Digital Image Processing

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Lecture - 3

Image Digitization - II

Hello, welcome to the course on digital image processing.

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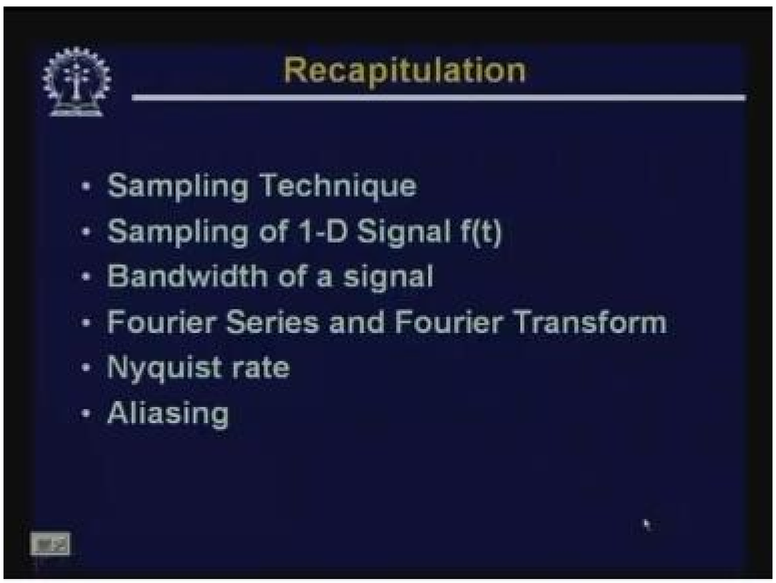

In the last class we have seen different sampling techniques particularly the sampling of 1 dimensional signal f (t) which is a function of a single variable t. We have also talked about what is meant by the bandwidth of a signal and to find out the bandwidth of a signal, we have made use of the mathematical tools like Fourier series and Fourier transform.

We have used Fourier series if the 1 dimensional signal f (t) is a periodic signal and if f (t) is an aperiodic signal, then we have made use of the Fourier transform to find out the bandwidth or the frequency spectrum of that signal.

Then we have talked about the sampling of this 1 dimensional signal and when we talked about the sampling, we have said that the sampling frequency must be greater than twice of the bandwidth of the signal to ensure proper reconstruction of the original signal from the sampled values and this particular minimum sampling frequency which has to be the twice of the bandwidth of the signal is known as Nyquist rate. And, you have also said that if the sampling frequency is less than nyquist rate that is less than twice the bandwidth of the signal, then what occurs is known as aliasing.

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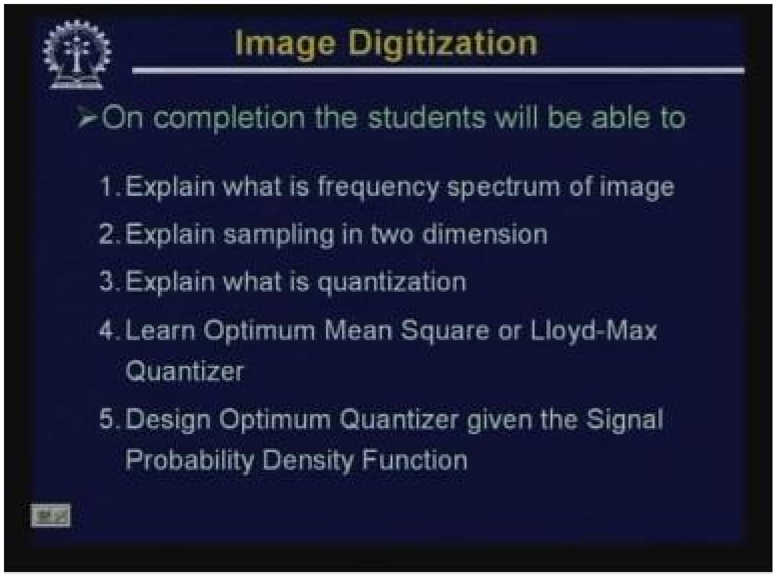

In today's lecture we will see the frequency spectrum of an image and we will also explain that how to sample the image in 2 dimension and then we will go to the second stage of the digitization process.

We have already said that image digitization consists of 2 faces. In the first face, we have to go for sampling and in the second face we have to go for quantization of each of the samples. Then we will talk about that what is meant by this quantization, we will also talk about the optimum mean square error or Lloyd-max quantizer. Then we will also talk about that how to design an optimum quantizer which with the given signal probability density function.

Now, let us briefly recapitulate that what we have done in the last class.

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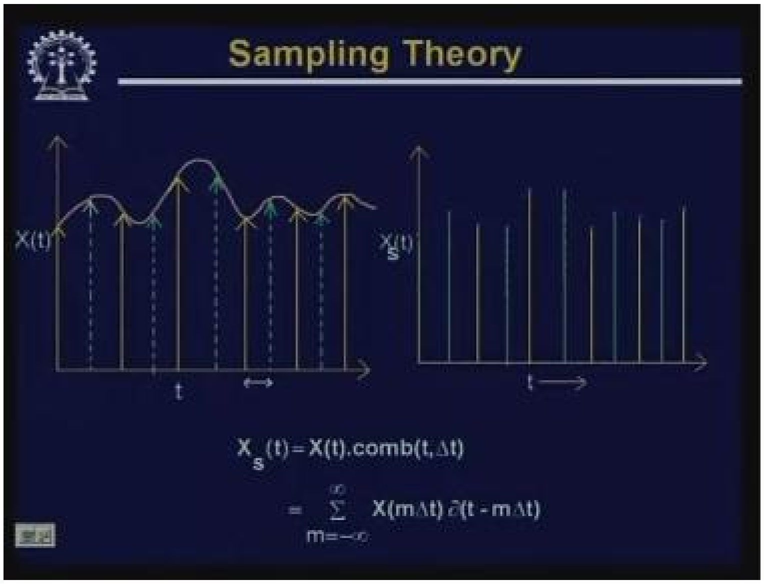

This is a signal X (t) which is a function of a single variable say t. And then, what we have done is we have sampled these 1 dimensional signal with a sampling function which is represented in the form of comb function say comb of t delta t and we get the sample values as represented by X_S (t) and we have we have also said that this X_S (t) can be represented in the form of multiplication of X (t) by comb function of t delta t.

Now, the same function can also be represented in the form of summation of X (m delta t) into delta (t minus m delta t) where this t minus m delta t, when you take the summation from m equal to minus infinity to infinity; this gives you what is the comb function.

So, this X_S of t that is the sampled value, that is the sampled version of the signal X (t) can be represented in the form of X (m delta t) into delta (t minus m delta t) where m varies from minus infinity to infinity.

Now, our problem is that given these sample values; how to reconstruct the original signal X (t) from the sampled values of X (t) that is X_S (t)?

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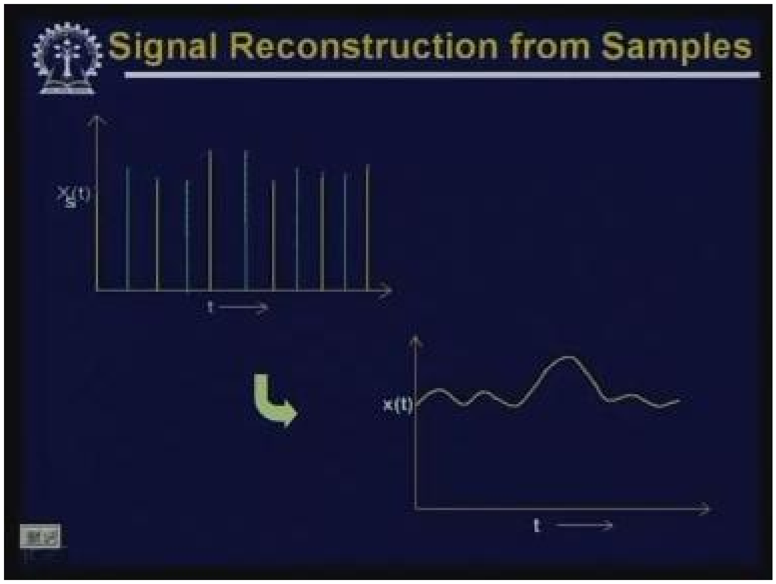

And for this purpose, we have introduced what is known as convolution theorem.

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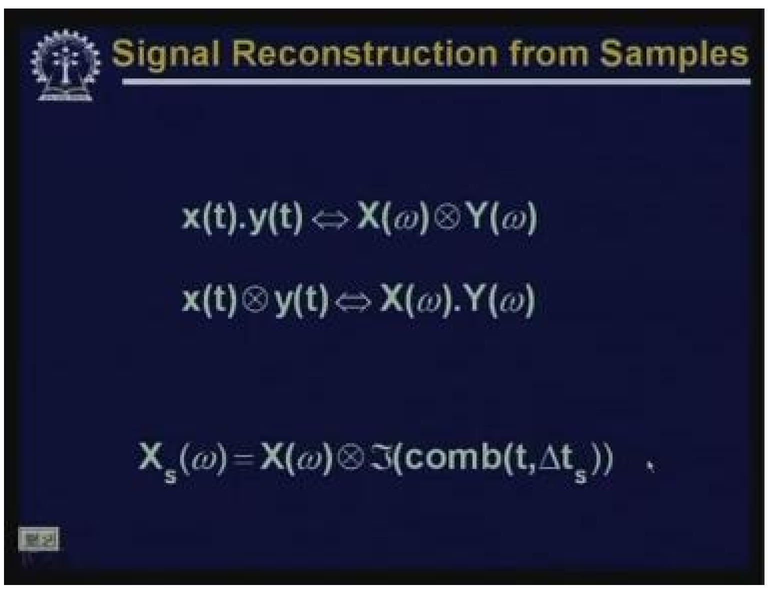

The convolution theorem says that if we have 2 signals X (t) and y (t) in time domain, then the multiplication of X (t) and y (t) in time domain is equivalent to if you take the convolution of the frequency spectrum of X (t) and frequency spectrum of y (t) in the frequency domain. So, that is to say that X (t) into y (t) is equivalent to X omega convoluted with Y omega.

Similarly, if you take the convolution of X (t) and y (t) in time domain, that is equivalent to multiplication of X omega and Y omega in the frequency domain. So, by using this concept of the convolution theory, we will see that how to reconstruct the original signal X (t) from the sampled values of X $_S$ (t).

Now, as per this convolution theorem, we have seen that X_S of t is nothing but multiplication of X (t) into the comb function comb of t delta t. So, in the frequency domain that will be equivalent to X $_S$ of omega is equal to X omega convoluted with the frequency spectrum of comb of t delta t $_S$ where delta t $_S$ is the sampling interval.

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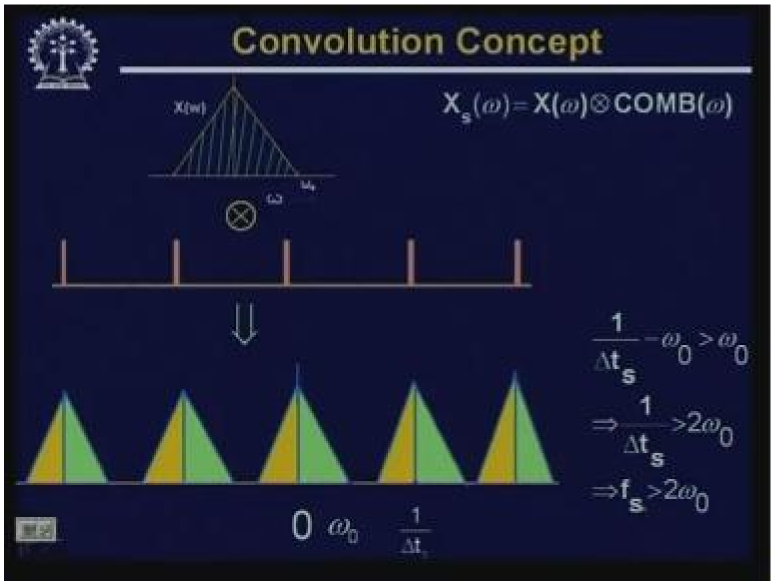

We have also seen that if X omega is the frequency spectrum or the bandwidth of the signal, a frequency spectrum of the signal which is presented here and this is the frequency spectrum of the sampling function; then when these 2 I we convolute, the convolution result will be like this where the original frequency spectrum of the signal gets replicated along the frequency axis at an interval at a interval of 1 upon delta t_S where 1 upon delta t_S is nothing but the sampling frequency f of s.

