Digital Image Processing

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Lecture - 24

Image Restoration - III

Hello, welcome to the video lecture series on digital image processing. For last few classes, we were discussing about restoration of blurred images.

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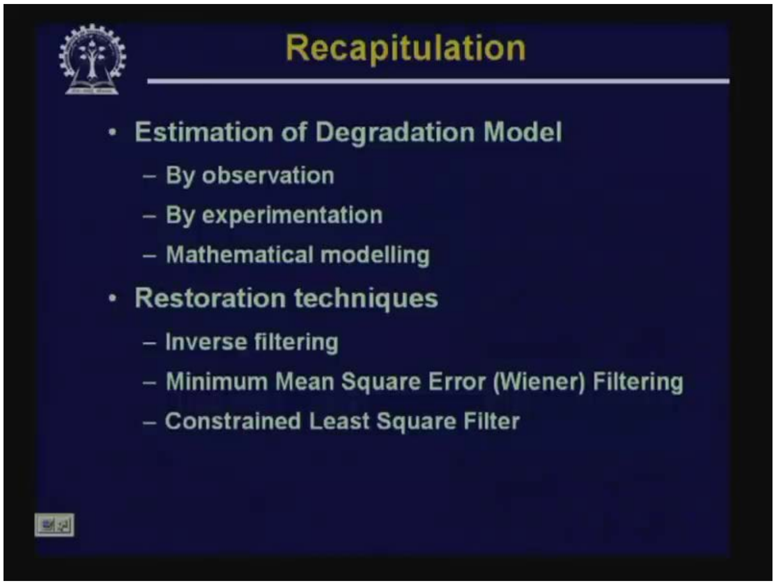

So, what we have done in our last class is estimation of degradation models. We have seen that whatever restoration technique we use, knowledge the restoration techniques mainly used the knowledge of the degradation model which degrades the image. So, estimation of the degradation model degrading an image is very very important for the restoration operation.

So, in our last class, we have seen 3 methods for estimation of the degradation model. The first method we discussed is the estimation of the degradation model by observation. The second technique that we have discussed is the estimation by experimentation and the third technique that we have discussed is the mathematical modeling of degradation. Then, we have also seen what should be the corresponding restoration technique and in our last class, we have talked about the inverse filtering technique and today we will talk about the other restoration technique which also makes use of the estimated degradation model or the estimated degradation function.

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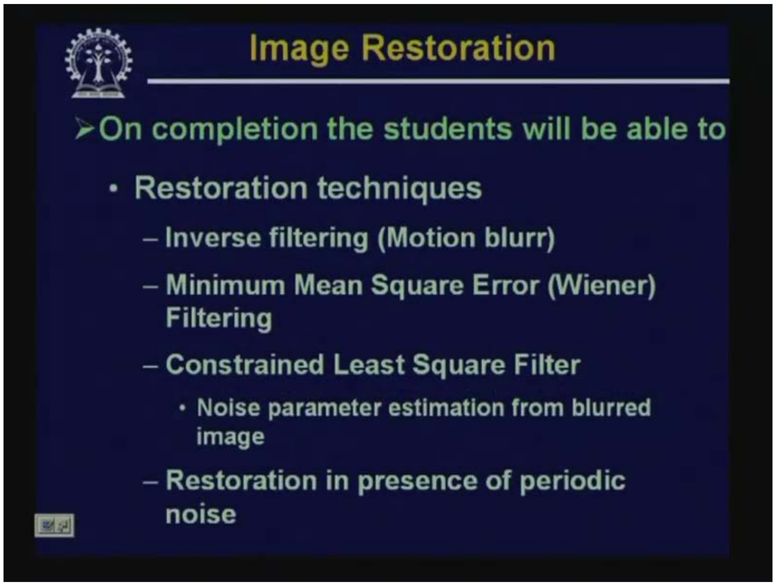

So, in today's lecture, we will see the inverse filtering, the restoration of the motion blurred image using the inverse filtering technique. In our last class, we have seen inverse filtering technique where the image was degraded by turbulence, atmospheric turbulence model. We will also talk about the minimum mean square error or Wiener filtering approach for restoration of a degraded image.

