Digital Image Processing

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Lecture - 30

Image Segmentation - II

Hello, welcome to the video lecture series on digital image processing. In today's class, we will continue with our discussion on image segmentation.

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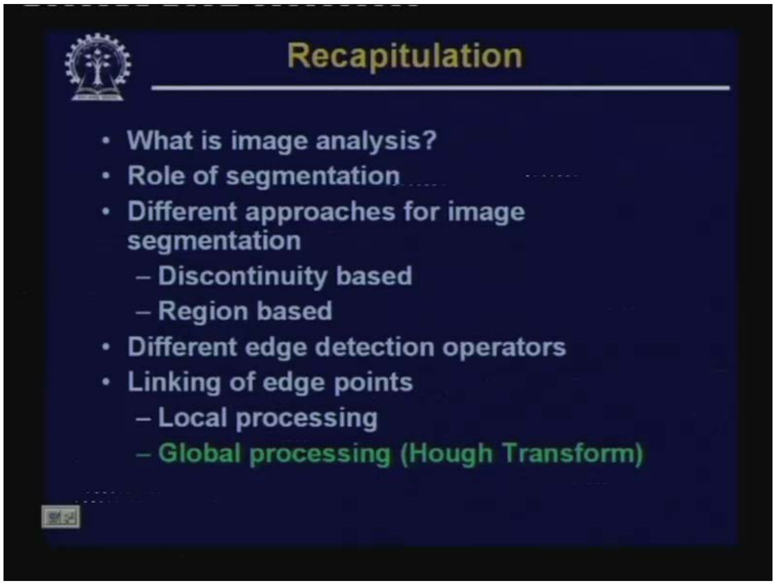

In the last class, what we have seen is we have discussed about the image analysis, we have seen what is image analysis and the role of image segmentation in image analysis process. We have seen that there are mainly 2 approaches for image segmentation; the first approach is discontinuity based image segmentation technique and the second approach is region based image segmentation technique.

What we are discussing now is the discontinuity based image segmentation technique. Then, we have seen that to implement discontinuity based image segmentation technique; first, what we have to do is we have to detect the edges present in the image. Edge means it is a region where there is a variation either from the low intensity value to a high intensity value or from a high intensity value to a low intensity value.

So in such transition regions, we have to detect the position of an edge and by this, what is expected is to get the boundary of a particular segment. But the problem we have discussed in this process that because of the problem of illumination, if the illumination is non uniform or if the image is noisy; in that case, the boundary points or the edge points that we get after the edge detection operation, those points are not continues.

So, take care of this problem, what we have to do is after we detect the edge points, we have to link the edge points. So, we have said that there are 2 approaches for linking the edge points. The first approach that we have discussed in the last class is the local processing approach and the second approach that we will be discussing today that is a global processing approach which is also called Hough transformation.

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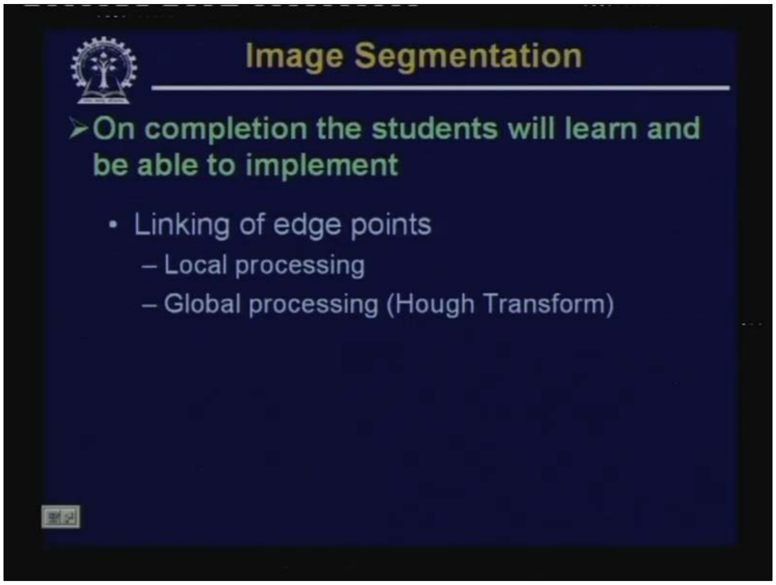

So, after today's lecture, the students will be able explain and implement the local processing technique for linking the edge points and also the global processing technique that is the students will be implement the Hough transformation to link the edge points.

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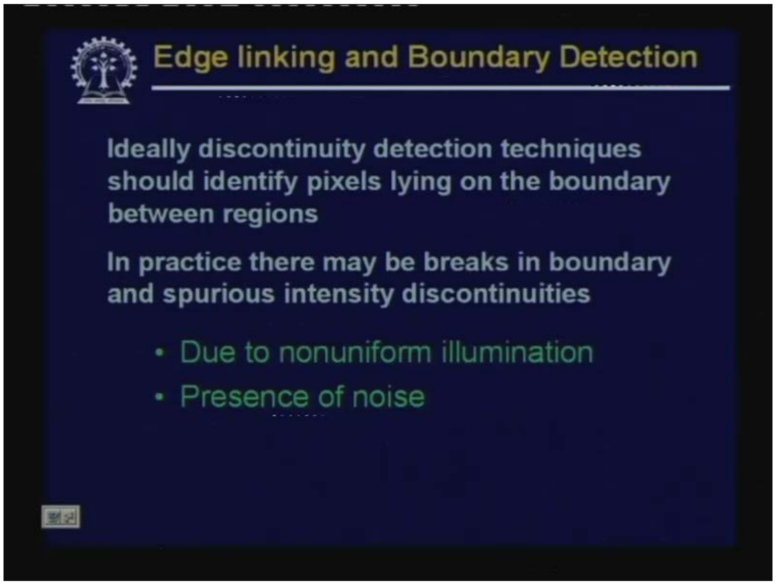

So, let us just see that what we have seen in the last class. We have said that ideally, this edge detection technique should identify the pixels line on the boundary between the regions. We say it is the boundary between the regions because we assume that it is a transition, this region is a transition region from a low intensity values, from low intensity region to a high intensity region or from a high intensity region to a low intensity region.

But while trying to implement this, it has been found that the edge points which we expect to be continues to give us a meaningful boundary description of a segment that cannot be achieved in practice. Now, this is mainly due to 2 reasons; first one is due to non uniform illumination of the scene. If the scene is not uniformly illuminated that leads to detection of edge points where the boundary points will not be continues and the second reason for getting this non continues boundary points is the presence of noise. That is if the image is noisy, then after doing the edge detection operation either the boundary points will not be continues or there may be some spurious edge points which are not actually edge points of the boundary points of any of the regions.

So, to tackle this problem, we have to go for linking of the edge points so that after linking, we get a meaningful description of the boundary of a particular segment.