And here you find that for proper reconstruction, what we have to do is this original spectrum, the spectrum of the original signal has to be taken out and if you want to take out this, then we have to make use of a filter which will only take out this particular band and the remaining frequency components will simply be discarded. And for this filtering operation to be successful, we must need that 1 upon delta t_S minus omega naught where omega naught is the bandwidth of the signal or the maximum frequency component present in the signal X (t).

So, 1 upon delta t_S minus omega naught must be greater than or equal to omega naught and that leads to the condition that the sampling frequency f $_S$ must be greater than twice of omega naught where omega naught is the bandwidth of the signal and this is what is the Nyquist rate.

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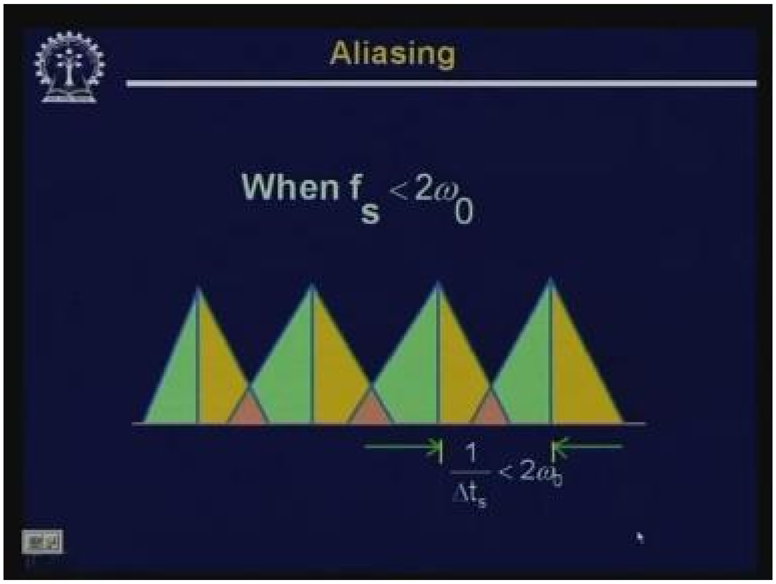

Now, what happens if the sampling frequency is less than twice of omega naught? In that case, as it is shown in this figure; you find that subsequent frequency bands after sampling, they overlap and because of this overlapping, a single frequency band cannot be extracted using any of the low pass filters.

So effectively, as a result, what we get is after low pass filtering the signal which is reconstructed is a distorted signal, it is not the original signal. And, this effect is what is known as aliasing. So, now let us see, what happens in case of 2 dimensional image which is a function of 2 variables x and y.

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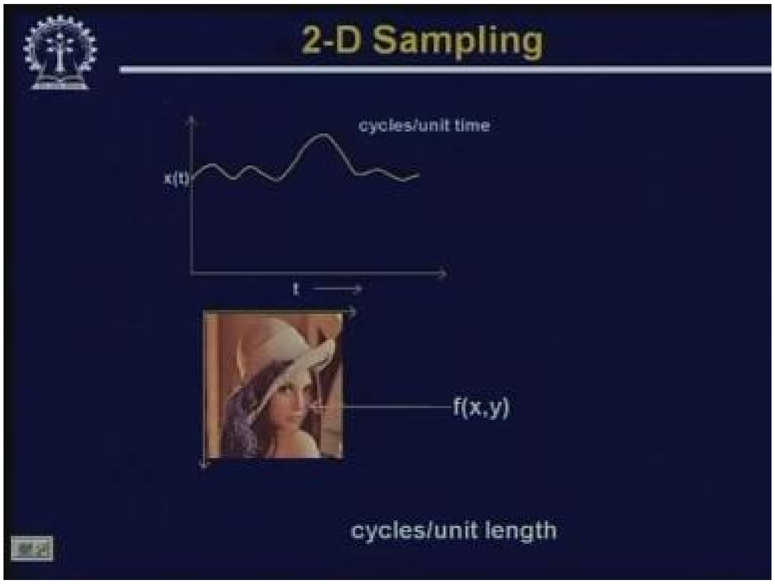

Now, you find here, in this slide, we have shown 2 figures. On the top, we have shown the same signal X (t) which we have used earlier which is a function of t and the bottom figure, is an image which is a function of 2 variables x and y.

Now, if t is time, in that case X (t) is a signal which varies with time and for such a signal, the frequency is measured as you know in terms of hertz which is nothing but cycles per unit time. Now, how do you measure the frequency in case of an image?

You find that in case of an image, the dimension is represented either in the form of say 5 centimeter by 5 centimeter or say 10 centimeter by 10 centimeter and so on. So, for an image, when you measure the frequency; it has to be cycles per unit length, not the cycles per unit time as is done in case of a time varying signal.

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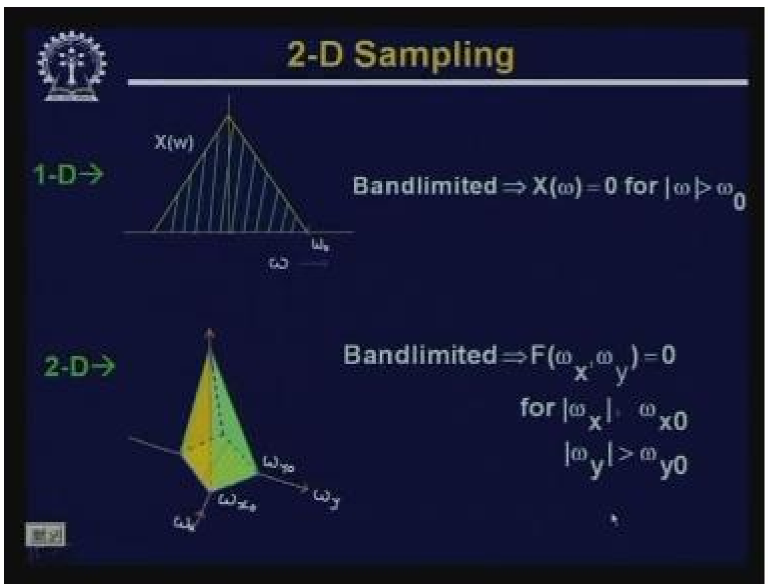

Now, in this figure we have shown that as in case of the signal X (t), we had its frequency spectrum represented by X of omega and we say that the signal X (t) is band limited if X of omega is equal to 0 for omega is greater than omega naught where omega naught is the bandwidth of the signal X (t).

Similarly, in case of an image, because the image is a 2 dimensional signal which is a variable, which is a function of 2 variables x and y; so it is quite natural that in case of image, we will have frequency components which will have 2 components - one in the x direction and other in the y direction. So we call them, omega x and omega y.

So, you see that the image is band limited if f of omega x omega y is equal to 0 for omega x greater than omega x_0 and omega y greater than omega y_0. So in this case, the maximum frequency component in the x direction is omega x_0 and the maximum frequency component in the y direction is omega y_0.

And, this figure on the bottom left shows how the frequency spectrum of an image looks like and here you find that the base of this frequency spectrum on the omega x omega y plane is what is known as the region of support of the frequency spectrum of the image.

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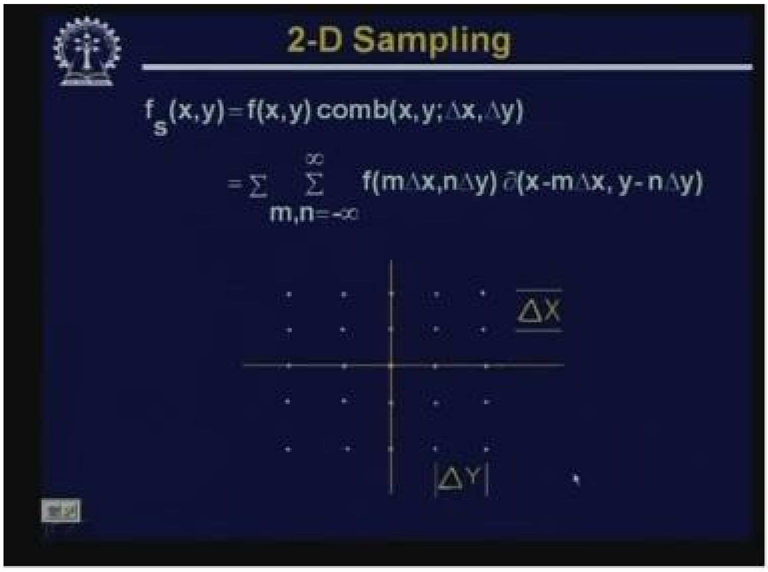

Now, let us see what will happens in case of 2 dimensional sampling or when we try to sample an image. The original image is represented by the function f of x y and as we have seen, in case of a 2 1 dimensional signal that if x of t is multiplied by comb of t delta t for the sampling operation; in case of image also f (x, y) has to be multiplied by comb of x y delta x y to give you the sampled signal f_S (x, y).

Now, this comb function because it is again a function of 2 variables x and y is nothing but a 2 dimensional array of the delta functions where along x direction, the spacing is delta x and along y direction, the spacing is del y.

So again, as before, this f_S (x, y) can be represented in the form f (m delta x n delta y) multiplied by delta function x minus m delta x, y minus n delta y where both m and n varies from minus infinity to infinity.

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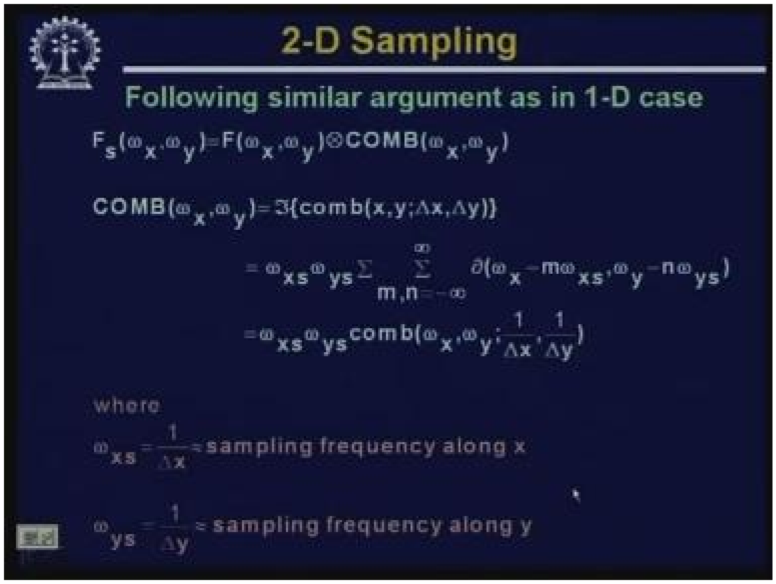

So, as we have done in case of 1 dimensional signal; if we want to find out the frequency spectrum of this sampled image, then the frequency spectrum of the sampled image f_S omega x omega y will be same as f omega x omega y which is the frequency spectrum of the original image f (x, y) which has to be convoluted with comb omega x omega y where comb omega x omega y is nothing but the Fourier transform of comb x y delta x delta y.

And if you compute this Fourier transform, you find that comb omega x omega y will come in the form of omega xs omega ys comb of omega x omega y, 1 upon delta x, 1 upon delta y where this omega xs and this omega ys, omega xs is nothing but 1 upon delta x which is the sampling frequency along the x direction and omega ys is equal to omega 1 upon delta y which is nothing but the sampling frequency along the y direction.

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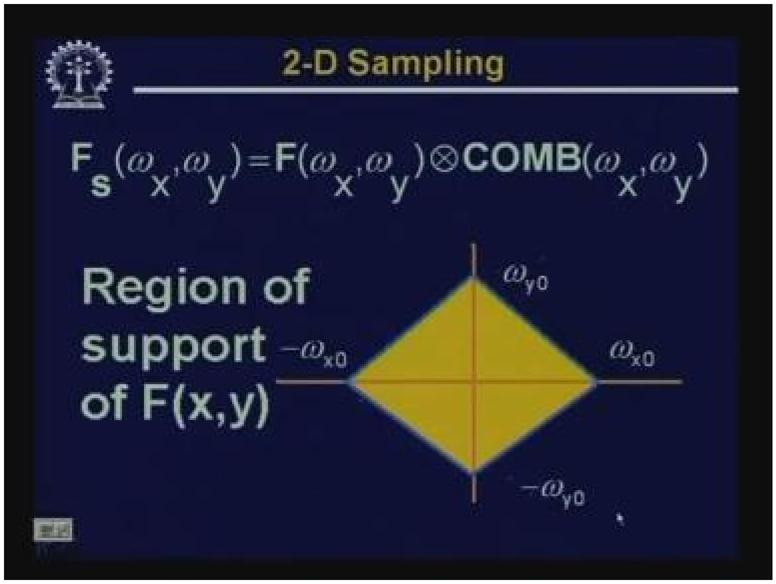

So, coming back to similar concept as we have done in case of 1 dimensional signal X (t) that F_S omega x omega y which is now the convolution of F omega x omega y which is the frequency spectrum of the original image convoluted with comb omega x omega y where comb omega x omega y is the Fourier transform of the sampling function in 2 dimension.