We will also talk about another technique called Constant Least Square Filter where the constant least square filter mainly uses the mean and standard deviation of the noise which contaminates the image, the degraded image and then we will also talk about the restoration techniques where the noise present in the image is a periodic noise.

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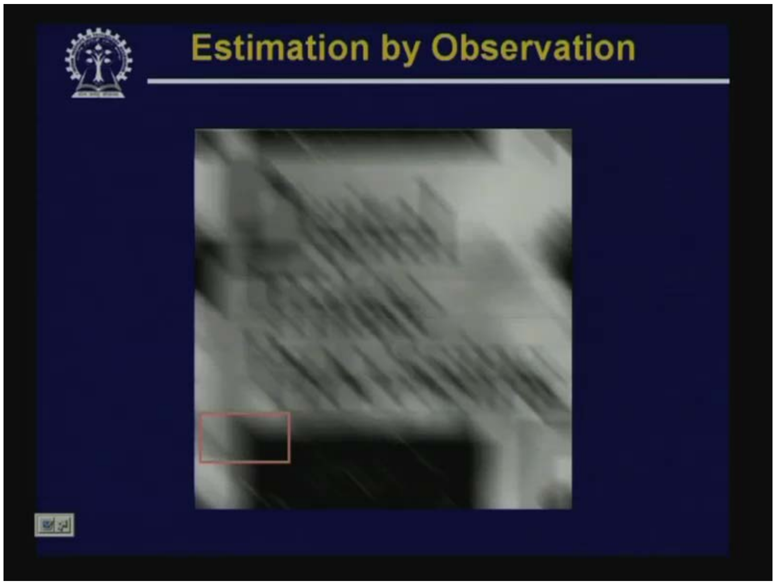

So firstly, let us quickly go through what we have done in our last class. So, we are talking about the estimation of the degradation model because that is a very very basic requirement for the restoration operation. So, the first one that we have said is the estimation of the degradation model by observation. So in this case, what is given to us is the degraded model and by looking at the degraded model, we have to estimate that what is the degradation function. From the degraded image, we have to estimate what is the degradation function.

So here, we have shown one such degraded image and we have said that once a degraded image is given, we have to look for a part of the image which contains some simpler structure and at the same time, the energy content, the signal energy content in that part in that sub image should be very high to reduce the effect of the noise.

So, if you look at this particular degraded picture, you find that this rectangle, it shows an image region in this degraded image which contains a simple structure. And from this, it appears that there is a rectangular figure present in this part of the image and there are 2 distinct gray levels; one is of the object which is towards the black and other one is the background which is a grayish background.

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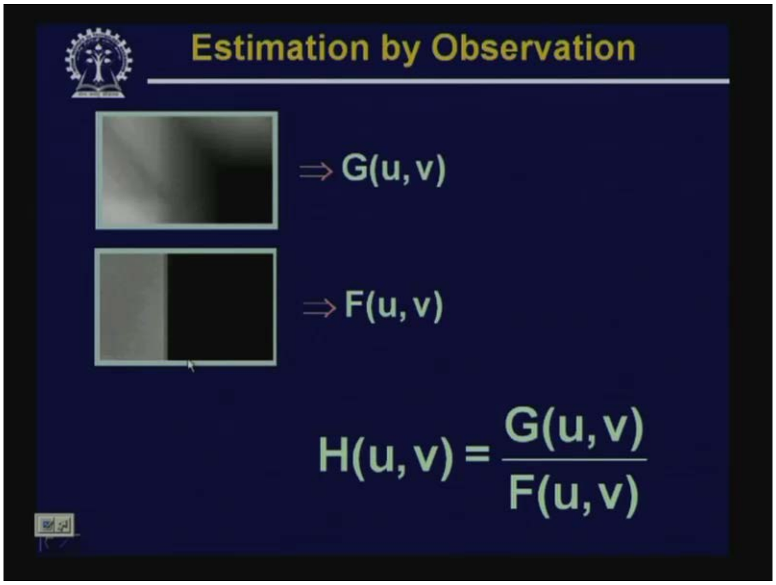

So, by having a small sub image from this portion, what I do is I try to manually estimate that what should be the corresponding original image. So, as shown in this slide; the top part is the degraded image which is cut out from the image that we have just shown and the bottom part is the estimated original image.

