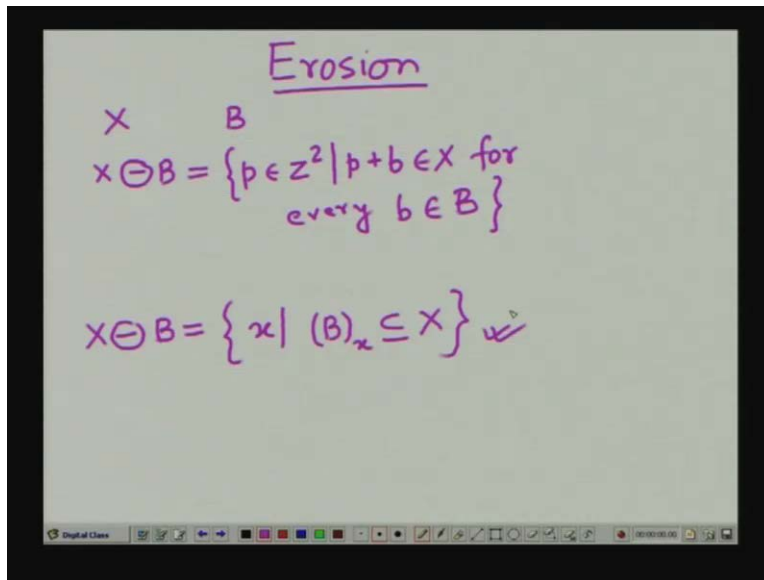
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Say,  $X$  eroded with  $B$ , the structuring element  $B$  can be defined as the set of points or the set of vectors  $X$ ; as we said that in two dimensional space a point and a vector is equivalent where the vector is drawn from the origin of the coordinate system, so it is the set of vectors or set of points  $x$  such that the structuring element  $B$  translated by vector  $x$  is a subset of our image capital  $X$ . So, this second definition is very very interesting that it simply says as we have told in our previous lecture that the way we compute the morphological transformation is similar to the way we compute convolution.

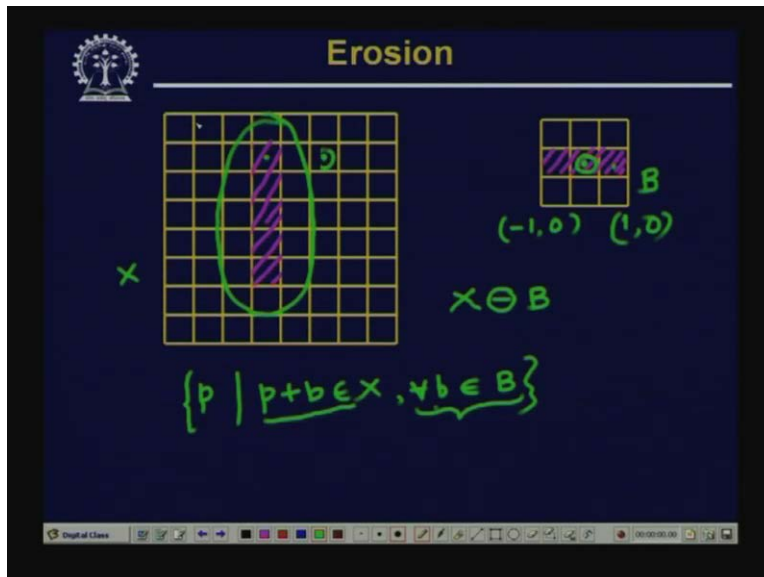
That is you try to translate you translate or shift the structuring element over the image in a systematic fashion and this one says that for those shifts of the structuring element or of those translations of the structuring element in the image such that the shifted or the translated structuring element is fully contained within the point set capital  $X$ , then those translations or those points by which the structuring element has been translated are part of erosion.

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So, that is what is meant by this particular definition that it is the set of all the translations such that the structuring element translated by that particular translation vector will be a subset of our given image  $X$ . Now, let us see what does this actually mean.

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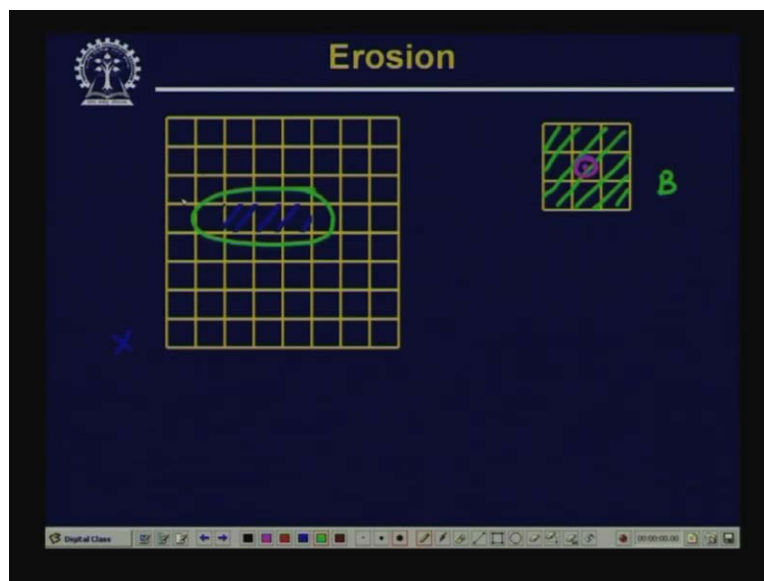
So now, let us take another binary image, say I have a binary image like this. So, these are the points in our binary image and I take a structuring element  $B$  which is given by these points and I assume that the local origin is at the center of the structuring element. So, this is the origin of the structuring element marked in green colour and what I want to find out is this is our given image  $X$  and this is the structuring element  $B$ , I want to find out the erosion of  $X$  with the structuring capital  $B$ .

Now, if I follow our first definition that says that this is the set of all the points  $p$  or all the vectors  $p$  such that  $p$  plus  $b$  belong to  $x$  where  $b$  is an element in the structuring element capital  $B$ ; so, following this definition, you find that if I consider this particular point and I take a particular vector say this vector which is say  $(-1, 0)$  from this structuring element and I translate or and I add this particular value of  $B$  to this particular point, then after addition you find that this  $p$  plus  $b$  gets translated to this particular point. So, this point is **so this point** should be a part of the erosion.

But if I consider this particular point in our structuring element  $B$ , the point in this case is  $(1, 0)$  and I consider this  $b$ , now you find that this  $p$  plus  $b$ , in this case is  $(1, 0)$ ; so  $p$  plus  $b$ , the point get shifted to this particular location and this point is not a member of  $x$ . So, infer that because our definition was that this condition should be true for all  $b$ , this is for all  $b$  belonging to the structuring element capital  $B$ . So though this condition is true for  $b$  equal to  $(-1, 0)$  but this condition is not true for  $B$  equal to  $(1, 0)$ .