And, as we have seen earlier that such a type of convolution operation replicates the original the frequency spectrum of the original signal in along the omega axis in case of 1 dimensional signal; so here again, in case of 2 dimensional signal this will be replicated, the original spectrum will be replicated along both x direction and y direction.

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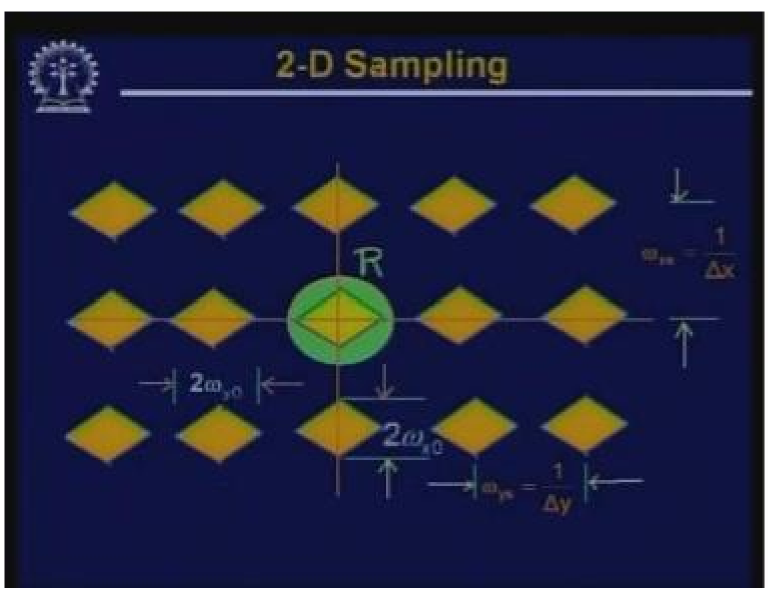

So as a result, what we get is a 2 dimensional array of the spectrum of the image as shown in this particular figure. So here again, you find that we have simply shown the region of support that getting replicated. You find that along y direction and along x direction, the spectrum gets replicated and the spacing between 2 subsequent frequency band along the x direction is equal to omega xs which is nothing but 1 upon delta x and along y direction, the spacing is 1 upon delta y which is nothing but omega ys but which is the sampling frequency along the y direction.

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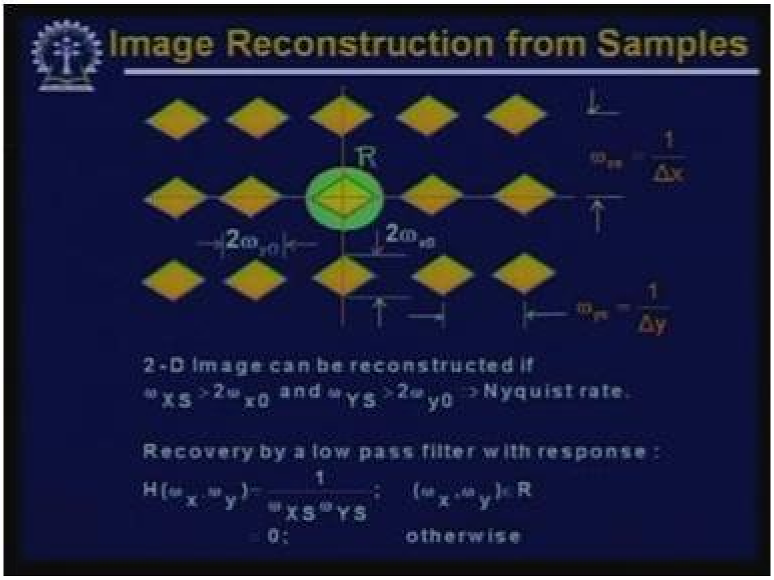

Now, if you want to reconstruct the original image from this particular spectrum, then what we have to do is we have to take out a particular frequency band. So, a frequency band which is around the origin in the frequency domain and if you want to take out this particular frequency band, then as we have seen before that these signal has to be low pass filtered and if we pass this through a low pass filter whose response is given by H omega x omega y is equal to 1 upon omega xs into omega ys for omega x omega y in the region R where region R just covers this central band and it is equal to 0 outside this region R.

In that case, it is possible that we will be able to take out just this particular frequency component within this region R by using this low pass filter and again for taking out this particular frequency region, the same condition of the Nyquist rate applies. That is sampling frequency in the x direction must be greater than twice of omega x naught while which is the maximum frequency component along x and sampling frequency along the y direction again has to be greater than twice of omega y naught which is the maximum frequency component along direction y.

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So, let us see some result. This is here we have shown 4 different images. So, here you find that the first image which is shown here was sampled with 50 dots per inch or 50 samples per inch. Second one was sampled with 100 dots per inch, third one with 600 dots per inch and the fourth one with 1200 dots per inch.

So, out of these 4 images, you find that the quality of the first image is very very bad. It is very blurred and the details in the image are not at all recognizable. As we increase the sampling frequency, when you go for the second image where we have 100 dots per inch; you find that the quality of the reconstructed image is better than the quality of the first image. But here again, still you find that if you study this particular region or wherever you have edges, the edges are not really not continuous; they are slightly broken.

So, if I increase the sampling frequency further, you find that these breaks in the edges have been smoothed out. So, at with a sampling frequency of 600 dots per inch, the quality of the image is quiet acceptable. Now, if we increase the sampling frequency further, when we go for 600 dots per inch to 1200 dots per inch sampling rate; you find that the improvement in the image quality is not that much as the improvement we have got when you moved from say 50 dots per inch from 100 dots per inch or 100 to 600 dots per inch.

So, it shows that when your sampling frequency is above the Nyquist rate, you are not going to get any improvement of the image quality. Whereas, when it is less than the Nyquist rate, the sampling frequency is less than the nyquist rate the reconstructed image is very bad.

So till now, we have covered the first face of the image digitization process that is quantization and we have also seen through the examples of this reconstructed image that if we vary the sampling frequency below and above the Nyquist rate; how the quality of the reconstructed image is going to vary. So, now let us go to the second face that is quantization of the sampled values.

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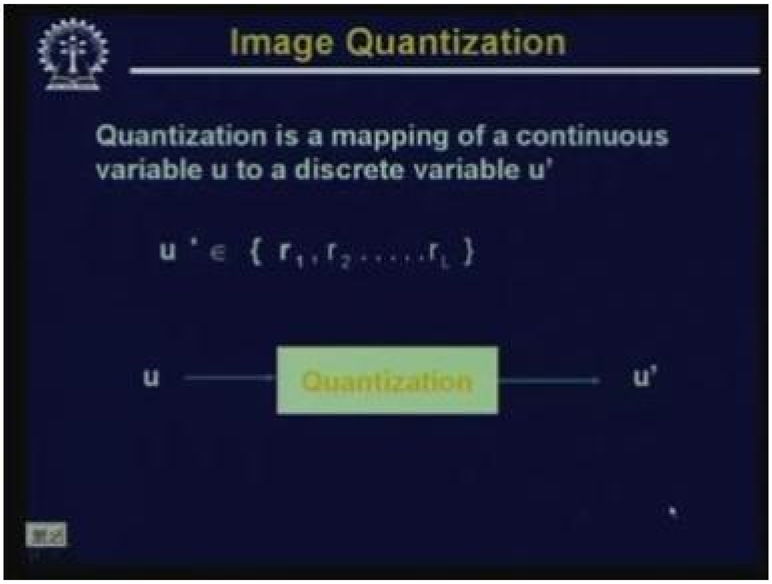

Now, this quantization is a mapping of the continuous variable u to a discrete variable u prime where u prime takes values from a set of discrete variables.

So, if your input signal is a u, after quantization the quantized signal becomes u prime where u prime is one of the discrete variables as shown in this case as r_1 to r_L. So, we have L number of discrete variables r_1 to r_L and u prime takes a value of one of these variables.

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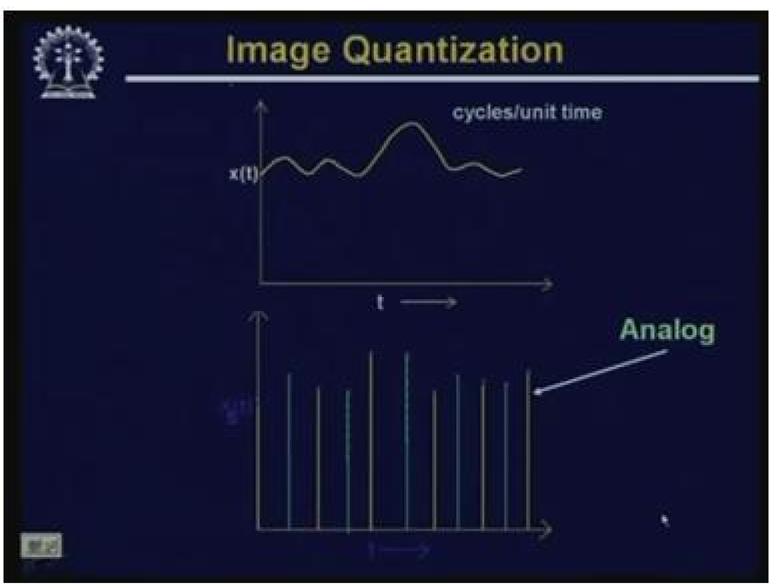

Now, what is this quantization? You find that after sampling of a continuous signal, what we have got is a set of samples. These samples are discrete in time domain. But still, every sample value is an analog value; it is not a discrete value.

So, what we have done after sampling is instead of considering all possible time instants, the signal values at all possible time instants; we have considered the signal values at some discrete time instants and at each of this discrete time instants, I get a sample value. Now, the value of this sample is still an analog value.

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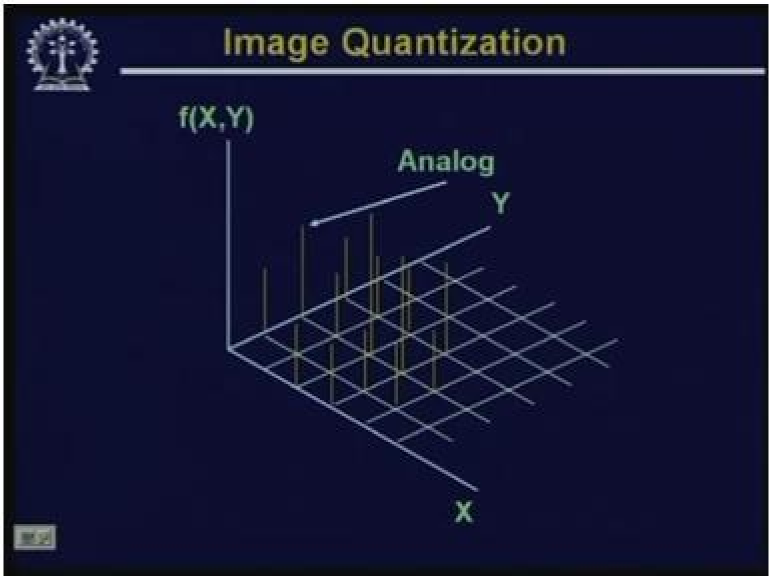

Similar is the case with an image. So here, in case of an image, the sampling is done in 2 dimensional grids where at each of the grid locations, we have a sample value which is still analog.

Now, if I want to represent a sample value on a digital computer, then this analog sample value cannot be represented. So, I have to convert this sample value again in the discrete form. So, that is where the quantization comes into picture.