Now, what we do is from this if you take the Fourier transform of the top one, what I get is the G (u, v) as we said that it is the Fourier transformation of the degraded image and the lower one, we are assuming this to be original. So, if I take the Fourier transform of this, what I get is F (u, v) that is the Fourier transform of the original image and obviously here, the degradation model or the degradation function in the Fourier domain is given by H (u, v) which is equal to G (u, v) by F (u, v) and you remember that in this case, the division operation has to be done point by point. So, this how the degradation functions can be estimated by observation when only the degraded images are available.

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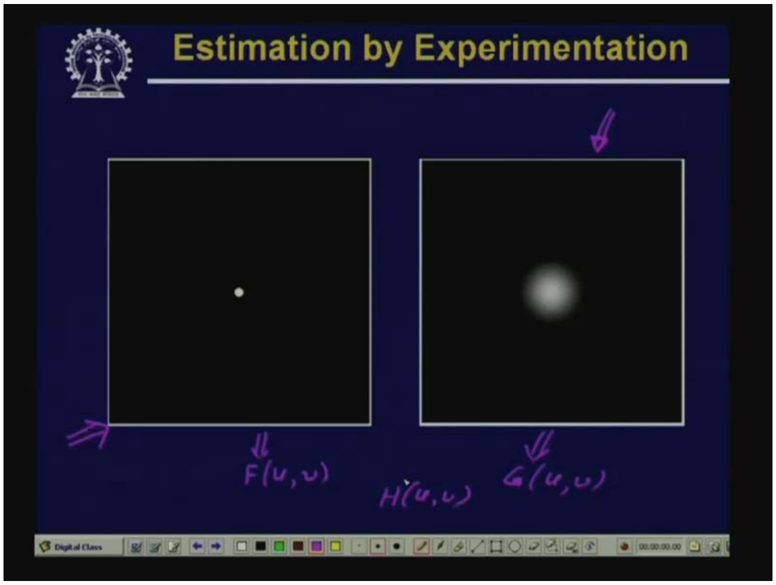

The other approach for estimation of the degradation function we have said is the estimation by experimentation. So, our requirement is that whichever imaging device or imaging setup that has been used for getting a degraded image which has been used to record a degraded image in our laboratory for experimental experimentation purpose; we will have a similar such imaging setup and then we try to find out that what is the impulse response of that imaging setup. As we have already discussed that it is the impulse response which completely characterizes any system.

So, if we know what is the impulse response of the system; we can identify, we can always calculate what is the response of the system to any type of input signal. So, by experimentation, what we have done is we have taken a similar imaging setup and then you simulate an impulse by using a very narrow strong beam of light. So, as has been shown in this particular diagram.

So, on the left hand side, what is shown is one such simulated impulse in this particular diagram. So, the left hand side, this one shows such simulated impulse and on the right hand side, what we have is the response of this impulse as recorded by the imaging device. So now, if I take the Fourier transform of this, this is going to give me F (u, v) and if I take the Fourier transform of this, this is going to give me G (u, v) and you see, that because the input the original is an impulse, the Fourier transform of an impulse is a constant.

So, if I simply take the Fourier transform of the response which is the impulse response or in this case it is the point spread function, then this divided by the corresponding constant will give me the degradation function which is H (u, v). So, this is how we estimate the degradation function or the degradation model of the imaging setup to experiment.

