Digital Image Processing

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Lecture - 25

Image Registration

Hello, welcome to the video lecture series on digital image processing. For our last few classes, we have talked about different types of image restoration techniques. In today's lecture, we are going to talk about another topic which is called image registration.

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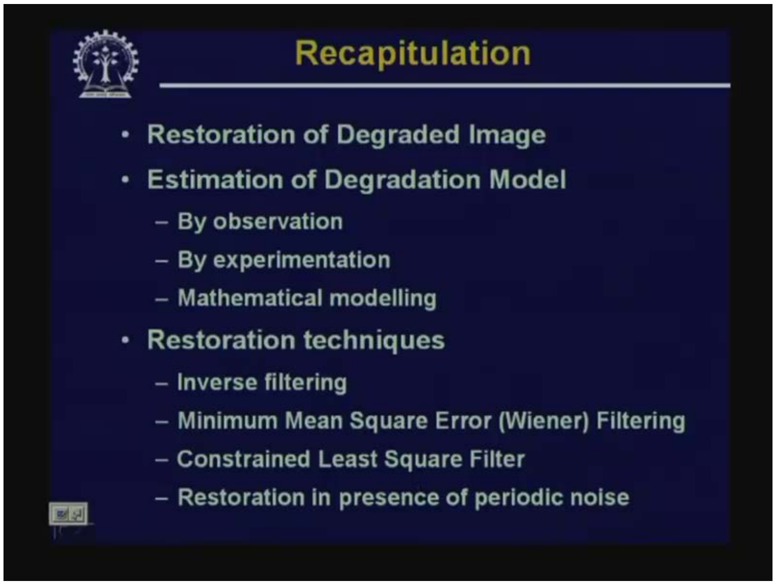

So, in our last few lectures, we have talked about restoration of degraded image, we have seen different techniques for estimation of the degradation model and the different model estimation techniques, we have discussed the estimation of the degradation model by observation, estimation of the degradation model by experimentation and the mathematical modeling of degradation and once you have the model the degradation model, then we have talked about the restoration techniques for restoring a degraded image.

So, among the different restoration techniques, we have talked about the inverse filtering technique, we have talked about minimum mean square error or Wiener filtering technique, we have talked about the constraint least square filtering approach and we have also talked about the restoration of an image if the image is contaminated by periodic noise.

So in such case, we have seen that if we take the Fourier transform of the degraded image, if the image is actually degraded by periodic noise or a combination of periodic noise; then those noise components, the noise frequencies appear as very bright spots, very bright dots in the Fourier transformation or in the frequency plane.

So there, we can apply a band reject filter or sometimes notch filter to remove that particular frequency component of the Fourier transform and after transforming the band reject operation, whatever is remaining, if we take the inverse Fourier transformation of that; then we get the restored image which is free from the periodic noise. So, in today's lecture, we will talk about the image registration techniques.

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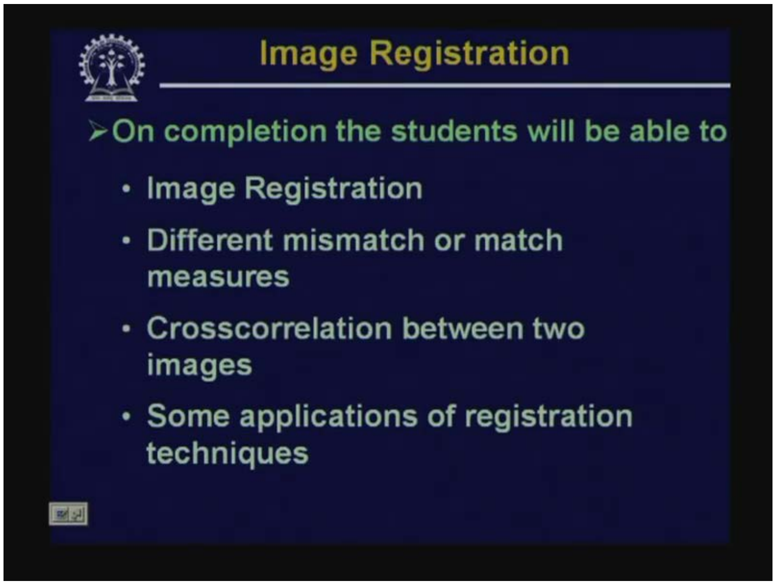

So we will see, what is image registration. Then, when we go for image registration, then we have to think of the mismatch or match measures. So, we will talk about the different mismatch or match measures or similarity measures. We will see that whether the cross correlation or we will see what is the cross correlation between 2 images and we will also see whether this cross correlation can be used as a similarity measure when we go for image registration and then we will talk about some applications of this image registration techniques with examples.

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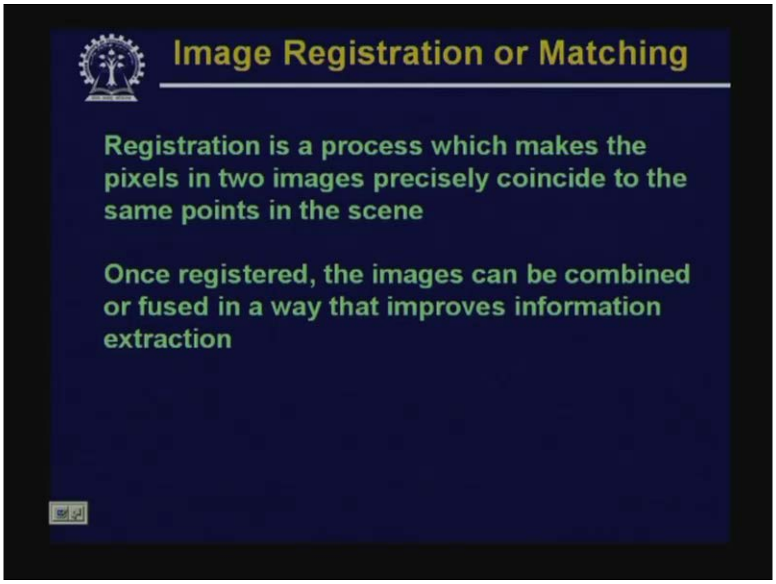

So, by image registration what we mean is that the registration is a process which makes the pixels in 2 images precisely coincide to the same points in the scene. So, by registration what we mean is if we are having 2 images of the same scene may be the images 2 or more images are acquired with different sensors located at different positions or may be the images are acquired using the same sensor but at different time instance, at different instance of time; so in such cases, if we can find out for every pixel in one image, the corresponding pixel in the other image or other images that is the process of registration or the process of matching.

And, this has various applications that we will talk about a bit later. So, once registered, the images can be combined or fused. This is called fusion or combination. So, once we have the images from different sensors, may be located at different locations or may be if it is a remote sensing image taken through a satellite where we have images taken in different bands of frequencies; then if we register all those images, then those images can be combined or they can be fused so that the information extraction or the image, the fused image becomes more rich in the information content.

So once registered, the images can be combined or fused in a way that improves the information extraction process because this fused image, now they have combination of the information from different image or from different bands of frequencies. They will have more information or they will be more rich in the information content.

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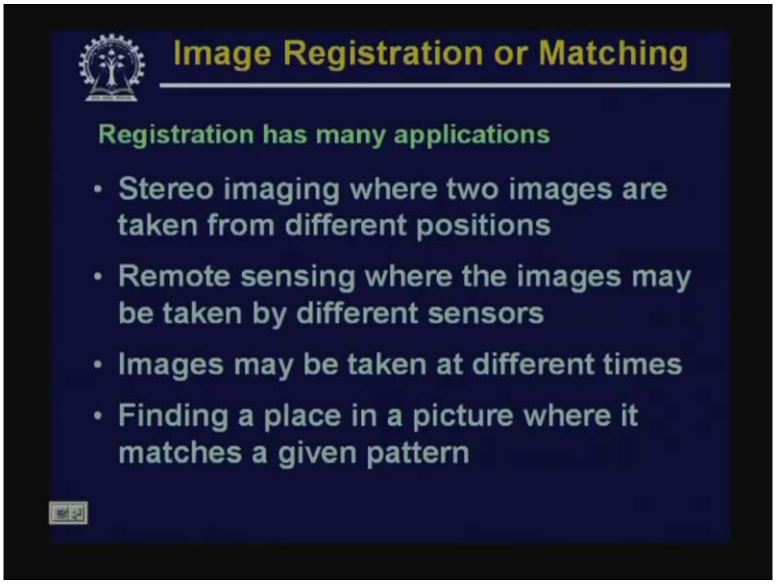

So, this image registration technique has many applications. The first application, we have already said that the stereo imaging technique, in stereo imaging what we do is we take the image of the same scene or the images of the same object by 2 cameras which are slightly displaced and there we have assumed that the cameras are otherwise identical that is apart from the displacement along a particular axis say y axis or x axis, the features of the cameras are identical that is they have the same focal length same view angle and all these things.

