Digital Image Processing

Prof. P.K.Biswas

Department of Electronic and Electrical Communication Engineering

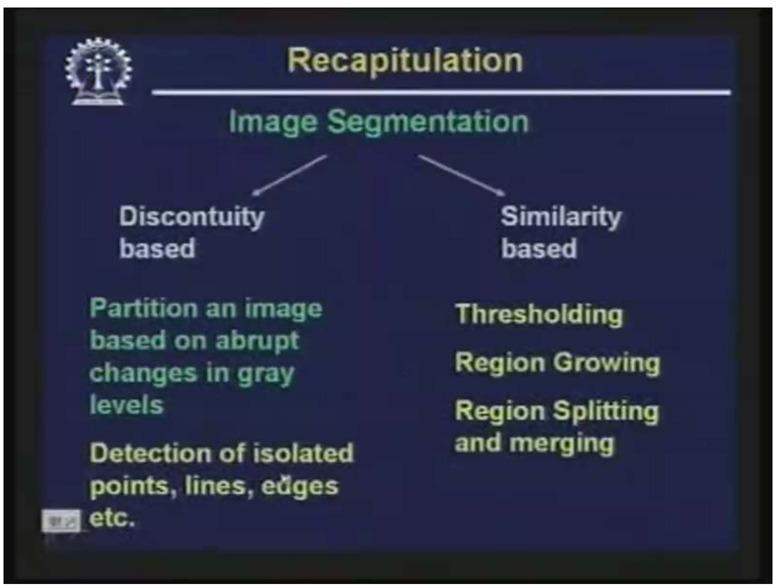
Indian Institute of Technology, Kharagpur

Lecture - 33

Mathematical Morphology - I

Welcometo the video lecture series on digital image processing. Till our last class, we have talked about various image processing techniques and for last few lectures, wewere talking about image segmentation techniques and as we have seen that image segmentation is nothing but a process of partitioning a given image into a number of sub images or into a number of sub partitions.And,we have talked about various techniques for partitioning the image into a number of partitions or into a number of components.

(Refer Slide Time: 01:39)



So, we have seen that image segmentation techniques can basicallybe divided into 2 different categories.One of the categoriesis discontinuity basedimage segmentation techniques and the other category is similarity based image segmentation techniques.In discontinuity based techniques, what we look for is some abrupt changes in the image gray levels or if it is a colour image some abrupt changes in the colour information in the images and based on these abrupt changesin the intensity values in the images, we partition the images into different components.

Soessentially, in discontinuity based image segmentation technique,what we look for is some isolated points or we look for some lines or we look for some edges and similar such discontinuities present in the image.And based on this discontinuity, once we detect some such isolated points or say points lying on a line,line obviously is a boundary between a lighted image region to a darker image regionandonce we identify those points, then by some post processing technique, we try to link those points and after that we get partitioning of the image into different regions and naturally in this case, the partitioning is based on the discontinuity approach.

Inthe other case,in similarity based technique, we have seen different type of techniques.The first one that you have seen is the thresholding and we have said that it is one of the simplest approaches for segmentation of an image.

So, in thresholding based approach, what we have done is by some means we try to find out a threshold and thresholdingan image means all the intensity values,the regions with the intensity values which are less than the threshold is put to some partition, some segment and the image regions where the intensity values are greater than the threshold, they are put to some other region.

So, when I put these image pixels to different regions based on their intensity values compared to the threshold, what we essentially do is we partition the image into a number regionswhere the intensity values indifferent regions are differentand intensity values in certain regionsare similar in the sense that either they are greater than the threshold for a particular region or the intensity values of all the pixelsin another regions is less than the threshold.

Theother kind of segmentation technique based on this similarity measurewe have done is the region growing operation.So, in region growing, what we have done is again by some means, we try to identify some seed points and then we try to grow the region from the seed points based some similarity measure.So, when you grow the region, you ascertain that a point will be included into a particular region if the intensity is similar to that region and at the same time,the point is also connected to that particular region.

So,you start from a number of seed points and from each seed point, you try to grow the region based on similarity of intensity value as well as the connectivity property and that is how we get an image segmentation based on region growing techniquesand the last approach under this category that is similarity based approach that we have seen is region splitting and merging operation.

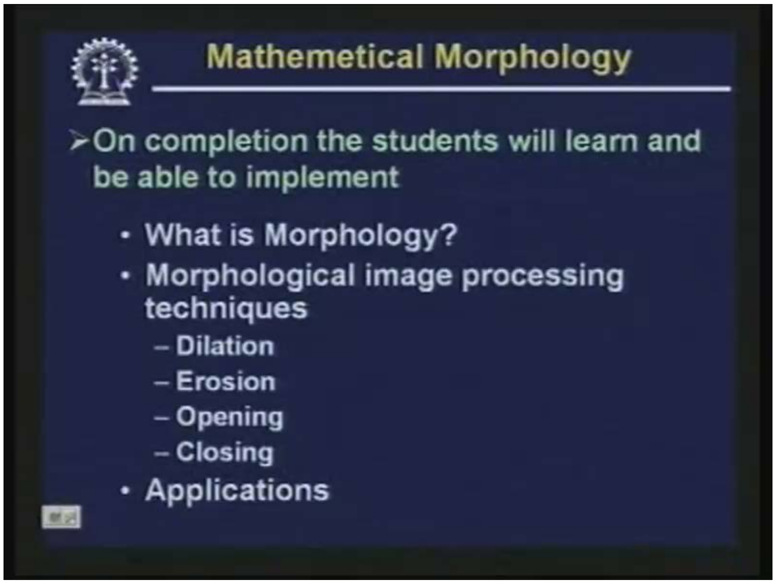
Sohere, you start with the entire image,then if the image intensities if the pixels intensities over the image is not similar; then you partition the image into four different partition or you make four different quadrants and then you check for the similarity in each of the quadrants.If in any of the quadrant,you find that all the pixels have similar intensity values; you donot partition itanymore but a particular quadrant where if the image pixel intensities are not similar,you partition again,you sub partition it again in four different quadrants and this process of splitting continues until and unless we find that a quadrant size is less than a given size or the intensities in each of the quadrants image intensities, pixel intensities in each of the quadrantsare similar.