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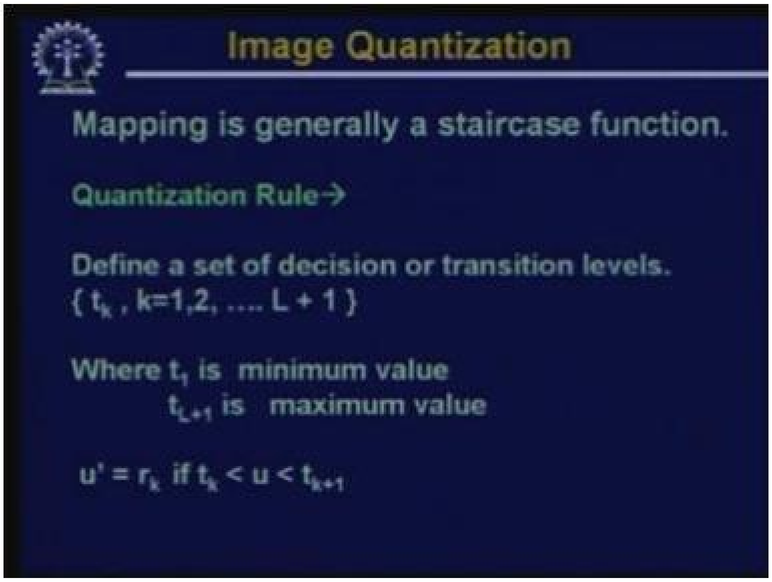

Now, this quantization is a mapping which is generally a staircase function. So, for quantization what is done is you define a set of decision or transition levels which in this case has been shown as transition level t_k where k varies from 1 to L plus 1.

So, we have defined a number of transition levels or decision levels which are given as t_1, t_2, t_3, t_4 upto t_L plus 1 L plus 1 and here t_i is the minimum value and t_L plus 1 is the maximum value and we also defined a set of the reconstruction levels that is r_k.

So, what we have shown in the previous slide that the reconstructed value r prime u prime takes one of the discrete values r_k. So, the quantized value will take the value r_k if the input signal u lies between the decision levels t_k and t_k plus 1. So, this is how you do the quantization.

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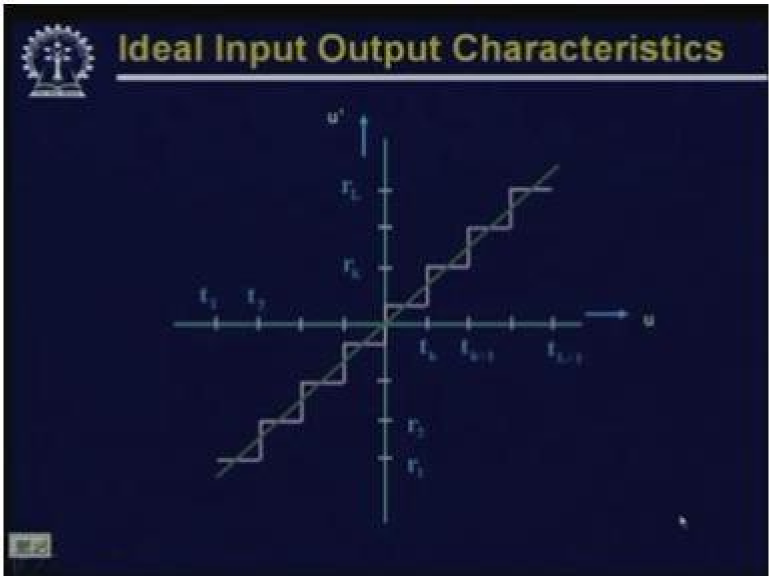

So, let us come to this particular slide. So, it shows the input output relationship of a quantizer. So, it says that whenever your input signal u, so along the horizontal direction we have put the input signal u and along the vertical direction we have put the output signal u prime which is the quantized signal.

So, this particular figure shows that if your input signal u lies between the transition levels t_1 and t_2; then the reconstructed signal or the quantized signal will take a value r_1. If the input signal lies between t_2 and t_3, the reconstructed signal or the quantized signal will take a value r_2. Similarly, if the input signal lies between t_k and t_k plus 1, then the reconstructed signal will take the value of r_k and so on.

So, given an input signal which is analog in nature, you are getting the output signal which is discrete in nature. So, the output signal can take only one of these discrete values, the output signal cannot take any arbitrary value.

Now, let us see that what is the effect of this. So, as we have shown in the second slide that ideally we want that whatever is the input signal, the output signal should be same as the input signal and that is necessary for a perfect reconstruction of the signal. But whenever we are going for quantization, your output signal as it takes one of the discrete set of values is not going to be same as the input signal always.

So, in this in this particular slide, again we have shown, the same staircase function where along the horizontal direction we have the input signal and in the vertical axis we have put the output signal. So, this peak staircase function shows what are the quantization function that will be used and this green line which is inclined at an angle of 45 degree with the u axis, this shows that what should be the ideal input output characteristics.

So, if the input output function follows this green line; in that case, for every possible input signal. I have the corresponding output signal. So, the output signal should be able to take every possible value. But when we are using this staircase function; in that case, because of this staircase effect, whenever the input signal lies within certain region, the output signal takes a discrete value. Now because of this staircase function, you are always introducing some error in the output signal or in the quantized signal. Now, let us see that what is the nature of this error.

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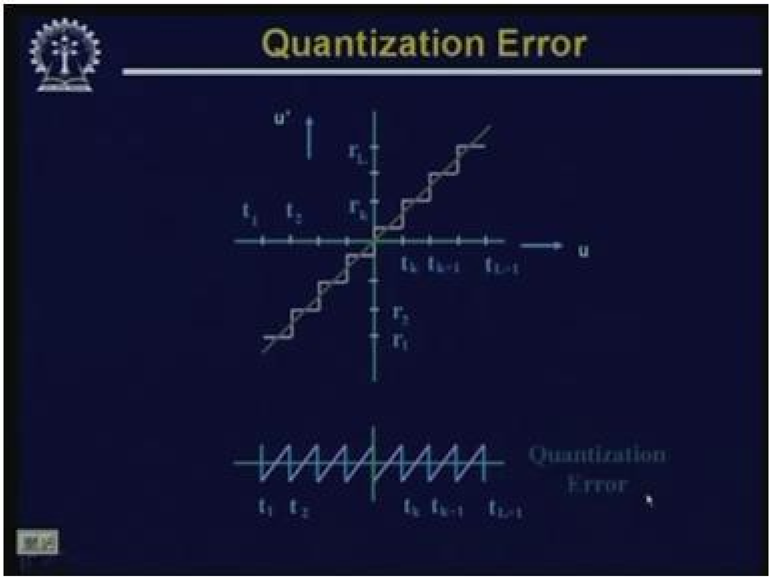

So, here we have shown the same figure. Here you find that whenever this green line which is inclined at 45 degree with the u axis crosses the staircase function; at this point, whatever is your signal value, it is same as the reconstructed value.

So, only at these cross over points, your error in the quantized signal will be 0. At all other points, the error in the quantized signal will be a non zero value. So, at this point, the error will be maximum, which will maximum and negative which will keep on reducing. At this point, this is going to be 0 and beyond this point, again it is going to increase.

So, if I plot this quantization error, you find that the plot of the quantization error will be something like this between every transition levels. So, between t_1 and t_2 the error value is like this, between t_2 and t_3 the error continuously increases, between t_3 and t_4 the error continuously increases and so on. Now, what is the effect of this error on the reconstructed signal?

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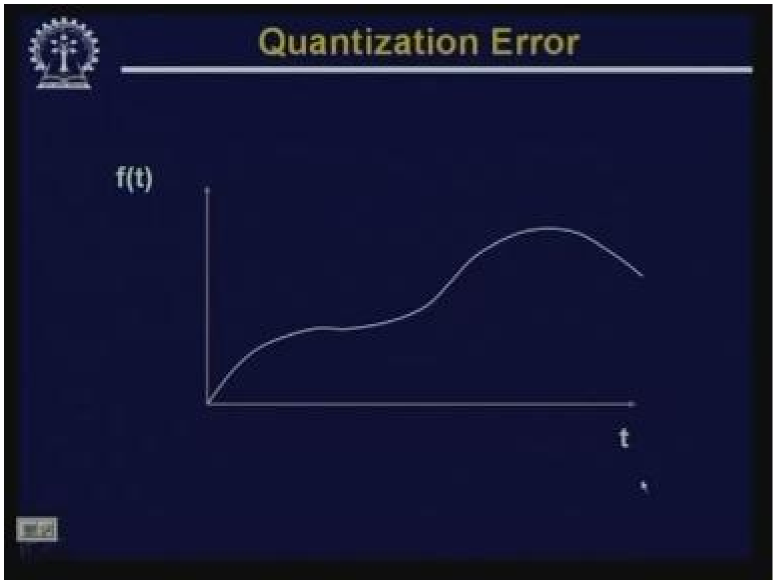

So for that, let us take again a 1 dimensional signal f (t) which is a function of t as is shown in the slide. And, let us see that what will be the effect of quantization on the reconstructed signal.

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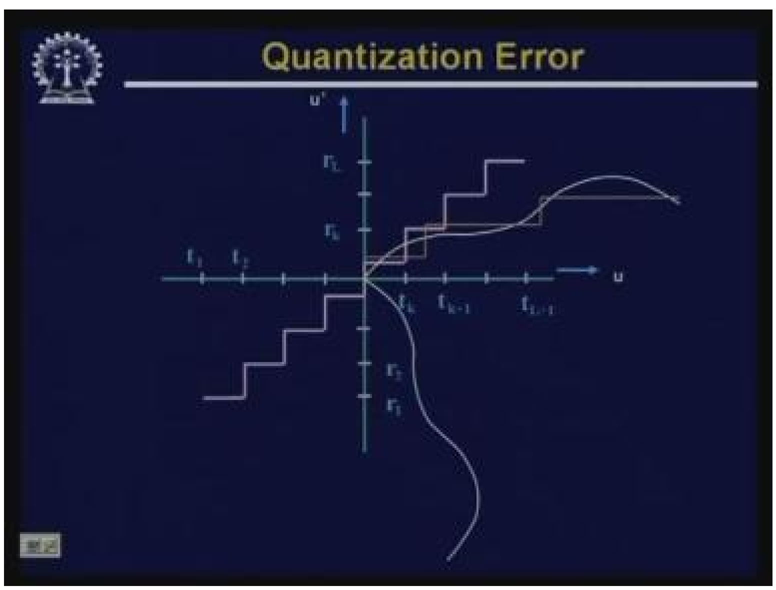

So, here we have plotted the same signal. So, here we have shown the signal is plotted in the vertical direction so that we can find out what are the transition levels or the part of the signal which is within which particular transition level. So, you find that this part of the signal is in the transition levels at t_k minus 1 and t_k.

So, when the signal, input signal lies between the transition levels t_k minus 1 and t_k; the corresponding reconstructed signal will be r_k minus 1. So, that is shown by this red horizontal line.

Similarly, the signal from this portion to this portion lies in the range t_k and t_k plus 1. So, corresponding to this, the output reconstructed signal will be r_k. So, which is again shown by this horizontal red line and this part of the signal, the remaining part of the signal lies within the range t_k plus 1 and t_k plus 2 and corresponding to this, the output reconstructed signal will have the value r_k plus 1.

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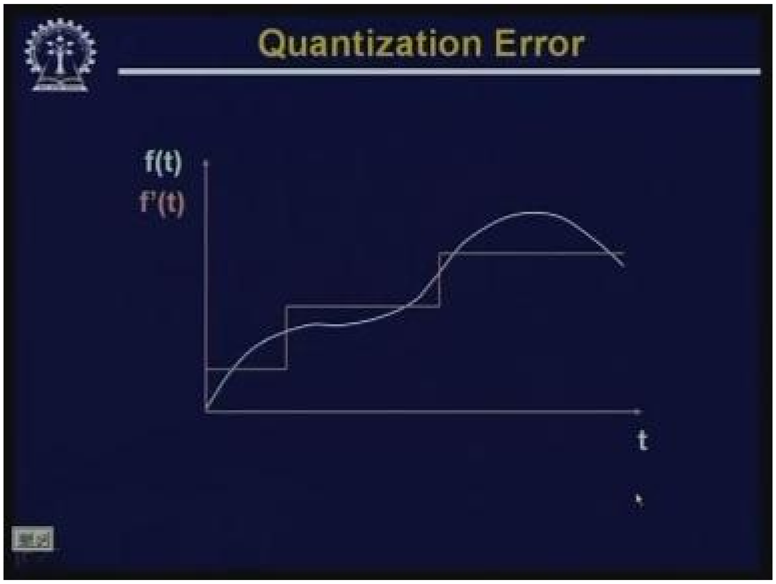

So, to have a clear figure, you find that in this, the green curve, it shows the original input signal and this red staircase lines, staircase functions, it shows that what is the quantization signal quantized signal or f at f prime t.