So, once I acquire these 2 images that one of them we call as left image, the other one is called as right image; then if I go for this point by point correspondence that is for a particular point in the left image if I can find out what is the corresponding point in the right image, then from these 2, I can find out what is the disparity for that particular point location and if this disparity is obtained for all the points in the image, then from the disparity, we can find out what is the depth or the distance of the different object points from the camera. So, that is in case of stereo imaging, we have to find out this point correspondence or the point registration or this is also called point matching.

The other application that we have just said that in remote sensing where the images may be taken by different sensors working in different bands, even the sensors may be located in different locations; so the images of the same scene are taken by different sensors working in different bands, they are also in different geometric locations, there also if we go for image registration that is point by point correspondence among different images, then we can fuse those different images or we can combine those different images so that your fused image become more rich in terms of information contents. So, the information extraction from such fused images becomes much more easier.

The other application, again, the images may be taken at different times. So, if the images are taken at different times and in all those images if we can register those images that is we can find out which is the point for a particular point in a given image, what is the corresponding point in the other image which is taken at some other time instant; then by this registration, we can find out what is the variation at different point locations in the scene and from this variations, we can extract many informations like say vegetation growth or may be the land erosion, deforestation or the occurrence of a fire. So, all these different informations can be obtained when we go for registration of the images which are taken at different instance of time.

There are other applications like finding a place in a picture where it matches our given pattern. So here, we have a small image which is known as a pattern or a template and we want to find out that in another image which is usually of bigger size, where does this template match the best.

Now, these has various applications like automated navigation where we want to find out that what is the location of a particular object with respect to a map. So, in such automated navigation applications, this pattern matching or template matching is very very important. So, this image registration technique or the image registration methods has various other applications and which can be exploited once the registration techniques are known.

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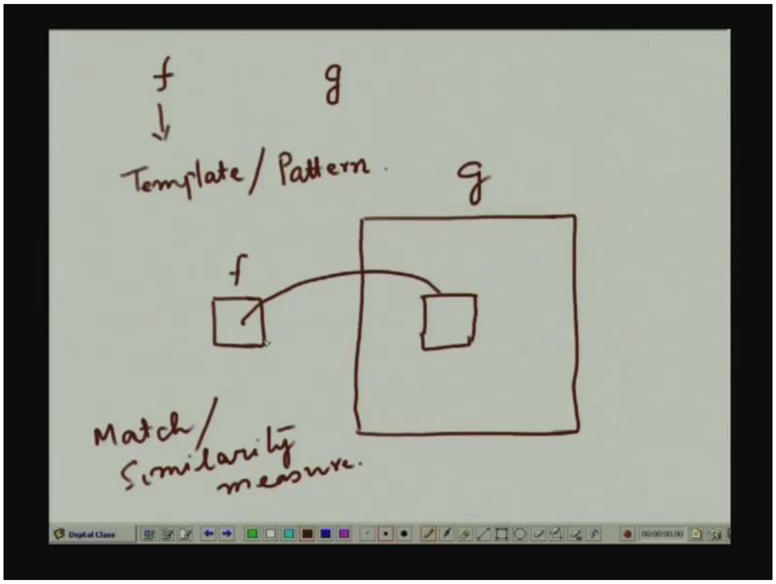

Now, to explain the registration techniques, let us take the first example that example of the template matching. So for this template matching, we take a template of a smaller size. We call the template to be a template f, say f is a 2 dimensional image of a smaller size and we have an image g which is of bigger size.

So, the problem of template matching is to find out where this f, the template f matches best in the given image g. So, this f is called a template or it also called a pattern. So, our aim is that given a very big image say g or 2 dimensional image g and a template f which is usually of size smaller than that of g; we want to find out that where this template f matches the matches best in the image g.

So, to find out where the template f matches best in the given image g, we have to have some measure of matching which is called which may be termed as mismatch measure or the opposite of this that is the match measure or the similarity measure. So, we have to take different match or similarity measures; so you call them as match measure or similarity measure to find out where this template f matches the best in the given image g.

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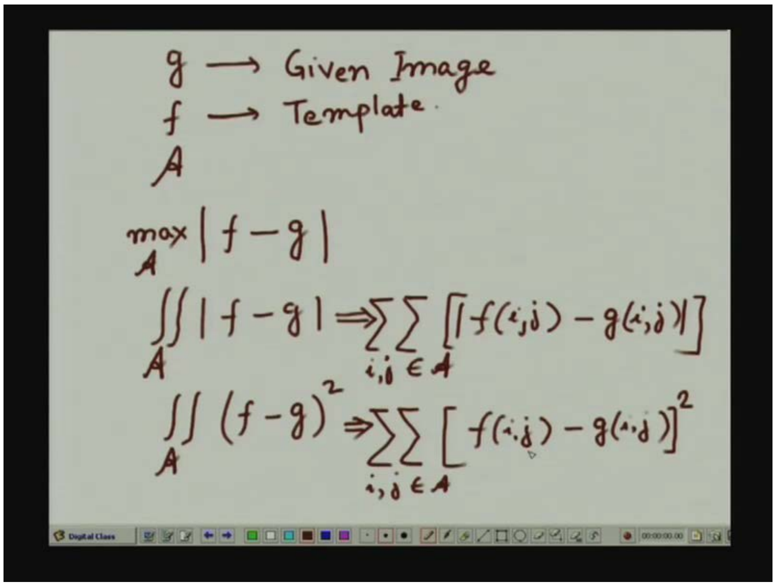

So, there are various such match or mismatch measures and let us see that what are the different measures that can be used. So, we have the given image g, g is the given image and we have the template f, so f is the template. So, we have to find out the measure of similarity between a region of the image g and the template f. So, there are various ways in which this similarity can be measured so that we can find out the match between the f and g over a certain region say given by A.

So, one of the similarity measure, the simplest similarity measure is we will take the difference of f and g, so you take the absolute difference between f and g and find out the maximum of the absolute difference and this maximum has to be computed over the region say A. The other similarity measure can be that again we take the absolute difference of f and g and then integrate this absolute difference over the same region A.

The other similarity measure can be you take f minus g, the difference between f and g and the square of this and take the integration of this over the same region say A. So, you find that in the first case when it is the difference between f and g, so when I am talking about the difference, it is pixel by pixel difference. So, it takes the difference between the f and g, take the absolute value and the maximum of that computed over the given region A.

In the second case, it is f minus g, again the absolute value and this is integrated over the given region A. So, this is in the analog case. If I convert this in the digital form, this will take the form as f (i, j) minus g (i, j) where i j is pixel location, absolute value of this and you take the double summation for all i and j in the given region A.

So, you find that this is nothing but what is called the sum of the absolute difference. So, sum of absolute difference between the image g and the template f over the region A and if I convert this expression again in the digital form, this becomes f (i, j) minus g (i, j) square of this and take the double summation for all i, j belonging to that region A. So, this is nothing but sum of the difference squares.

So, if I say that f (i, j) minus g (i, j) is the difference between the 2 images or the error; so this last expression that is f minus g square integration over A double integration over A in the digital domain, it becomes f (i, j) minus g (i, j) square take the double summation for all i j in the region A, this is something equivalent to sum of square error. Now, out of these 3 different measures, it is the last one - g f minus g square integration, this is very very interesting.