Now, while doing so, it is possible that a connected regionof similar intensity will be dividedinto a number of different partitions. So, in the second phase that is the first phase which we have done is the splitting that is an image is split into a number of quadrants and this process continuous until and unless we find thatimage intensities in every quadrant is similar or the quadrant size which is a minimum specified size. So, after splitting operation, as a connected region may be split into a number ofsub regions; so I go for the second phase which is the merging operation.

So, in the merging phase, what we do is we take the neighboring regions which might have been split into two different regions but their intensity values are similar. So, in merging phase, you try to merge them in a bottom upfashionand once your splitting and merging these two phases are complete, you get partitions of the image or segmentation of the image into different segments where the intensity values in different segments will be similar, in each segment will be similar and the intensity values will be in the different segments will not be similar.

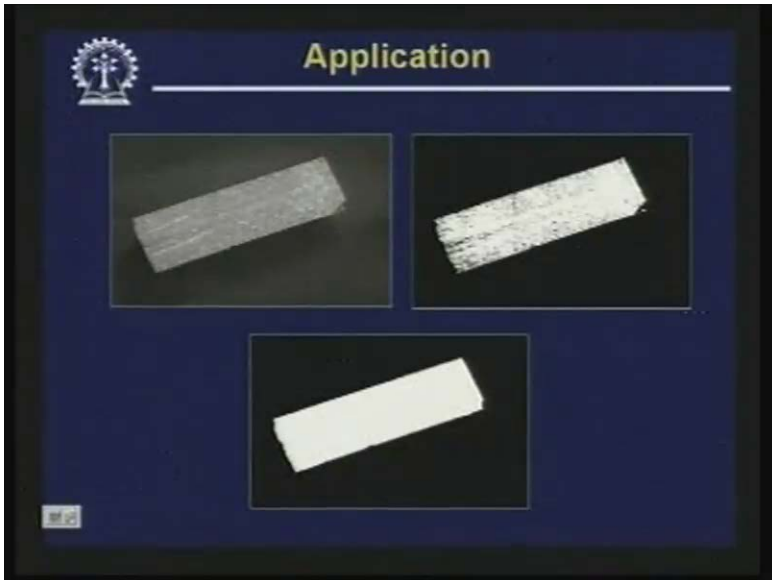
So, these are the different segmentation techniques that we have talked about during our last few lectures.Today, what we are going to talk about is something different.That is we are going to talk about a topic called mathematical morphology and we will see its application in image processing.

(Refer Slide Time: 8:29)



So,in today’s lecture,we will see what is mathematical morphology and we will talk aboutdifferent morphological image processing techniques and in particular, in today’s lecture, we will try to cover the topics ofdilation, erosion, opening and closing operations.We will come to each ofthese techniques, discuss each of these techniques one by one and then we will see what are the applicationsof these techniques in our image processing operations.Sofirstly, let us see that what is morphology.

(Refer Slide Time: 9:09)



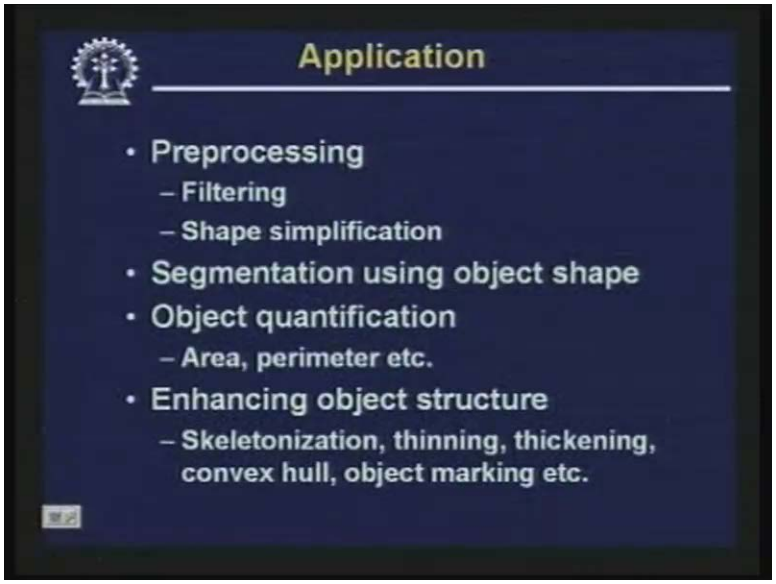
Now, morphology is a term which is widely used in the field of biology and it tells you that it discusses about the shape and structure of different animals different plants and so on.In our application, we will talk about morphology to discuss about the image processing techniques which take into consideration the structure and shape of objects.

So, the image processing techniques based on the structure and shape of the objects are classified asmorphological operationsor this is nothing but the application of mathematical morphology in image processing.Now, when we talk about such morphological operations this have a number of applications.

Forexample, in one of ourearlier lectures, we had shown this examples where we have an imageof an object; then what we do is we try to separate the object region from the background region and for separation of the object region from the background region, we have applied a simple threshold operation and by applying the threshold the kind of image that we get is shown on the top right corner and hereyou find that all the white pixels which are supposed to belong to theobject region, within this white region we have a number of black dots.

Now, these block dots we consider them to be noise.This noise can be filtered by the morphological operations and after doing the morphological operations,the filtered image that we get is shown on the bottom andwe will see through our discussion today and our subsequent few lectures that how such filtering operations can be done by using the mathematical morphological techniques.

(Refer Slide Time: 11:23)



So, these are various applications;onesimple application that I have shown earlier.The other applications of this mathematical morphology is in preprocessing of images that is in case of filtering; the example that I havejust shown is nothing but a filtering operation because here what we have done is all the black patches which are present in the segmented imageif I consider that those black patches arrives because of the presence of noise;then by morphological operation, we are eliminating those effects those effects of those noise by using the morphological filters.

So, it is a preprocessing kind of application where the application is to filter the noise present in the image.The other application of the image processing techniques can be for shape simplification.We can have a situation that we have an object of very very complicated structure but it is possible to break that complicated structure into a number of sub structures where each of the substructureswill be simple in nature so that it is easy to describe or easy to quantifythe shape attributes of that particular structure.