So, from this we can infer that this point will not be a member of the erosion and if you continue the same operation for all other points in the given image  $x$ , you will find that these are the points which will not belong to the erosion. Similarly, these points will also not belong to the erosion. So, the given  $x$ , the given image  $x$  when eroded with this particular point, this particular structuring element capital  $B$ ; the eroded output will be given only by these points and the other points will be removed from the eroded output. So, this is what we get in erosion.

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Now, let us see how this erosion can be applied in the image processing operation. Again, let us take a particular image of this form say I have an image like this. Then say, I have few pixels over here and I have some distributed pixels like this. So, this is our given image capital  $X$  and I assume that my structuring element is a 3 by 3 structuring element capital  $B$  and the center of the structuring element is taken as the origin.

Now, if I erode this image with this particular structuring element; in that case, you will find that all these points following the same operation that we have just done now, you will find that this point will be removed from the eroded image, this point will also be removed from the eroded image, all these points will be removed from the eroded image, all these points will be removed from the eroded image, these points will be removed from the eroded image and these points will also be removed from the eroded image. So, your eroded image output will simply be only these 3 pixels.

So, what we have done in this case is we have taken a binary image where there was an extrusion in the binary image, just one pixel white and there were few pixels distributed over the image which actually should have been in the background. But because of noise, they have been converted to object pixels. So, if I erode such a kind of image with a 3 by 3 structuring element and we have assumed that the origin of the structuring element is the center, element is the center pixel; in that case, all those distributed points as well as the extrusion that gets eliminated but at the same time, we have a side effect. That is all the boundary pixels of the object region, they also get eliminated and because of this the area of the object region gets reduced as if the object has been shrunked.

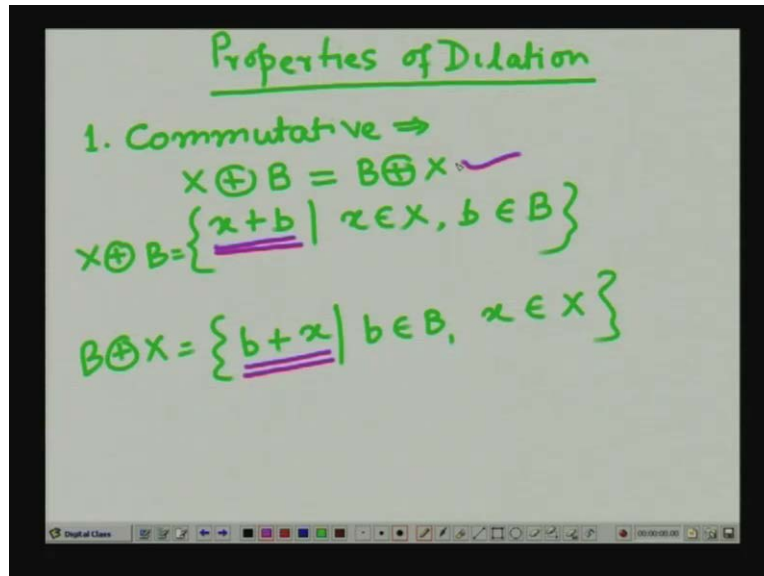
So, we find that this is just an opposite operation, opposite to what we have got in case of dilation. In case of dilation, the object region gets expanded whereas in case of erosion, the object region is reduced. So, these two are two opposite operations, opposite effects. So naturally, if I want to apply these morphological operations in image processing, I would like to retain the size of the object region. So, if I apply the dilation operation at any point, then that has to be compensated, the expansion of the object region has to be compensated by subsequent application of the erosion operation.

Similarly, if I apply erosion operation first, then since the object region gets reduced to compensate for this, this operation has to be followed by the corresponding dilation operation and this erosion and dilation has to be done with the same structuring element. So, we will see the combination of these two operations when we talk about the opening morphological opening and morphological closing operations.

Now, before we come to the morphological opening and morphological closing operations, let us see some of the properties of dilation and erosion operations.

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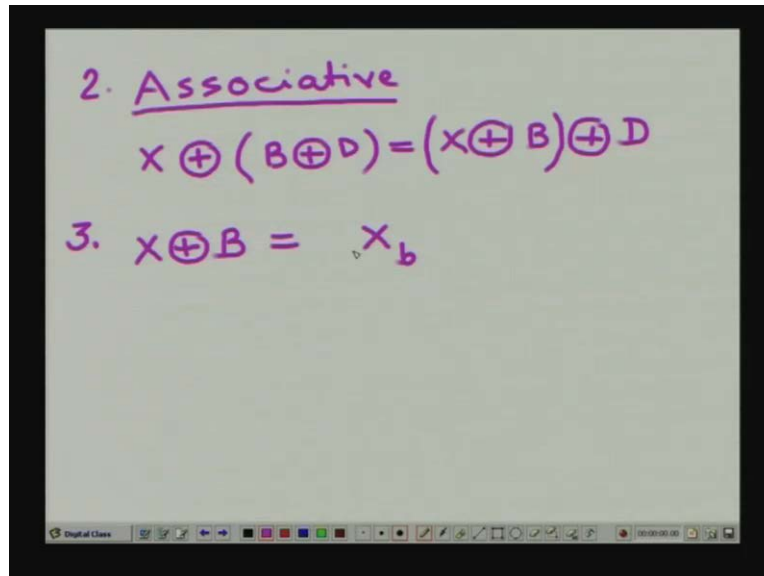


So first, let us see some properties of dilation. So, we will talk about say some properties of dilation first and then we will see some properties of erosion. The first property of the dilation operation is the dilation operation is commutative. So, what do you mean by saying that the dilation operation is commutative? So, this means that given an image say capital X and a structuring element say capital B; then if I dilate the given image capital X with the structuring element capital B, then the result that I get will be same as if I dilate the structuring element capital B with the given image capital X. So essentially, X dilated with B is same as B dilated with X.

Now, proof of this is quite trivial because as we have said that we can define this morphological dilation operation by vector addition; so this X dilation with B is nothing but all the resultant vectors when we compute x plus p where x belongs to our image capital X and b belongs to our structuring element capital B. So, this what is X dilated with B.

Now, if I take the other one that is B dilated with X; this is nothing but b, the vector b plus vector x again for b belonging to the structuring element capital B and x belonging to the given image capital X. Now obviously, this x plus b; thus this resultant vector is same as b plus x resultant vector. So, that clearly shows that the dilation operation is commutative that is X dilated with B is same as B dilated with X.