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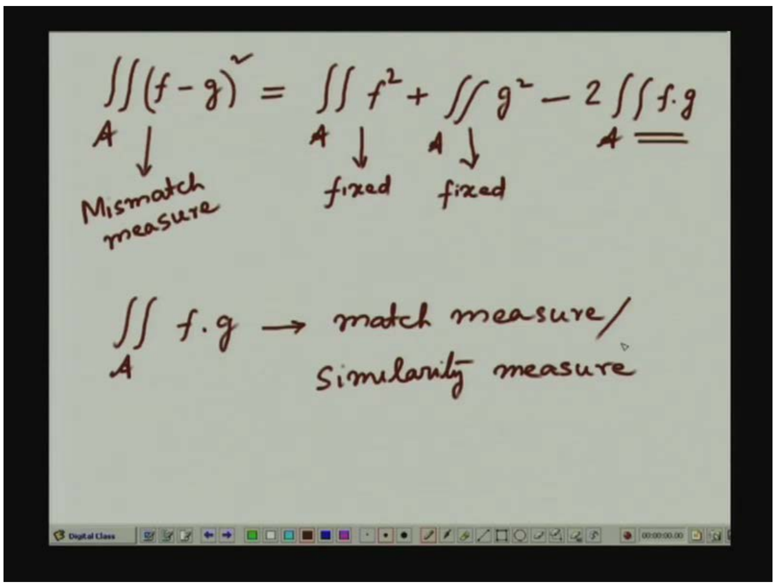

So, if I expand this term, f minus g square double integration over the region A, if I expand this, then this becomes double integration f square plus double integration g square minus 2 into double integration f into g. So, all this double integrations are to be taken over the given region A. Now, you find that for a given image, this double integration f square, this is fixed; for a given template, this is fixed and also for a given image over the region A, this double integration g square, this is also fixed.

Now, what is this f minus g square, sum of this? This is nothing but a sum of the square differences that means this gives the degree of mismatch or this is nothing but the mismatch measure. So, if this f minus g square integration over the region A is minimum because this is the mismatch measure; so wherever this is minimum, at that particular location, f matches the best over that particular region of g.

Now, when I expand this, this becomes integration, double integration f square plus double integration g square minus twice into double integration f into g and as I said that for a given template f square is fixed and for a given image and a given region g square is also fixed. That means this f minus g square integration, this will be minimum when this f into g double integration this term will be maximum.

So, whenever the mismatch measure is minimum that will lead to f into g double integration over the region A, this will be maximum. So, when f minus g square double integration is taken as the measure of mismatch, we can take the double integration f into g over the region given region A is to be the match measure or the similarity measure.

So, this means that whenever the given template matches the best in a particular region in a particular portion of the given image g; in that case, f into g double integration over the region A will have a maximum value and we take this as the similarity measure or the match measure.

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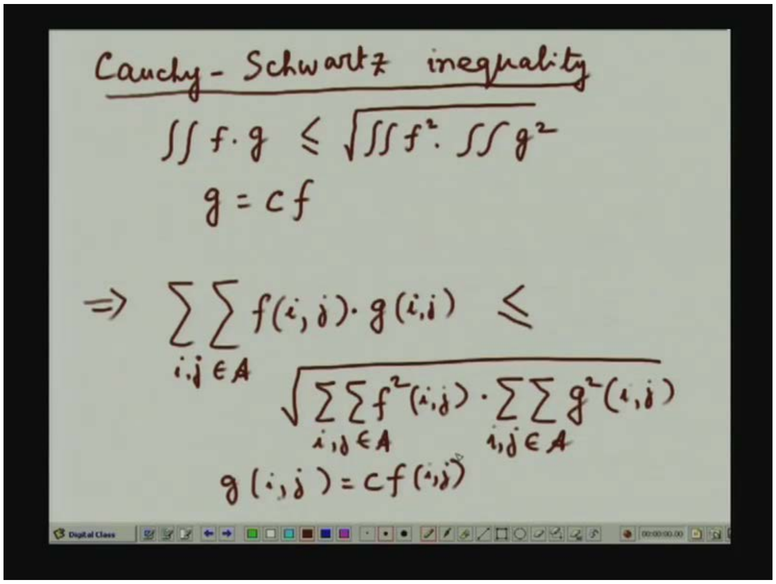

Now, the same conclusion can also be drawn from what is called Cauchy Schwartz inequality. So, this Cauchy Schwartz inequality says that double integration of f into g, this is less than or equal to square root of double integration of f square into double integration of g square and these 2 terms will be equal only when g is equal to some constant c times f.

So, this Cauchy Schwartz inequality says that f into g double integration will be less than or equal to square root of double integration f square into double integration g square and the left hand side and the right hand side will be equal whenever g is equal to c times f. Otherwise, left hand side will always be less than the right hand side.

So, this also says that whenever f or the template is similar to a region of the given image g within with a multiplicative factor of constant c, then this f into g integration will take on the maximum value. Otherwise, it will be less. If I convert this into the digital case, then the same expression is written can be written in the form that double summation f (i, j) into g (i, j) where i and j belongs the given region A, this should be less than or equal to square root of double summation f square (i, j) into g square (i, j).

Again, sorry this is has to be double summation j square (i, j) for this i and j belonging to the region A, here also for i and j belong to the region A and this left hand side and the right hand side will be equal only when g (i, j) is equal to some constant into f (i, j) and this has to be true for all values of (i, j) within the given region. So, for this template matching problem, we have assumed that f is the given template and g is the given image and we have also assumed that f is less than or the size of f is less than the size of the given image g.

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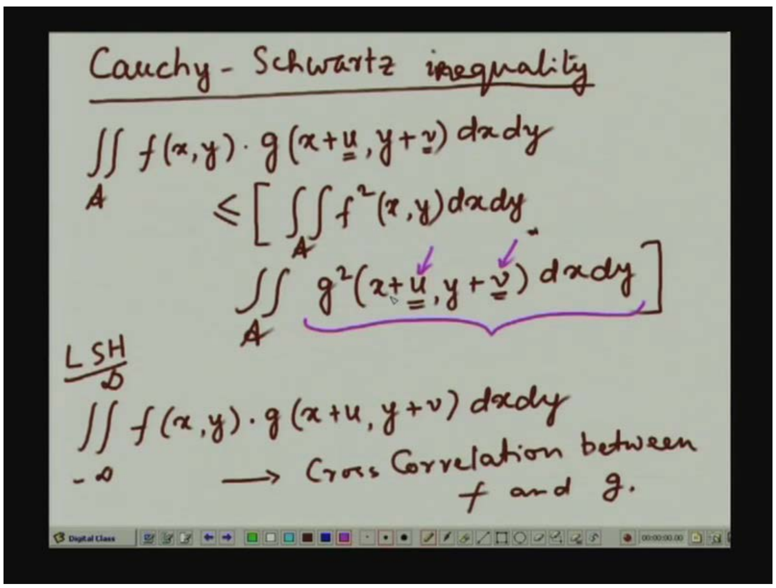

Now again, from the Cauchy Schwartz inequality, so I go back to this Cauchy Schwartz inequality, what we get that double integration f (x, y) into g (x plus u, y plus v) into dx dy should be less than or equal to double integration f square (x, y) dx dy into double integration g square (x, y) into dx dy sorry this should be this into double integration g square (x plus u, y plus v) dx dy.

Now, the reason we are introducing these 2 variables u and v is that whenever we try to match the given pattern f against the given image g, we have to find out what is the match measure or the similarity measure at different locations of g. So for this, f has to be shifted at all possible locations in the given image g and that shift, the amount of shift that has to be given to the pattern f to find out the similarity measure at that particular location, we introduce these 2 shift components u and v. So, this says that shift along x direction is u and shift along y direction is v.

So here, in this expression, the similarity between the given template f and the image g with shift u and v is computed and this is computed over the given region A. Now, because this f (x, y) is small and the value of f (x, y) is 0 outside the region A, so we can replace this left hand side by an integration of this form - f (x, y) into g (x plus u, y plus v) into dx dy and as I said that because f (x, y) is 0 outside the region A, so this definite integral the integral over A can now be replaced by an integral from minus infinity to infinity. So, this is what we get from this left hand side of the expression.