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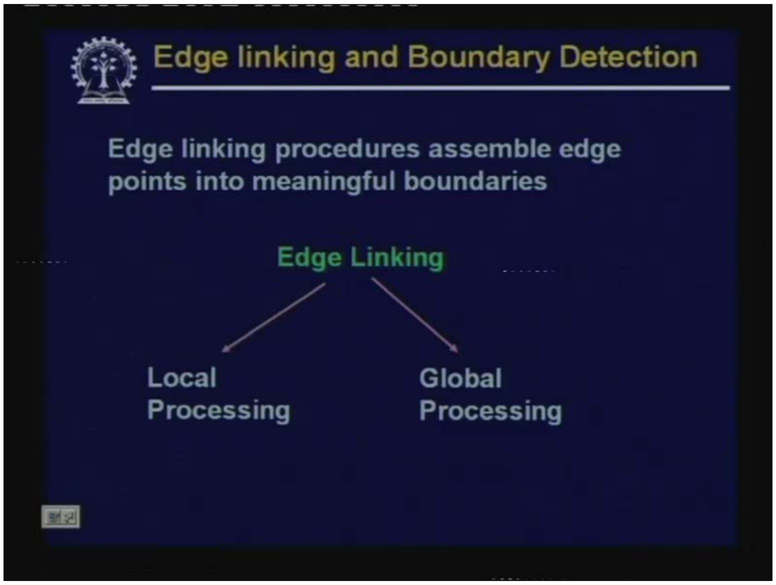

So, we have said that there are mainly 2 approaches for edge linking operation. The first approach is the local processing approach and the second approach is the global processing approach domain.

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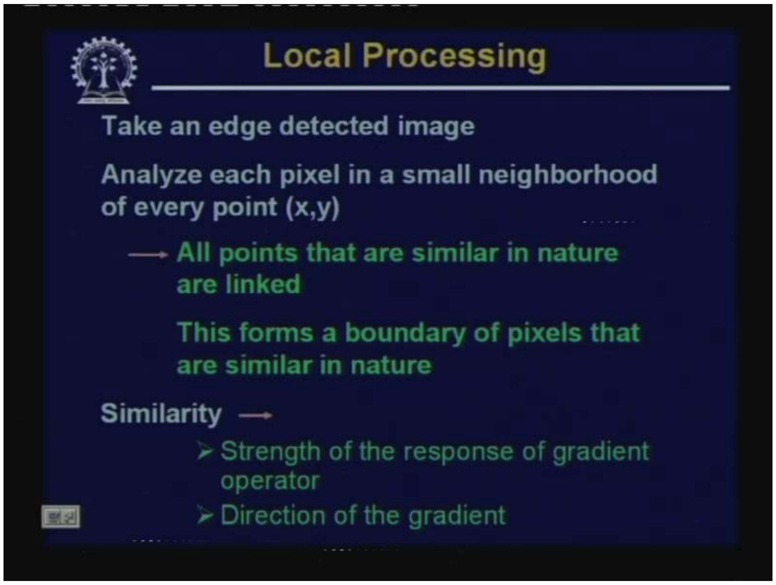

In the local processing approach, what we do is we take an edge detected image that is the image that we have as an input. This is an image containing only the edge pixels, so we assume that edge points will be white and all the non edge points will be black and in this edge detected image, we analyze each pixel in a small neighborhood.

So for every point (x, y) if that is an edge pixel; we take a small neighborhood of point (x, y) and we link the other edge points within this neighborhood with the point (x, y) if they are similar in nature.
So whenever, we find that within the neighborhood, we have 2 edge points which are similar in nature; then we link these edge points and after linking all such edge points, we get a boundary of pixels that are similar in nature. So basically, what we get is a well defined boundary of a particular segment.

So, when you say that we have to link the edge points which are similar in nature, then we have to have some similarity measure. So, we remember that after edge detection operation, for every edge point, we get 2 quantities. One is the boundary strength at that particular edge point and the second quantity that is the direction of edge at that particular edge point.

So, by comparing the boundary strength as well as the direction of a boundary at point (x, y) and at a point which is in the neighborhood of (x, y); we try to find out whether these 2 points are similar or not. So, if these 2 points are similar, we simply link that.

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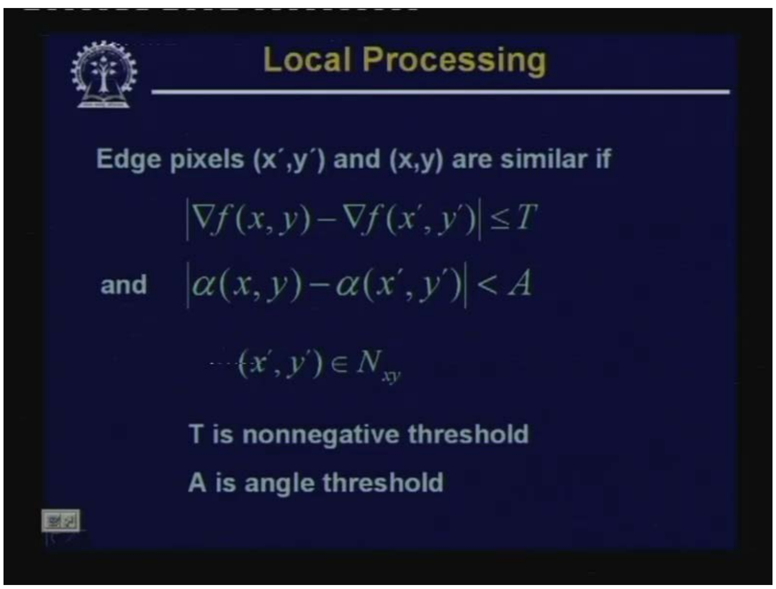

So for this, what you have to do is we take an edge point (x, y) and we find out a point (x dash, y dash) which is in the neighborhood of (x, y). So, what we are doing is we are taking this point (x, y) and considering the point (x dash, y dash) which is in the neighborhood of (x, y) and we find out the difference of edge strength.

We know that this gradient operator f (x, y) is the image function and f (x, y) gives the intensity value at location (x, y) in the image and grad f (x, y), this gives you the gradient of the intensity value at location (x, y).

So, you compute the gradient of at location (x, y) and also the gradient at location (x dash, y dash) and if the difference between these 2 is less than or equal to certain threshold T where T is a non negative threshold and at the same time so this gives you whether the strength is similar or not and at the same time, we also have to check whether the direction at this edge which is given by this alpha (x, y) at location xy and alpha x dash at location (x dash, y dash); so if their if the orientation of the direction of the edge is also similar that is the difference is less than or equal to some angle threshold value A, then we consider these 2 points (x, y) and (x dash, y dash) to be linked together.

So, in this particular case, our (x dash, y dash) has to be in the neighborhood of (x, y). So, that is what is represented by (x dash, y dash) belongs to neighborhood of (x, y). So, this is the local processing technique.

But as we said that we are going for the linking of the edge points because the edge points are discontinues and normally the neighborhood size that is taken is a small neighborhood. So, if (x dash, y dash) is not within the neighborhood of (x, y) over a given over a given definition of the neighborhood; in that case, (x dash, y dash) cannot be linked with the edge point (x, y).

So, to solve this problem because (x dash, y dash), the 2 edge points can be far apart depending upon the amount of that noise you have or the depending upon the lighting condition but we have to be we should be able to link those points as well. So, in such cases, the local processing technique does not help to link the edge points. What we have to go for is the global processing technique.

