

Digital Image Processing

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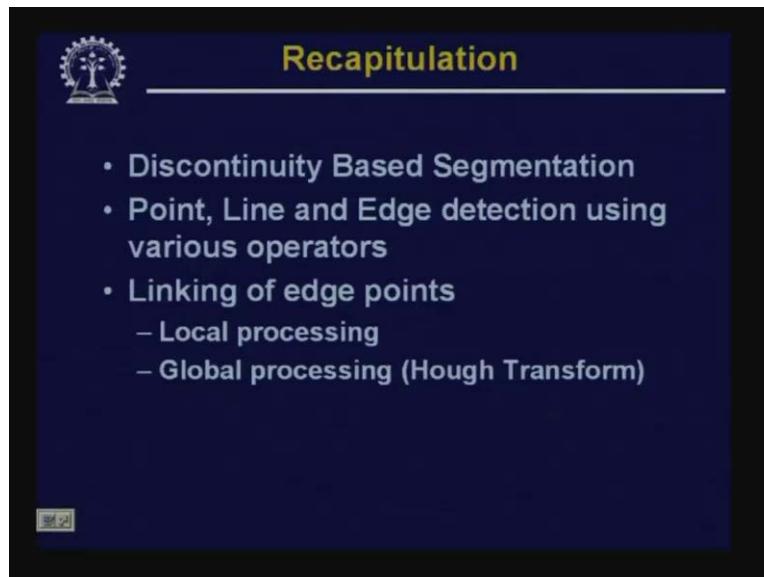
Indian Institute of Technology, Kharagpur

Image Segmentation - III

Lecture - 31

Hello, welcome to the video lecture series on digital image processing. For last few lectures, we were discussing about image segmentation operations and image analysis operations. So, in our last lecture, we have talked about the discontinuity based image segmentation.

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We have seen earlier or we have discussed earlier that there are mainly two approaches of image segmentation. One is the discontinuity based image segmentation and the other one is similarity based image segmentation. For last two classes, we have talked about the discontinuity based image segmentation and in discontinuity based image segmentation we have seen that the segmentation is done using the characteristics of variation of intensity values when there is a variation of intensity from say background to a foreground object.

So under this, we have seen various point and line and edge detection operations which are used in this segmentation process. Here, the basic purpose was that an object is to be described by its boundary or its enclosing boundary which is to be obtained using one of this discontinuity based operations and we have discussed that though we want that the object boundary should be continues or it should have a complete definition but because of noise or may be because of non

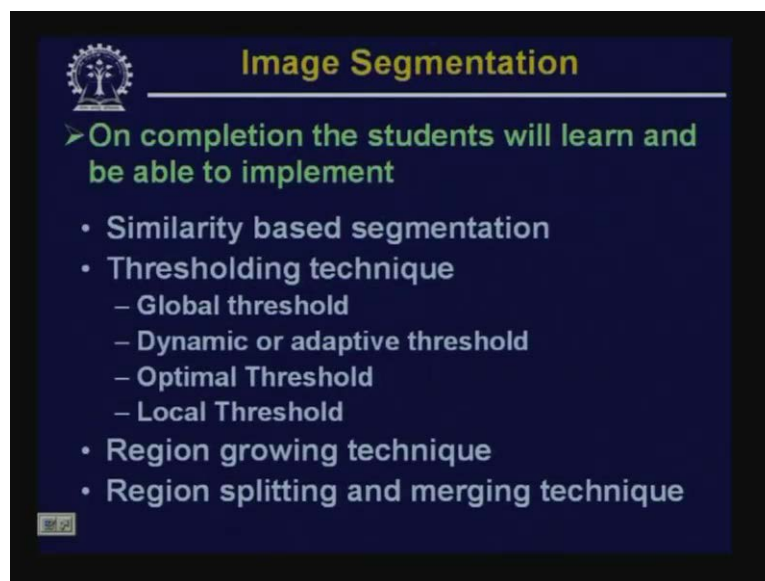
uniform illumination, after performing this different edge detection operations, the edge points that we get they are not normally continues.

So, to take care of this problem, after this edge detection operation; the edge points that we get, they are to be linked. So, for that we have discussed about two different approaches. One is local linking operation where the edge points in the neighborhood are linked together if we find that those two edge points are similar in nature and for that as similarity criteria, we have taken the strength of the gradient operator or strength of the edge operator as well as the direction of the edge at those points.

So, if we find that within a neighborhood, two edge points have the similar edge strength and also they have similar edge direction; in that case, those two points are linked together to be part of the same edge. Now, here again, the problem is that if the points are not in the small neighborhood which is defined but the points are at a larger distance; in that case, this local is linking operation does not help. So, in such cases, what we have to go for is the global link edge linking operation.

So, we have discussed a technique that is Hough transform. So, using Hough transform, we have been able to link the distant edge points and this is an operation which is called global edge linking operation or it is the global processing technique. Now today, we will start our discussion on the other type of segmentation which is the similarity based segmentation.

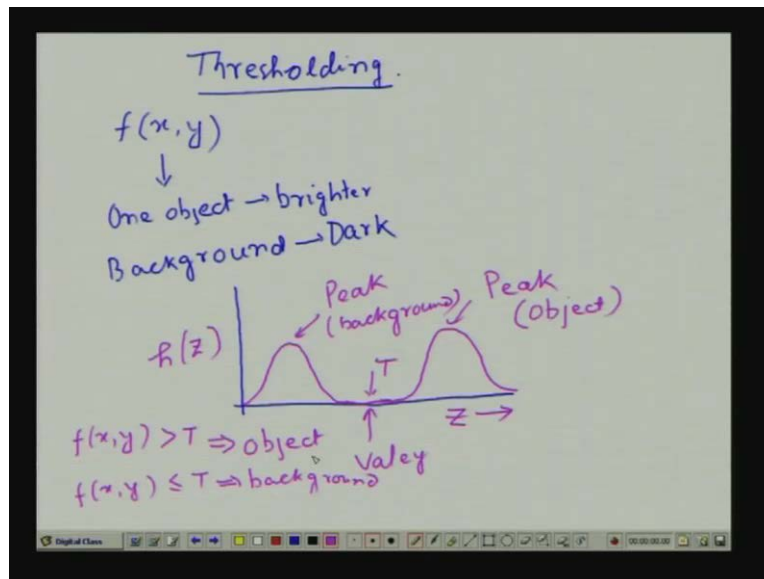
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So under similarity based segmentation, there are mainly three approaches. One is called thresholding technique, the second approach is region growing technique and the third approach is region splitting and merging technique. Under thresholding technique, again, we have four different types of thresholding. One is called global threshold, the other type of thresholding is called dynamic or adaptive thresholding, there is something called optimal thresholding and there is also a thresholding operation which is called local thresholding.