Now, if you look at this particular expression that is f (x, y) into g (x plus u, y plus v) dx dy double integral from minus infinity to infinity, this is nothing but the cross correlation between f and g and then if you look at the right hand side, this f square (x, y) dx dy integral over A, this is a constant for a given template whereas, this particular component that is g square (x plus u, y plus v) dx dy, this is not a constant because the value of this depends upon the shift u and v.

So, though from the left hand side, we have got that this is equivalent to cross correlation between the function f and g but this cross correlation directly cannot be used as a similarity measure because the right hand side is not fixed. Though f square (x, y) dx dy is fixed but g square (x, y) (x plus u, y plus v) dx dy integral is not fixed. It depends upon the shift u and v. So, because of this, the cross correlation measure cannot be directly used as a similarity measure or a match measure.

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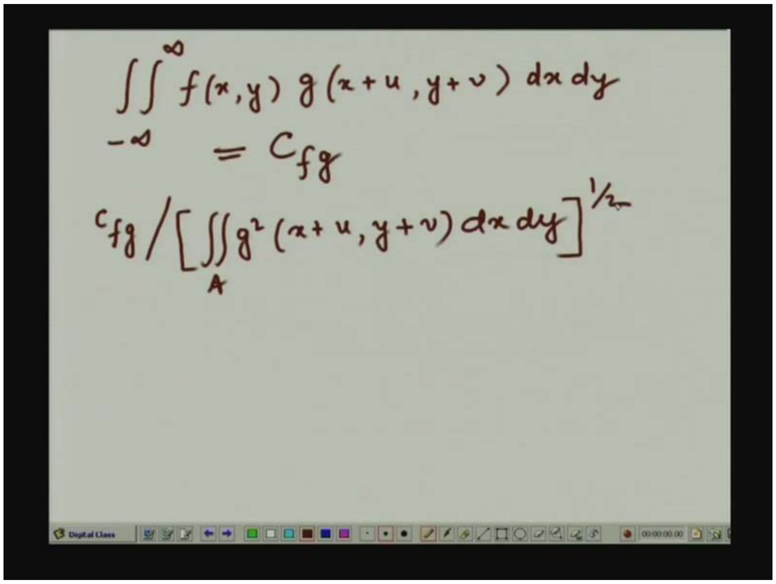

So, what we have to go for is what is called a normalized cross correlation. So, if we call this cross correlation measure f (x, y) g (x plus u, y plus v) dx dy integration from minus infinity to infinity, if I represent this as the cross correlation C_{fg}, then the normalized cross correlation will be given by C_{fg} divided by g square (x plus u, y plus v) dx dy double integral over the region A and square root of this.

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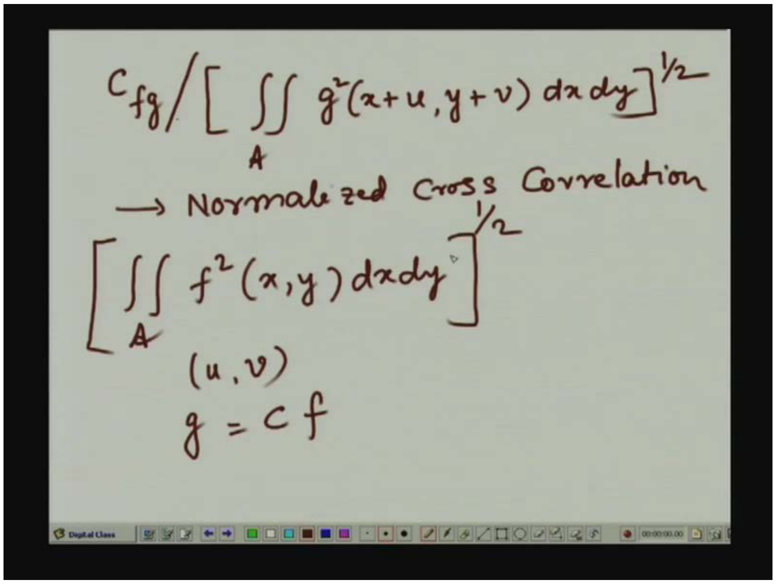

So, in our previous one, so what we said is that C_{fg} has to be, the normalized cross correlation will be C_{fg} divided by double integral g (x plus u, y plus v) dx dy where the integration is taken over the region A and square root of this and you see that once I take this normalized cross correlation, this is what we are calling as normalized cross correlation.

So, once we consider this normalized cross correlation, you find that C_{fg} will take the maximum possible value which is given by double integral f square (x, y) dx dy, take the integral over the region A and because this is fixed; so this region of integration is not very important, square root of this, this is the maximum value which will be attained by this normalized cross correlation for a particular value of u and v for the values of u and v for which this function g becomes some constant c times f.

So, for that particular value that particular shift (u, v) where g is equal to sum constant c times f, this normalized cross correlation will take the maximum value and the maximum value of this normalized cross correlation is given by double integral f square (x, y) dx dy and square root of this. So, now to illustrate this, let us take an example.

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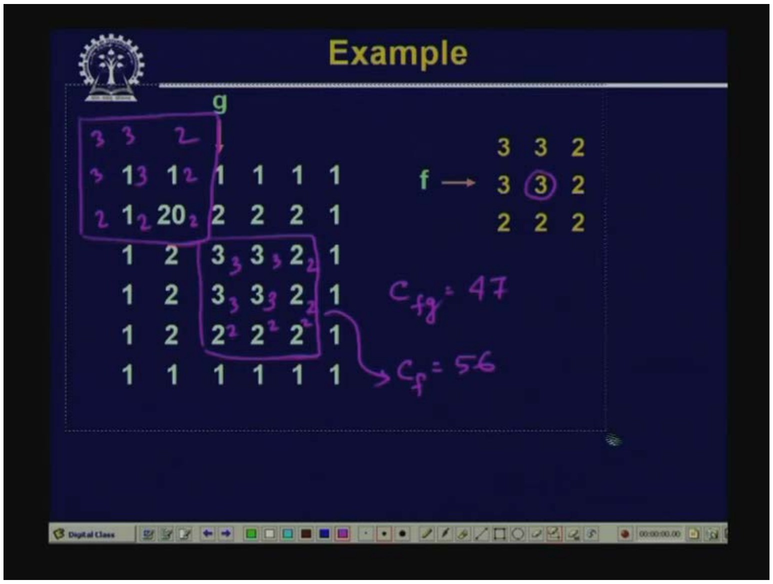

Say here, this is the image g which is given in the form of a 2 dimensional matrix, 2 dimensional array of size 6 by 6 and our template is a 3 by 3 matrix which is given on the right hand side. Now, if I calculate the C_{fg} for this; so what I have to do is to find out the match location, I have to take the template, shift it at all possible locations in the given image g and find out the cross correlation or the similarity measure for that particular shift.

So initially, let us put this template at the left most corner and what we do is because our template is 3, 3, 2 - 3, 3, 2 - 2, 2, 2, let us match let us place the center of the template at the location at which we want to find out the similarity measure. So because of this, 3 will be placed over here, this particular element will be placed over here, this 2 will go here, this 2 will come here, this 2 will come here and on the left hand side, the other part of the template will be like this – 2, 3, 3, 3, 2.

So, this will be the position of the template and at this location, we have to find out what is the similarity value. So, for that let us find out what is C_{fg}, the cross correlation between f and g for this particular shift. Now, here if you compute, you find that all these elements of the template are going beyond the image. So, if I assume that the image components are 0 beyond this boundary of the image, then these elements will not take part in the computation of cross correlation.

The cross correlation will be computed only considering these 4 elements and here if you compute you, will find that this C_{fg} for this particular position, for this particular shift will attain a value of 47 because here it is 40 plus 2 plus 2 plus 3. So, this becomes assumes a value of 47. Similarly, if I want to find out the cross correlation at this particular location over here where the template is placed, the center of the template is placed over here; the other components of the template will come like this, you will find that the cross correlation at this location is given by C_{fg} is equal to 56.

So like this, for all possible shifts within this particular image, I have to find out what is the cross correlation value and if I complete this particular cross correlation computation; in that case, you will find that finally I get a cross correlation matrix which is like this.