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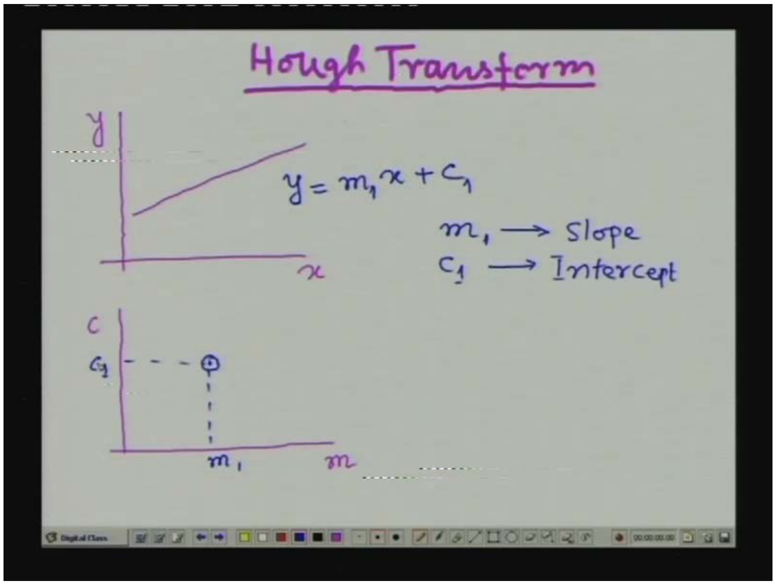

And, the global processing technique that we will discuss today is called the Hough transformation. So, it is Hough transform. So, what is this Hough transform? The Hough transform is a mapping from the spatial domain to a parameter space. So, let us take an example. Suppose, I have this (x, y) coordinate system and I have a single straight line in this (x, y) coordinate system and we know that in the slope intercept form, this straight line is described by a equation which is given by y is equal to mx plus c where m is the slope of the straight line and c is the intercept value.

Now, for a particular straight line, the values of m and c will be constant. So, I represent them by m_1 c_1 indicating that these 2 values - the slope and the intercept are constant for a particular given straight line in the xy plane. So, you find that this particular straight line is now defined, is now specified by 2 parameters. One of the parameters is m_1 which is the slope of the straight line and the other parameter is c_1 which is the intercept.

Now, if I map this straight line to the parameter space because I have 2 parameters m and c that is slope and intercept; so our parameter space will also be a 2 dimensional space. So, what I am trying to do is I am trying to map this straight line in the parameter space. So, I draw this mc plane, I will have the slope m along one direction and the intercept c along another direction and since for this given straight line y equal to m_1x plus c_1; m_1 that is the slope and c_1 that is the intercept is fixed. So, this particular straight line will be represented by a single point in the mc plane and this point is at location m_1 and c_1.

So, you find that when I map a given straight line in the spatial domain to the parameter space, a straight line gets map to a single point. Now, let us see what happens if we are given a point in the spatial domain that is in the xy plane, we are given a particular point. Let us see the situation; what will happen in this case.

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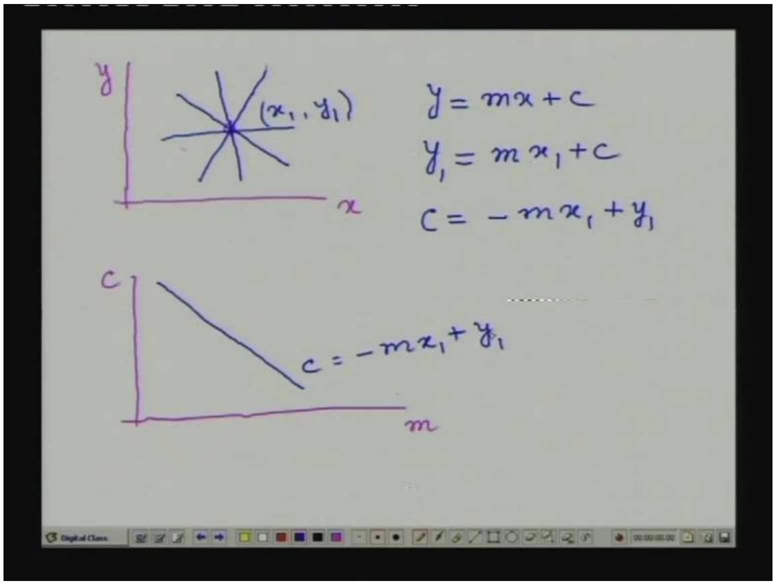

So now, what I have is I have again this xy plane and in the xy plane, I have a single point and let us assume the coordinate of this point is (x_1, y_1). Now, you find that equation of any straight line in the xy plane as we have seen earlier in the slope intercept form is given by y is equal to mx plus c. Now, if this straight line y is equal to mx plus c has to pass through this given point (x_1, y_1); then $(x_1\ y_1)$ must satisfy this equation.

So, in effect what I will get is I will get an equation that is y_1 is equal to mx_1 plus c because this line y is equal to mx plus c is passing through the given point (x_1, y_1) and this is the equation that has to be satisfied by all the straight lines that passes through this point (x_1, y_1).

Now, you find that ideally I can have infinite number of straight lines passing through this given point (x_1, y_1). So, there will be infinite number of straight lines like this and for each of these straight lines, the value of the slope that is m and the intercept c, it will be different.
So, if I now map this single straight line in our parameter space that is mc plane, you will find that this m and c, these 2 become the variable where as y y_1 and x_1, they are the constants. So now, what I can do is I can rewrite this equation that is y_1 equal to mx_1 plus c in this way, I can write it as c is equal to minus mx_1 plus y_1. So here, what I have is I have this x_1 and y_1, these 2 are constants and c and m are variable.

So, if I map this point (x_1, y_1) into our parameter space; so what I will have now is I will have this mc plane and in the mc plane, you will find that c equal to minus mx_1 plus y_1, this is now the equation of a straight line. So effectively, what I get is I get a straight line in the mc plane following the equation c is equal to minus mx_1 plus y_1. So, we have seen 2 cases that in one case, a straight line in the xy plane is mapped to a point in the mc plane and in the other case, if we have a point in the xy plane that is mapped to a straight line in the mc plane and this is the basis of the Hough transformation by using which we can link the different edge points which are present in the image domain which is nothing but the spatial domain or we can say that this is nothing but our xy plane.

So now, let us see that what happens if I have 2 points in our spatial domain or the xy plane.