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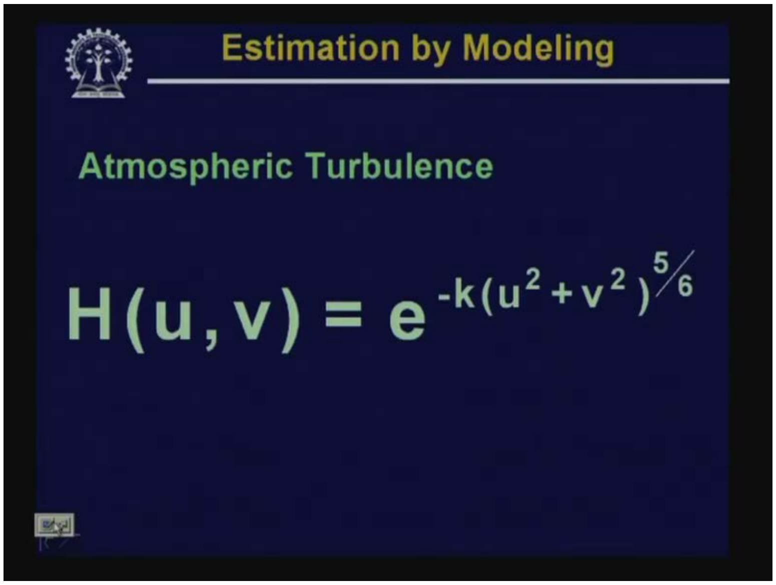

The third approach for obtaining the degradation model is by mathematical modeling. So, in our last class, we have considered 2 such mathematical models. The first mathematical model that we have considered which try to give the degradation function corresponding to atmospheric turbulence and the function is given, the degradation function in the frequency domain is given like this that H (u, v) is equal to e to the power minus k (u square plus v square) to the power 5 by 6 and using this degradation model, we have shown that how a degraded image will look like.

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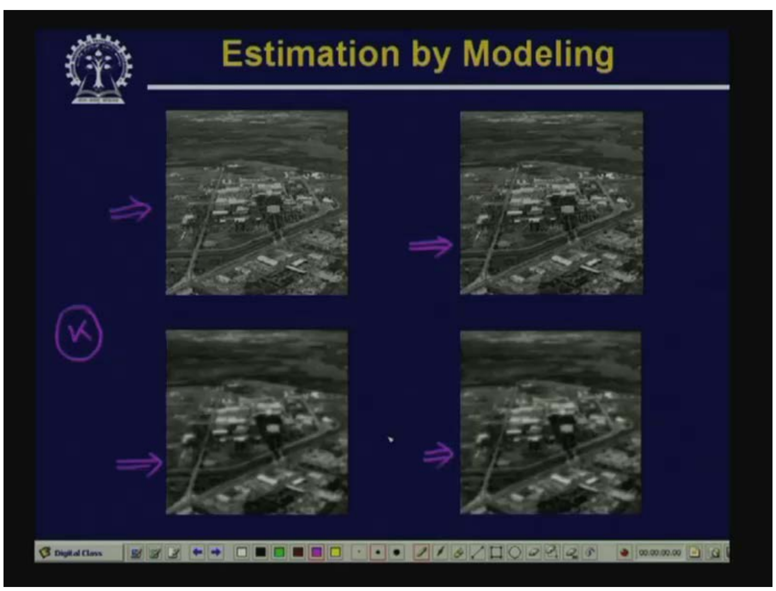

So, in this particular case, we have shown 4 images. Here, the top image, the top left image; this is the original one and the other 3 are the degraded image which have been obtained by using the degradation model that we have just said. Now, in that degradation model when I said that e to the power minus k (u square plus v square) to the power 5 by 6, it is the constant k which tells you that what is the degree of the degradation or what is the intensity of the disturbance. So, low value of k indicates the disturbance is very low. Similarly, higher value of k indicates that the disturbance is very high.

So here, this image has been obtained with a very low value of k, this image has been obtained with a very high value of k and this image has been obtained with a medium value of k. So, you find that depending upon the value of k, how the degradation of the original image changes.