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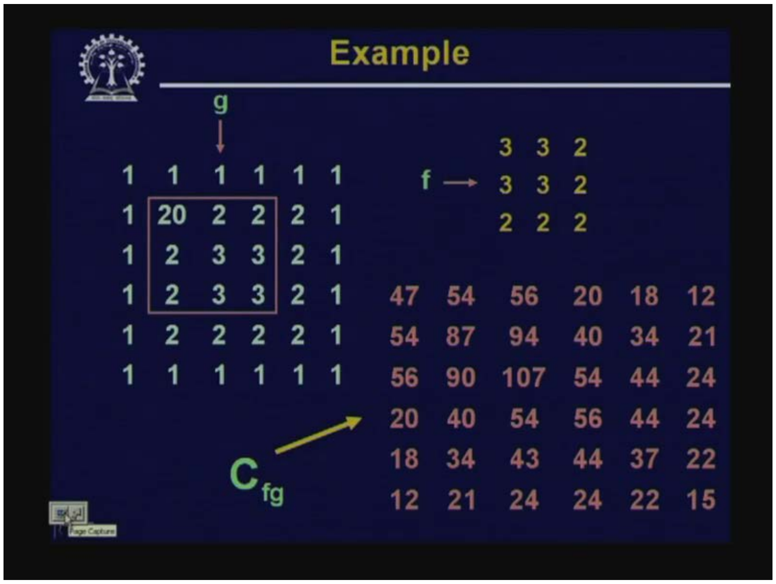

So, these gives the complete cross correlation matrix when this template is shifted to all possible locations in this given image and the cross correlation value is computed for all such possible shifts. Now from this, you find that here the maximum cross correlation value is coming at this particular location which is given by 107 and if I take this 107 to be the similar or the cross correlation values to be the similarity measure that means this is where I get the maximum similarity and because of this, it gives a false match that it appears that the template is matching the best in this particular location as shown by this red rectangle. But that is not the case because; if I just checking it visually, we can see that the template is best match in this location. So, that is why we say that the cross correlation measure directly cannot be used as a similarity measure.

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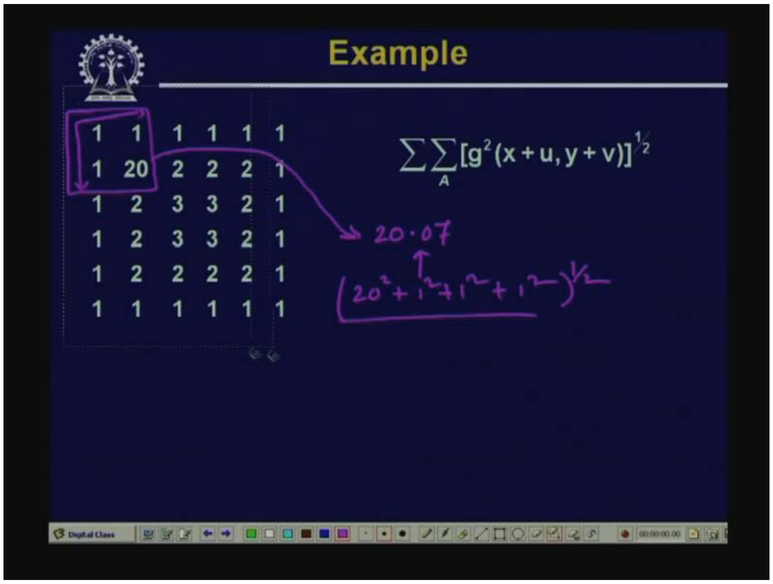

So, over here, as we have said that we cannot use the cross correlation measure directly as a similarity measure; so, what we have to do is we have to compute the normalized cross correlation value. So, for computation of the normalized cross correlation, we have to compute this particular component that is g square (x plus u, y plus v) double summation over the region A and square root of that. So, this is the one that we have to compute for all possible shifts in the given image g and we have to normalize the cross correlation that we compute with the help of this quantity.

So, if I compute this, again let us take the same location; if I compute this g square (x plus u, y plus v) summation over the region A, square root of that summation over the region A, then you will find that this particular value will come out to be something like 20.07. Because this is nothing but 20 square plus 1 square plus 1 square plus 1 square other elements for this shift within this 3 by 3 window is equal to 0; so this is what we get, I have to take the square root of this. So, if I compute this component, I get the value of 20.07.

So, this is how if I compute this quantity that is g square (x plus u, y plus v) over the region A, square root of this and summation over the region A for all possible u and v that is for all possible shifts; then finally, this normalization component that I get is given by this particular matrix.

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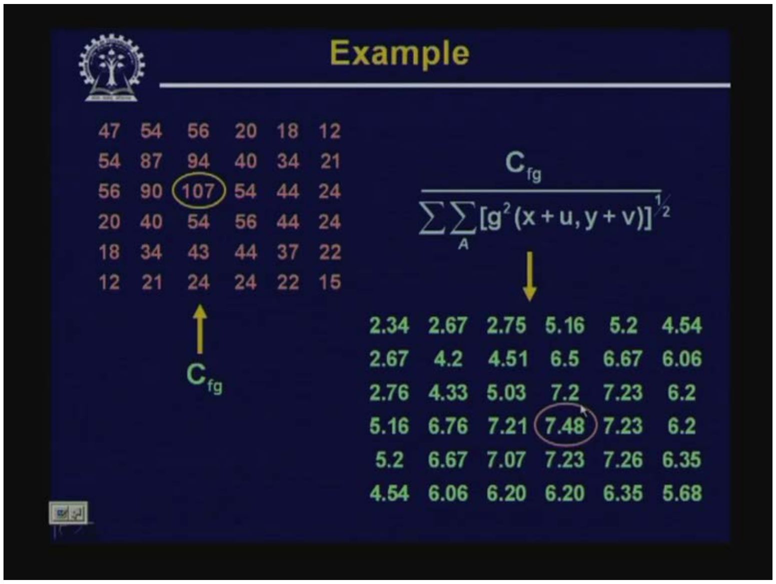

So, this cross normalization coefficient have been computed for all possible values of u and v and then what we have to do is we have to normalize the cross correlation with these normalization factors.

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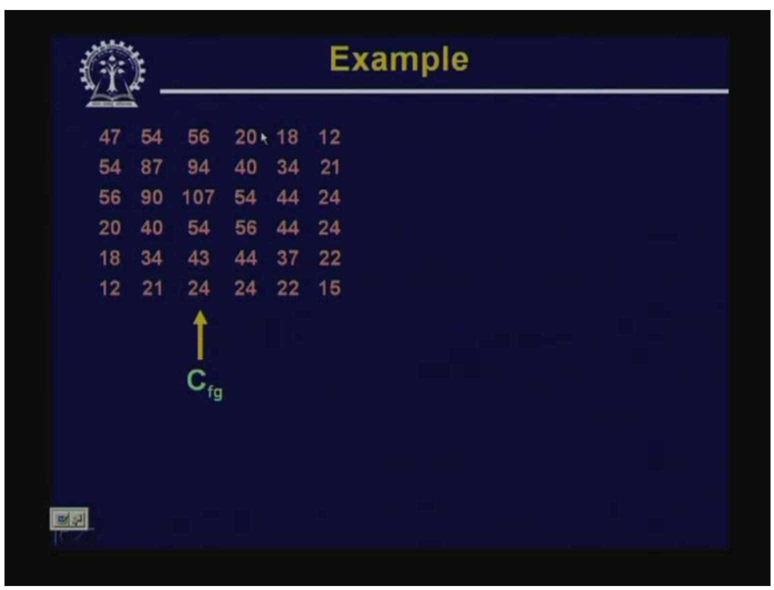

So, if I do that normalization, this is my original cross correlation coefficient matrix that I have computed earlier and by using the normalization, what I get is the normalization normalized cross correlation coefficient matrix which comes like this. And now, you find the difference. In the original cross correlation matrix, the maximum was given in this location which is 107 and in the normalized cross correlation matrix; the maximum is coming at this location which is 7.48.