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The second property of the dilation operation is the dilation operation is associative. What do you mean by that? So, this associative property says that if we have two structuring elements say we have a structuring element capital B and we have a structuring element capital D, then this image capital X dilated with capital B dilated with capital D. So, here we have said that we have two structuring elements; one structuring element is capital B, the other structuring element is capital D.

So, the operation that we are doing in this case is first we are dilating capital B with the second structuring element capital D, so this dilation output gives me a point set and if I take this resultant point set as a structuring element and then we dilate our given image capital X with this resultant structuring element, then what should we get? So, this X dilated with B dilated with D, this is same as first you dilate X with the structuring element capital B and then you dilate this resultant with the second structuring element capital D.

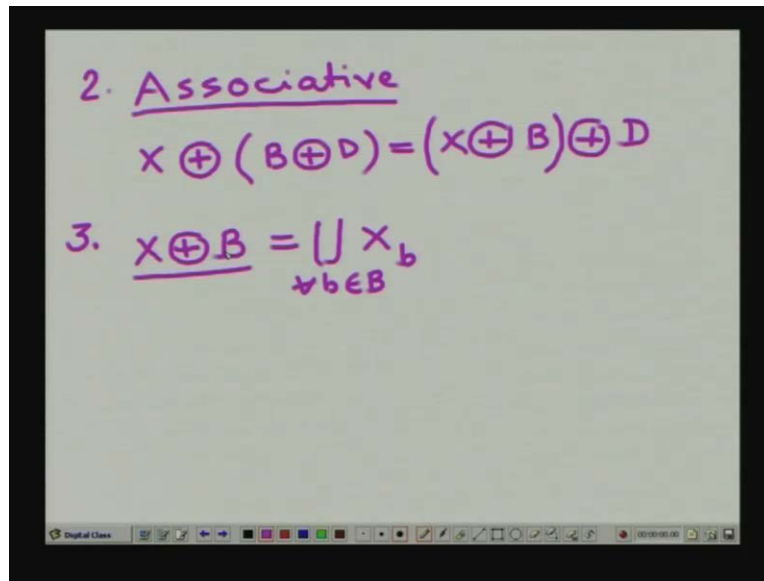
So, this is what is the associative property of dilation that is X dilated with B dilated with capital D is same as X dilated with capital B dilated with capital D where capital B and capital D are two structuring elements and X is our assumed image.

Now, the third property is I should not call it a property but this is an implementation, how we can implement the dilation operation more easily. So, from the definition of the dilation operation we have said that if I want to dilate a given image capital X with a structuring element capital B, then what I do is I take an element a vector from capital X, take a vector from capital B; just do vector addition of these two vectors and the resultant vector will belong to the dilation output and this I have to do for every element, for every vector in the image capital X which is to be added with every vector in a structuring element capital B.

Now, addition of a vector to a point is nothing but translation of the point by that vector. So, if I add a vector; a vector b from the structuring element capital B to all the points in x that is equivalent to or that is same as translating x by that vector b in structuring element capital B. So, this simple definition follows that I can simply interpret the dilation operation in another way.

So, I can interpret it like this that X dilation with B is nothing but given X is translated by a vector b, so this b is a vector in our structuring element capital B and I have to consider all the vectors present in the structuring element capital B; so for all the vectors, I will get for every vector in capital B, I will get one translated point set X b. So, what I will do is I will take the union of all these translated point sets.

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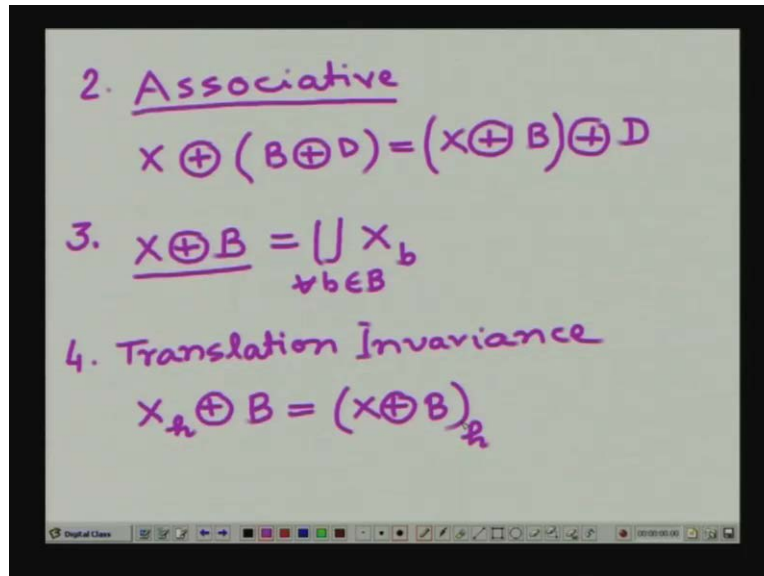


The image shows a whiteboard with two mathematical properties of dilation. The first property is labeled '2. Associative' and is written as  $X \oplus (B \oplus D) = (X \oplus B) \oplus D$ . The second property is labeled '3.' and is written as  $X \oplus B = \bigcup_{b \in B} X_b$ . The whiteboard has a toolbar at the bottom with various drawing tools and a timer showing 00:00:00.

So, if I take the union of such  $x_b$  for all b belonging to our structuring element capital B, then what I get is this X dilated with the structuring element capital B and in fact, following this interpretation, this dilation operation can be implemented very easily. So, what I have to do is given a point set capital x, I translate this point set by every vector in our structuring element capital B and all these translated point sets, I take the union of all these translated pointsets and this union output is nothing but x dilation with x dilated with the structuring element capital B.

So, this is the third property. I will say that this is an interpretation and following this interpretation, our implementation of the dilation operation becomes very easy.

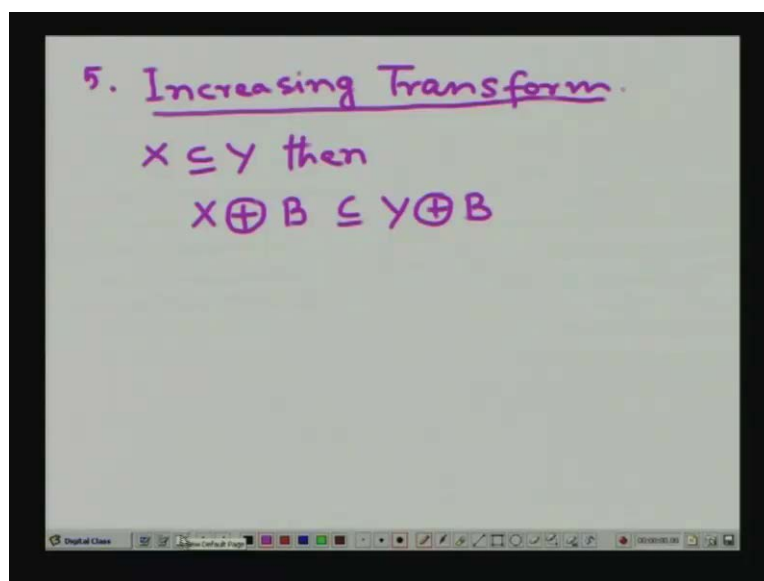
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Then, the fourth property of the dilation operation is translation invariance. What it says is that if the given image  $x$  is translated by a vector say  $h$  and this translated pointset is dilated with the structuring element capital  $B$ , this will be same as  $x$  dilated with capital  $B$  and this dilation output is translated with the same vector  $h$ .