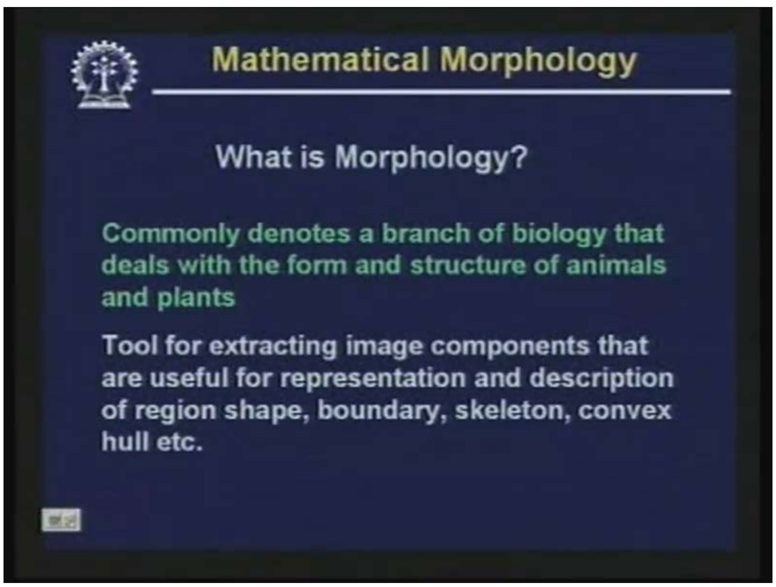
So, in such cases, these morphological operations will also help us to simplify the shape of a complicated structure.Sonaturally, in this case,the complicated structure will be broken into a number of substructures where each of the substructures is veryvery simple in natureso that we can describe them very easily.Then as we said that morphology is a topic which deals with the shape or structure of the objects.So, we can use the object shape for segmentation operations as well.

So, the segmentation using the object shape is another major application of these morphological techniques.Morphological techniques can also be used forimage quantification.By quantification, what I means is we can find out the area of the object region, we can find out the perimeter of the object region,we can find out the boundary of the object region and many such related attributes of the object regions and this is whatI mean by object quantification and later on we will see that the morphological operations or morphological transformations help us to a great extendfor quantification of objects.

Morphological operations can alsobe used in other applications in other cases;for example,enhancing the object structure.By enhancing what we means is in one of our earlier lectures we have seen that given a shape, a two dimensional shape; in many cases, that two dimensional shapes can be better described by usingthe skeleton and one of the skeletons we have seen is the medial access.So,these morphological operations are also very very useful to find out theskeleton of an object.Similarly, these are also useful for thinning operation, these are also useful for thickening operation, these are also useful to find out the convex hull of a set of points - we will come later what is meant byconvex hull - these are also useful for object marking and there are various such applications of the morphological operations.

Naturally, it is not possible to coverentire morphological operations in this particular course and that is beyond the scope of this course but we will look at some basic image processing operations which employ morphological techniques.

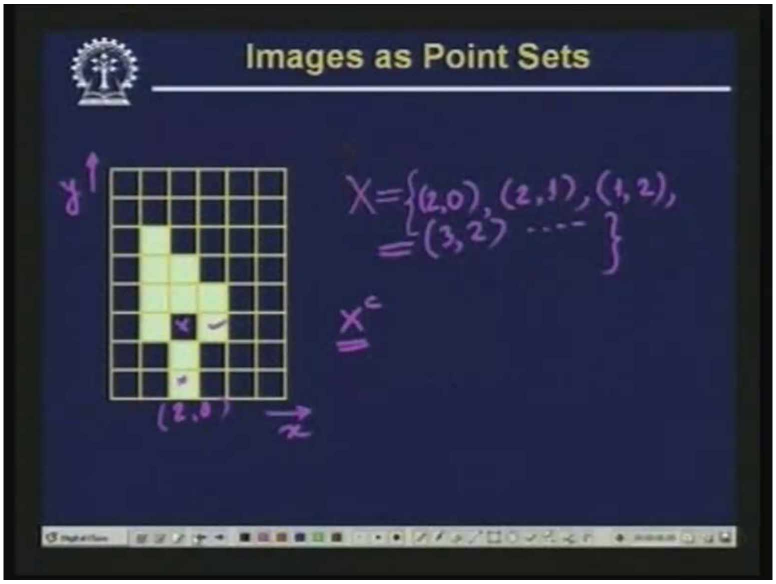
(Refer Slide Time: 15:27)



So,as we said earlier that by morphology what youmean is it commonlydenotes a branch of biology,we said that morphology is very very a common term in the biology and itdeals with the form and structure of animals and plants whereas in our case, when we mean use of mathematical morphology in images processing, sometimes it is also called image morphology.Bythis we mean the mathematical tool whichhelps us to extract image components which are useful for representation description of region shape, boundary, skeleton convex hull etc.

Now, whenever we go for such mathematical morphology in image processing or image morphology,the basic assumption is an image can be presented by a point set.That is we can give an image, we can represent an image by a set of points.What do you mean by that?

(Refer Slide Time: 16:34)



Letus look at this particular figure.So, as you find that we can say that this is nothing but a binary image where the shaded regions represent object pixels and the black regions represent the background pixels.So, this is nothing but abinary image.So, when we talk about this mathematical morphology; initially,initially we shall concentrate on application of mathematical morphology to binary images which we also called binary morphology.Later on, towards the end of this topic,we can also see that how this binary morphology can also be extended to take care of graylevelimages and we will called that as gray level morphology.

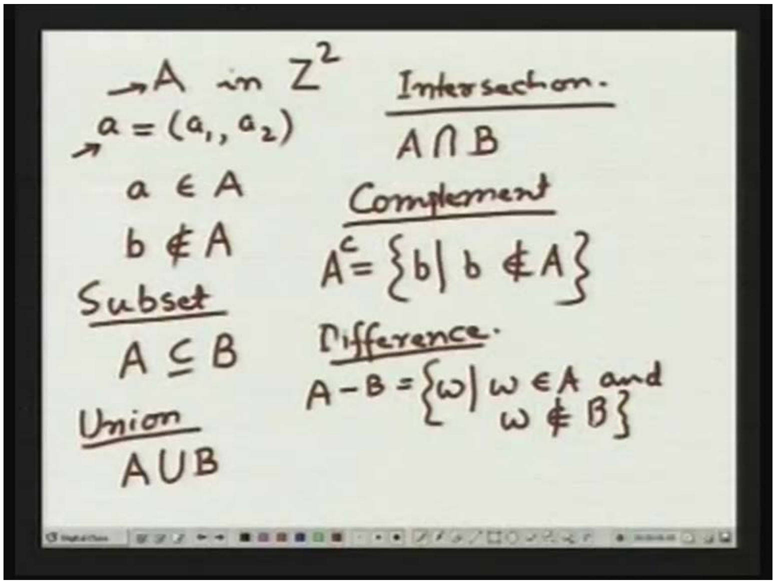
So, as I was saying that the basic assumption in application of mathematical morphology in image processing is that the images should be represented by point sets or I should be able to represent an image by a set of points.So, looking at this example where I have shown a binary image and as I said this shaded regions represents the object pixels and the black regions represents the background pixels and here, you find that this particular binary image can be represented by a set of point.