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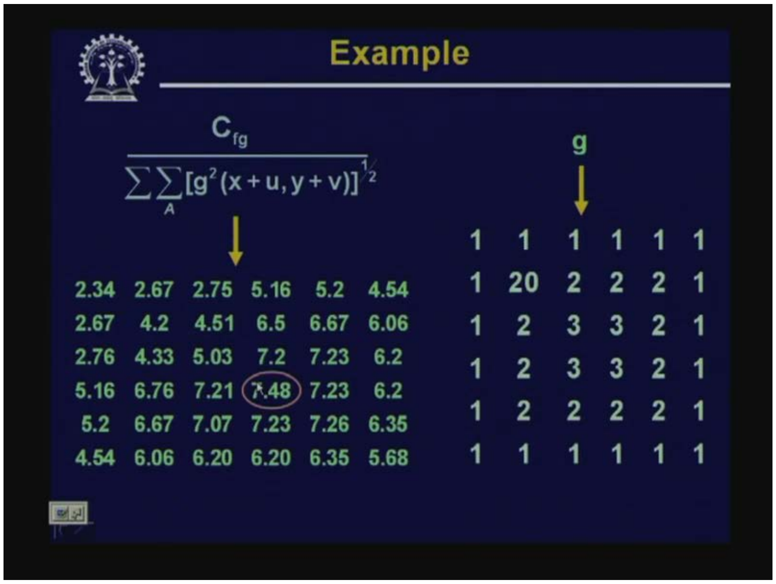

So now, let us see that using this as the similarity measure that is normalized cross correlation as the similarity measure; what happens to our matrix? So, this is the same matrix, the normalized cross correlation matrix; the maximum 7.48 is coming at this location and for this maximum, the corresponding matched location of the template within the image is given over here as shown by this red rectangle.

So now, you find that this is a perfect matching where the template is where the similarity measure gives the perfect measure where the template matches exactly. So, that is possible with the normalized cross correlation but it is not possible using the simple cross correlation. So, the simple cross correlation cannot be used as the similarity measure but we can use this normalized cross correlation as a similarity measure.

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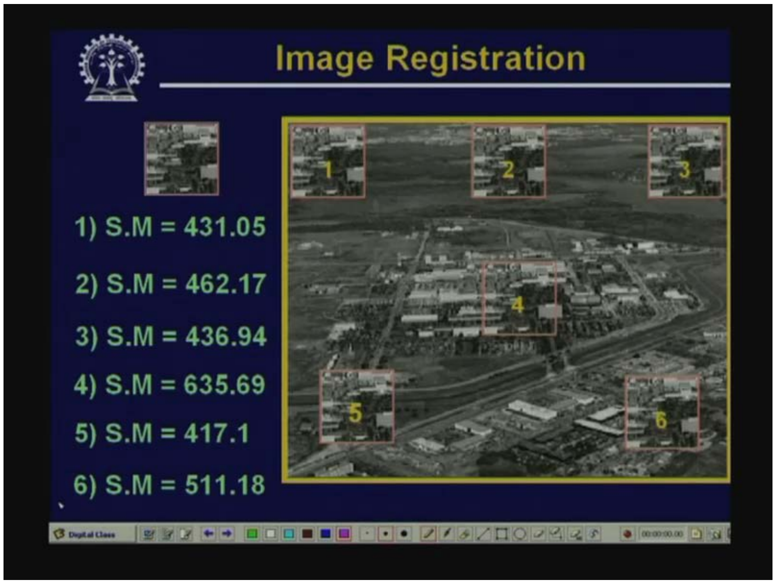

Now, coming to the application of this: so here, we have shown that application over a real image, so this is an aerial image taken from the satellite and the smaller image on the left hand side which is a part of the image which is cut out from this original image and we are using this as a template, the smaller image we are using as a template. So, this is our template f which we want to match against this image g. So, we want to find out where in this given image g, this template f matches the best.

So, for do doing that, as we have said earlier that what we have to do is we have to paste this template f that is we have to shift this template f at all possible locations in the given image g and for all possible such locations, we have to find out what is the normalized cross correlation and wherever we get the normalized cross correlation to be maximum, that is the location where this template matches the best.

So, let us place this template at different locations in the image and find out what is corresponding similarity measure we are getting. So, this is the template and this is the given image. If I place this template over here, we are calling this as location 1; so this location is given by this red rectangle, then the similarity measure that we are getting is a value 431.05. If I place it at location 2 which is over here, the similarity measure or the normalized cross correlation you get is given by 462.17.

If I place it at location 3; then at location 3, the similarity measure is 436.94. If I place the template at location 4, the corresponding similarity measure is coming out to be 635.69. If I place it at location 5, the corresponding similarity measure is coming out to be 417.1. If I place it at this location 6, the corresponding similarity measure comes out to be 511.18.

So, you see that from these similarity measure values and if I compute for all possible locations; of course for all possible locations, I cannot show it on the screen, so if I compute this for all possible locations, then you will find that this similarity measure value is coming out to be maximum that is 635.69 for this location which is location 4 in the given image and exactly this is the location from where this particular template f was cut out and if you look at this picture after placing this template, you will find that there is almost a perfect match.

So, for a given image and a given template, if I find out the normalized cross correlation for all possible shifts u and v, then for the shift u and v where you get the normalized cross correlation to be maximum, that is the location where the template matches the best. So obviously, this is a registration problem where we want to find out where that given template matches the best in a given image.

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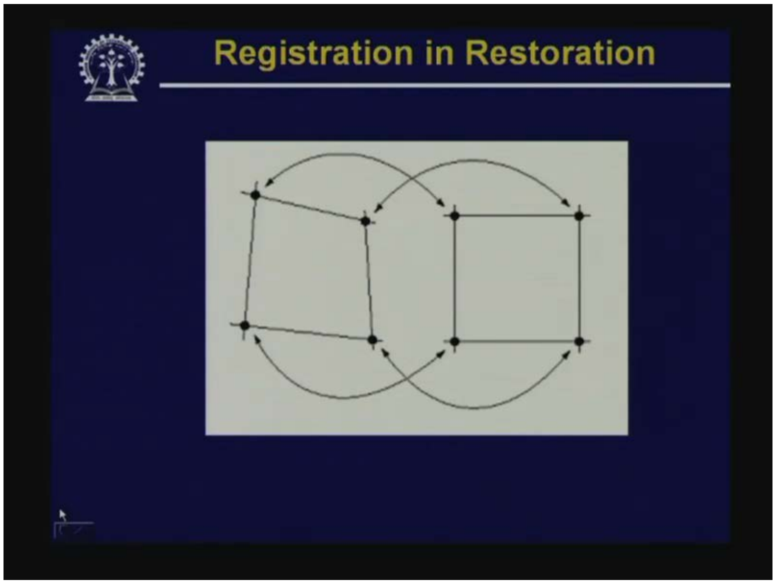

Now, to come to the other applications of this registration, you will find that this registration is also applicable for image restoration problem. Earlier, we have talked about the image restoration problem where you have to estimate that what is the degradation model which degrades the image and by making use of the degradation model, we can restore the image by different types of operations like inverse filtering, Wiener filtering, constant least square estimation and all those different kinds of techniques.

Now here, we are talking about a kind of degradation where the degradation is given in the form of a geometric distortion and that is a distortion which is introduced by the optical system of the camera. So, if you take an image of a very large area, you might have noticed that as you move away from the center of the image, the points try to become closer to each other. So, that is something which leads to a distortion in the image as the point goes away from the center of the image. So here, we have shown one such distortion.

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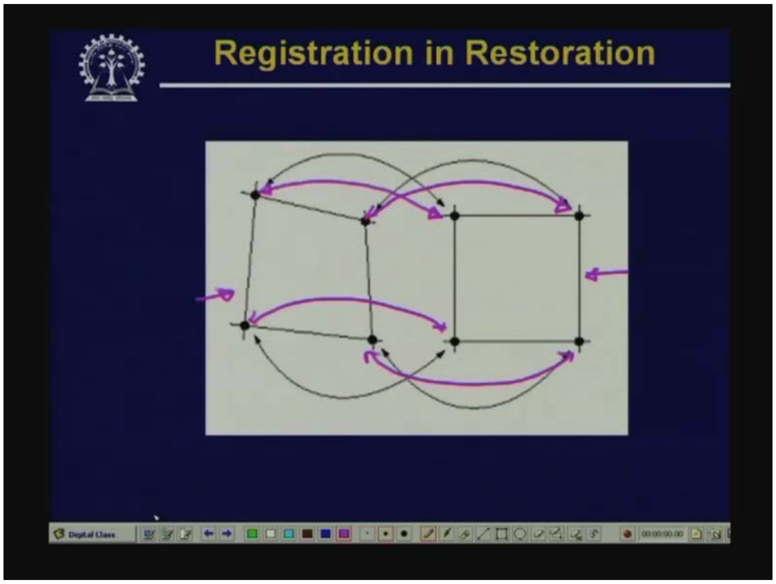

So suppose, this is the figure that we want to of which we want to take the image but actually the image comes in this particular form; then this distortion which is introduced in the image, that can be corrected by applying this image registration technique. So, for doing this, what we have to do is we have to register different points in the expected image and the different points in the degraded image or the restored image.