So, we will discuss about these different region based segmentation operations either thresholding or region growing and the region splitting and merging techniques one after another. Now, let us first start our discussion with the thresholding technique.

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So, first we will discuss about the thresholding technique for segmentation. Now, thresholding is one of the simplest approaches of segmentation. Suppose, we have an image and as we have said earlier that an image is described by or represented by a two dimensional function $f(x, y)$ and let us assume that this image contains a dark object against a light background. So, in such cases, if there is a dark object against a light background or if it is the reverse that we have a light object against a dark background; then you will find that the intensity values, they are mainly concentrated near two regions or we call them two modes. One of region will be towards the darker side or towards the lower intensity values and other one other mode will be towards the brighter side or towards the higher intensity values.

So, if you plot the histogram of such an image; so here we are assuming that we have one object and let us assume that the object is brighter and the background is dark. So, if we plot the histogram of such an image, the histogram will appear something like this. So, on this side, we put say intensity value z and this side says our histogram of z . So, as we said that because we are having one object and we are assuming that the object is bright which is placed against a dark background; so the intensity values will try to be accumulated, the histogram will give rise to bimodal histogram where the intensities will be concentrated on dark side as well as on the brighter side.

So, for such a bimodal histogram, you find that there are two peaks. One peak here and the other peak here and these two modes or these two peaks are separated by a deep value. So, this is the valley and this one peak and this is the other peak and as we have assumed that our object is

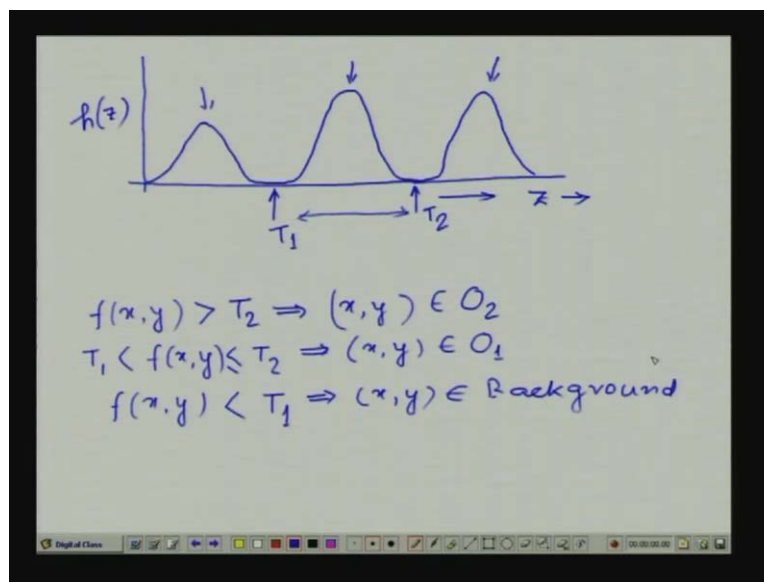
bright and the background is dark; so all these pixels which are grouped in the lower intensity region, these pixels belong background and the other group of pixels they belong to the object.

Now, the simplest form of the segmentation is if we can choose a threshold value say T in this valley region and we take a decision that if a pixel at location xy have the intensity value $f(x, y)$ which is the greater than T ; then we say that these pixel belongs to object whereas if $f(x, y)$ is less than or equal to the threshold T , then these pixel belongs to the background.

So, this is our simple decision role which is to be used for thresholding purpose. So, what we have to do is we have to choose a threshold in the valley region and then check the image the segmentation is simply testing each and every pixel to check whether its intensity value is less than the threshold or the intensity value is greater than the threshold.

So, if the intensity value is greater than the threshold, then we will say that **it belongs** the pixel belongs to an object whereas if the intensity value is less than or equal to threshold, we say that the pixel belongs to the background.

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Now, the situation can be even more general. That is instead of having bimodal histogram, we can have multimodal histograms. That is our histogram can even be of this form, like this. So, this is our pixel intensity z and on this side is the histogram. So, here you find that the histogram has three different modes which are separated by two different values. So now, what we can do is we can choose one threshold say T_1 in the first value region and the other threshold T_2 in say second value region.

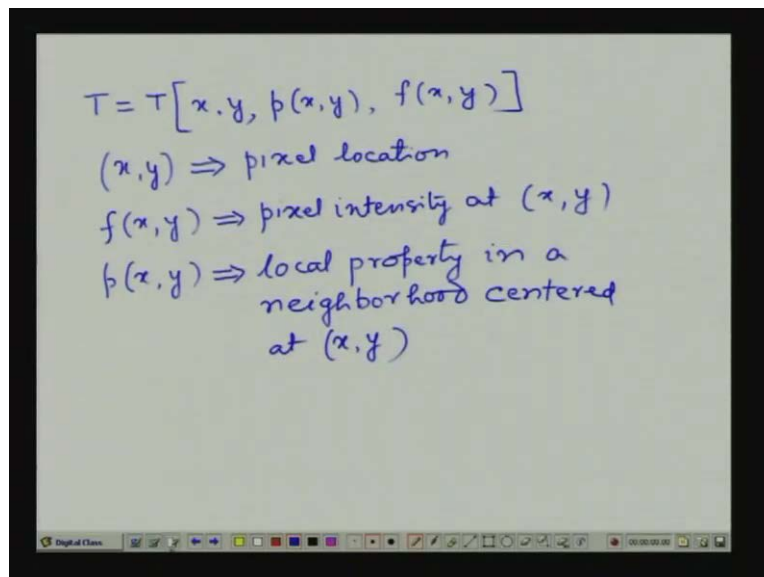
So, what this histogram indicates is that there are three different regions or three different intensity regions which are separated by some other intensity band and those three different intensity regions are represented or gives rise to these three different peaks in the histogram. So here, our decision role can be something like this, that if we find that the intensity value $f(x, y)$ at a pixel location (x, y) is greater than threshold T_2 , then we say that the point (x, y) belongs to

say object O_2 . So, all the intensity values of all the pixels having intensity values greater than T_2 , these pixels, we say that they belong to the object O_2 .

In the other case, if a pixel has an intensity value in this region that is **greater than T_2** greater than T_1 and less than T_2 , then we will say that this particular pixel belongs to object O_1 . So, our decision rule will be that T_1 less than $f(x, y)$ less than or equal to T_2 , then this indicates that the corresponding pixel (x, y) , it belongs to object O_1 .