So, whether I translate the given pointset  $x$  first and then dilate with the structuring element capital  $B$  or I first dilate  $x$  with capital  $B$  and then I translate by the same vector  $h$ ; the output remains the same. So, this is what is meant by the translation invariance.

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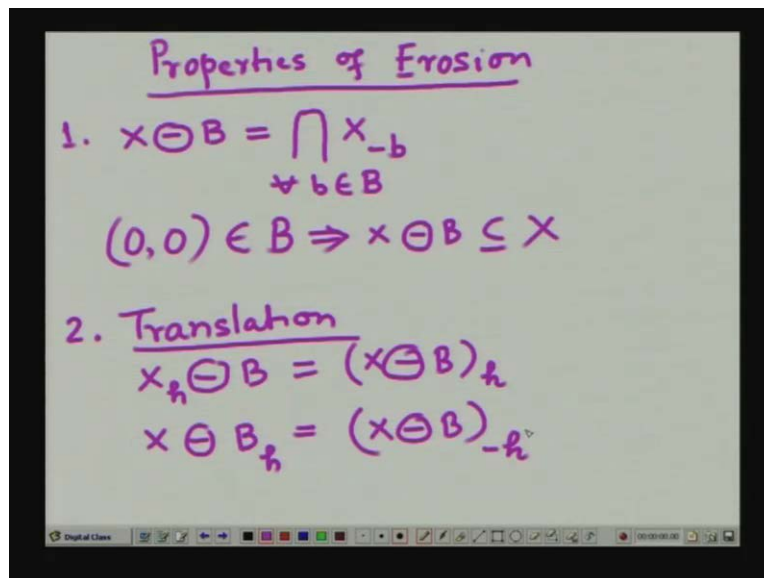


The next property of dilation, this is also very interesting which is called the dilation is an increasing transformation. It is an increasing transformation. So, what do we mean by this? This

increasing transform, by increasing transform what we mean is say if we are given two point sets; capital X and capital Y such that capital X the point set X is a subset of the point set capital Y, so if it is so, then if I dilate both this point sets capital X and capital Y by the same structuring element say capital B, then X dilated with capital B, X dilation B will be a subset of Y dilation B. So, this is what is meant by an increasing transform.

So, if one set is a subset of the other, then if both of them are dilated by the same structuring element, then the corresponding dilations, the subset relation; for the corresponding dilations, the subset relation will also hold. So, these are some of properties of the dilation operation.

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Now, let us see that what are the properties of the erosion operation and then we will see that where the dilation operation and the erosion operation, they differ. So now, we will see some properties of erosion. So here again, the first one we will see that we will say that it is an interpretation as we have seen in case of dilation that given a point set x and erosion of point set x with structuring element capital B, this can be interpreted as x translated by minus b and you take the intersection of all these translated point sets for every b belonging to the structuring element capital B.

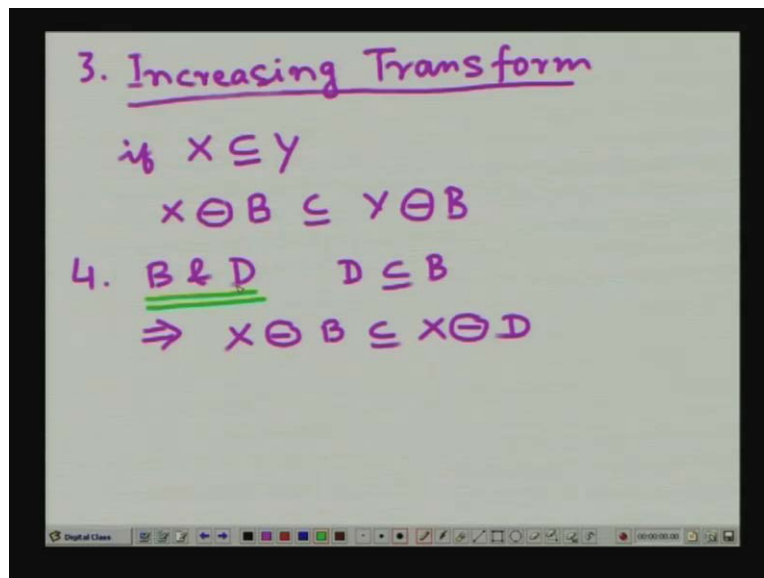
So, it means that I take every point or every vector belonging to the structuring element capital B; then negate it, translate x, the given image x by that negated vector. So, I get a translated point set x minus b and for all these translated point sets if I take the intersection of all these translated point sets, then the point set that I get that is nothing but my erosion output or x erosion B.

Now, following this interpretation, you find that if the origin that is (0, 0) is a member of the structuring element B; in such case, it is always true that x dilation B will always be a subset of X. Here again, the proof of this is very very trivial. You can take a very simple example and try to do this and you will always find that if the origin, local origin of the structuring element is the

member of the structuring element; in that case,  $x$  dilated with  $B$  whatever the dilation output that you get that will always be a subset of the original set, the original point set capital  $X$ .

The next property is translation. So, I will put this as one. So, as in case of dilation, we have translation invariance property in this case which says that  $x$  translated by vector  $h$  dilated with the point set  $B$  will be  $x$  dilated with structuring element  $B$  and this is translated by the same vector  $h$  whereas  $x$  dilated with  $B$  translated by the vector  $h$  will be  $x$  dilated with  $B$  which is translated by the vector minus  $h$ . So, these two are the translation properties of the erosion operation.

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As in case of dilation, the erosion is also an increasing transformation. So, the increasing transformation property in case of erosion, increasing transform; in case of erosion, the increasing transform property is something like this that again for two pointsets - capital  $X$  and capital  $Y$ ; if  $X$  is a subset of  $Y$ , then if I dilate this capital  $X$  and capital  $Y$  by the same structuring element capital  $B$ , then the relation will be that  $x$  dilated with capital  $B$  will also be a subset of  $y$  dilated with capital  $B$ .