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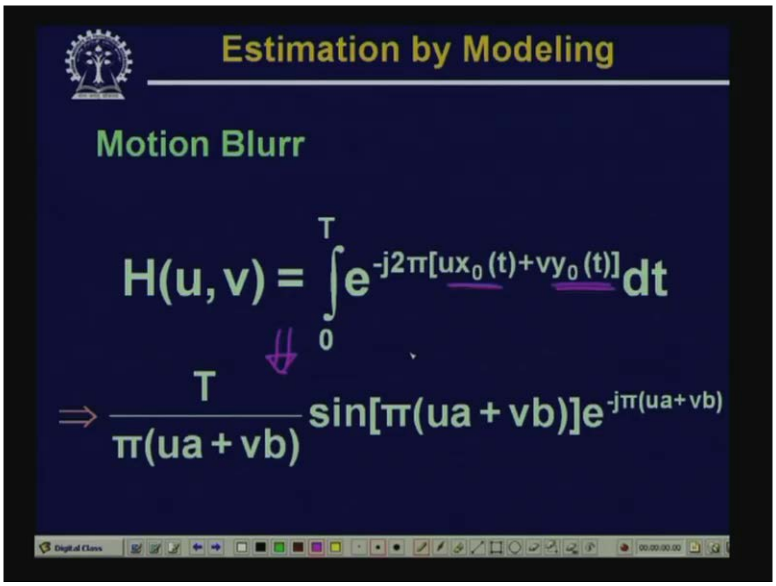

The second mathematical approach that or the second model that we have considered is a motion blurred or the blurring which is introduced due to motion. And we have said that because the camera shutter is kept open for a finite duration of time, the intensity which is obtained at any point on the imaging sensor is not really coming from a single point of the scene but the intensity at any point on the sensor is actually the integration of the intensities or light intensities which are falling to that particular point from different points of the moving object and this integration has to be taken over the duration during which the camera shutter remains on.

So, using that concept, what we have got is motion mathematical model for motion blur which in our last class we have derived that it is given by H (u, v) equal to integration zero to T e to the power minus j two pi [u x_0 (t) plus v y_0 (t)] dt where this x_0 is the movement, this x_0 (t) indicates movement along x direction and y_0 (t) indicates movement along the y direction.

So, if I assume that x_0 (t) is equal to a and y_0 (t) equal to b, then this particular integral can be computed and we get a final degradation function or degradation model as given in the lower equation.

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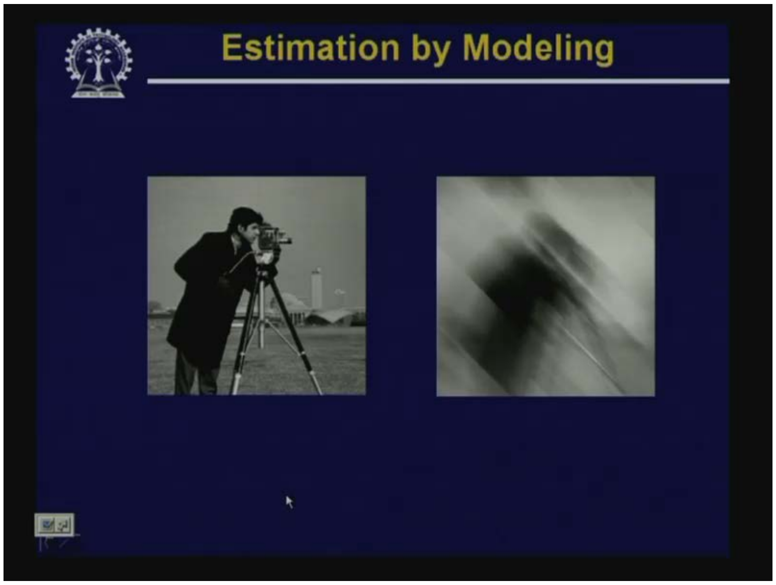

And after this, in our last class what we have done is we have used the inverse filtering and using this motion blurring model, this is the kind of blurring that we obtained, this is the original image; on the right hand side, we have shown the motion blurred image. Then we have seen that once you have the model for the blurring operation, then you can employ inverse filtering to restore a blurred image.