Now from this, it is quite obvious that I can never get back the original signal from this quantized signal because within this region, the signal might have might have had any arbitrary value and the details of that is lost in this quantized one in this quantized output.

So, because from the quantized signal I can never get back the original signal; so we are always introducing some error in the reconstructed signal which can never be recovered and this particular error is known as quantization error or quantization noise.

Obviously, the quantization error or quantization noise will be reduced if the quantizer step size that is the transition intervals say t_k to t_k plus 1 reduces; similarly, the reconstruction step size r_k to r_k plus 1 that interval is also reduced.

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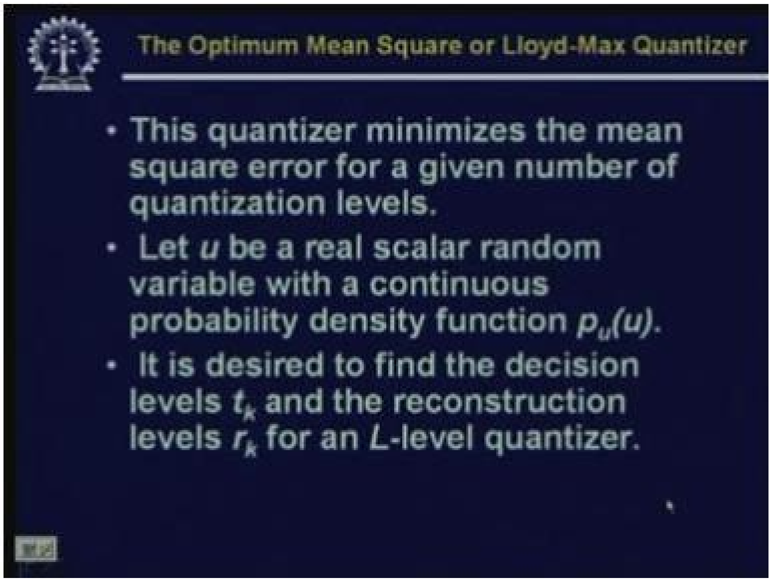

So, for quantizer design, the aim of the quantizer design will be to minimize this quantization error. So accordingly, we have to have an optimum quantizer and this optimum mean square error quantizer known as Lloyd-Max quantizer, this minimizes the mean square error for a given number of quantization levels and here we assume that let u be a real scalar random variable with a continuous probability density function p_u of u and it is desired to find the decision levels t_k and the reconstruction levels r_k for an n L level quantizer which will reduce or minimize the quantization noise or quantization error.

Let us see how to do it. Now, you remember that u is the input signal and u prime is the quantized signal. So, the error of reconstruction is the input signal minus the reconstructed signal.

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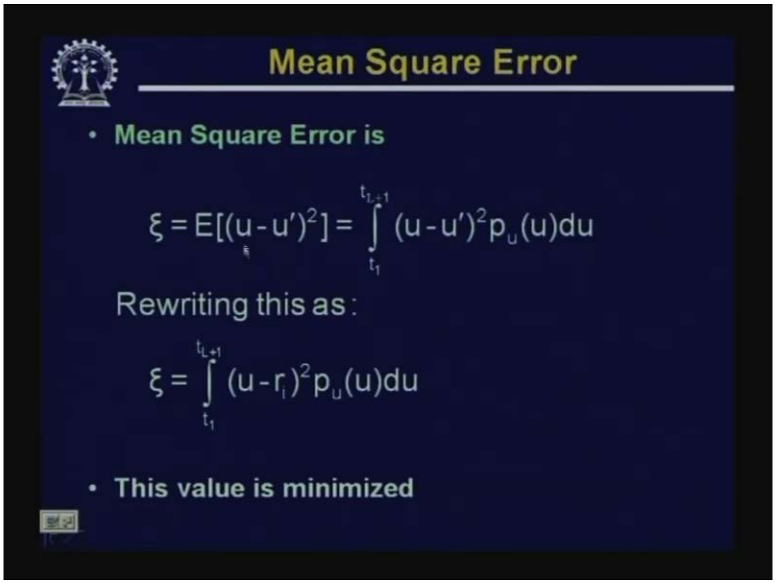

So, the mean square error is given by the expectation value of u minus u prime square and this expectation value is nothing but if I integrate u minus u Prime Square multiplied by the probability density function of u du and I integrate this from t_1 to t_L plus 1. You will find that you remember that t_1 was the minimum transition level and t_L plus 1 was the maximum transition level. So, if I just integrate this function, u minus u prime square p_u (u) du over the interval t_1 to t_L plus 1, I get the mean square error.

This same integration can be rewritten in this form as u minus r_i square because r_i is the reconstruction level or the reconstructed signal in the interval t_i to t_i plus 1. So, there is an error. It is not t_1 to t_L plus 1; it should be t_i to t_i plus 1.

So, I integrate this u minus r_i square p_u (u) du over the interval t_i to t_i plus 1; then I have to take a summation of this for i equal to 1 to L. So thus, this modified expression will be same as this and this tells you that what is the square error of the reconstructed signal and the purpose of designing the quantizer will be to minimize this error value.

So obviously, from school level mathematics we know that for minimization of the error value because now we have to design the transition levels and the reconstruction levels which will minimize the error. So, the way to do is do that is to differentiate the error function, the error value with t_k and with r_k and equating those equations to 0.

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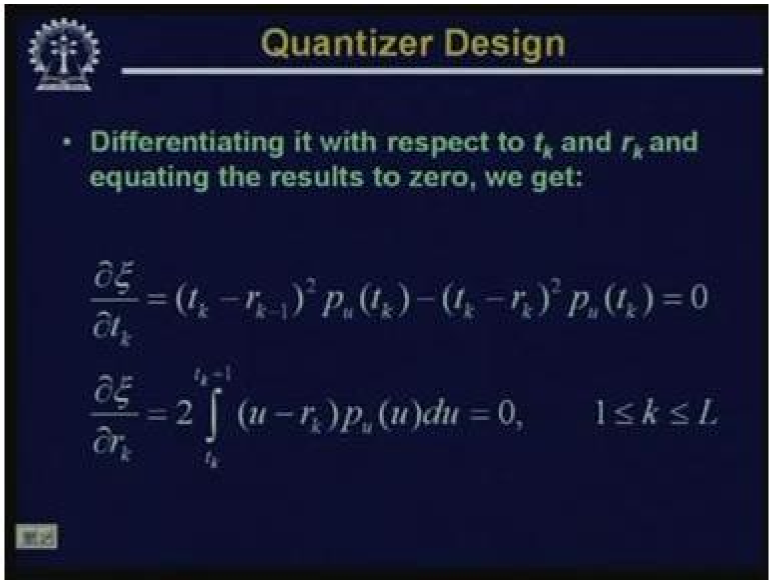

So, if I differentiate this particular error value - u minus r_i square p_u (u) du, integration from t_i to t_i plus 1; in that case what I get is zeta is the error value del zeta del t_k is same as t_k minus r_k minus 1 square pu t_k minus t_k minus r_k square pu t_k and these has to be equated to 0.

Similarly, the second equation - del zeta del r_k will be same as twice into integral u minus r_k p_u(u) du equal to 0 where the integration has to be taken from t_k to t_k plus 1.

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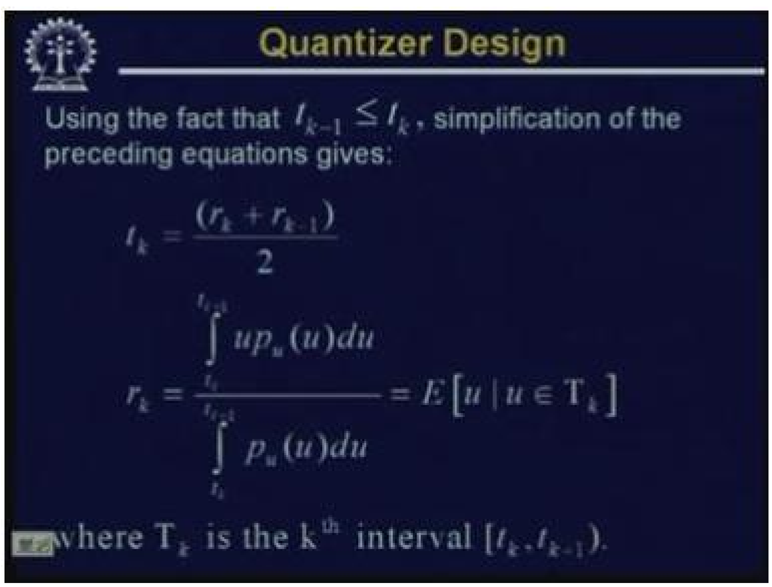

Now, by solving these 2 equations and using the fact that t_k minus 1 is less than t_k; we get 2 values. One is for transition level and the other one is the for the reconstruction level. So, the transition level t_k is given by r_k plus r_k minus 1 by 2 and the reconstruction level r_k is given by integral t_k to t_k plus 1 u p_u (u) du u p_u (u) du divided by integral from t_k to t_k plus 1 p_u (u) du.

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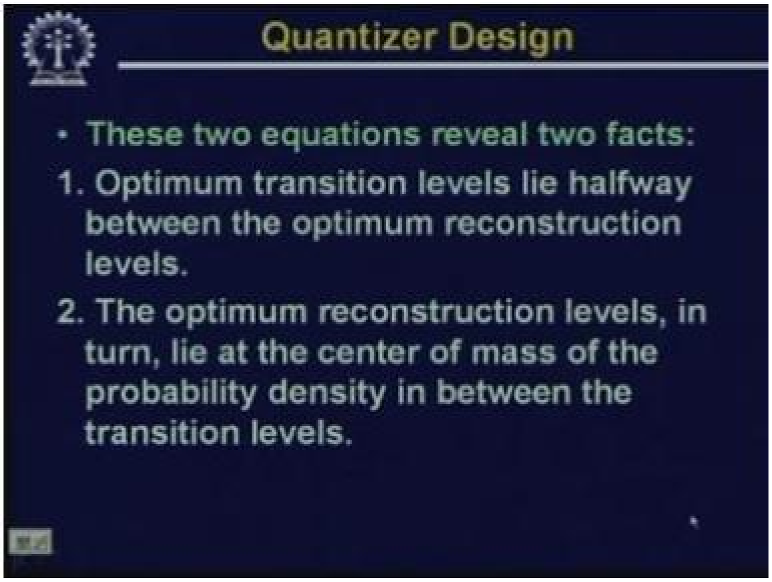

So, what we get from these 2 equations? You find that these 2 equations tell that the optimum transition level t_k lie halfway between the optimum reconstruction levels. So, that it is quite obvious because t_k is equal to r_k plus r_k minus 1 by 2. So, this transition level lies halfway between r_k and r_k minus 1 and the second observation is that the optimum reconstruction levels in turn lie at the center of mass of the probability density in between the transition levels; so which is given by the second equation that is r_k is equal to u puu du integral from t_k to t_k plus 1 divided by p_u (u) du integral again from t_k to t_k plus 1. So, this is nothing but the center of mass of the probability density between the interval t_k and t_k plus 1.

So, this optimum quantizer or the Lloyd-Max quantizer gives you the reconstruction value, the optimum reconstruction value and the optimum transition levels in terms of probability density of the input signal.

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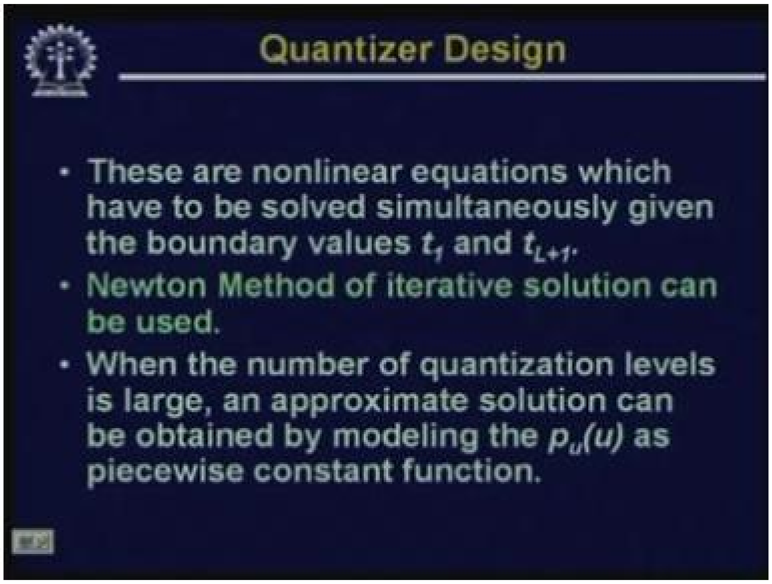

Now, you find that these 2 equations are non linear equations and we have to solve these non linear equations simultaneously, given the boundary values t_1 and t_L plus 1. And for solving this, one can make use of the Newton method, Newton iterative method to find out the solutions.