So,I can write a setX which is nothing but a set of points, the points belonging to the object region.So, in this particular case,if I say that this is my x direction and this is my ydirection, you find that the coordinate of this point is nothing but(2, 0).So, this point (2, 0) is a member of this set X.Similarly, the point (2, 1) is also a member of this set X.The point (1, 2) is also a member of this set X but point (2, 2) is not a member because this pixel is a black pixel and it belongs to the background whereas the point (3, 2), it is a member of this set which is this point.Solikewise,I can consider the other points which belong to the object region to be member of this particular set X.

So, you find that this object is now represented by a set of points where a member of this set represents the corresponding point is an object point.Similarly,the background pixels in this particular image can be represented byX compliment because all the points which does not belong to this set, thispoint set X, they belong to the complement of set X or X complement.So,the background pixels is also nothing but a point set which is complement of the set Xwhere set X is a point set representing the object pixels.

So, this is what I mean by an image should be represented by a point set and once an image is represented by a point set, then all the morphological operations on the images are nothing but some set operationson those point sets.So, because this morphological operation will be set operations; let us quickly review some basicset operation technique.

(Refer Slide Time: 20:59)



So,as we all know that suppose I consider a setin a two dimensional phase;so I consider a set A,point set A insaytwo dimensional space Z^2 and if a point say a because we are considering two dimensional phase, soa point in thistwo dimensional spacewill be represented by on order pair.So, this point a ifI represent it by an order pairs (a_1 a_2); then our first definition is if this point a is a member of this point set capital A, thenwe will write as a belongs to capital A.So, this is what says that A belongs to the point setcapital A.

Similarly, given another point in this two dimensional phasesay b; if b does not belong to the point set A, we write it as b does not belong to the point set A.So, these are some basic set operations or set notations.Then, we can also define a subset operation.That is we say that a set A is a subset of set B where both the points setscapital A and capital B are in the two dimensional space - z square.So, it definethe point set capital A is a subset of the point set capital B if every element inset A also belong to the set capital B.

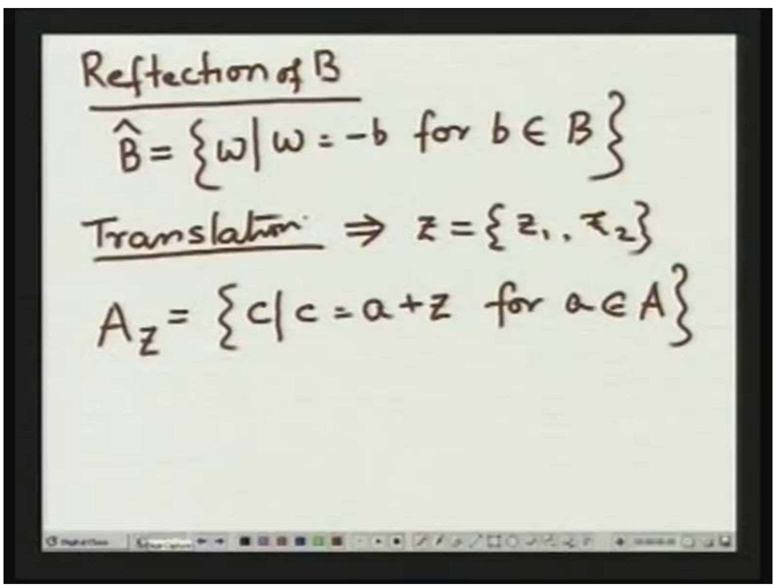
However,the reverse may not be true.If I have an element inset B, that may or may not belong to set capital A.But every element belonging to set capital A must be a member of set capital B.Inthat case, the set, the point set capital A is a subset of thepoint set capital B.

Wealso have set union operation.So, for 2 sets - capital A and capital B, the union operation A union B is the set of elements taken from set A and set B.So,if I combine all the elements of set capital A and all the element capital B and take them together, then the set that I get is the union of the two sets - capital A and capital B.We can also have set intersection operations.So, by intersection,theintersection of two sets capital A and capital B means that all the elements in A and B which are common that is if an element belongs to set A and that also belongs to set B, then that particular common element will be a member of A intersection B. So,A intersection B,really represents the common elements which are common in both setA and set B.

Similarly, we can have set complementation.So, by complementation,what we mean is aset or complement of a set capital A,this is nothing but all the elements b such that b does not belong to set capital A.So, set of all the elements which are not members of the set A, forms the set A complement and we have seen in our last example that if I represent all the object pixels by a set say capital X, then all the background pixels will be represented by X complement because the background pixels are not object pixels and the vice versa,the object pixels are not background pixels.

We can also have set difference operations.That is we define set difference A minus B and which is nothing but it is the set of all elements say w such that w belongs to A and this w does not belong to B. So essentially, if we have some common elements in the sets A and B; so if I remove the common elementsfrom set A, then whatever setI get, that is the set of elements which are in the set difference A minus B.