And obviously, the third condition will be that if $f(x, y)$, the intensity value at a location (x, y) is less than threshold T_1 ; in that case, we say that the corresponding pixel (x, y) , it belongs to the background. So, even in cases, we can have histograms which are even which will have even more number of peaks more than three peaks; such cases also, similar such classification is possible. But what we have to do for this thresholding based segmentation technique is that we have to choose proper threshold values.

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Now, this threshold value of the thresholding operation can be considered as an operation that involves testing against a function T where this function T is of the form T is equal to $T[x, y, p(x, y)$ and $f(x, y)]$. So, this thresholding operation, what we are doing is we are considering or this can be viewed as an operation to test the image pixels against a function T where this function T is of this form this, function T is a function of (x, y) which is nothing but the pixel location in the image, $f(x, y)$ which is nothing but the intensity value at location (x, y) ; so this is pixel intensity at location (x, y) and $p(x, y)$ it is some local neighborhood property some local property in a neighborhood centered at (x, y) .

So, in general, this threshold T is a function can be a function of pixel location, the pixel value as well as the local property within a neighborhood around a pixel location (x, y) . So, any combination of these three that is pixel location, pixel value and **neighbor property** neighborhood property, this neighborhood property can even be the average intensity values within a

neighborhood around pixel (x, y) ; so **any combination of this** T can be a function of any combination of these 3 terms and depending upon the combination, this T can be either a global threshold or a local threshold or it can even be an adaptive threshold.

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The image shows handwritten notes on a whiteboard. The first three lines define thresholding functions: $T[f(x,y)] \Rightarrow \text{Global}$, $T[f(x,y), p(x,y)] \Rightarrow \text{Local}$, and $T[(x,y), f(x,y), p(x,y)] \Rightarrow \text{Adaptive/Dynamic}$. The fourth line defines a binary image $g(x,y)$ as a piecewise function: $g(x,y) = \begin{cases} 1 & \text{if } f(x,y) > T \rightarrow \text{object} \\ 0 & \text{if } f(x,y) \leq T \rightarrow \text{background} \end{cases}$. At the bottom, there is a software toolbar with various drawing tools.

So, in case, **the T is** the threshold T is only a function of $f(x, y)$, we say that the threshold is a global threshold whereas if T is a function of $f(x, y)$ and the local property that is $p(x, y)$, then we say that the threshold T is a local threshold and if in addition to all this, T is also a function of the location of the pixel that is in the more general case, if T is a function of (x, y) , $f(x, y)$ as well as $p(x, y)$, then we say that this threshold T is an adaptive or dynamic threshold.

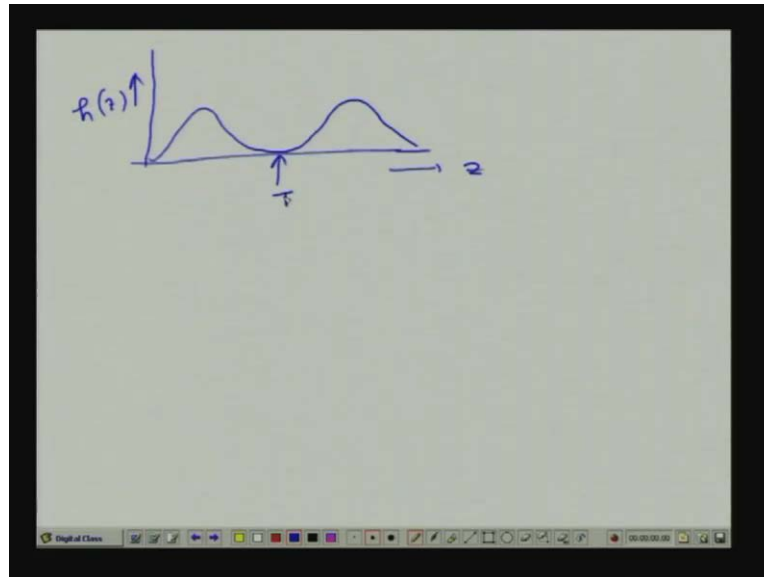
Now, whichever the nature of the threshold T is whether it is local or global or adaptive; our thresholding operation is by using this threshold, we want to create a thresholded image say $g(x, y)$ from our input image $f(x, y)$ and we said the value of $g(x, y)$ is equal to 1 if the corresponding function or the intensity of the image at that location that is $f(x, y)$ is greater than the threshold T . Now, this threshold T can be either global or local or adaptive and we said $g(x, y)$ is equal to 0 if $f(x, y)$ is less than or equal to the chosen threshold T .

So, you will find that the basic aim of this thresholding operation is we want to create a thresholded image $g(x, y)$ which will be a binary image containing pixel values either 0 or 1 and this value will be set to 0 or 1 depending upon whether the intensity $f(x, y)$ at location (x, y) is greater than T or it is less than or equal to T .

So, if we have a bright object against a dark background; in that case, $g(x, y)$ equal to 1, this indicates that the corresponding pixel is an object pixel whereas $g(x, y)$ equal to 0, this will indicate the corresponding pixel is a background pixel. On the contrary, if we have dark objects against bright background; in that case, what we will do is we will set $g(x, y)$ equal to 1 if $f(x, y)$ is less than or equal to T again indicating that in the thresholded image, a pixel location having an intensity value of 1 that indicates the corresponding pixel belongs to the object and in

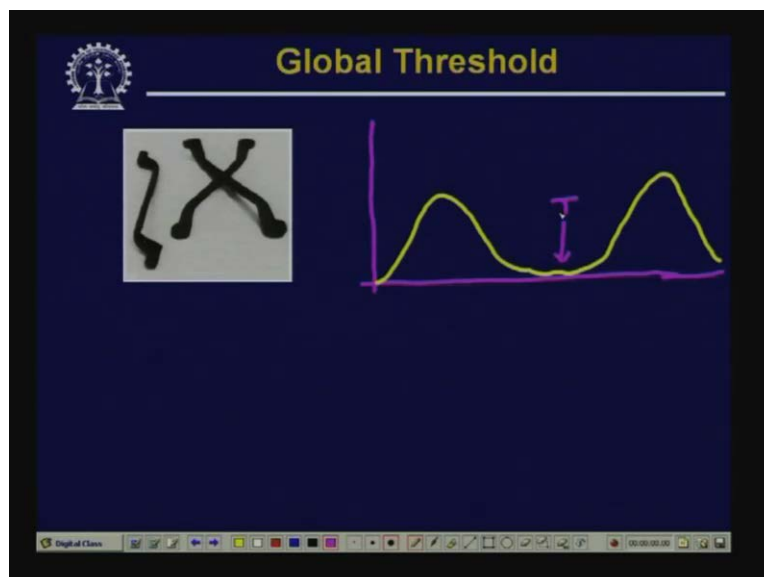
such case, we will put $g(x, y)$ equal to 0 if $f(x, y)$ is greater than T again indicating that a pixel in the thresholded image $g(x, y)$ if it is equal to 0, the corresponding pixel is a background pixel. Now, the question is how to choose this threshold value?