An approximate solution or an easier solution will be when the number of quantization levels is very large. So, if the number of quantization levels is very large; you can approximate p_u (u) the probability density function as piecewise constant function.

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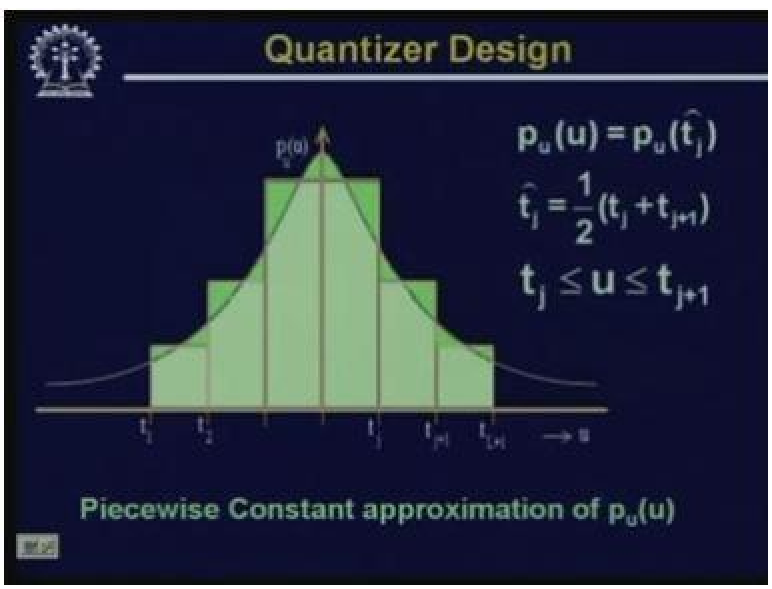

So, how do you do this piecewise constant approximation? So in this figure, you see that there is a probability density function has been shown which is like a Gaussian function. So, we can approximate it this way that in between the levels t_j and t_j plus 1, we have the mean value of this as t_j hat which is halfway between t_j and t_j plus 1 and within this interval; we can approximate p_u (u) where p_u (u) is actually a nonlinear one, we can approximate this as p_u (t_j hat).

So, in between t_j and t_j plus 1 that is in between every 2 transition levels, we approximate the probability density function to be a constant one which is same as the probability density function at the midway halfway between these 2 transition levels. So, if I do that, this continuous probability density function will be approximated by the staircase functions like this.

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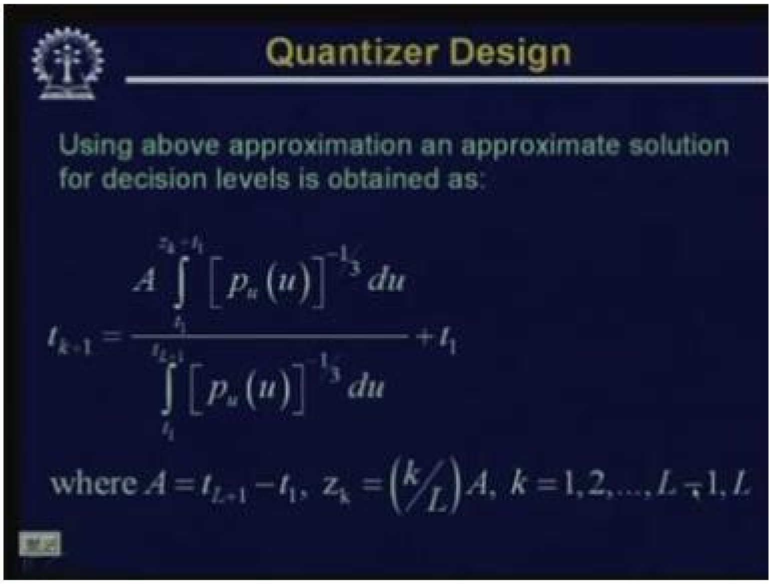

So, if I use this approximation and recompute those values, we will find that this t_k plus 1 can now be computed as p_u (u) cubic root of that du integral from t_1 to z k plus t_1 multiplied by A divided by again p_u (u) to the power 1 third, minus 1 third du integration from t_1 to t_L plus 1 plus t_1 where this A the constant A is t_L plus 1 minus t_1 and we have said that t_L plus 1 is the maximum transition level and t_1 is the minimum transition level and z_k is equal to k by L into A where k varies from 1 to L.

So, we can find out t_k plus 1 by using this particular formulation when the continuous probability density function was approximated by piecewise constant probability density function and once we do that, after that we can find out the values of the corresponding reconstructed reconstruction levels.

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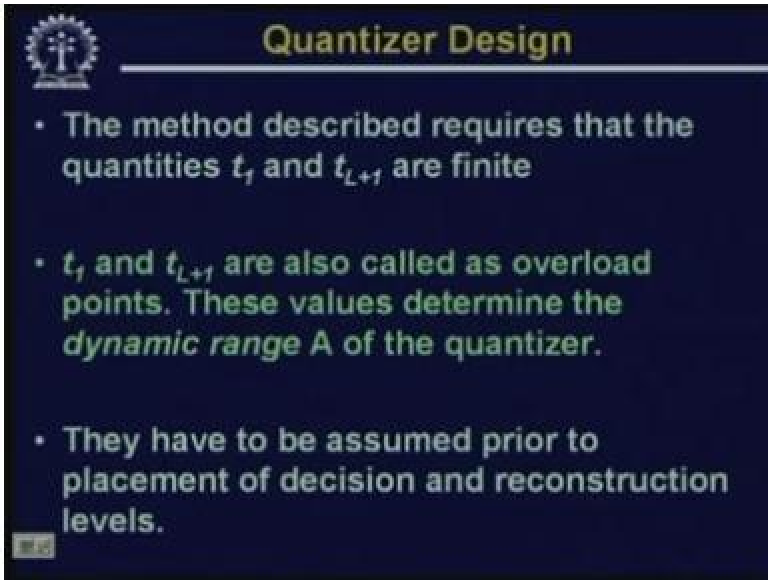

Now, for solving this particular equation, the requirement is that we have to have t_1 and t_L plus 1 to be finite. That is the minimum transition level and the maximum transition level, they must be finite. At the same time, we have to assume t_1 and t_L plus 1 a priory before placement of decision and reconstruction levels.

This t_1 and t_L plus 1 are also called as called as valued points and these 2 values determine the dynamic range A of the quantizer. So, if you find that when we have a fixed t_1 and t_L plus 1; then any value less than t_1 or any value greater than t_L plus 1, they cannot be properly quantized by this quantizer. So, this represents that what is the dynamic range of the quantizer.

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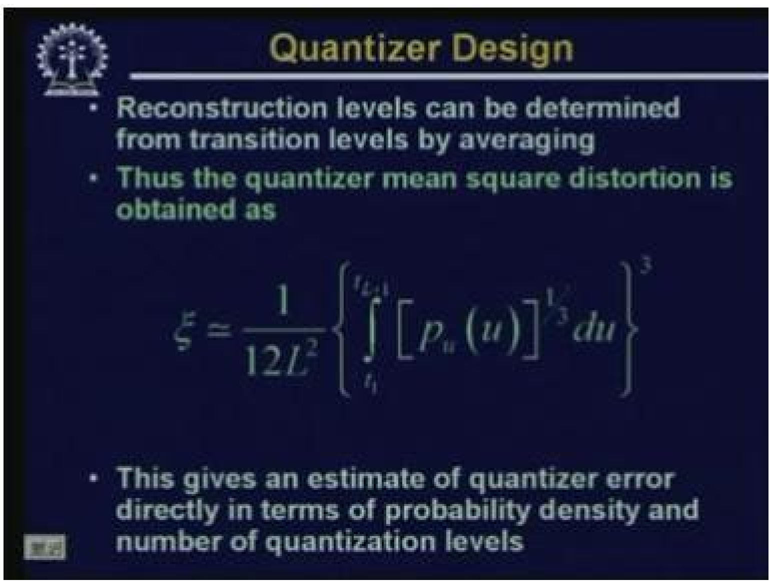

Now, once we get the transition levels, then we can find out the reconstruction levels by averaging the subsequent transition levels. So, once I have the reconstruction levels and the transition levels, then the quantization mean square error can be computed as this. That is the mean square error of this designed quantizer will be 1 upon 12 L square into p_u (u) to the power 1 third du integration between t_1 to t_L plus 1 and cube of this whole integration. And this expression gives an estimate of the quantizer quantizer error in terms of probability density and the number of quantization levels.

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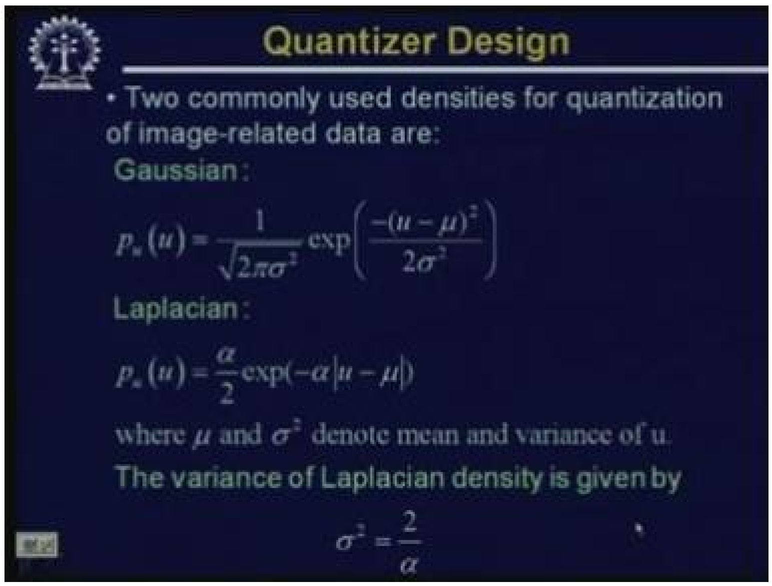

Normally, 2 types of probability density functions are used. One is Gaussian where the Gaussian probability density function is given by this well known expression p_u (u) equal to 1 upon root over 2 pi sigma square exponentiation of minus u minus mu square by twice sigma square and the laplacian probability density function which is given by p_u (u) equal to 1 upon alpha into exponentiation of minus alpha u minus mu absolute value where mu and sigma square denote the mean and variance of the input signal u. The variance in case of laplacian density function is given by sigma square is equal to 1 upon alpha.

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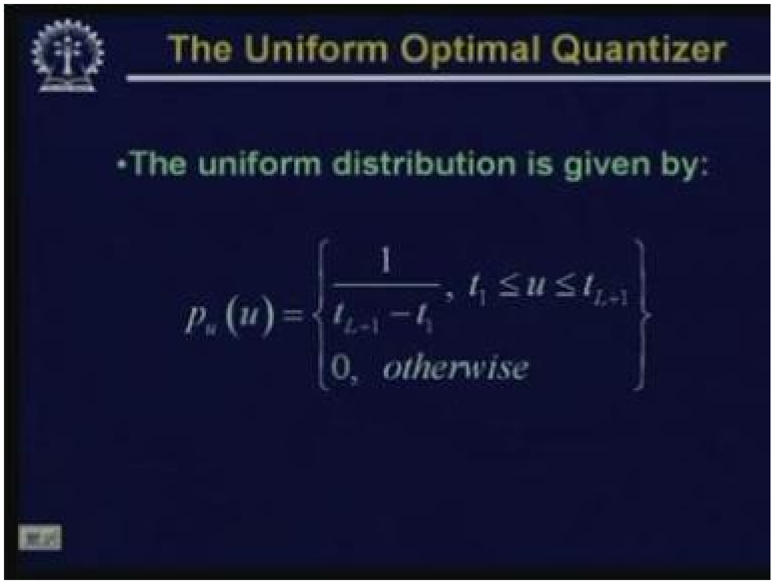

Now, find that though the earlier quantizer was designed for any kind of probability density functions; but it is not always possible to find out the probability distribution function of a signal a priori.