So, you find that this is similar to what we have done; the property that we have seen in case of dilation. So, if  $x$  is a subset of  $y$ , then  $x$  dilated with  $B$  is a subset of  $y$  dilated with  $B$ . So similarly, in this case, if  $x$  is a subset of  $y$ ; then  $x$  erosion  $B$  will also be a subset of  $y$  erosion  $B$ . The next property is like this that if we have two structuring elements  $B$  and  $D$ , say these are the two structuring elements and structuring elements are such that  $D$  is a subset of  $B$ ; so if it is such and then a given pointset  $X$  is dilated with  $B$  and  $D$  separately, then  $X$  eroded with  $B$  and  $D$  separately, then  $X$  eroded with  $B$  will be a subset of  $X$  eroded with  $D$ .

So, you find that in this particular case, we have these two structuring elements  $B$  and  $D$  such that the structuring element  $D$  is a subset of structuring element  $B$ . So, in such case, if  $X$  is eroded with structuring element  $B$  which is a superset, this will be a subset of  $X$  erosion  $D$  where

D is a subset. So, following the properties of the erosion and dilation, you can find that the erosion and dilation, they are different in certain cases.

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The image shows a whiteboard with handwritten mathematical properties of dilation and erosion. The text is written in green ink. The first two lines are grouped by a right-facing curly brace, and the next two lines are grouped by a left-facing curly brace. At the bottom of the whiteboard, there is a software interface with a toolbar and a timestamp '00:00:00.00'.

$$\left. \begin{aligned} X \oplus B &= B \oplus X \\ X \ominus B &\neq B \ominus X \end{aligned} \right\}$$

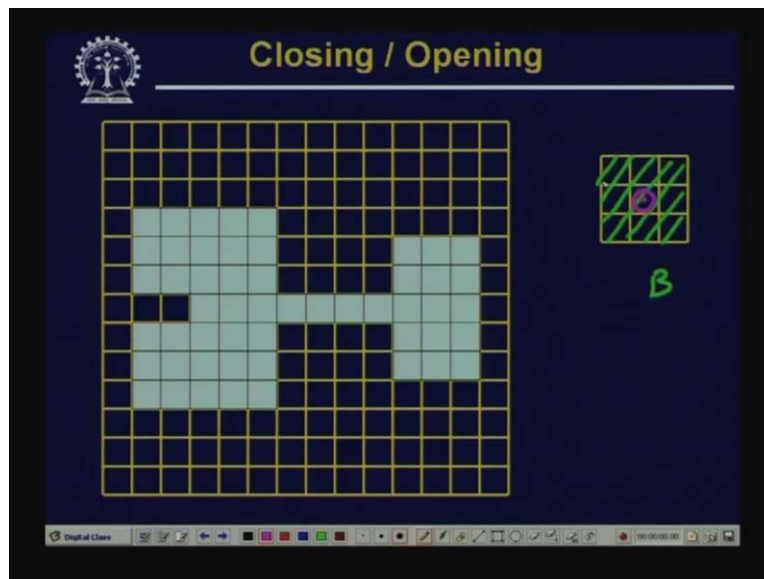
$$\left\{ \begin{aligned} X \oplus (B \oplus D) &= (X \oplus B) \oplus D \\ X \ominus (B \ominus D) &\neq (X \ominus B) \ominus D \end{aligned} \right.$$

In case of dilation, we have seen that  $X$  dilation  $B$  is same as  $B$  dilation  $X$  whereas in case of erosion,  $X$  erosion  $B$  is not same as  $B$  erosion  $X$ . So here, the properties of erosion and the property of dilation, they differ and the other one was the associated property that is in case of dilation, we have said that  $X$  dilation  $B$  dilation  $D$ , this is same as  $X$  dilation  $B$  dilation  $D$  in case of dilation. However, this is not so in case of erosion that is  $X$  erosion  $B$  erosion  $D$  is not same as  $X$  erosion  $B$  erosion  $D$ . So, these are the cases where the properties of erosion and the properties of dilation, they are different.

Now, given these properties, we have just said earlier that if I apply the dilation operation, then the area of the object region increases; if I apply the erosion operation, then the area of the object region decreases. However, both the dilation and erosion operations are useful to remove the noise present in the image. The dilation operation removes the noisy pixels belonging to the object region; at the same time, it reduces the object size; it increases the object size.

On the contrary, the erosion operation removes the noise present in the background and while doing so, it reduces the size of the object region. So, if I apply one that has to be compensated by the other. So accordingly, as we have said that we have two different operations; one is called closing operation and the other one is called opening operation.

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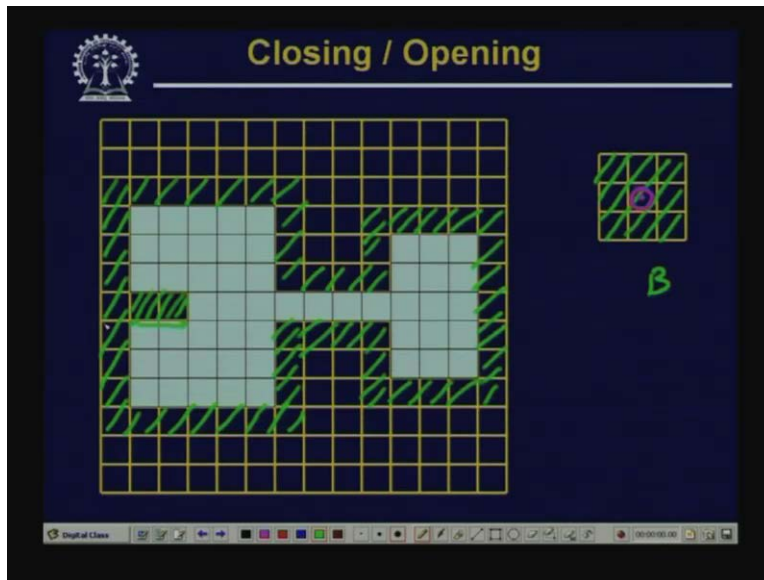
Now, let us see what are those opening operation and closing operation. So, I take this particular binary image. So here, you find that apparently, this image contains two different object regions and there are two noisy regions in this image. One is this one, it appears that this interior region should have been object but may be due to noise, these two object pixels have turned out to be background pixels and on this side, this is another object region another object where internally we do not have any noise but these two object regions are connected by a thin line which is just one pixel white. So generally, we can assume that this connection is nothing but because of the presence of noise.

Now, let us see that how we can apply the morphological operations to remove these noises present in the image. So here again, I assume that my structuring element is a 3 by 3 structuring element. So, this is my structuring element – B, capital B and the center of the structuring element, the center pixel is taken as the local origin. So, the operation that I want to perform first is dilating this given image with the given structuring element, the 3 by 3 structuring element - capital B.