(Refer Slide Time: 26:59)

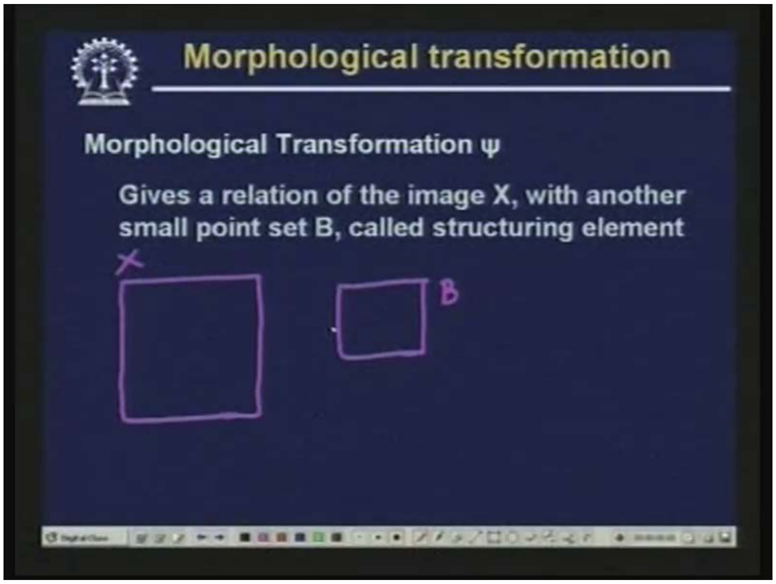


Similarly,we can have other set operation, like set reflection. So, what is this reflection?So, reflection of a set say B, reflection ofset B;so, this is usually represented as B hat and B hat is nothing but say it is the set of w such that w is equal to minus b for b belonging to set capital B. So, if I have a set ofpointswhich I call as capital B, then all the points belonging to set B if I negate them; then all those negated points, set of all those negated points is actually the reflection of set B and this reflection of set B is represented by B hat.

Similarly, we can have a translation operation on a set. So, if we translate by a vector say z which is equal to (z_1,z_2)components of this vector z_1 z_2,then the translation of a set A by vector z which is represented by A with subscript z is given by it is set of all point c such that c is equal to a plus z for a belonging to set capital A.

So,if I shift all the points belonging to set A by this vector capital Z, then the set of all those points all those shifted points formthe set$A_z$orthe set A which is translated by vector z. Now, as we are considering about consideringthe point sets and we know that in a two dimensional phase,a point can also be represented by vector; so, a point set we can also say that it is a set of two dimensional vectors and as we have said that all our morphological operations will be based on set operations and some of the set operations which are usually used in morphological transformation that we have just briefly described.Now, let us see that what we mean by a morphological transformation.

(Refer Slide Time: 30:02)



So, we say a morphological transformation; say psi; so this morphological transformation psi givesa relation of the image X capital X and as we have just said that this image capital X is nothing but a point set or set of point.So, this morphological transformation gives a relation of the image X with another small point set or another small image say capital B which is called a structuring element.

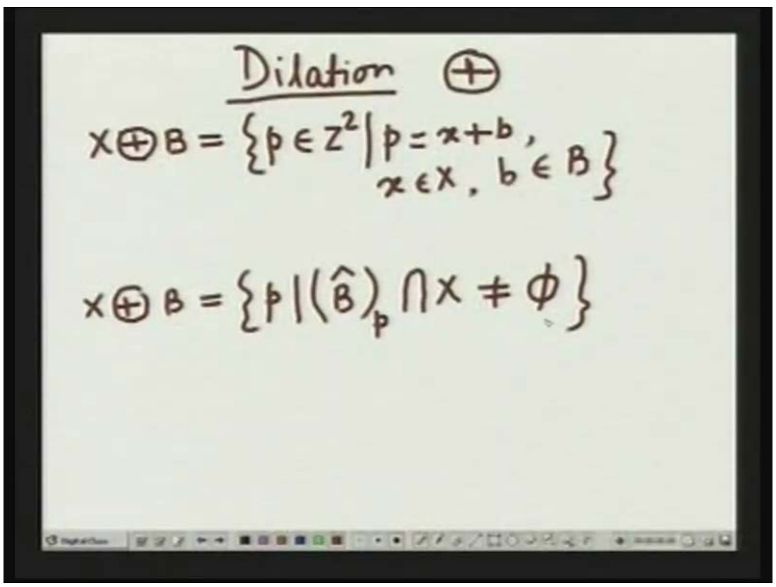
So, in this case, what we have?We have an image say X - capital X and we have a small structuring element or a small image capital B.So, this morphological transformation actually describes or gives a relation between this image X and the small point set capital B which we are calling as a structuring element.Now, the way we compute this relation is similar to the way we have done the convolutionoperation or the way we havedone the image registration operation.

So, incase of convolution operation, what we have done is we are given a big image and we have given a small image.So, there what we do is you take this small image and move this small imagesystematically over the big image and for each such shift or eachsuch translation of thesmall image over the biggerimage,you do some computation which is nothing but the convolution operation.

Sohere also, for morphological transformation of a morphological operation,what we will do is we will translate or we will move the structuring elements systematically over the given image X and for each translation of the structuring element in the given image,we will perform some operations which the type of operation will depend upon what type of transformation or morphological transformation that we want to do.

Sofirstly, we will talk about some simple morphological transformations and one of them, we call as dilation. So, what is dilation?There are two different definitions of the morphological dilation operation.However, we will see that both these definitions lead to the same result or in the other sense that two different definitions are identical.So, what are those definitions?

(Refer Slide Time: 32:58)



So first, what you consider isthe dilation operation and this dilation operation is denoted by a symbol you have across within a circle. So, this is the symbolwhich is used to represent dilation operation and this dilation operation is actually implemented is defined in terms of setadditionor vector addition.So,if I have an image or a point set X and I have a structuring elementsay capital B, then as we said that all the morphological operations actually define a relation of the given set X with the structuring element B; so this dilation operation has to be implemented or has to be defined with respect to the structuring element capital B over the image capital X which is nothing but a point set.

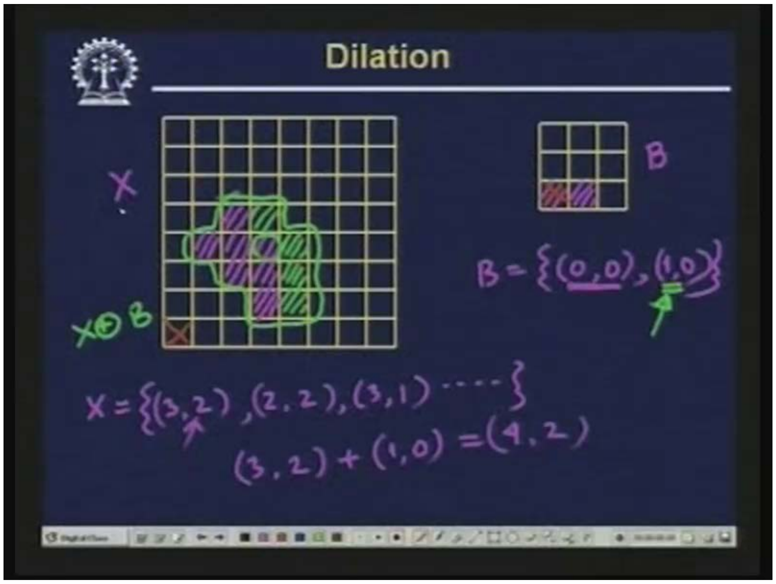
So, we represent X dilation B like this and this X dilation B is defined as it is the set of points p obviously in the two dimensional space.Sothe two dimensional space we have written as z square such that p is equal to x plus b where x belongs to capital X and bbelongs to capital B. So, this is our first definition of the dilation operation.So, what it says that I have been given two sets, two points sets; the first one is capital X which represents an image and the second one is capital B which represents our structuring element. Now, each of these sets are nothing but as we have said the set of points or set of vectors in two dimensionalspace.