So, what is in practice is you assume an uniform distribution, uniform probability distribution which is given by p_u (u) equal to 1 upon t_L plus 1 minus t_1 where u lies between t_1 and t_L plus 1 and p_u (u) equal to 0 when u is outside this region t_1 to t_L plus 1.

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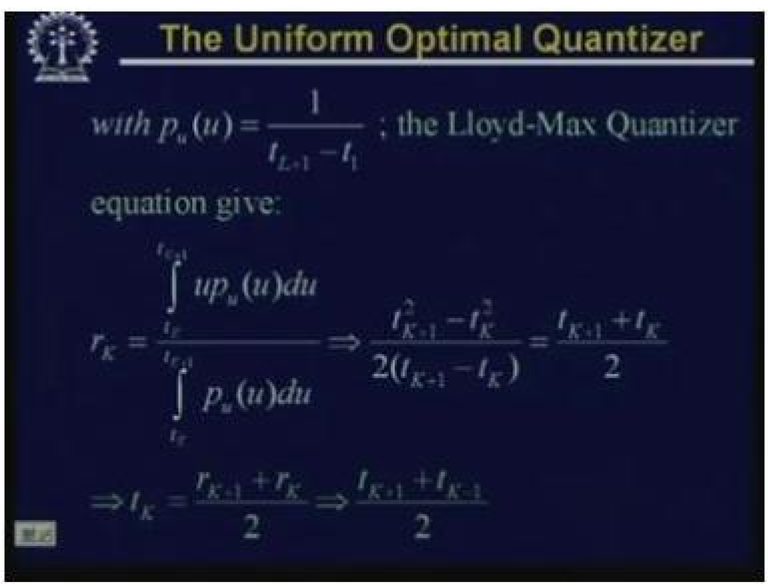

So, this the uniform probability distribution of the input signal u and by using this uniform probability distribution, the same Lloyd- max quantizer quantizer equations give r_k as if I compute this, then you will find the reconstruction level r_k will be nothing but t_k plus 1 plus t_k by 2 where t_k will be r_k plus 1 plus r_k by 2 which is same as t_k plus 1 plus t_k minus 1 by 2. So, I get the reconstruction levels and the decision levels for a uniform quantizer.

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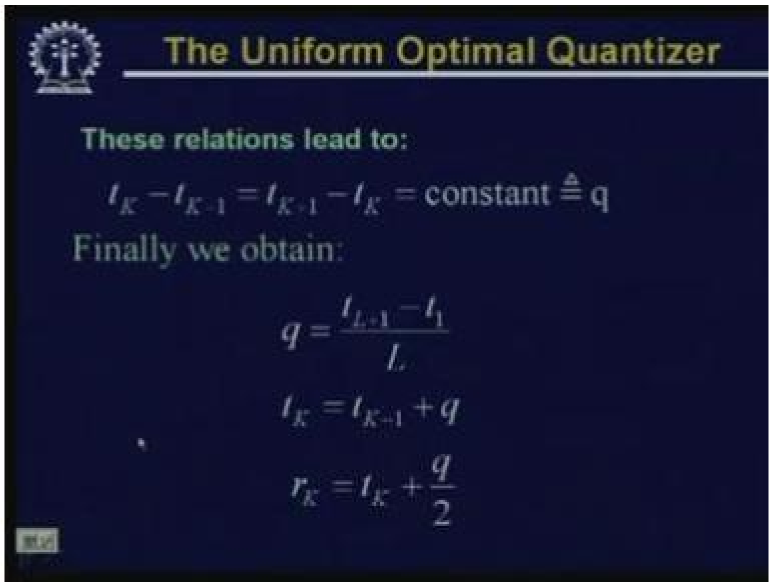

Now, these relations leads to t_k minus t_k minus 1 is same as t_k minus 1 minus t_k and that is constant equal to q which is known as the quantization step. So finally, what we get is the quantization step is given by t_L plus 1 minus t_1 by L where t_L plus 1 is the maximum transition level and t_1 is the minimum transition level and L is the number if quantization steps.

We also get the transition level t_k in terms of transition level t_k minus 1 as t_k equal to t_k minus 1 by plus q and the reconstruction level r_k in terms of the transition level t_k as r_k equal to t_k plus q by 2. So, we obtain all the related terms of a uniform quantizer using this mean square error quantizer design which is the Lloyd-Max quantizer for a uniform distribution.

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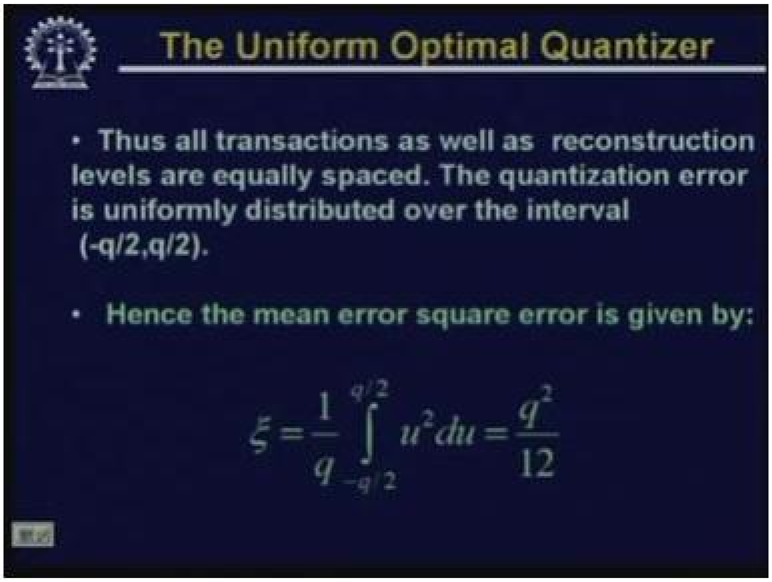

So, here you find that all the transactions, all the transition levels as well as the reconstruction levels are equally spaced and the quantization error in this case is uniformly distributed over the interval minus q by 2 to q by 2. And the mean square error in this particular case if you compute, will be given by 1 upon q u square du you take the integral from minus q by 2 to q by 2 which will be nothing but q square by 12.

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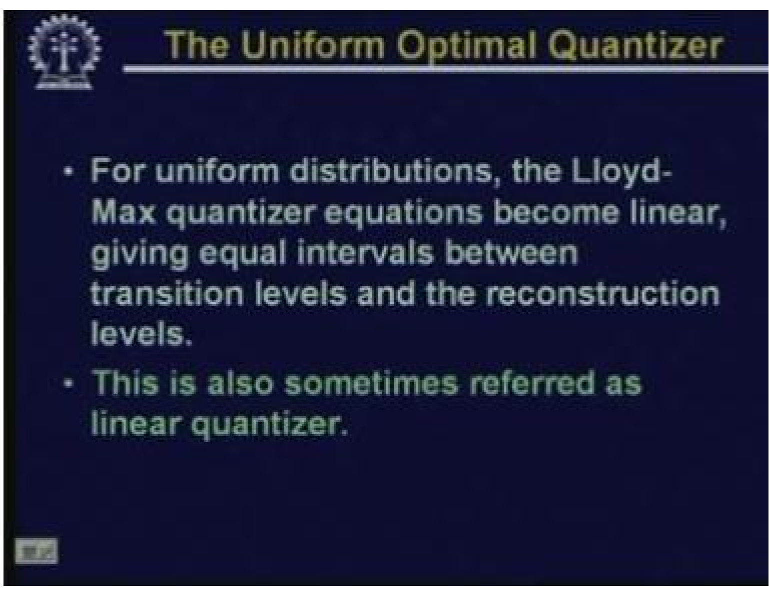

So, for uniform distributions, the Lloyd-Max quantizer equations becomes linear because all the equation that we have derived earlier they are all linear equations giving equal intervals between transition levels and the reconstruction levels and so this is also sometimes referred as a linear quantizer.

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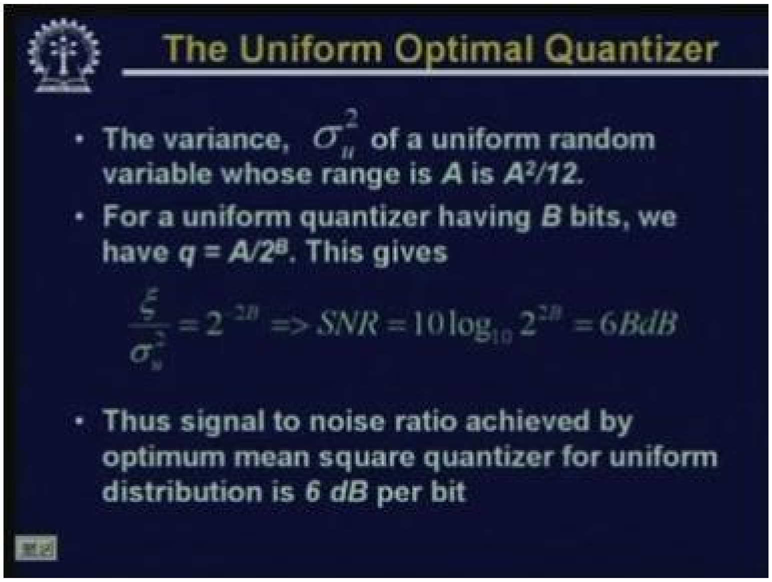

So, there are some more observations on this linear quantizer. The variance sigma u square of a uniform random variable whose range is A is given by A square by 12. So for this, you find that for a uniform quantizer with B bits. So, if we have a uniform quantizer where every level has to be represented by B bits; we will have q equal to A by 2 to the power B because the number of steps will be 2 to the power B number of steps. And thus, the quantization step will be q equal to A upon 2 to the power B and from this you will find that the error decided by sigma u square will be equal to 2 to the power minus 2B.

And from this, we can compute the signal to noise ratio in case of an uniform quantizer where the signal to noise ratio is given by 10 log 2 to the power 10 where 2 to the power twice B or the logarithm has to be taken with base 10 and this is nothing but 6 BdB. So, this says that signal to noise ratio that can be achieved by an optimum mean square quantizer for uniform distribution is 6 dB per bit.

That means if I increase the number of bits by 1, so if you increase the number of bits by 1, that means the number of quantization levels will be increased by 2 by a factor of 2. In that case, you gain a 6 dB in the signal to noise ratio in the reconstructed signal. So, with this we come to an end on our discussion on the image digitization process.

So, here we have seen that how to sample an image or how to sample a signal in 1 dimension, how to sample an image in 2 dimension. We have also seen that after you get the sample values where each of the sample values are analog in nature; how to quantize those sample value so that you can get the exact digital signal as well as exact digital image?

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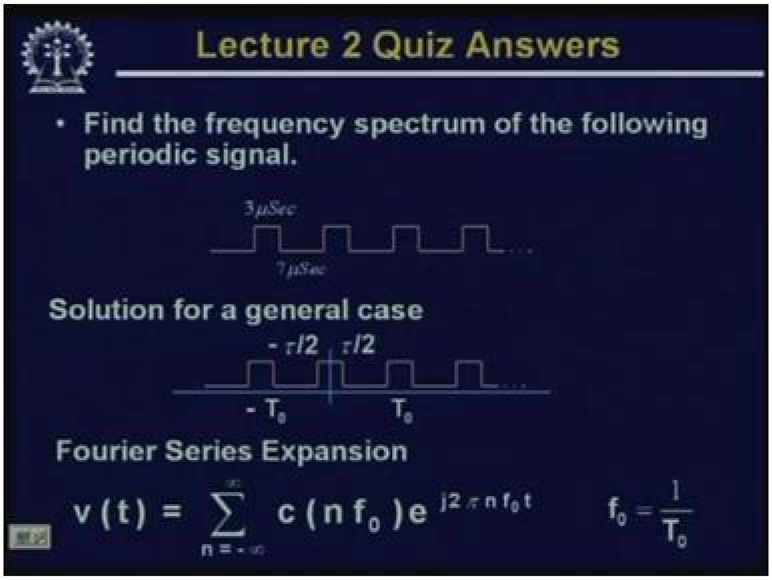

So now, you remember that in the previous class we had given some tutorials, tutorial problems. I will discuss only few of the tutorial problems which are bit critical the tutorial problems some of them are very simple.