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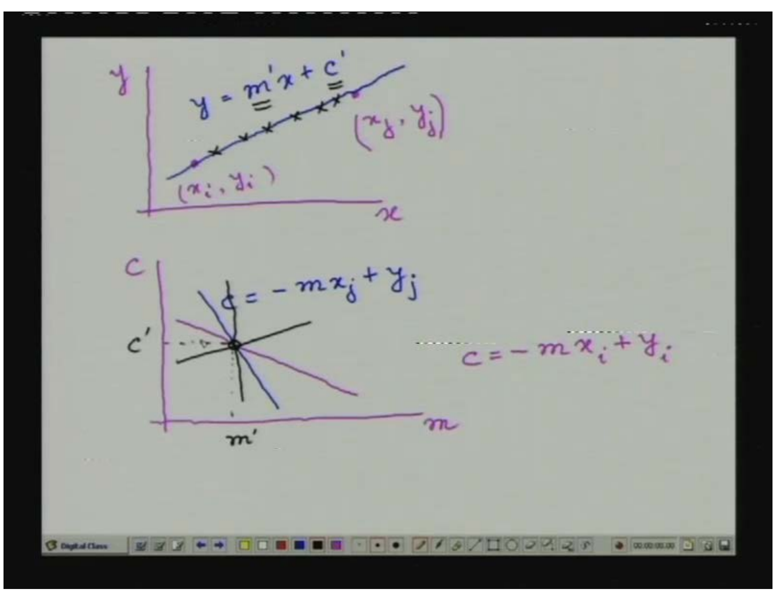

So again, I go to our spatial domain or xy plane; so this is my x axis and this is my y axis and suppose, I have 2 points - one is say (x_i, y_i) and the other point I have in this spatial domain is (x_j and y_j. Now, if I draw straight line passing through these points – (x_i, y_i) and (x_j, y_j); say this is the straight line which passes through these 2 points (x_i, y_i) and (x_j, y_j) and we know that this straight line will have an equation of the form - y equal to say m dash x plus c dash.

So, what we have to do by using the Hough transformation is that given these 2 points- (x_i, y_i) and (x_j, y_j), we have to find out the parameters or the equation of the straight line which is passing through these 2 points - (x_i, y_i) and (x_j, y_j).

Now, as we have seen earlier that a point in the xy plane is mapped to a straight line in the mc plane or in the parameter space. So here, in this case, since we have 2 points in the xy plane, this will be mapped to 2 different straight lines in the mc plane. So, if I draw in the mc plane, if I find the mapping in the mc plane, it will be something like this. So, I have this parameter space or mc plane, the first straight line the first point (x_i, y_i) will be mapped to a straight line like this where equation of this straight line will be given by c equal to minus mx_i plus y_i and the second point will be mapped to another straight line say something like this where the equation of this straight line will be given by c equal to minus mx_j plus y_j.

And, you will find that the point at which these 2 straight lines meet that is this particular point, this is the one which gives me the values of m and c. So, this will give me the value of m dash and c dash and this m dash and c dash are nothing but the parameters of the straight line in the xy plane which passes through these 2 given points (x_i, y_i) and (x_j, y_j).

Now, if I consider that there are infinite numbers of points or there are a large number of points lying on the same straight line in the xy plane; each of these points will be mapped to a particular straight line in the mc plane like this. But each of these straight lines will pass through this single point m dash c dash in the parameter space. So by this, what we have seen is as we know that if there is a single point in the parameter in the spatial domain or in the xy plane that is mapped to a single straight line in the parameter space that is mc plane.

So, if I have a set of collinear points in the xy plane, each of these collinear points will be mapped to a single straight line in the parameter space or in the mc plane. But all these straight lines corresponding to the points which are collinear in the xy plane will pass through, will intersect at a single point and this point in this case is m dash c dash the values of which that is m dash and c dash, these give us the slope and intercept values of the straight line on which all those collinear points in the spatial domain lie and this is the basic essence of using Hough transformation for linking the edge points.

Now, how do you compute this Hough transformation? So far, what we have discussed, this is in the continues domain that is our assumption is the values of m and c or m and c are continues variables. But in our implementation, since we are considering the digital cases; we cannot have the continues values of m and c. So, we have to see that how this Hough transformation can really be implemented. So for implementation, what we have to do is this entire mc space has to be subdivided into a number of accumulator cells. So, as has been shown in this particular figure.

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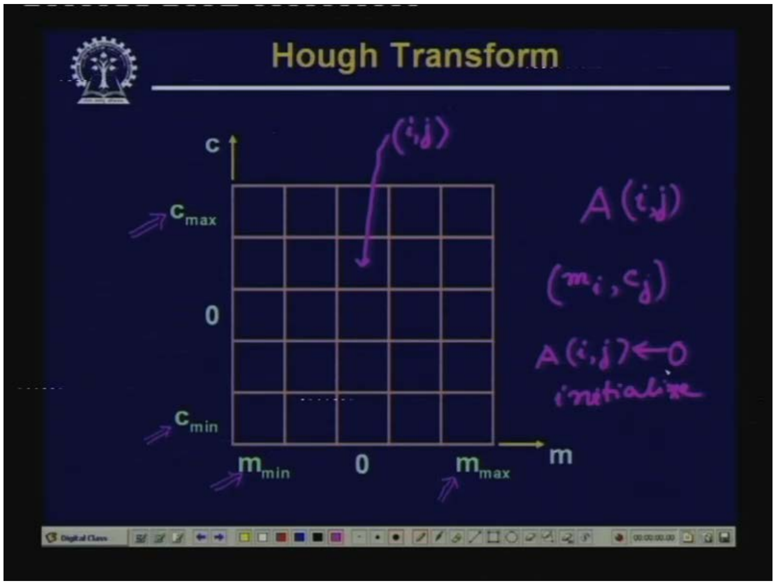

So here, you will find that what we have done is this mc space mc plane is divided into a number of smaller accumulator cells. So here, we have a range of the slopes which are the expected range of slopes in a particular application and the range is from a minimum that is at the minimum slope to a maximum slope. So, this is the minimum slope value - m minimum and this is the maximum slope value - m maximum.

Similarly for c; this is also subdivided and the total range is within an expected a maximum value and a minimum value. So, c minimum is the expected minimum value of the intercept c and c maximum is the expected maximum value of the intercept c. So, within this minimum and maximum within this range, this space is divided into a number of accumulator cells and this array, this 2 dimensional array; let us name this as an array say A and each of these accumulator cell locations can be indexed by say index i and j.

So, a cell location at location say (i, j), I may call this location as is a cell location (i, j) and the corresponding accumulator cell will have a value say A (i, j). So, this A (i, j), this particular cell the ij'th cell corresponds to the parameter values let us say m_i and c_j. So, an ij'th, an accumulator cell (i, j) having an accumulator value A (i, j) corresponds to the corresponding parameter values m_i and c_j.

And for implementation, what we do is we initialize each of these accumulators to 0. So initially, A (i, j) is set to 0 and that is our initialization. So, once we have this array of accumulator cells; then what we do? In the spatial domain, we have a set of boundary points and in the parameter space; we have a 2 dimensional array of accumulator cells.