So, if once I can do that kind of registration; so in this case, this is a point which corresponds to this particular point. So, if somehow, we can register these 2 that is we can find out we can establish the correspondence between this point and this point, we can establish the correspondence between this point and this point, we can establish the correspondence between this point and this point and we can establish the correspondence between this point and this point; then it is possible to estimate the degradation model.

So here, you find that for estimating this degradation, we have to go for registration. So, this registration is also very very important for restoring a degraded image or the degradation is introduced by the camera optical system. So, the kind of restoration that can be applied here is something like this.

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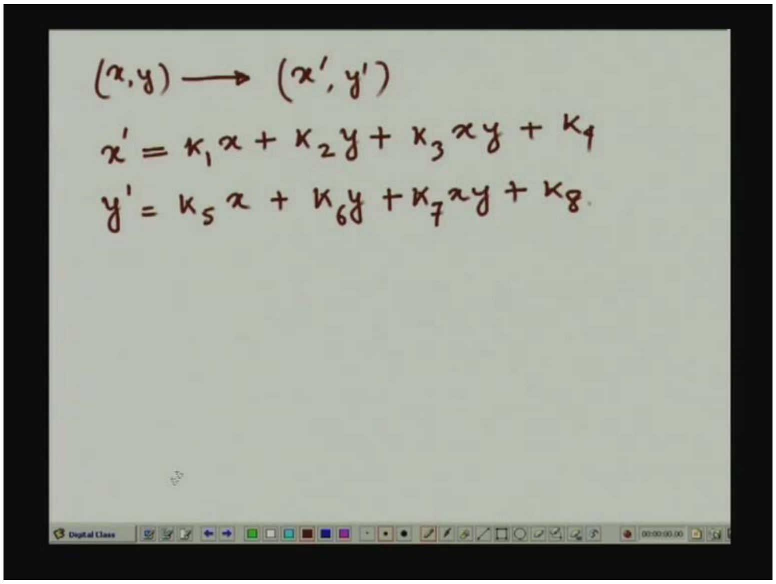

Say, if I have an original points say (x, y) in the original image and this point after distortion is mapped to location say (x prime, y prime); then what we can do is we can go for estimation of a polynomial degradation model in the sense that I estimate that x prime is a polynomial function of x and y.

So, I write it in this form that x prime is equal to say some constant k_1 times x plus a constant k_2 times y plus a constant k_3 times xy plus a constant k_4. Similarly, y prime can be written as some constant k_5 times x plus k_6 times y plus k_7 times xy plus say k_8. So from this, you find that if you can estimate this constant coefficients k_1 to k_8, then for any given point in the original image, we can estimate what will be the corresponding point in the degraded image. So, for estimation, for computing this k_1 to k_8, this constant coefficients because there are 8 such unknowns, I have to have 8 such equations and those equations can be obtained by 4 pairs of corresponding points from the 2 images.

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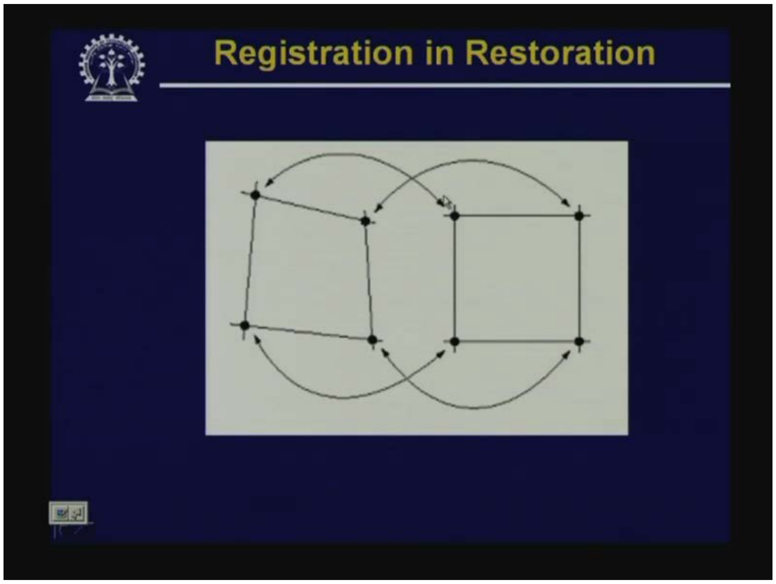

So, that is what if I look at this figure; I have to get 4 such correspondence pairs. So, once I have four such correspondence pairs, I can generate 8 equations and using those 8 equations, I can solve for all those constant coefficients k_1 to k_8 and once I get that, then what I can do is I can say that I want an original image, an undistorted image and to a particular point to that undistorted image; I apply that distortion, find out a point in the distorted image and whatever is the intensity value at that location in the distorted image, I simply copy that in my estimated location in the original image. So, I can find out a restored image from the distorted image.

Obviously, while doing this, we will find that there will be some location where I do not get any information. That is for a particular location say (p, q) in the estimated undistorted image, when I apply the distortion, then in the distorted image, I do not get any point at that particular location. So in such cases, we have to go for interpolation techniques to estimate what will be the intensity value at that point location in the distorted image. So, for that the different interpolation operations that we have discussed earlier can be used.

So this is how this image registration technique is also playing a major role in restoration of distorted images.

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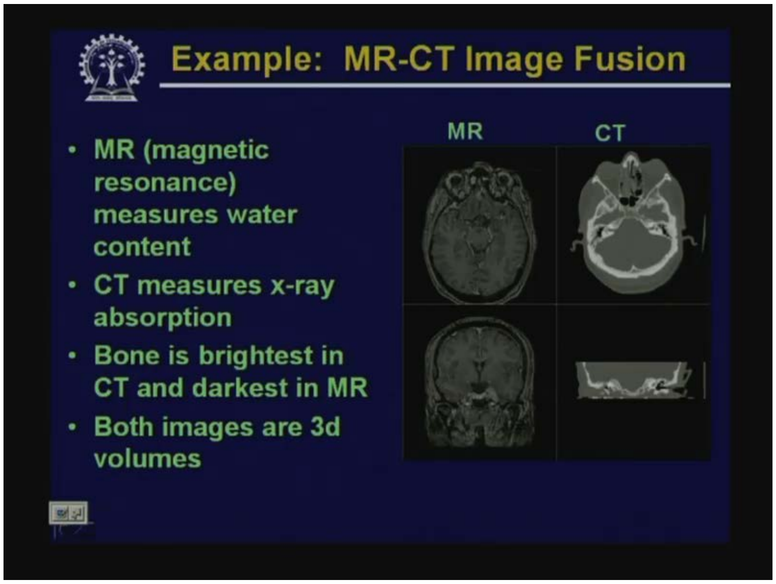

This image registration technique is also very very useful as I said; in image fusion or combining different images and for that also we have to go for image registration. Say for example, here we have given shown 2 types of images; one is magnetic resonance images and CT scan images. Now, magnetic images that gives you a measure of water content whereas in case of CT images, CT x-ray images that gives you the brightest region for the bone regions.

So, if I combine if I can combine the magnetic resonance image along with the CT x-ray image, the fused image that you get where you can get both the informations that is the water content as well as the nature of the bone in the same image. So naturally, the information extraction is much more easier in the fused image.

Now again, for doing this, the first operation has to be image registration because the alignment of the images of the MR images and the CT images, even if they are of the same region; they may not be properly aligned or they may not be properly scaled, they may not be proper there may be some distortion in those 2 different images.