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Now, for that let us come to the case; again considering the histogram we have said that if my histogram is a bimodal histogram of this form, then what I can do is by looking at the histogram, so this is our intensity value z and on this side we have $h(z)$, by inspecting this histogram, we can choose a threshold in this deep value region and using this threshold, I can go for the segmentation operation. Now by doing this, I will show you one particular result.

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Say for example, in this particular case, here, you find that we have an image where the objects are dark whereas the background is bright. So naturally, in this case, I will have a histogram where the histogram will be a bimodal histogram. So, the nature of the histogram will be like this. So, this will be bimodal histogram. So here, if I choose a threshold T in this region and using this threshold, I segment this image, then the kind of segmentation that we get is as given here.

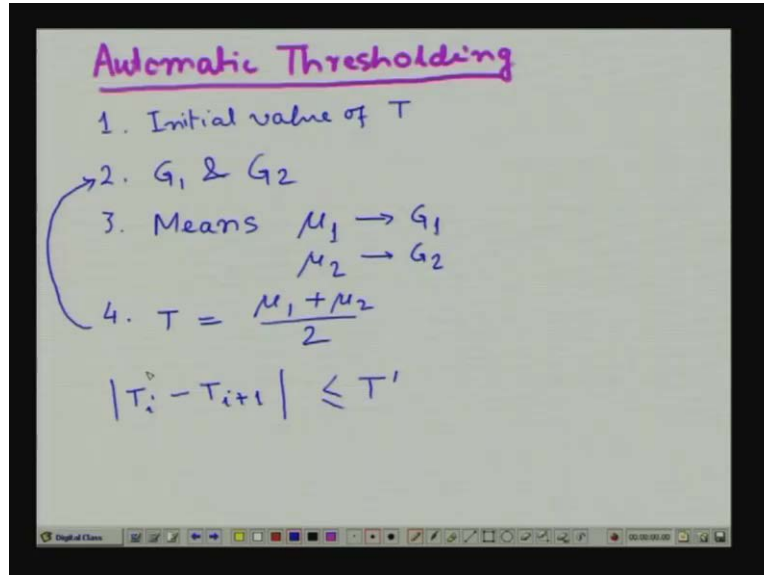
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So, here you find, in the second image in the segmented image that your background and object regions have been clearly separated; even the shadow which is present in the original image that has been removed in the segmented image. So, this segmentation, though it is a very very simple operation if you choose the threshold in the value region between the two modes in a bimodal histogram; then this segmentation, this simple segmentation operation can clearly take out the object regions from the background.

But here what we have done is we have chosen the histogram to choose the threshold. That is you inspect the histogram and then from inspection of the histogram, you have to choose the threshold value. But is it possible to automate this process? That is **instead of finding the histogram** by instead of finding the threshold value by looking at the histogram; can we automatically determine what is the threshold value which should be used for segmenting an image? So, this operation can be done by using an iterative procedure.

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So, automatic threshold; so here again for detecting this threshold automatically, what we can do is we can first choose an initial value of threshold. So, arbitrarily or by or somehow, we can choose an initial value of threshold and using this initial value of threshold, what we can do is we can have a segmentation of the image.

So, when you segment the image using this initial value of threshold, the segmentation operation basically will partition your histogram into two partitions or the image will be divided into two groups of pixels. So, you can say that one group of pixels, we term them as group G_1 and the other group of pixels we term them as group G_2 . So, the pixel intensity values in group G_1 will be similar and the pixel intensity values in group G_2 will also be similar but these two groups will be different.

Now, once I separate or partition the image intensities into these groups G_1 and G_2 , the next step that what we will do is you compute the means or the average intensity values μ_1 for group G_1 and the average intensity value μ_2 for group of pixels G_2 . So, once I get this μ_1 and μ_2 that is the average intensity value in the group of pixels G_1 and also the average intensity value for the group of pixels G_2 , then in the fourth step, what I do is I choose a new threshold T which is equal to μ_1 plus μ_2 divided by 2.

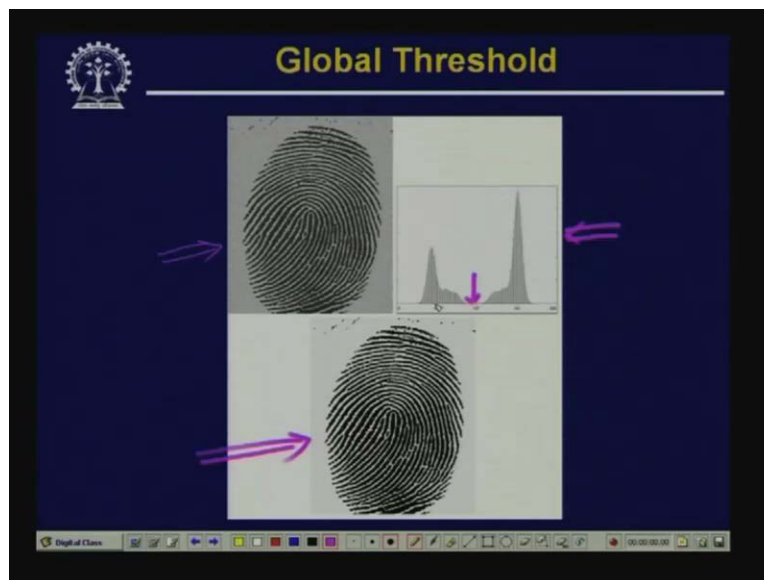
And, after doing this, you go back to step two and perform the operation, thresholding operation once again. So, what we are doing is we are choosing an initial value of threshold, using that initial value of threshold we are thresholding the image, by thresholding what we are doing is we are separating the intensity values into two groups G_1 and G_2 ; for group G_1 , I find out the average intensity value μ_1 , for group G_2 , I also find the average intensity value G_2 , then I find out a new threshold which is the mean of these two averages that is μ_1 plus μ_2 by 2 and using this new threshold, I threshold the image again so there by these groups G_1 and G_2 will be modified and I repeat this process that is thresholding to grouping, then finding out the intensity averages in the two different groups two separate groups, recalculating the threshold; this entire

process will be repeated until and unless I find that the variation in two successive iterations in the computed value of T is less than some pre specified value.