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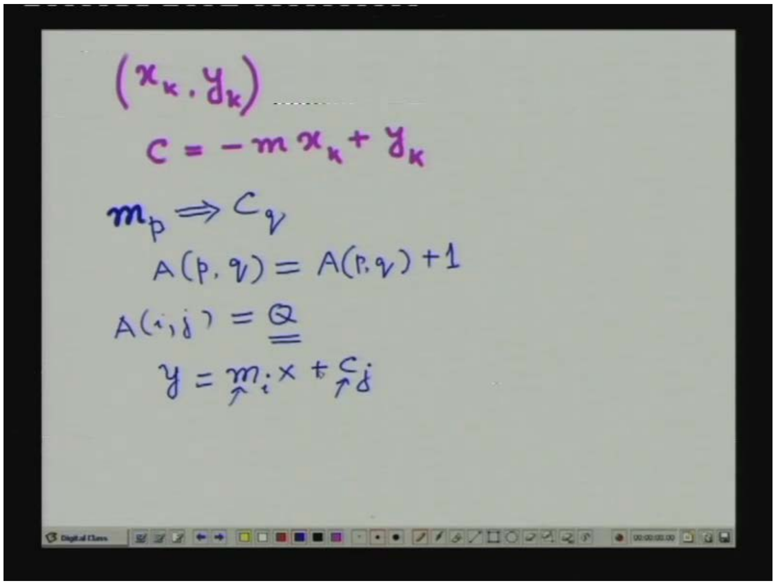

So, what we have to do is we have to take a boundary point say (x_k, y_k), a single boundary point in the spatial domain and we have seen earlier that this boundary point (x_k, y_k) is mapped to a straight line in the parameter space that is in the mc plane and the equation of this straight line is given by c is equal to minus m x_k plus y_k. So, what we have to do is we have to find out the values of m and c from this particular equation.

Now, in our case, what we have is the values of m and c is not continuous but they are discrete and as we have said that an ij'th accumulator cell corresponds to the corresponding parameter values m_i and c_j. So, to solve for values of m and c, our basic equation is c equal to minus m x_k plus y_k. So here, what we do is we allow the value of m to vary from the minimum to the maximum as we have said that we have chosen a range a minimum to a maximum; so, we allow the value of m to take all possible values or all allowed values as specified in our accumulator cell ranging from the m minimum to m maximum and for each of these values of m, we solve for the corresponding value of c following this particular equation c is equal to minus m x_k plus y_k.

Now, the value of c that you get by solving this particular equation for a specific value of m, that may be a real number whereas, we have to deal with the discrete case. So whenever, we get a value which may be a real number or may be the value of c that we get which is not allowed as per our definition of the accumulator cell, then what you have to do is this value of c has to be rounded off to the nearest allowed value of c as specified in the accumulator cell.

So, if I have say n number of such possible values of m, I will get n number of corresponding values of c by solving this equation - c equal to minus m x_k plus y_k. So, suppose by following this a particular choice of m say m_k or we have already used k for something else instead following it m_k, let us call it say particular value of m, say m_p; when I put this value of m_p, this mp in this equation, suppose the corresponding value of c I get after solving this equation is a C_q and you remember that we have initialized our accumulator cells to all the cells having a value of 0.

So whenever, for a particular value of m_p, I get a value of C_q that is the intercept value, then the operation I do is the corresponding accumulator cell A(p, q) is incremented by 1. So, I make A (p, q) is equal to A (p, q) plus 1. So, this I have to do for all the points, all the boundary points in our spatial domain that is in the xy plane and for each of this boundary points, I have to compute it for every possible or every allowed value of m as allowed in our parameter space. So, at the end, you will find because for every computed value or computed values m_p C_q, I am incrementing the accumulator cell by 1 or the accumulator cells were initialized to 0; so you find that at the end of the process if an accumulator cell A (i, j) contains a value say capital Q. So, we are considering this after we consider all the boundary points in the spatial domain.

For each of these, we compute the corresponding say m_p and C_q, for all allowed values of m, I find out the corresponding allowed values of c and for each such pair of m_p and C_q, I do this incrimination operation on the accumulator cell. So, at the end if an accumulator cell say A(i, j) contains a value of say capital Q, this indicates that there are Q number of points lying on a straight line whose equation is given by y is equal to m_i x plus C_j because as we said that this accumulator cell (i, j) corresponds to the slope value of m_i and the intercept value of c_j and for every point, wherever I get a corresponding value of m and c, the corresponding accumulator cell is incremented by 1.

So, at the end of the process if a particular accumulator cell A (i, j) contains a value capital Q, this is an indication that in the spatial domain, I have capital Q number of points or boundary points which are lying on the straight line y is equal to m_i plus c_j. Now, the question is what is the accuracy of this particular procedure? That is how accurate is this estimation of m_i and c_j? That depends upon how many number of accumulator cells I will have in the accumulator array.

So, if I have a very large number of accumulator cells in the accumulator array, then the accuracy of the computed m and c will be quite high whereas, if I have small number of accumulator cells in our accumulator array, then the accuracy of this computed value will be quite less. Now, the question is how can we use this to detect the number of straight lines present in the spatial domain?

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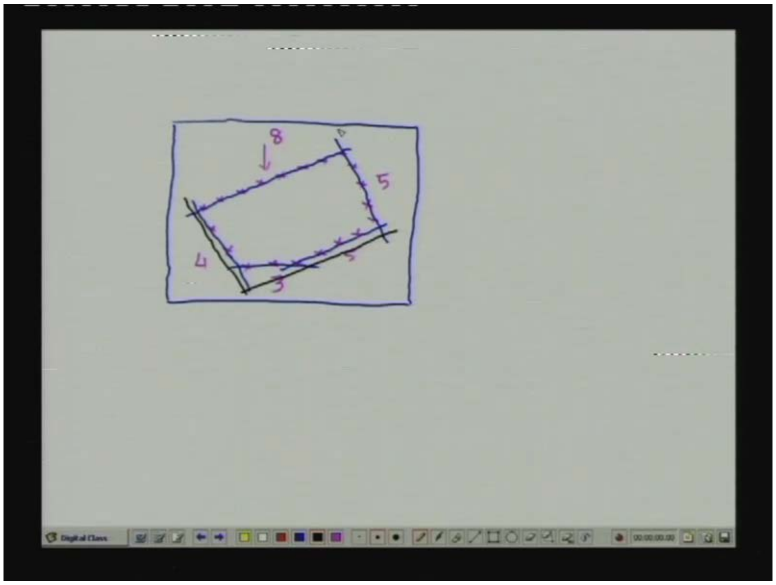

Let us consider a case like this. Say, I have an image in the spatial domain and the boundary points of this image are something like this. So, these are the boundary points in the straight line, so when I compute this Hough transformation, I get an accumulator cell. So, you will find that in this particular straight line there are 1, 2, 3, 4, 5, 6, 7, 8; 8 number of points on this side, on this straight line there are 1, 2, 3, 4, 5, 5 number of points, on this part of the straight line there are again 5 number of points, on this part of the straight line there are 4 number of points and if I consider that this part is also a straight line, so on this straight line there are 3 number of points.

So, in our accumulator cell at the end of this Hough transformation operation, I will get one cell with value equal to 8, I will get another cell with value equal to 5, I will get one more cell with value equal to 5, I will get one cell with value equal to 4 and I get another cell with value equal to 3 and if I choose all these values say if I say that I will consider a straight line to be significant if the corresponding accumulator cell contains a value greater than or equal to 3; then by this process, the number of straight lines that I will be able to detect is this straight line, this straight line, then this straight line, this straight line, as well as this straight line.