So, our first definition of dilation operation says that I take every element from set X- capital X and every other element and every element from the set capital B and add them vectorwise. So, you take vector addition of every element from set capital X with every element from the set capital B of the structuring element and set of this resultant vectors that you get, that gives you the dilation X dilated with the structuring element B.

Theother alternate definition of the same operation is given like this.We can also define X dilation with B, the structuring element B in such a way that again it is the set of points p such that if I take the reflection of Band translate this B with vector p, then take the intersection of this with set X,this should not be equal to null.

So, I have this structuring element and you have defined what we mean by the reflection of a set. So, I take thereflection of the structuring element capital B and then translate it by a vector say p then I take the intersection of this translated reflection of set B with the set of points capital Xand if this intersection is not null; in that case, the translation vector p is a member ofXdilation with capital B and we will see that though we have two different definition of dilation but both this definitions are equivalent in the sense that theyproduceidentical results. Nowfirst, let us see that what we mean by this dilation operation.

(Refer Slide Time: 37:41)



So, let us take an image say like this, say I have a binary image which are represented by say these points. Say, these are the points which belong to the binary image.So, this is my set of points capital X and I define a structuring element say capital B where this structuring element contains only these two points and obviously,I have to have somecoordinatesystem for both myimage as well as the structuring element. So,I assume say this is the origin of my point set capital X and suppose this is the origin of the structuring element B.

So, considering these two origins, now you find that thispoint set X is nothing but a set of points where this X can be represented by this point set consisting of points say(3,2) points (2, 2) points (3, 1). So similarly,I can consider the other points in it.So, I get the image capital X as the set of points or set of vectors. Similarly, this structuring element is also nothing but a point set where you find that this point set isnothing but the point (0, 0) and it includes the point (1,0).

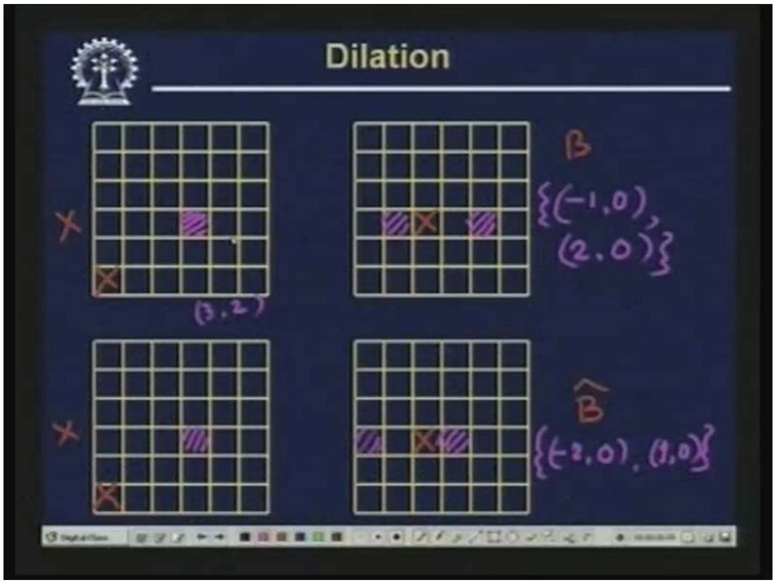
So, as we have seenthe definition of dilation operation that is I take every point fromset X and I take every point set B and you do the vector addition; take a point from set X, take a point from set B,do the vector addition of these two points and this vector addition the resultant vector is a member of X dilation B. So, by doing this,when I add all these points in setsay X with the first vector or the first point in ourstructuring element B which is nothing but (0, 0); so these points will be retained because every vectorfrom setX when it is added with (0, 0), the resultant vector remains the same.

So, all the points inset Xare also membersofset X dilation A. Similarly, if I take a point from this set Xsay (3, 2) and add to this a vector from our structuring element B which is a (1, 0), this is nothing but (4, 2).So, this point (4, 2) which is over here, this is also a member of adilationwith B. Similarly, when I take this point(2, 2), add to this the vector (1, 0), what I get is (3, 2). Now,(3, 2) is already member of X. So, this point remains a member of X dilation with B.

If I consider this point (3, 3), this point; add to this, this vector (1, 0), then my resultant vector becomes (4, 3).So, this is my resultant vector which is also a member ofthis a dilation withB. So, if I continue like this you find that all the points which were belonging to set capital X, theyremain in capital X dilation with B.Inaddition to that I get other additional points becauseof addition with this vector (1, 0); so, these additional points are nothing but these points. So, these are the points which will be added to X dilationwith B.So,our X dilation B will be set of all these points.So, this gives us what is Adilation with B.

Now, let us see whether we get identical result, the same result if I apply two different definitions.