So, this operation has to continue until you will find that in one at iteration T_i and the next iteration T_i plus 1, the threshold value in the i 'th iteration T_i and **in the i plus** iteration T_i plus 1; the difference between these two is less than or equal to some pre specified value say T prime. So, when I attain this condition, I stop my thresholding operation.

So, here you will find that we do not have to go to the histogram to choose the threshold. Rather, what we do is we choose some initial value of threshold, then go on modifying this threshold value iteratively; finally you converse, you come to a situation where you find that in two subsequent iterations, the value of the threshold does not change much and at that position whatever the thresholded image that you have got that is your final thresholded value.

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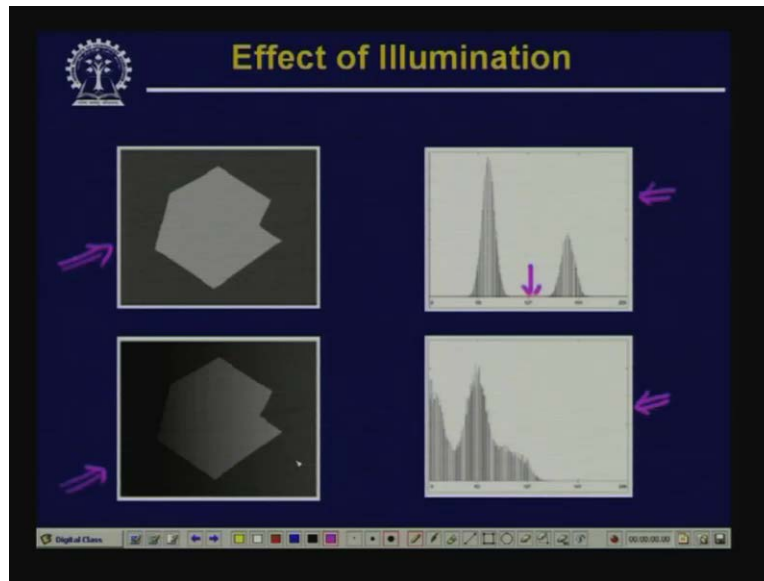
So, using this kind of adaptive threshold, the kind of result that can be obtained is something like this. So, here you will find that this is one input image and you can identify this that this is a finger print image. This is the histogram of that particular image. So obviously, from this histogram also, I can choose a threshold somewhere here.

But this thresholded output that has been obtained is not by choosing a threshold from the histogram but this is by automatic threshold selection process that is by doing this iterative process and it can be observed that from this histogram whatever threshold you choose by this automatic process, the threshold will be similar to that and here you will find that since the threshold that you have chosen, this does not consider the pixel location or the local neighborhood of the pixel intensity values.

Here, the threshold is a global one that is for the entire image, you choose one particular threshold and using that threshold, you go for segmenting the image. So, the kind of thresholding

operation that we have done in this particular case, this is called a global thresholding operation. Now, you find that in this particular case, this global thresholding will give you very good result if the intensity of the illumination or the scene is uniform. But there may be cases where the scene illumination is non uniform and in case of such non uniform illumination, getting a global threshold which will be applicable over the entire image is very very difficult.

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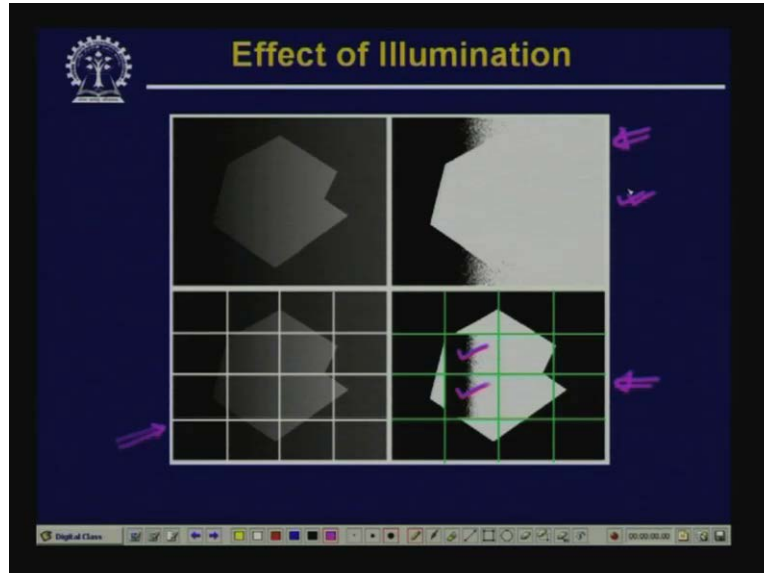


So, let us take one particular example, say in this particular case; on the top, we have an image and you can easily find out that for this image if I plot the histogram, the histogram will be as shown on the right hand side. Clearly, this histogram is a bimodal histogram and there is a valley in between the two modes. So, these modes are separated by a deep value. So obviously, for such a kind of histogram, I can always choose a threshold inside the valley and segment this image successfully.

But what happens if the illumination is not proper? If the background illumination is not uniform, then this image, because of this non uniform illumination may turn out to be an image like this and whenever I have such an image with poor illumination and you find that the histogram of this image appears as being on the right hand side and here you find that though the histogram appears to be a bimodal one, but the valley is not well defined. So, this simple kind of thresholding operation or the global thresholding operation is likely to fail in this particular case.

So, what should we do for segmenting these kinds of images using the thresholding operation? Now, one approach is you subdivide this image into a number of smaller sub images. Assuming that in each of this sub image, the intensity will be more or less uniform or the illumination is more or less uniform; then for each of the sub image, we can find out a threshold value and using this threshold value, you can threshold the sub images and then the combination of all of them or the union of all of them will give you the final thresholded output.