But if I say that I will consider only those straight lines to be significant where the number of points, number of collinear points lying on the straight line is greater than or equal to 4; then I will be detecting only this straight line, this straight line, this straight line and this straight line. So again, here you will find that by choosing that how many points i should consider to lying on a straight line so that the straight line will be significant and I consider this to be a boundary straight line, that also can be varied or tuned depending upon the application by choosing the threshold on this number of points lying on the straight line.

Now here, you find that in the mc plane, though we are able to find out the straight line segments but this particular formulation of Hough transformation that is mapping from xy domain to the parameter domain that is the mc plane has a serious problem. The problem is in mc plane, what we are trying to do is we are trying to find out the slope intercept value of the straight line in the spatial domain.

Now, the problem comes when this straight line tries to be vertical that is parallel to x axis. If the straight line is parallel to x axis, then the slope of the straight line that is the value of m tends to be infinity and in this formulation, we cannot tackle the value of m which becomes very large or which tends to become infinite. So, what should we do to solve this problem? So, to solve this problem, instead of considering the slope intercept form, what we can do is we can make use of the normal representation of a straight line.

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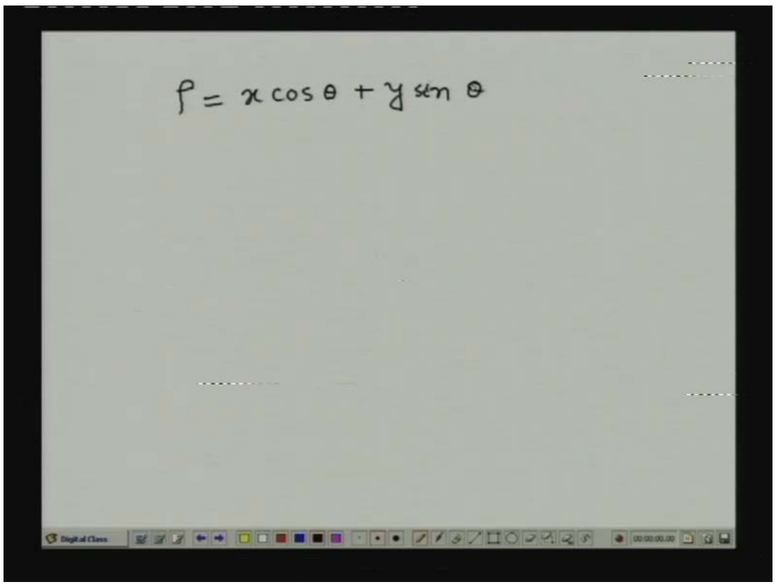

So, the normal representation of a straight line is given by this. The formula is rho equal to x cosine theta plus y sin theta and what we get in case of this normal representation? The line that we get is something like this.

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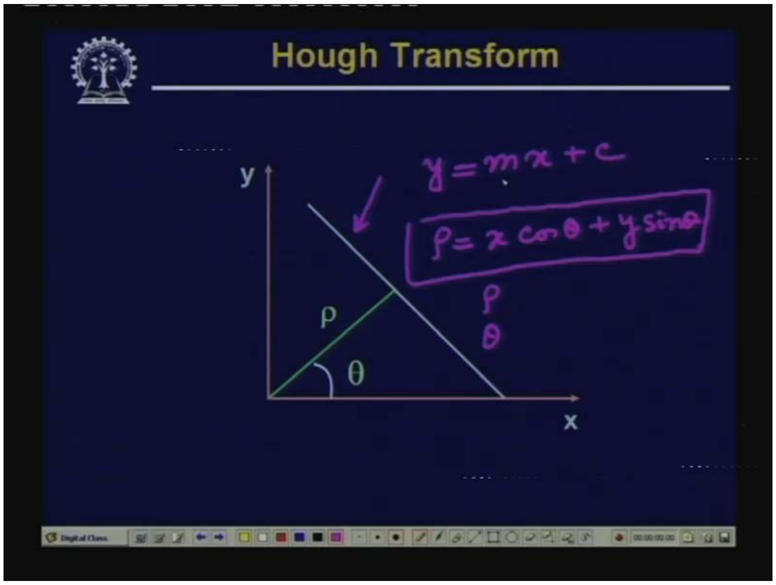

So here again, I have this straight line in the xy frame. But instead of taking the slope intercept form where the equation was given by y is equal to mx plus c, I take the normal representation where the equation of the straight line is given by rho is equal to x cosine theta plus y sin theta.

What is rho? Rho is the length of the perpendicular, it is the length of the perpendicular dropped on the straight line drawn from the origin of the xy frame and theta is the angle made by this perpendicular with the x axis.
So, you will find that the parameters of the straight line which is defined in this normal form - rho is equal to x cosine theta plus y sin theta, the parameters are rho which is the length of the perpendicular drawn from the origin to the given straight line and theta which is the angle formed by this perpendicular with the x axis. So, unlike in the previous case where the parameters were the slope m and c, now parameters become rho and theta.

And now, when I have these 2 parameters rho and theta, then the situation is quite manageable. That is I do not have the situation of leading to a parameter which can take an infinite value. So, we find that in this particular case, what can be the maximum value of rho and the maximum value of theta.

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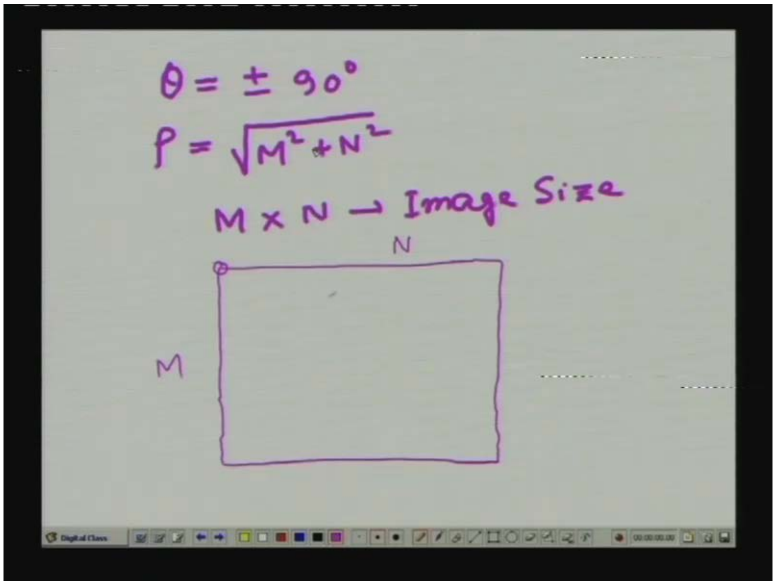

We consider the value of theta to be ranging in the range of plus minus 90 degree and the value of rho that is the length of the perpendicular to the straight line from the origin to be square root of m square plus n square where m by n is the image size and this is quite obvious because if I have an image of dimension say m by n; so here, the image dimension is there are m number of rows and say n number of columns and this is the origin of the image frame and in this, you can find that I cannot draw any straight line for which the value of theta will be beyond this range plus minus 90 degree and the value of rho will go beyond this plus minus. This should be plus minus square root of M square plus N square. But what is the difference of our earlier formulation with this formulation?