(Refer Slide Time: 43:05)

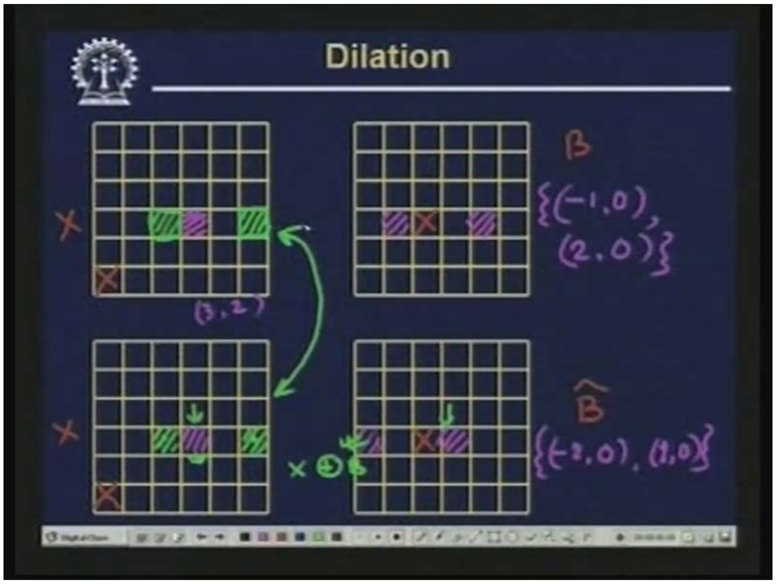


So, for that I consider, a similarkind of situation and for clear explanation,let us take a very simple situation.Againas before,I consider say this is the origin of the point set X; so,I am having two different instances. So, this is my point set X, this is also point set X and I consider say this is the originof ourpoint set B. So, this is the origin of ourpoint set B and I take a single point in our image.

Say, this X contains the single point and what I do is I take the structuring element consisting of only two points like this.So, you will findthat this X contains only the point (3, 2) and the structuringelement contains two points; one is (minus 1, 0) and other one is (2, 0). So, these are the two points which belong to the structuring elements.So, you find that these two points are because I have taken the origin at this location and over here what I consider is the reflection of B.

So, in this case, the reflection ofB will be this particular set.So, the reflection of B will be nothingbut (minus 2, 0) and (1, 0) and this set X is again because it is (3, 2); so this is the only point which belongs to set X. So, in the first case if I apply our first definition that is dilation as a resultof vector addition.

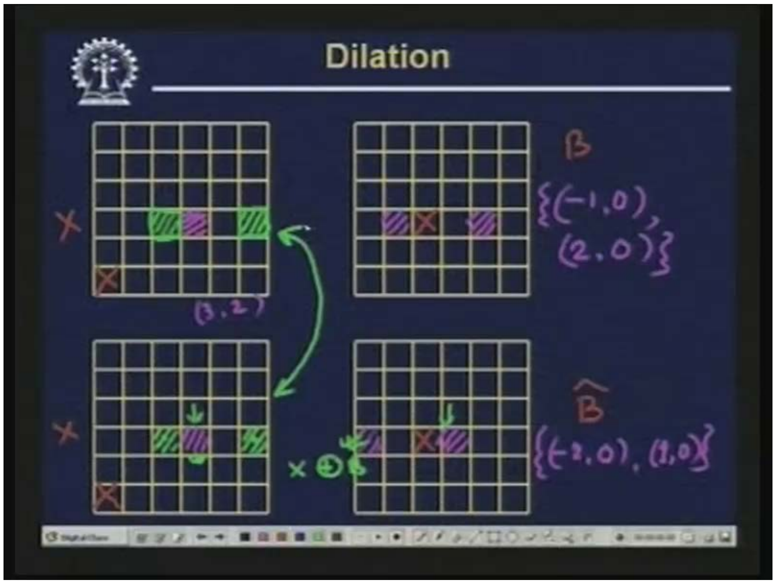
(Refer Slide Time: 45:38)



So, what I have to do is I have to take the point form the point set X and add to it every vector from points setcapital B. So, by doing that what I will get in this particular case ismy dilated image will consist of only these green boxes. So, these are the only two points because if I add (minus 1, 0) to this particular point,I come to this green box and if I add (2, 0) to this point,I come to this green box.So, these are the two points which will be members of X dilation B following our first definition.

Now, what will happen if I apply the second definition? The second definition is in terms oftranslationof the reflection of set capital B and we have said that for all those translations where the reflection ofcapital B or thetranslated reflection of capital Bintersection with X does not become null, those translations are members of X dilation B.

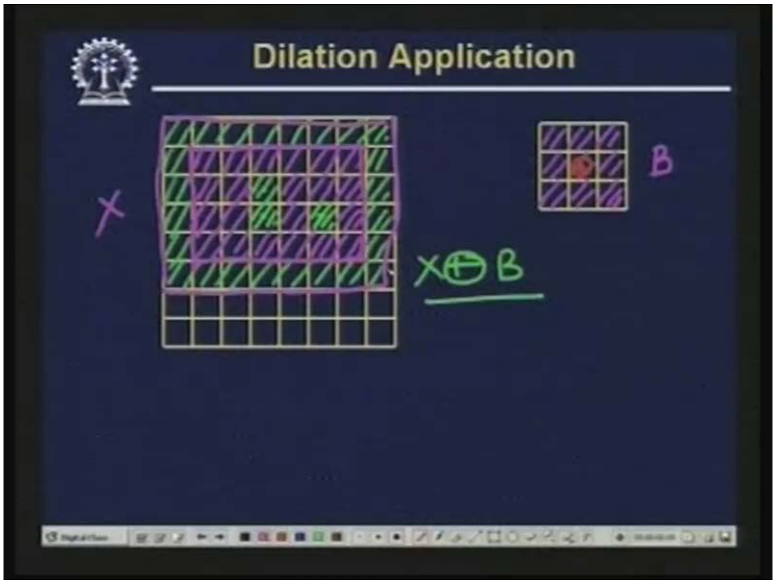
(Refer Slide Time: 46:53)



So, if I do that in that case, you find given this uh reflection ofcapital B if I translate this to this particular location; in that case, this point of B translation will beintersecting with thispoint in our set capital X. Similarly, if I translate this B reflection to this particular location, then this particular point in B translation will be intersecting with this point in our set X.Sohere again, you find that these are the two points or two translation vectors which will be members of X dilation with B.

Sonow, if you compare this result with this result; you find that we have got identical results in both of these two cases.So, whichever definition wewill follow, both the definitionsare equivalent definitions in the sense that they give as identical results when we talk about dilation ofa given set X with respect to a given structuring element say capitalB.

(Refer Slide Time: 48:24)



Now, what will be the application of this dilation operation?Let us take again a particular image like this, say I have an image which consists of say this points. Now, in between,I have saynoisy points say something like this. So, theseare my object pointsand I consider that these black pixels, these three black pixels that I havewithin this object region, these black pixels are because ofthe noise and suppose I consider a structuring elements; so this is my set X and I consider a structuring elements say capital B which is something like this.So, all these elements are members ofstructuring element capital Band when I have this structuring element, the origin of the structuring element,I can consider to be this particular point.So, this is the origin ofthe structuring element.