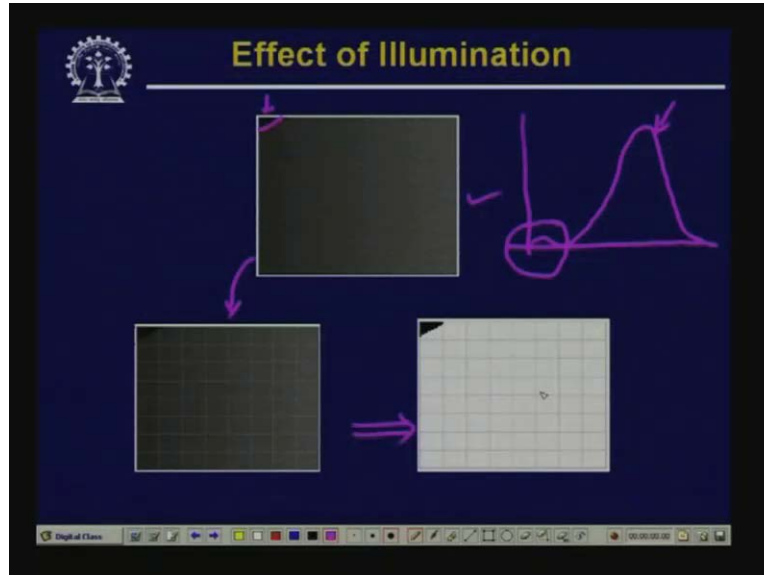
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So, let us see what we get in the case. As we said that for these kinds of images where the illumination is non uniform, if I apply a single global threshold; then the kind of output, the thresholded output that we are going to get is something like this. So, here you find that the thresholding has failed miserably whereas if I subdivide this image into a number of sub images as given on this left hand bottom and then for each of these sub images, I identify the threshold and using that threshold, you go for segmenting that particular sub image and the thresholded output that you get is given on this right hand side.

Here, you will find that excepting these two, rest of the sub images has been thresholded properly. So, at least your result is better than what you get with a global threshold operation. So, now because we are going for a thresholds, lecture of a threshold which is position dependent because every sub image has a particular position; so now because this threshold selection is position dependent, it becomes an adaptive thresholding operation. Now, let us try to analyze that why this adaptive threshold has not been successful for these two sub regions.

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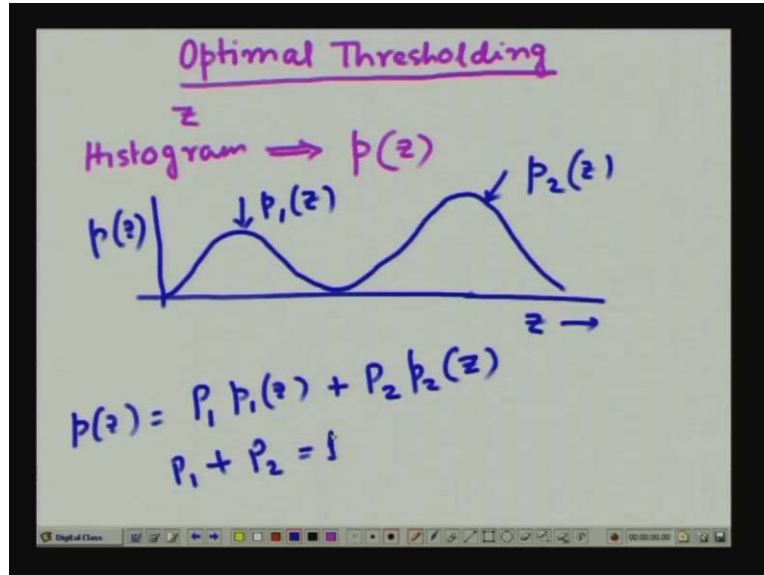
So, if I look at the nature of the image; here, if you look at this top image, you will find that in this top image, here is a boundary where this small portion belongs to the background and this large portion of the image belongs to the object. Now, if I plot the histogram of this, the histogram will be something like this; because the number of pixels in the background is very very small, so the contribution of those pixels to the histogram that is within this region is almost negligible.

So, instead of becoming a bimodal histogram, the histogram is dominated by a single peak and that is the reason why this thresholding operation has not given good result for this particular sub region. So, how to solve this problem? Again, our solution approach is same. You subdivide this image into smaller sub division, so you go for sub dividing further and for each of these smaller subdivisions, now you try to find out the threshold and segment each of the subdivisions with each of these sub **subdivisions** using this particular threshold. So, if I do that you will find that a kind of result that we get is here and here, the segmentation output is quite satisfactory.

So, if the scene illumination is non uniform, then a global threshold is not going to give us a good result. So, what we have to do is we have to subdivide the image into a number of sub regions and find out the threshold value for each of the sub regions and segment that sub region using this estimated threshold value and here, because your threshold value is position dependent, it depends upon the location of the sub region; so the kind of thresholding that we are applying in this case is an adaptive thresholding.

Now, in all these thresholding whether it is global thresholding or adaptive thresholding that we have discussed so far, none of these cases we have talked about the accuracy of the thresholding or how accurate or what is the error that has been involved that is by this thresholding process. So, we can go for a kind of thresholding by making use some statistical property of the image where the mean error of the thresholding operation will be minimum.

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So, that is a kind of thresholding operation which is called optimal thresholding. So, what is this optimal thresholding? Again, let us assume that the image contains two principle gray levels, intensity regions; one intensity region corresponding to the object and the other intensity region corresponding to the background and we use a variable and we assume that these intensity variables can be modeled as a random variable and this random variable is represented by variable say z .

Now, once we represent the random variable by this z , then the histogram of this particular image or the normalized histogram can be viewed as a probability density function of this random variable z . So, the normalized histogram can be viewed as a probability density function $p(z)$ of this random variable z .

Now, as we have assumed that the image contains two major intensity regions to dominate intensity vales; so our histogram is likely to be a bimodal histogram. So, the kind of histogram that we will get for this image is a bimodal histogram. So, it will be something like this and as we said that the histogram, we are assuming to be a probability density function of the intensity variable z .

So, this bimodal histogram can be considered as a combination of two probability density functions or combination of two periods. So, one of them is say probability distribution function $p_1(z)$, the other one is probability density function say $p_2(z)$. So, $p_1(z)$ indicates the probability distribution function, the probability density function of the intensities of pixels which belong to say background and $p_2(z)$ is the probability density function of the pixel intensity values which belong to say object.