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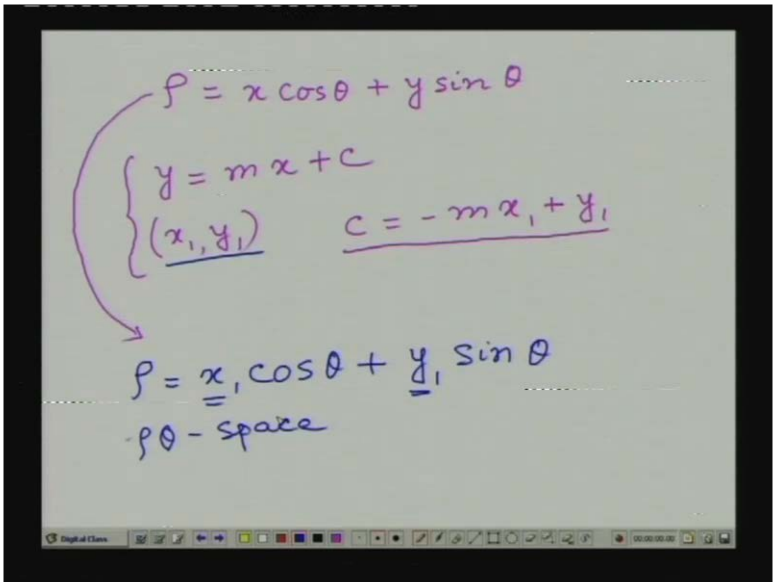

Now, in this particular case, our equation of the straight line is taken as rho equal to x cosine theta plus y sin theta whereas in our earlier case, our equation was y is equal to mx plus c. So, here, you will find that in this particular case, given a single point say (x_1, y_1) in the spatial domain in the parameter domain in the mc plane, the corresponding equation becomes c is equal to minus m x_1 plus y_1 which is again the equation of a straight line.

So, when you consider the parameter space to be mc space or we represent a straight line in the slope intercept form; in that case, a point is mapped to a straight line in the parameter space whereas in this particular case, for the same given point (x_1, y_1), in the parameter space, the equation that we get is rho is equal to x_1 cosine theta x_1 cosine theta plus y_1 sin theta.

So here, you find that this x_1 and y_1, these 2 are constants for a given point (x_1, y_1) in the parameter space whereas the rho and theta, they are variables. So, a particular point in the spatial domain is now mapped to a sinusoidal curved in the parameter domain or in the rho theta space. However, if I have q number of collinear points in the xy plane which will be mapped to q number of straight lines in the mc plane but all those straight lines will pass through a single point; in this case, the same q number of collinear points in the xy plane will be mapped to q number of sinusoidal curve in the rho theta plane but all those sinusoidal curve will intercept at a particular point, at the single point which gives us the values of rho and theta which are the parameters for the straight line on which all those q number of collinear points lie.

Now, this is the only difference between the mc plane and the rho theta plane. Apart from this, the formulation is exactly same as before.

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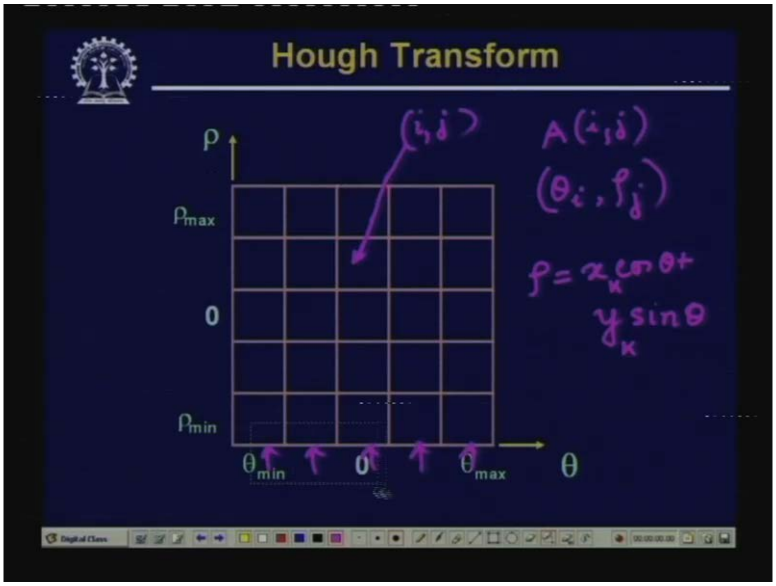

So for computation, again here, what we have to do is we have to define it, the rho theta space into a number of accumulator cells. So, the accumulator cells are given like this as in this figure. So, here again, any ij'th accumulator cell, an accumulator cell say (i, j) this ij'th accumulator cell which will have an accumulator value of say A (i, j) corresponds to our parameters theta $_i$ and rho$_j$.

So again, as per as we have done in our previous formulation that for a given point, as we have seen that our equation becomes rho is equal to x cosine theta plus y sin theta and for a given point say (x_k, y_k) in the spatial domain, our equation becomes rho is equal to x_k cosine theta plus y_k sin theta.

What we do is we allow the value of this variable theta to assume any of this allowed values as given in this accumulator cell. So, the theta can assume any of this allowed values between this given maximum and minimum and we solve for the corresponding value of the rho and because the solution of rho that you get may not be one of the allowed values, so what we have to do is we have to round off the value of rho to one of the nearest allowed values in our rho axis.

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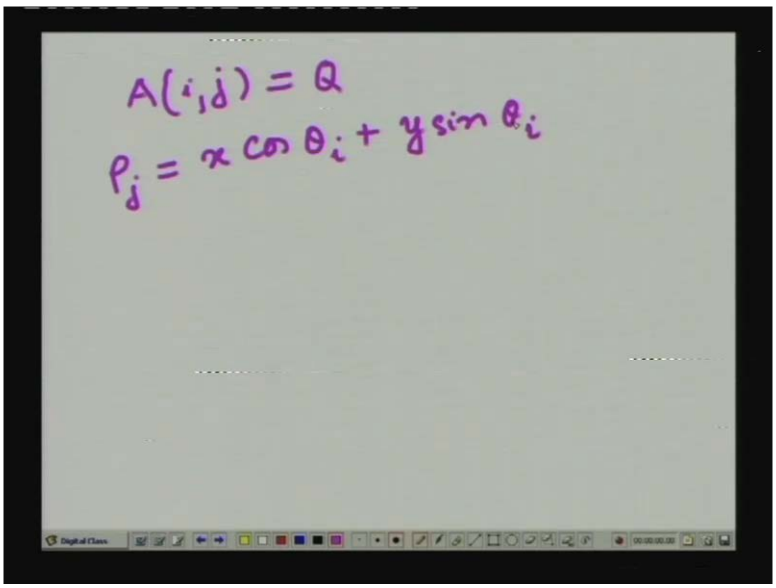

So again, as before that at the end of the process, if an accumulator cell say A (i, j) contains a value equal to Q, this means that there are Q number of collinear points in the spatial domain which in the spatial domain lying on the straight line which satisfy the equation - rho $_j$ is equal to x cosine theta $_i$ plus y sin theta $_i$.