So, if I dilate this image with this structuring element capital B, then what is the effect that we are going to have?

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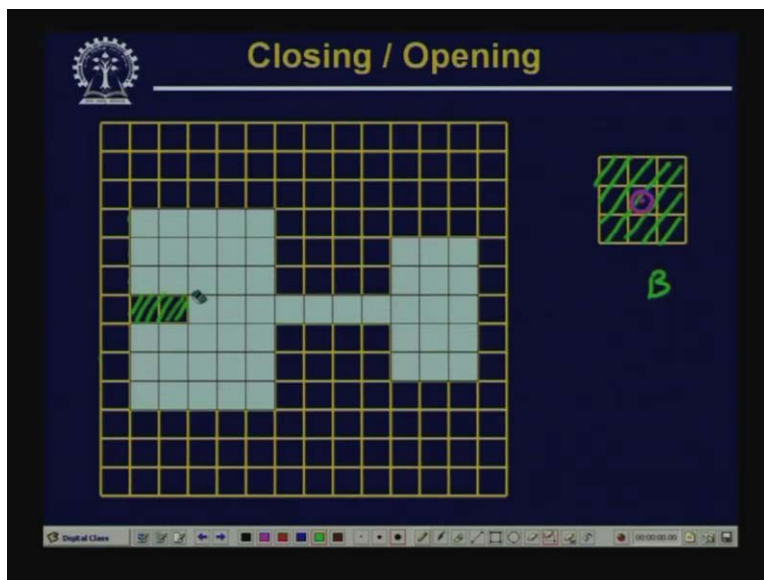




The effect will be something like this that these internal noisy points as we have assumed, these will be filled up but simultaneously, the boundary of the object will also be expanded by one pixel all around the boundary. So, this is the dilation output that I am going to have. So, this is the dilation output, all these pixels will turn out to be object pixels.

So, I have achieved an unintended operation. That is I wanted to fill this gap, I wanted to convert these two background pixels into object pixels which has been done but at the same time, there is a side effect that the objects have become expanded by one pixel all along the boundary.

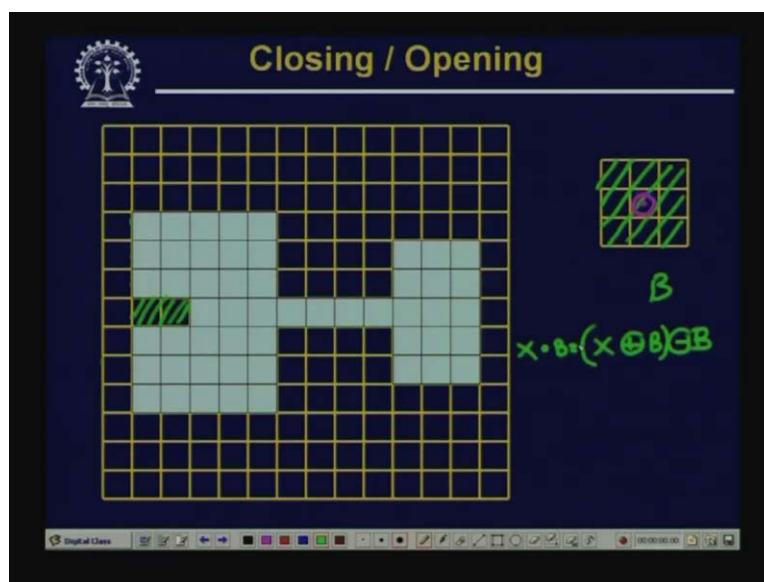
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So, to negate this expansion, what I do is this dilated image is now eroded by applying the same structuring elements. So, if I erode this by applying the same structuring element, then after erosion, as we have said that the effect of erosion is to shrink the boundary, to reduce the object region all along the boundary; so because of this shrinking, all these additional pixels which have been introduced by the dilation operation will be removed.

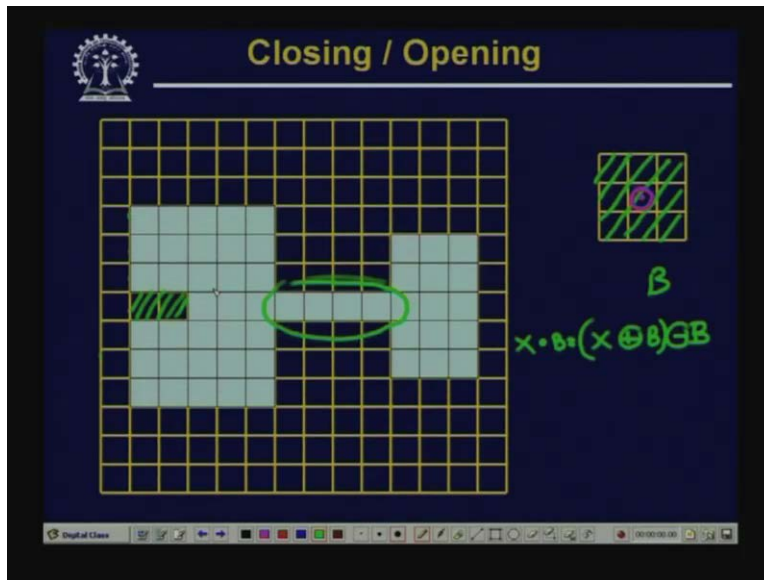
So, all these pixels will be removed but the effect that I have is that internal noisy pixels where an object pixel was turned to be a background pixel that has been reconverted to object pixel. So, all the internal noise, the internal noise within an object region has been removed by the dilation operation and the expansion of the area has been converted by the following erosion operation.

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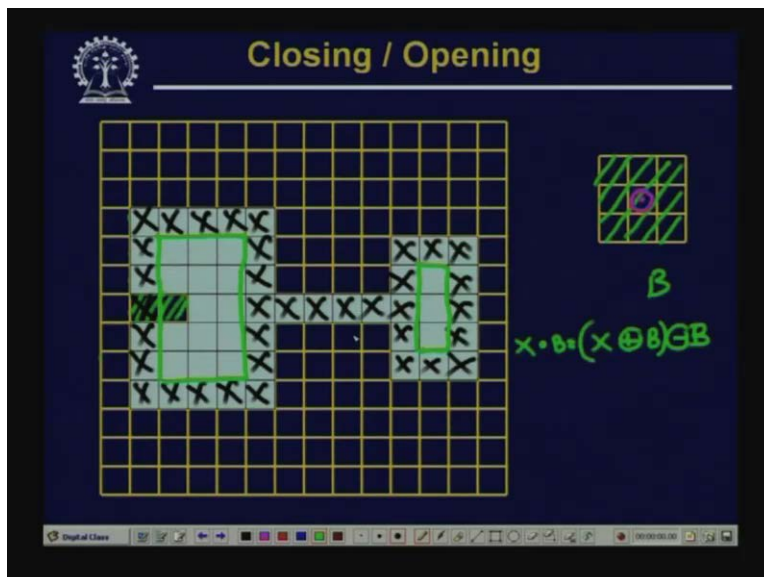
So here, the operation that I have done is first  $x$  is  $x$  is first dilated with  $B$  and this dilated output is eroded with the same structuring element  $B$ . So first, we are doing the dilation operation followed by the erosion operation and this is an operation which is called morphological closing operation and the closing operation is represented by this symbol. So here, what we have done is a morphological closing operation. So, the effect is quite clear. After doing the morphological closing operation, we have removed the internal noise.