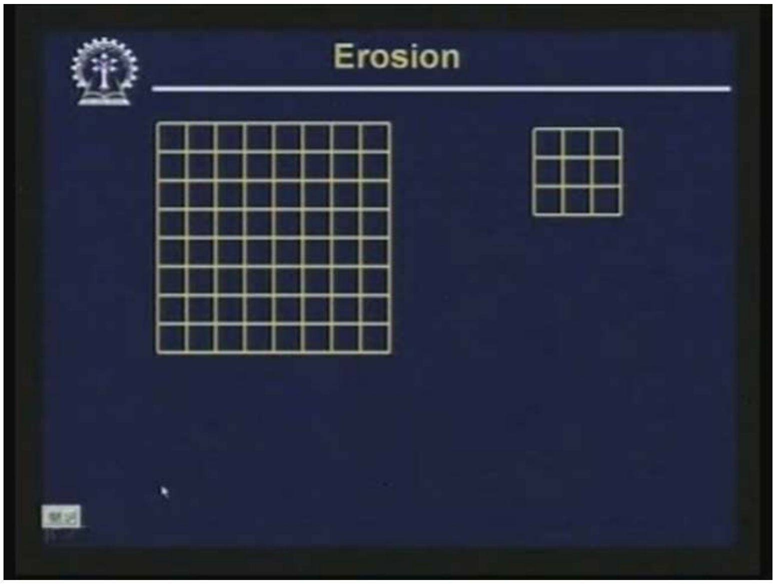
Now, findthat ifI dilateif I go for dilation of this given image with the given structuring element, then the dilationresult will be I mean whichever definition you follow that whether it is vector addition definition or thetranslated version or the translation of the reflection of b tovarious locations in our given image X, as we seenthat we get identical, relations identical outputs; so if I go fordilation of this ah image X with our structuring element capital B, then you will findthat all these points will be in our dilated output.So, all these points will be in the dilated output and these noisy points that we hadwithin the object region, they will also be filled up.

So,naturalapplication of this dilation operation as we have seen in one of the applications for segmentation by thresholding operation that we had a number of black spotswithin the object region which has supposed to be awhite. So, for filling those black spots, we can make use ofthis dilation operation.Aswe have seen herethat all these black pixels that these threeblack pixels in the object region can be filled up by dilation with respect to a structuring element like this. So, this output is X dilation with B.

Nownaturally, in this particular case, we have a side effect.The side effect is it is good that all these internal noisy points, they have been filled up by white pixels or object pixels but at the same time, you find that our original boundary the original object region boundarywas this,this was the object boundary. Now, because of this dilation operation, the boundary gets expanded.So, in addition to this, all these points are also included in the object region.Soin effect, what we have is the object boundary is expanded.So, this is naturally aside effect of this dilation operationand we have other operations by which this expansion, the boundary expansion can be compensated.

So, let us now talk about some other operation which is another morphological operation that is image erosion operation.

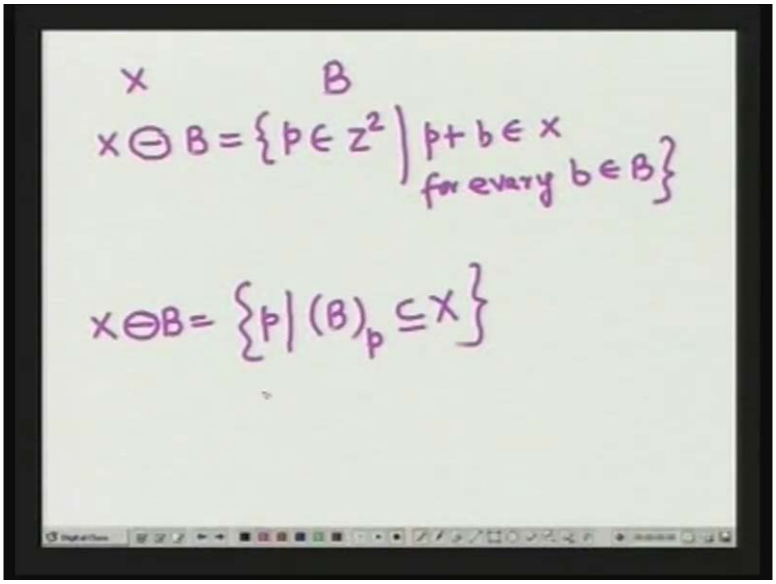
(Refer Slide Time: 52:41)



So, as we have seen that in case of dilation, what we have is we havea given image X and we have a given structuring elements say capital Band what we are doing is we are dilating this given image X with respect to the given structuring element B and this dilation operationis definedin terms ofvector addition.In the same manner, we can have an erosion operation where erosion can be defined in terms of vector subtraction.

So, as we have seen in case of dilation, we can have two alternate definitions though the definitions are equivalent.Similarly, incase of erosion, we can also have two different definitionsand the definitions are equivalent.So, the definition of the erosion operation can be something like this.

(Refer Slide Time: 53:39)



So, again here,we will have a set Xand a structuring element B because for all the morphological operations, it has to be defined with respect to a structuring element.So, we have a point set X - capital X representing our image and we have a point set capital B representing the structuring element and the erosion of the point set X with the structuring element capital B, this can be defined as as per our first definition, it is the setof points set P belonging to our two dimensional phase Z square wherep plus b belongs to X for every b belonging to the structuring element capital B.

So, this is ourfirst definition of the erosion operation.The second definition of erosion operation can be that the same operation, X minus B,X erosion with B this can bedefined as the set p such that I take the structuring element B,translate this structuring element by vector p,this must be our sub set of our set X.

So, you find that this second definition it says that you take the structuring element, then translate the structuring element by a vector say p and if this translated structuring element is a sub setof this set X,then the corresponding vector p or the corresponding pointp will be a member of X eroded with capital B or in other words,for all those translations of the structuring element capital B, if the structuring element is contained fully within the set of pointsX,then the corresponding translation or the corresponding point will be a member ofX eroded with B.

So, we will stop our discussion here today.We will consider this erosion operation further and in our subsequent lecture,in your next lecture, we will see some of the properties of the dilationand erosion operation.

Thank you.