So, one of the tutorial problems was that find the frequency spectrum of the following periodic signal where this periodic signal is a square wave, where the on period is 3 micro second and the off period is 7 micro second. Let us see, what is the solution to this particular problem.

Let us try to solve a general case. That is again we have a square wave whose time period is say T_0 and the on period is tau and we divide this between minus tau by 2 and plus tau by 2. We have also said earlier and you will find that this signal is a periodic signal and we have said that for a periodic signal, the frequency spectrum is obtained by Fourier series expansion.

So, if we expand a signal v (t) with Fourier series, then the expansion will be c n f_0 e to the power j 2 pi n f_0 t where f_0 equal to 1 upon t_0 that is the fundamental frequency.

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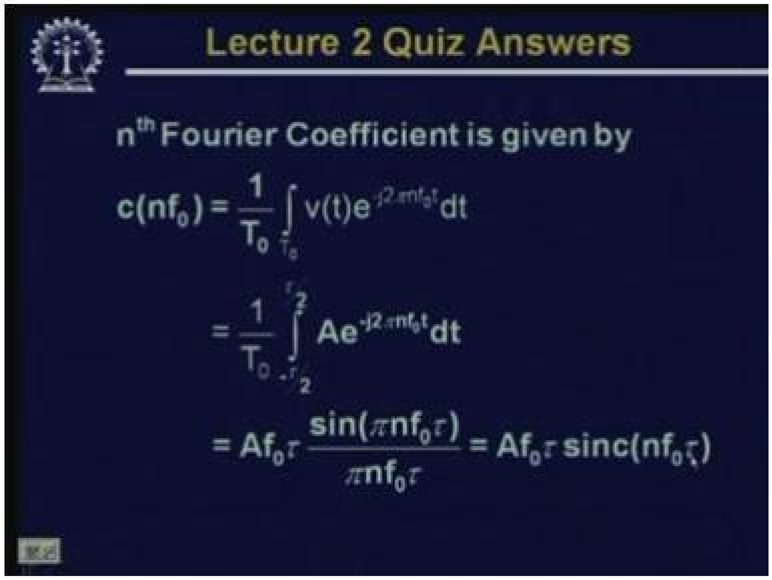

And this term, you have to take the summation from n equal to minus infinity to infinity where the c n f_0 that is the n'th Fourier coefficient is given by the expression c n f_0 equal to 1 upon t_0 then integral v (t) e to the power minus j 2 pi n f_0 t dt where this integration has to be taken over the period t_0.

And if you take this integration, you will find that the final expression will come in this form A f_0 tau sin of pi f pi n f_0 tau divided by pi n f_0 tau which is represented in the form A f_0 tau; then you have a sin c function or sinc function n f_0 tau.

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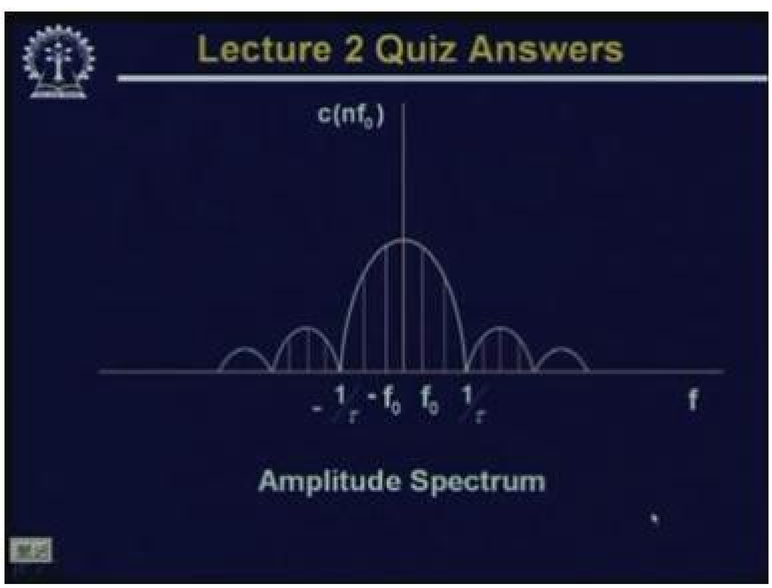

And if I plot this, the plot would be something like this where we have plotted c n f_0 versus A f and this will be a line spectrum where I get lines at f equal to 0, f equal to $f_{0,}$ f equal to twice f_0 and so on and the those components will vary the envelop of this frequency components will follow the sin c function.

Now in this, if we put tau equal to 3 micro second and t_0 equal to 10 micro second; then you will get the solution to the problem that was given in the last class.

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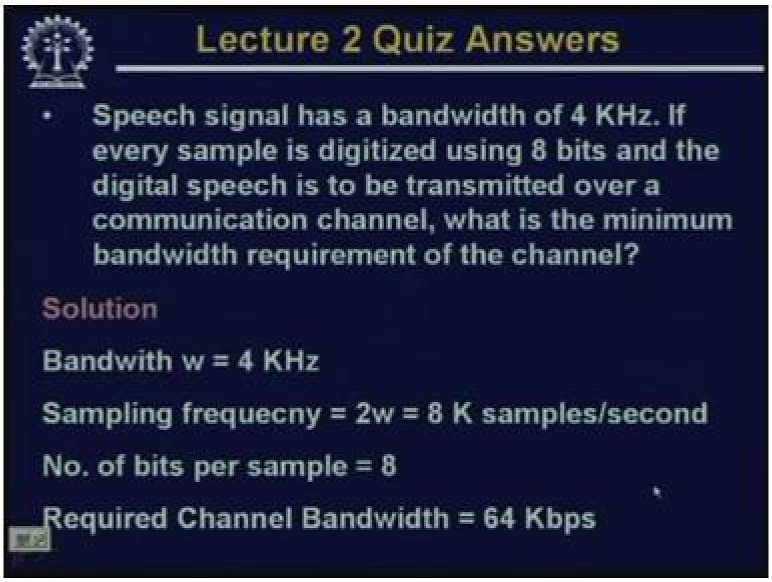

Let me put the other one. That is a speech signal has a bandwidth of 4 kilo hertz. If every sample is digitized using 8 bits and the digital speech is to be transmitted over a communication channel; what is the minimum bandwidth requirement of the channel?

Again, this problem is a very simple problem because the bandwidth of the speech signal is stated to be 4 kilo hertz, so minimum sampling frequency following the nyquist rate will be twice of omega which is equal to 8 samples per 8 k samples per second. And they stated that number of bits per sample equal to 8. So, the number of bits generated by this sampled signal will be 8 into 8 k that is 64 kilo bits per second.

So, if we want to transmit this digitized speech over a channel, then the minimum channel bandwidth requirement will be 64 kilo bits per second. So again, this is a simple problem.

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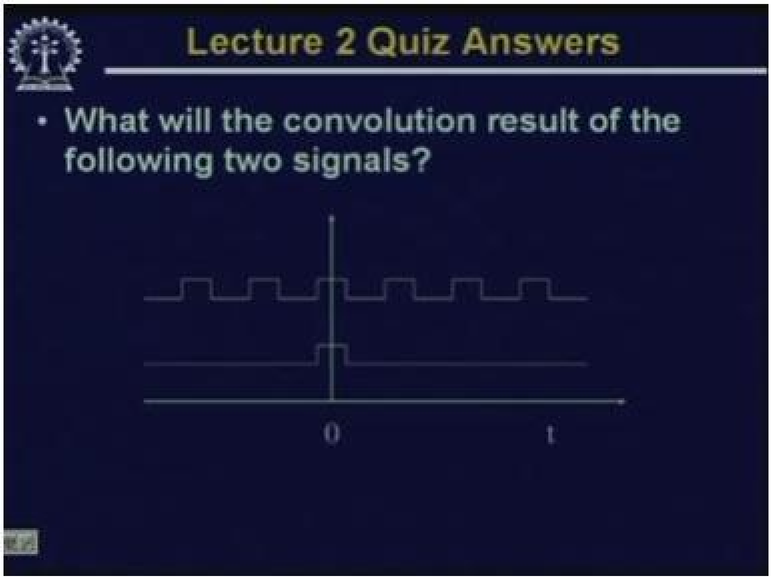

Coming to the next problem which was given, we had given 2 different signals and you have to find out what will be the convolution of these 2 these two different signals.

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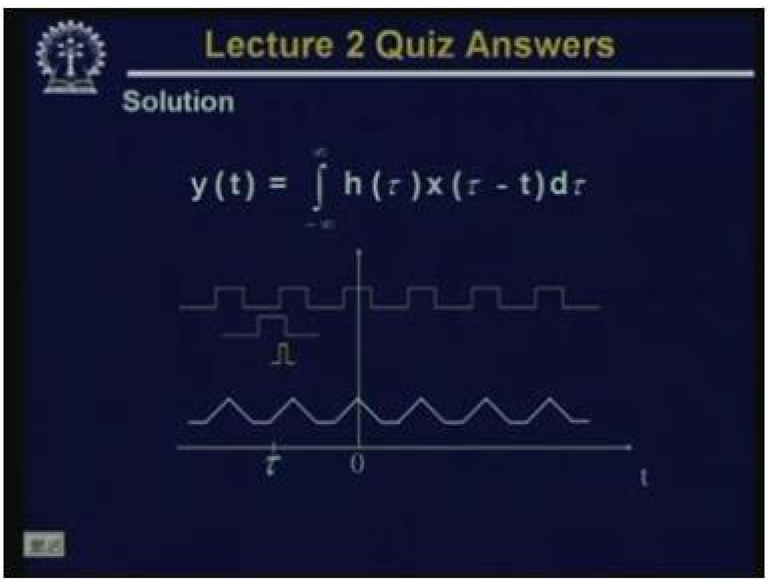

Again here, you will find that if I simply follow the convolution equation which is y (t) equal to ht h tau x tau minus td tau where the integration has to be taken from minus infinity to infinity, you will find that final convolution that you will get is a triangular wave of this form.

Of course, the shape of the triangular wave form will depend upon what is the on period and off period of these 2 different signals. You can work it out and you will find that the final convolution output will be rectangular wave like this.

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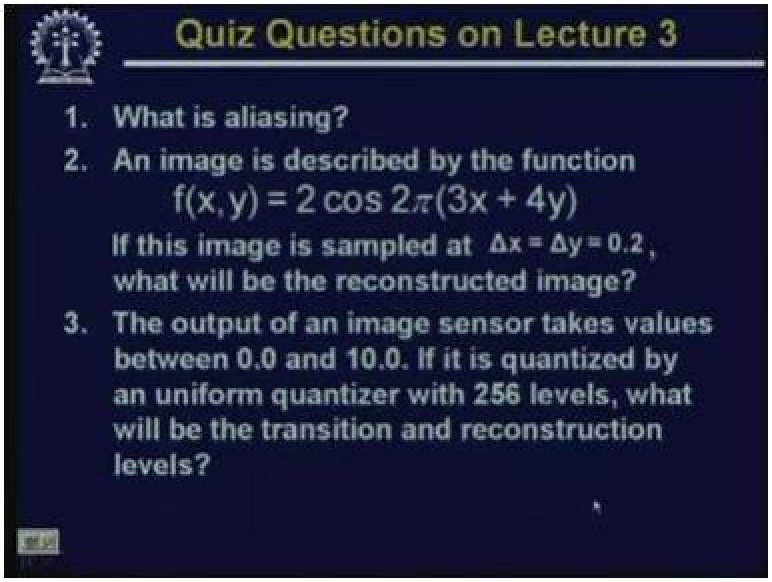

Now, let us come to today’s tutorial questions. So, today I give you 3 tutorial questions. The first one is what is aliasing? The second question is an image is described by the function f (x, y) equal to 2 cos 2 pi into 3x plus 4y. If this image is sampled at delta x equal to delta y equal to 0.2; then what will be the reconstructed image?

So, you have been given an image which is represented by a function that is f (x, y) equal to twice cos 2 pi into 3x plus 4y, the image is sampled both in x direction and y direction where the sampling interval in both the directions is 0.2; then you have to find out that what will be the reconstructed image.

And the third problem - the output of an image sensor takes values between 0.0 and 10.0. If it is quantized by in uniform quantizer with 256 levels; what will be the transition and reconstruction levels? So, you have an image sensor, the image sensor produces analog values between 0 and 10. These analog values are to be quantized by uniform quantizer which is having 256 numbers of levels. Then you have to find out that what will be the optimum transition and reconstruction levels.

Thank you.