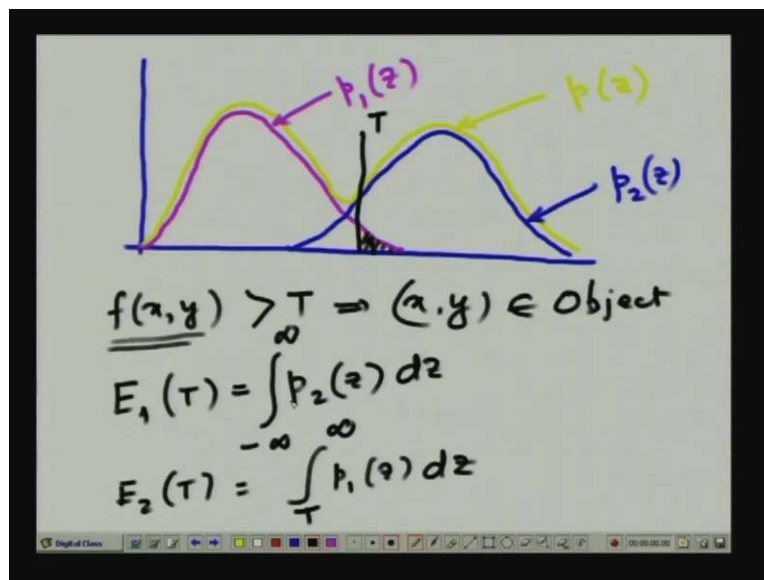
Now, this overall histogram that is $p(z)$ can now be represented as the combination of $p_1(z)$ and $p_2(z)$. So, this overall $p(z)$, we can write as capital P_1 into $p_1(z)$ plus capital P_2 into $p_2(z)$ where

this capital P_1 indicates the probability that a pixel will belong to the background and capital P_2 indicates that indicates the probability that a pixel belongs to an object.

So obviously, this capital P_1 plus capital P_2 , this will be is equal to 1. So, these are the pixel probabilities which belong to either foreground or the background. So here, our assumption is that we have a bright pixel against a dark background because we are saying that capital P_1 , sorry the capital P_1 , it is the probability that a pixel belongs to the background and capital P_2 is the probability that a pixel belongs to the foreground of the object.

Now, what is our aim in this particular case? Our aim is that we want to determine a threshold T which will minimize the average segmentation error.

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Now, you find that since this over all probability is modeled as a combination of two different probabilities; so it is something like this. I can say that I have one probability distribution function which is given by this and the other probability distribution function is say given by this so that my overall probability distribution function is of this type, this is my over all probability distribution function.

So, this blue colour, this indicates $P_2(z)$ and the pink colour this indicates $P_1(z)$ and the yellow colour indicates my overall probability density function that is $p(z)$. So, in this particular case, if I choose a threshold T somewhere here, so this is my threshold T and I say that if $f(x, y)$ is greater than T , then (x, y) belongs to object. Now here, you find that though we are taking a hard decision that if $f(x, y)$ is greater than T , then (x, y) belongs to object but the pixel with intensity value $f(x, y)$ also has a finite probability; say given by this that it may belong to the background. So, while taking this decision, we are incorporating some error. The error is the area given by this probability curve for the region intensity value greater than T .

So, the probability of considering a background point as an object point or **the error** leads to an error that is a background point may be classified as an object point. So, the error that you encounter in that particular case is given by say $E_1(T)$ because this error is threshold T dependent; so write this as $E_1(T)$ is equal to say $\int_T^\infty P_2(z) dz$, take the integral of this minus infinity to infinity.

So, what is this? This is the probability that this is the error incorporated that an object pixel may be classified as a background pixel. Similarly, if a background pixel is classified as an object pixel, then the corresponding error will be given by $E_2(T)$ is equal to integral $\int_{-\infty}^T P_1(z) dz$ where the integral has to be taken from T to infinity.

So, these give you the two error values. One of them gives the error that you encounter if you classify a background pixel as an object pixel and the other one if you segment object pixel as a background pixel.

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The image shows a whiteboard with the following handwritten equations:

$$E(T) = P_2 E_1(T) + P_1 E_2(T)$$

$$\frac{\partial E(T)}{\partial T} = 0$$

$$\Rightarrow P_1 p_1(T) = P_2 p_2(T) \leftarrow$$

$$p(z) = \frac{P_1}{\sqrt{2\pi}\sigma_1} e^{-\frac{(z-\mu_1)^2}{2\sigma_1^2}} + \frac{P_2}{\sqrt{2\pi}\sigma_2} e^{-\frac{(z-\mu_2)^2}{2\sigma_2^2}}$$

So, from these two error expressions, the overall error probability can now be represented as $E(T)$ is equal to capital P_2 into $E_1(T)$ plus capital P_1 into $E_2(T)$. So, you find that this E_2 was the probability, it was the error of classifying a background pixel as a foreground pixel and $E_1(T)$ was the error of classifying an object pixel as a background pixel and P_1 is the probability that a pixel belongs to background and capital P_2 is the probability that a pixel belongs to the object. So, the overall probability of error will be given by this expression capital P_2 into $E_1(T)$ plus capital P_1 into $E_2(T)$.

Now, for minimization of this error, what we have to do is we have to take the derivative $\frac{\partial E(T)}{\partial T}$ and equate this two 0. So, whatever the value of T that you get that is what is going to be will the minimum error. So, if put this restriction, then this above expression; we are not going into the details of mathematical derivation, I will just give you the final result. This can be given by capital P_1 into $p_1(T)$ plus capital P_2 into p_2 **sorry** this is not plus this is equal. So,

we are going to get an expression of this form and the solution of this equation gives the values of T.

So, if we try to solve this, you will find that what I need is the knowledge of this probability density functions - $p_1(T)$ and $p_2(T)$. So, as we know that in most of the cases, we normally assume the Gaussian probability density function. So, if I assume that Gaussian probability density function; in that case, the overall probability $p(z)$ is represented by capital P_1 divided by square root of $2\pi\sigma_1$ e to the power minus z minus μ_1 square by $2\sigma_1$ square plus capital P_2 by square root of $2\pi\sigma_2$ e to the power minus z minus μ_2 square by $2\sigma_2$ square where μ_1 is the average intensity value of the background region and μ_2 is the average intensity value of the object region and σ_1 and σ_2 , they are the standard deviations of the intensity values in the background region and the intensity values in the object region.

So, by assuming this Gaussian probability density function, we get the overall probability density function as given by this expression.