So, the first operation we have to do is we have to go for registration; using registration, we have to get that what is the transformation that can be applied to align, properly align the 2 images and after the transformation, applying the transformation, when you align the 2 images, then only they can be fused properly.

So here, there are 2 MR images and there are 2 CT images and obviously you find that this MR image and this CT image; though they are of the same region but they are not properly aligned, similarly on the bottom row, this MR image and this CT image, they are not properly aligned.

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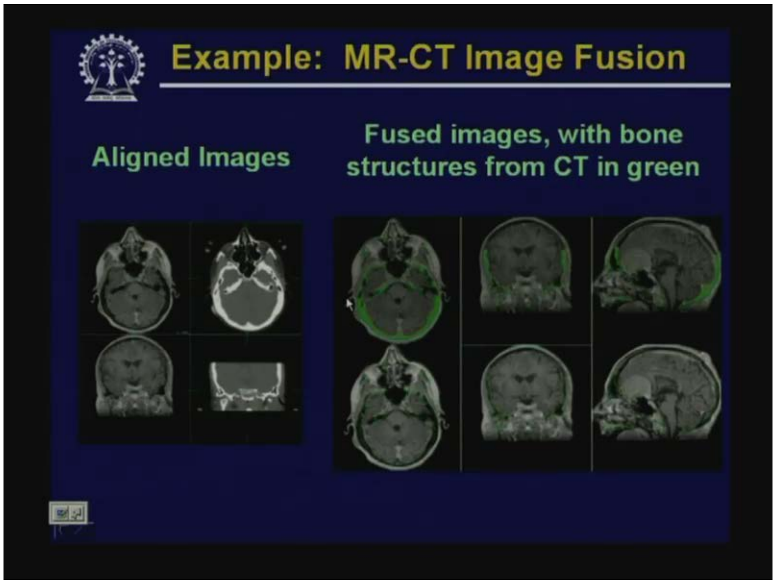

So, first operation we have to do is alignment. So, in this slide on the left hand side, what we have got is the result after alignment and on the right hand side, it is the result after fusion. So, in this fused image, you find that the green regions, this shows you what is the bone structure and this bone structure have been obtained from the CT image and the other regions contain the information from MR image. So, this is much more convenient to interpret than taking the MR image and the CT image separately.

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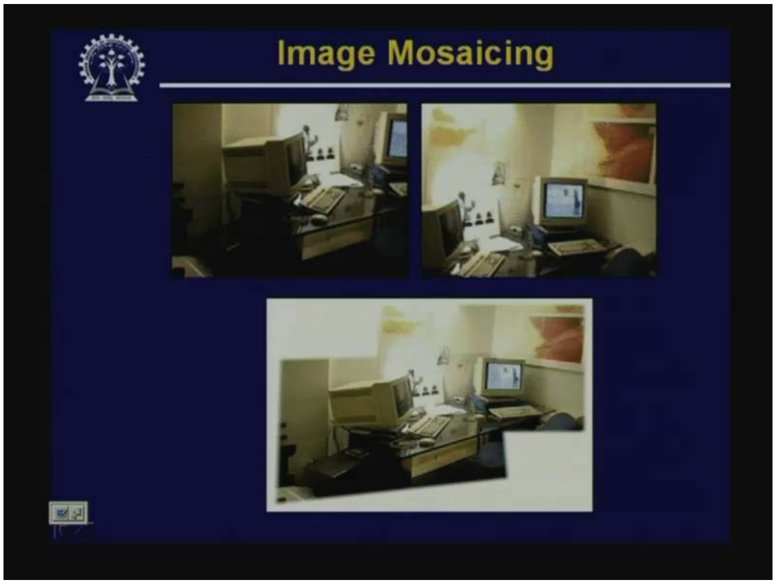

The other application of this is for image mosaicing. That is normally the cameras have very narrow field of view. So, using a camera which is having a narrow field of view, you cannot image a very large area. So, what can be done is you take the images, smaller images of different regions on in the scene and then you try to stitch those smaller images to give you a large field of view image.

So, that is what has been shown here; on the top, there 2 smaller images, these 2 images are combined to give a bigger image as shown on the bottom. So, this is a problem which is called image mosaicing and here again because there are 2 images; they may be scaled differently, their orientation may be different. So firstly, we have to go for normalization and alignment and for this normalization and alignment, again the first operation has to be image registration.

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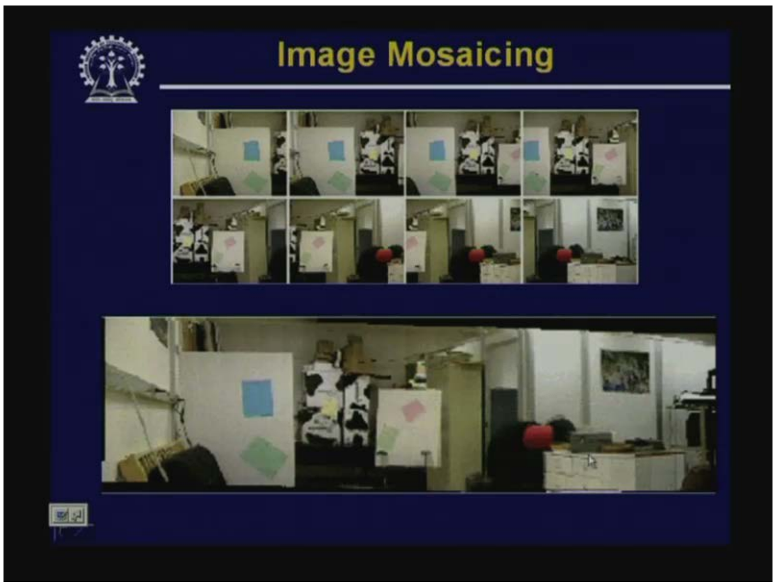

This shows another mosaicing example where this bottom image has been obtained from top 8 images. So, all these top 8 images have been combined properly to give you the bottom image. So, this is the mosaic image that we get. So, with this we complete our discussion on image registration. Now, let us have some questions on today’s lecture.

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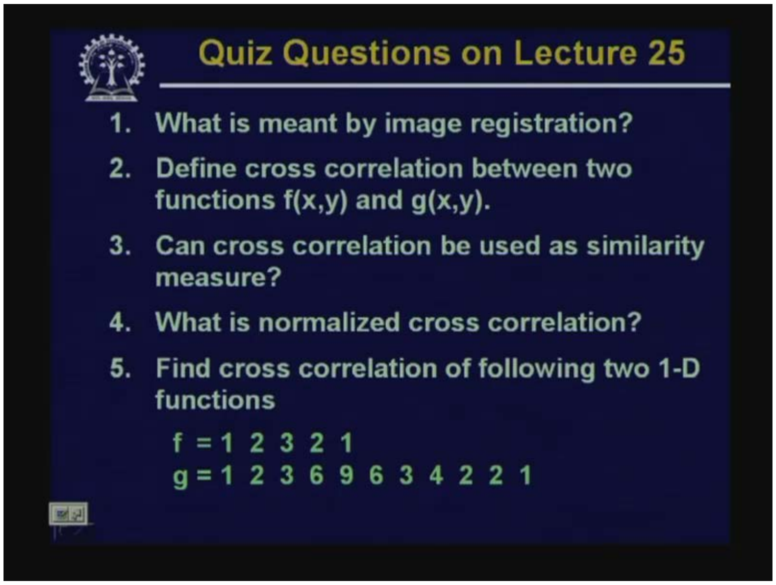

So, the first question is what is meant by image registration? Second question, define cross correlation between 2 functions f (x, y) and g (x, y). Third question, can cross correlation be used as similarity measure? Fourth question, what is normalized cross correlation? Fifth, find the cross correlation of the following 2 one - dimensional functions; one is given by f is equal to 1, 2 3, 2, 1 and g equal to 1, 2, 3, 6, 9, 6, 3, 4, 2, 2, 1. So here, you find that these 2 functions, one dimensional functions - f and g, they are represented in the form of sequence of samples. So, these are discrete functions and you have to find out the cross correlation of these 2 discrete functions.

Thank you.