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But the other noise that is this one where two different object regions have been connected by a thin line which is one pixel white that has not been removed. So, in order to remove this, I do the inverse operation. That is first I apply the erosion and by applying the erosion, I will have reduction in the object region. To compensate for this reduction, I will have the subsequent dilation operation. And, this erosion and dilation will also be done by the same with respect to the same structuring element.

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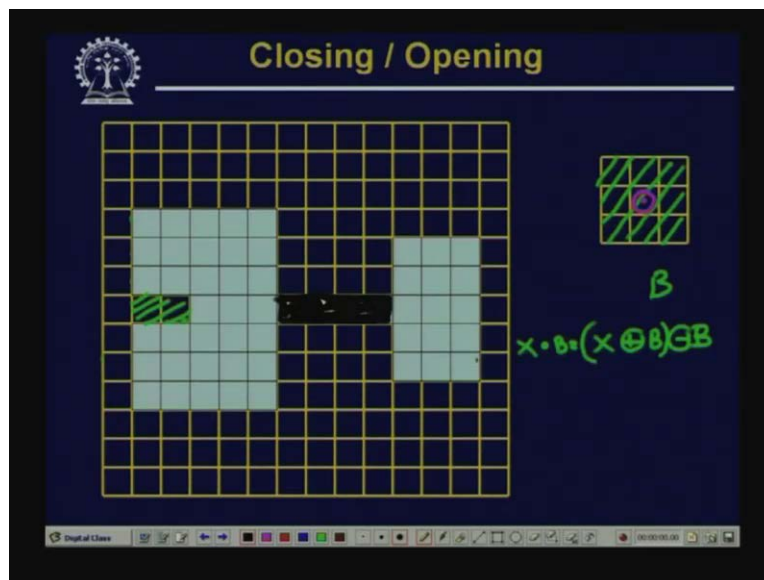


So, if I do erosion what I will get is something like this. By doing the erosion operation, these are the pixels which are going to be removed, all the boundary pixels which are going to be removed;

these pixels will be removed from this image. This one pixel white thin line will also be removed and at the same time, these boundary pixels are also going to be removed.

So, what I get at the end of this erosion operation is two object regions; one is this, a shrunk version of the object and here another shrunk version of the object. So, this shrinking has been done because first we have applied erosion. Now, to compensate for this shrinking, as we said that now will perform the inverse, the dilation operation. But in this case, the dilation operation will be done by the same structuring element capital B.

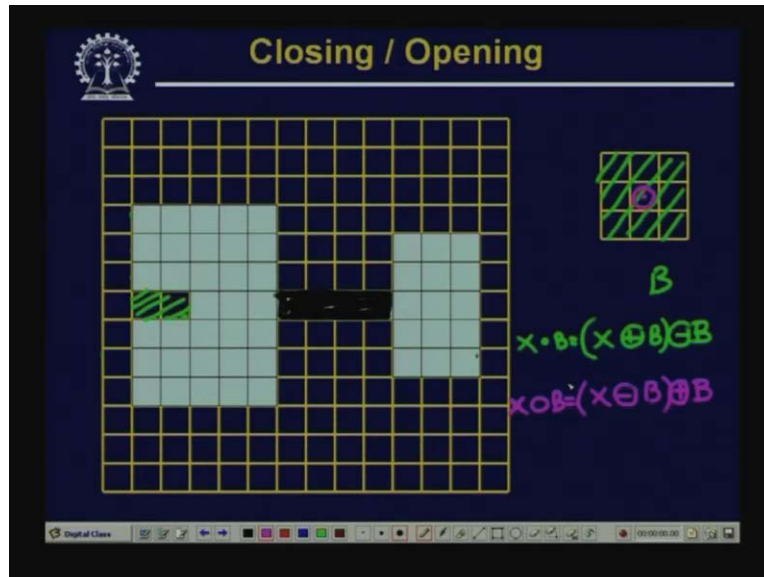
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So, if I do this dilation, then the output that I am going to get will be all these regions will be restored, these regions will be restored, this internal noise which was filled up earlier that will remain as it is. But what I remove is this thin pixel, this thin line of one pixel white which was connecting these two object regions, this I had been able to remove.

So, you find that this image has now been separated into two object regions and the noises present in the image which were present in the image that has been removed. So, in this case, what we have done is first we have done the erosion operation.

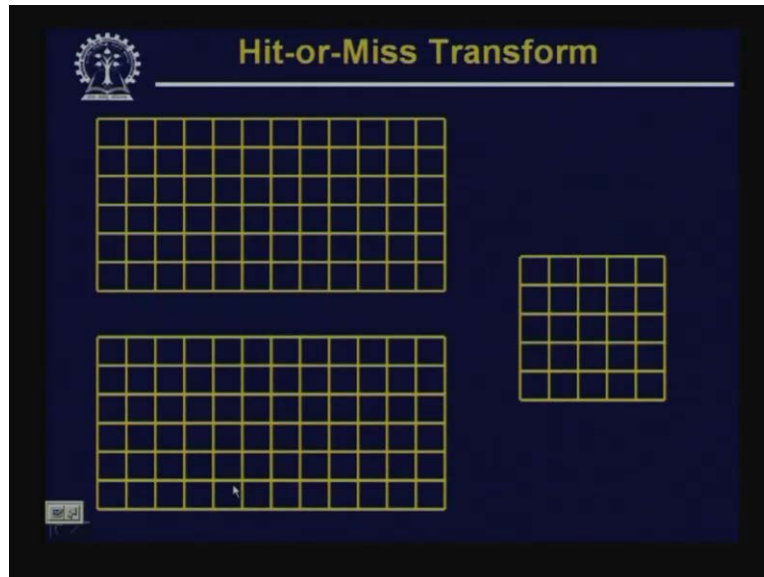
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So,  $x$  is eroded with  $B$  and subsequently, it has been dilated by the same structuring element capital  $B$  and this is an operation which is known as the opening operation and the opening operation is represented by a symbol like this a small circle  $B$ . So, you find that we have applied two different operations; one is the opening operation the other one is the closing operation. By using the closing operation, we remove all the internal noise present in the object region and by using the opening operation; we remove the external noise which are present in the background region.