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The image shows a whiteboard with handwritten mathematical equations. The equations are:

$$AT^2 + BT + C = 0$$

$$A = \sigma_1^2 - \sigma_2^2$$

$$B = 2(\mu_1\sigma_2^2 - \mu_2\sigma_1^2)$$

$$C = \sigma_1^2\mu_2^2 - \sigma_2^2\mu_1^2 + 2\sigma_1^2\sigma_2^2 \ln\left(\frac{\sigma_2 P_1}{\sigma_1 P_2}\right)$$

$$\sigma_1^2 = \sigma_2^2 = \sigma^2$$

$$T = \frac{\mu_1 + \mu_2}{2} + \frac{\sigma^2}{\mu_1 - \mu_2} \ln\left(\frac{P_2}{P_1}\right)$$

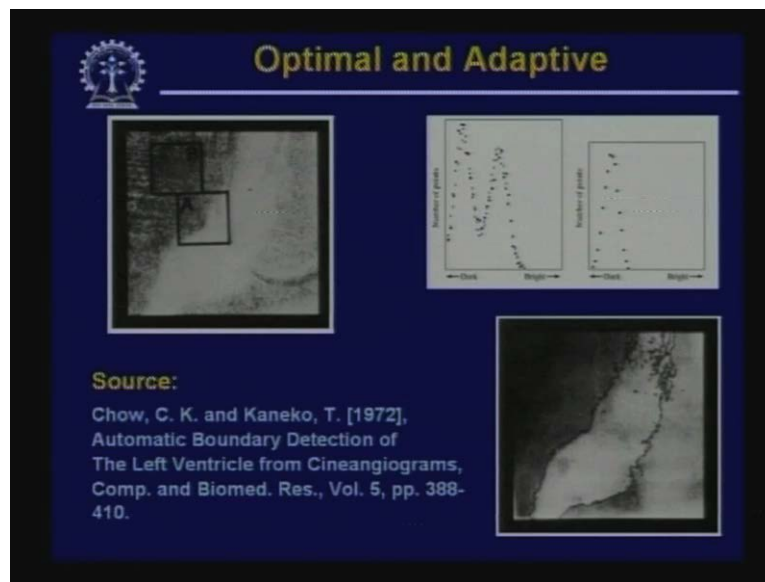
And, by assuming this and then from this particular expression, the value of T can now be found out as the solution for T is given by, solution of this particular equation - AT^2 plus BT plus C is equal to 0 where this A is equal to σ_1 square minus σ_2 square, B is equal to 2 into μ_1 σ_2 square minus μ_2 σ_1 square and C is given by σ_1 square μ_2 square minus σ_2 square μ_1 square plus 2 σ_1 square σ_2 square \ln σ_2 capital P_1 by σ_1 capital P_2 and here if we assume that σ_1 square is equal to σ_2 square is equal to say σ square; then the value of the threshold T comes out to be T is equal to μ_1 plus μ_2 divided by 2 plus σ square upon μ_2 μ_1 minus μ_2 \ln capital P_2 divided by capital P_1 .

So, this is a simple expression for the value of threshold that we can obtained in this optimal thresholding operation and this is optimal in the sense that these value of the threshold gives you

minimum average error and here again, you find that if the probability - the capital P_1 and capital P_2 , they are same; in that case, the value of T will be simply $\mu_1 + \mu_2$ by 2 that is the mean of the average intensities of the foreground region and the background region.

So, as we said that by estimating a threshold by this process, if we segment the image; then the average error of segmentation will be minimum. That is minimum number of foreground pixels will be classified as object pixels and minimum number of object pixels will be classified as foreground pixels.

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Now, let us see an example that where these optimal thresholding can give us good results. Let us take a very complicated case like this. This is the cardiogram and geography in which the purpose is to detect the ventricle boundaries. You find that the image that is given here is very very complex and though we can somehow figure out that there is a boundary somewhere here but it is not very clear.

So, the approach that was taken is this image was divided into a number of sub images; for every sub image, the threshold was estimated the optimal threshold was estimated and then the thresholding was done. So, for this optimal thresholding, what was done is for each of the sub image, say for example, this was divided into a number of sub images like this; for each of the sub image, what was computed is the histogram and the threshold was computed for those sub images which shows a bimodal histogram like this whereas you will find that if I take a sub image here, this normally shows a uni... histogram which is given here.

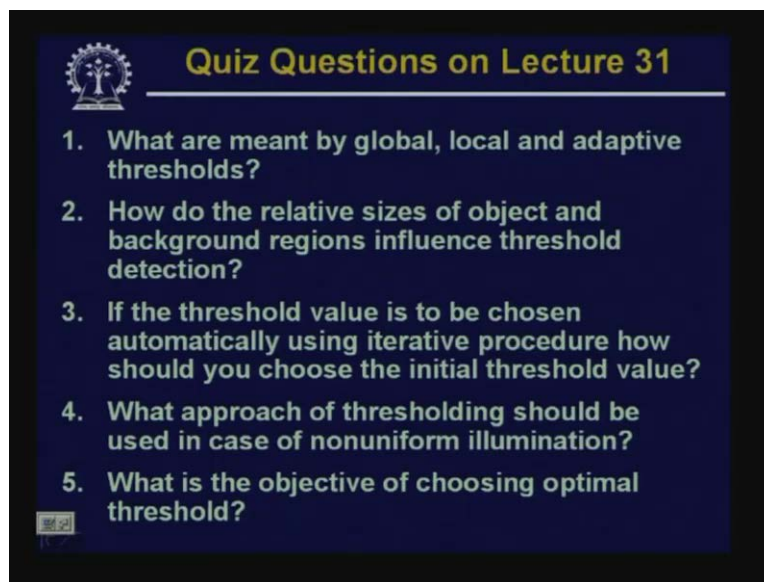
For these sub images, no threshold was detected. The threshold was detected only for those sub images which showed bimodal histogram and the threshold for other sub images were estimated by interpolation of the threshold of the regions having bimodal histogram and then a second level of interpolation was done, iteration was done to estimate the threshold value at each of the pixel locations and after doing that for each pixel location using that particular threshold, the decision

was taken whether the corresponding value should be equal to 0 or the in the threshold that you know the corresponding value should be equal to 1.

So, using this, the thresholded image was obtained and the boundary of such thresholded image when super imposed on this particular image, you will find that this one shows the boundary of the thresholded image. So, as was estimated that this was the estimated boundary, the boundary points are quite well estimated in this particular case.

So, with this, we stop this particular lecture on thresholding operations. Now, let us see some of the quiz questions on today's lecture.

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The first question is what is meant by global, local and adaptive thresholds? The second question, how do the relative sizes of object and background regions influence threshold detection? The third question, if the threshold value is to be chosen automatically using iterative procedure, how should you choose the initial threshold value? The fourth question, what approach of thresholding should be used in case of non uniform illumination? And, the last question, what is the objective of choosing optimal threshold?

Thank you.