So again as before, depending upon the number of points, putting a threshold on the number of points which we consider to be a significant point or not; I can determine how many straight lines in the given boundary image I will extract which will give me a meaningful boundary description. Now let us see, by applying this technique; what kind of result we can get.

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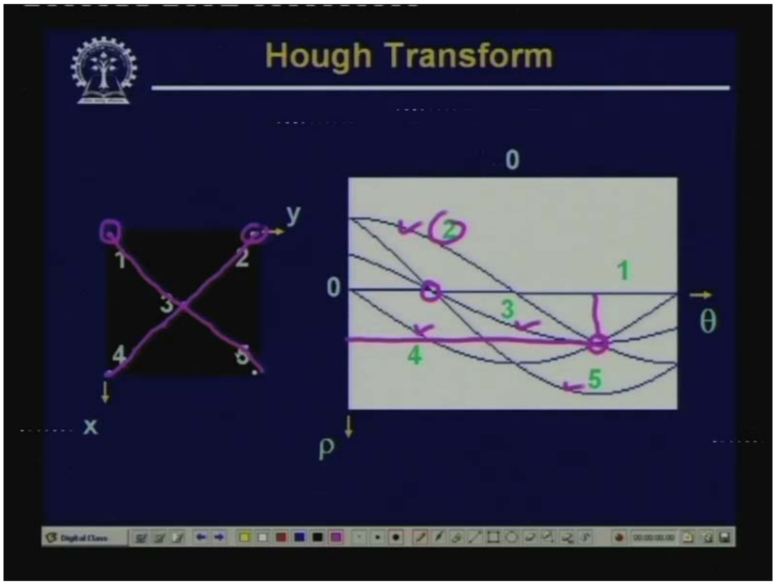

Here, what we have shown is as we have said that every point in the spatial domain is mapped to a sinusoidal curve in the parameter domain in the rho theta plane; so you find that this point 1, this point 1 over here has been mapped to a straight line, the point 2 has been mapped to a sinusoidal curve as has been given by this, the point 3 again has been mapped to this particular sinusoidal curve, 4 have been mapped to this particular sinusoidal curve and 5 have been mapped to this particular sinusoidal curve.

And now, you will find that if I want to find out the equation of the straight line passing through say 2, 3 and 4, these 3 points; then you will find that 2, 3 and 4, these 3 sinusoidal curves are meeting at this particular point in the rho theta plane. So, the corresponding cell will have a value equal to 3 indicating that there are 3 points lying on the straight line which satisfy this particular value of theta and this particular value of rho.

So from here, I can get the parameters of the straight line on which these 3 points 2, 3, 4, they are lying and same is true for other cases as well. For example, 1, 3 and 5; so you will find that this is the curve for 1, this is the curve for 3 and this is the curve for 5 and all of them are meeting at this particular point.

So here, whatever the value of theta and rho I get, that is the parameter of the straight line passing through these points - 1, 3 and 5.

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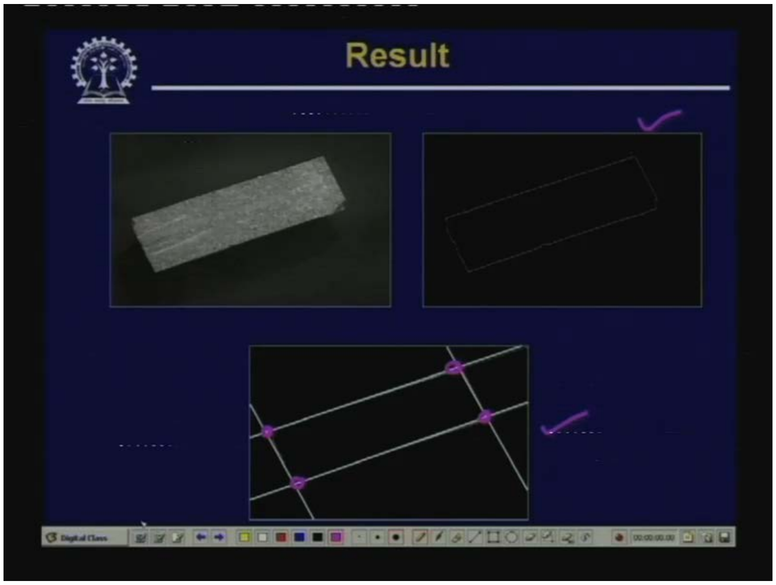

So, by applying this, you will find that in one of previous classes, we had shown this image. So here, we had shown that we have an image of a brick and after edge detection operation, what we get is the edge points are given on this right hand side. And now, if I apply the Hough transformation and try to detect the foremost significant straight lines, then by applying Hough transformation, I find these 4 straight lines which are most significant and the boundary which is specified by these 4 straight lines.

So, I can always find out that what are these vertex locations and this is a rectangular region which is actually the boundary of this particular object region. So, with this, we come to the end of our today's discussion that is the global edge linking operation where the global is linking is done by using Hough transformation and as we have said that this Hough transformation is nothing but a process of mapping from the special domain to the parameter space and now let us see some of the quiz questions on today's lecture.

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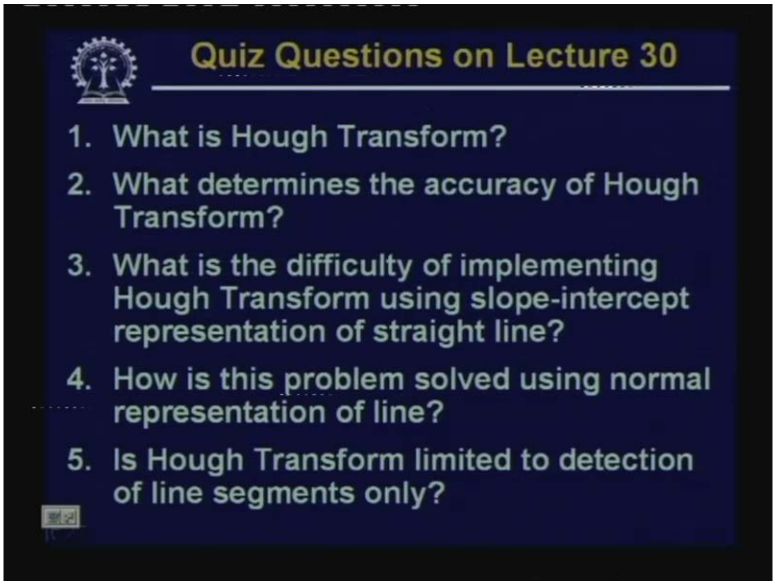

The first question is what is Hough transform? Second question, what determines the accuracy of Hough transform? Third question, what is the difficulty of implementing Hough transforms using slope intercept representation of straight line? Fourth, how is this problem solved using the normal representation of straight line? And fifth question: is Hough transform limited to detection of line segments only?

Thank you.