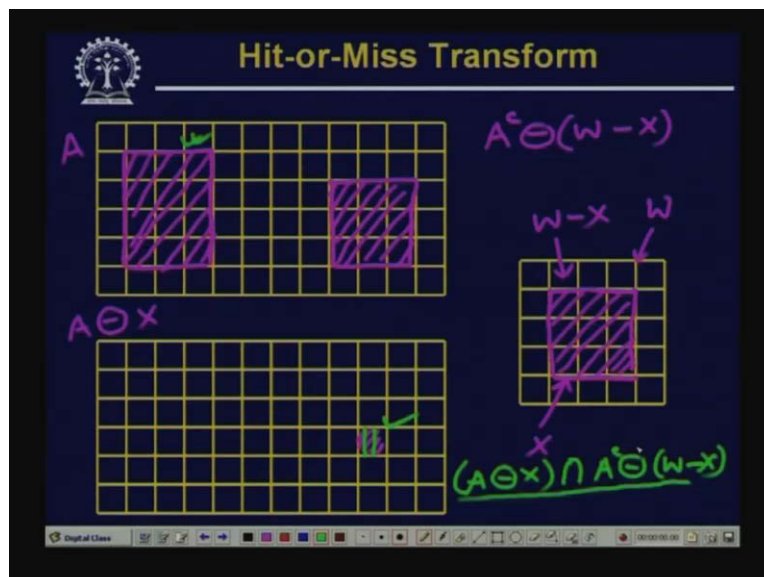
So initially, when we said that this morphological operations are also very very useful to remove the noise or for filtering operation, so by filtering what we mean is we have to use the opening and closing operation one after the other and by using this opening and closing operations, we can remove the noise present in the image.

(Refer Slide Time: 52:50)



Now, let us talk about another operation, another transformation which we call as hit or miss transform. So, what is this hit or miss transform? Hit or miss transform is normally a transformation which is used to detect or to locate an object of a given shape and size in an image.

(Refer Slide Time: 53:20)



So, let us assume that we have an object say something like this - a 3 by 3 square object. So, this is our object and the image that we have is something like this. So here, I have an object region which is 4 pixels by 3 pixels and here, I have say another object region which is 3 pixels by 3 pixels.

Now, to detect this object within this given image X, the kind of operation that have to do is let us assume that this is our image A and this is the object x that we are looking for.

Now, what we do is we embed this X into a higher into a larger size window and suppose that window is W. Sohere, we have two different sets; one is x and other one is the boundary of this X which is represented by W minus X and now, the kind of operation that we have to do is first let us do one thing that we will perform A erosion with X. So, here you find that if you erode A with X, then the output will be something like this. There will be two pixels in the eroded output and here we will have 1, 2, 3 -these pixelsin the eroded output.

So, this is our first operation when I do A erosion with X and the next operation that I will do is I will take complement of A which will be eroded with W minus X. So, if I erode A complement with W minus X, then I will get only one pixel which is this. So now, if I take the intersection of A eroded with X, this is one set and I take the intersection of this with A complement eroded with W minus X, then I get a single point which is only this point and you find that this point identifies that this particular object X is located in this particular location.

So, here you find that though this object region is a superset of this object X but still this has not been detected.What we have detected is only a single location where this particular object is present. So,this operation that is A erosion with X intersection with A complement erosion with X, this is what is known as hit or miss transform and thishit or miss transform is used to locate an object of a given shape and size within a given image.

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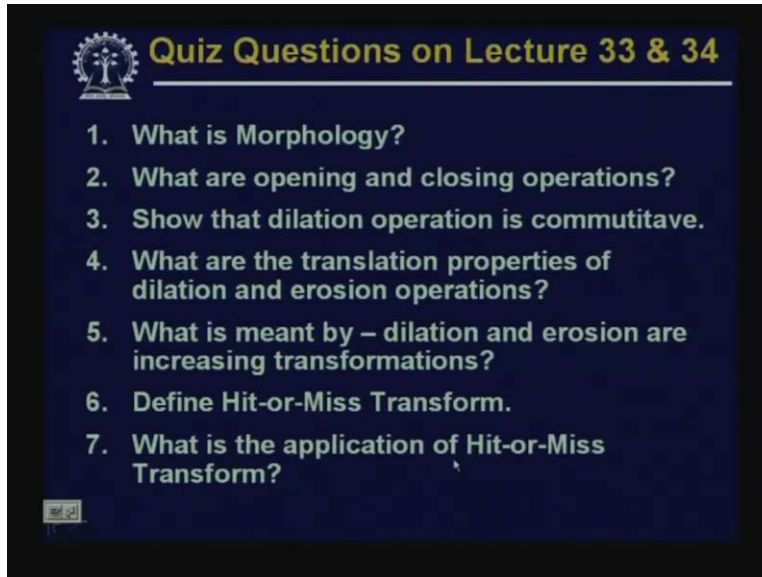
The image shows a whiteboard with two mathematical formulas written in green marker. The first formula is  $B = (B_1, B_2)$ . The second formula is  $A \circledast B = (A \ominus B) \cap (A^c \ominus B_2)$ , which is underlined. At the bottom of the whiteboard, there is a toolbar with various icons and the text 'Digital Class'.

Now, this operation can be generalized, this particular transform can be generalized and the generalized definition is something like this. Suppose, we have a structuring element B and if I represent this structuring element B if I can represent this as  $B_1$  and  $B_2$  if I can break the structuring element B into two structuring elements  $B_1$  and  $B_2$ , then for a given image A, the hit

or miss transform of A with B is given by  $A \ominus B = (A \ominus B) \cap (A^c \ominus B^c)$ . So, this is a general definition of hit or miss transform.

So, with this, we stop our lecture today. We will continue with our morphological operations in subsequent classes.

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The image shows a slide titled "Quiz Questions on Lecture 33 & 34". The slide has a dark blue background with a white border. At the top left, there is a small logo of a person standing under a gear. The title is in yellow text. Below the title, there is a list of seven quiz questions in white text. At the bottom left, there is a small icon of a presentation screen.

**Quiz Questions on Lecture 33 & 34**

1. What is Morphology?
2. What are opening and closing operations?
3. Show that dilation operation is commutative.
4. What are the translation properties of dilation and erosion operations?
5. What is meant by – dilation and erosion are increasing transformations?
6. Define Hit-or-Miss Transform.
7. What is the application of Hit-or-Miss Transform?

Now, let us see some of the quiz questions of today's lecture.

Thank You.