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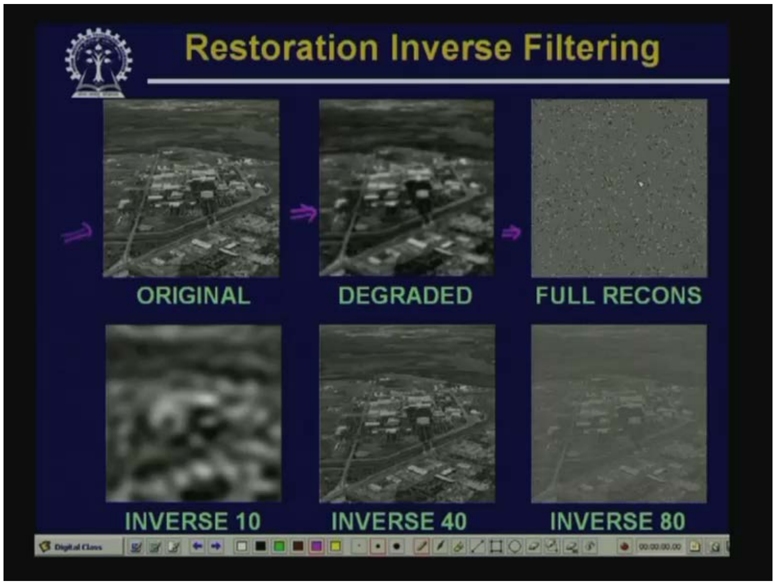

So, in your last class, we have used the inverse filtering to restore the images which are blurred by atmospheric turbulence and here again, on the left hand side of the top row, we have shown the original image. So, this is the original image, this is the degraded image and we have said in the last class because by inverse filtering, the function the blurring function H (u, v) comes in the denominator.

So, if the value of H (u, v) is very low, then G (u, v) upon H (u, v) that particular term becomes very high. So, if I consider all the frequency components or all values of (u, v) in H (u, v) for inverse filtering, the result may not be always good. And that is what has been demonstrated here that this image was reconstructed considering all values of u and v and you find that this fully reconstructed image does not contain the information that we want.

So, what we have to do is along with this inverse filtering, we have to employ some sort of low pass filtering operation so that the higher frequency components or the higher values of u and v will not be considered for the reconstruction purpose. So here on the bottom row, the left most image, this shows the reconstructed image where we have considered only the values of u and v which are within a radius of 10 from the center of the frequency plane and because this is a low pass filtering operation where the cut off frequency was very very low; so, the reconstructed image is again very very blurred because many of the low frequency components along with the high frequency components have also been cut out.

The middle image shows to some extent a very good image where this distance within which (u, v) values were considered for the reconstruction were taken to be 40. So, here you find that this reconstructed image this contains most of the information which was contained in the original image. So, this is a fairly good restoration.

Now, if I increase the distance function, the value of the distance; if I go to 80, that means many of the high frequency components also we are going to incorporate while restoration. And, you find that the right most image on the bottom row that is this one where the value of the distance was equal to 80, this is also a reconstructed image but it appears that the image is behind a curtain of noise.

So, this clearly indicates that if I go on increasing or if I take more and more u and v values, the frequency components for restoration using inverse filtering; then the restored image quality is going to be degraded. It is likely to be dominated by the noise components present in the image.

Now, though this inverse filtering operation works fine for this turbulence kind of blurring, the blurring introduced by the atmospheric turbulence but this direct inverse filtering does not give good result in case of motion blurring.

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So, you find that here we have shown the result of direct inverse filtering in case of motion blur. So, on the top most one, this is the original image. On the left, we have shown the degraded image and the right most one, on the bottom is the image restored image obtained using the inverse direct inverse filtering and the blurring which was considered in this particular case is the motion blur. Now, let us see why this direct inverse filtering does not give satisfactory result in case of motion blur.

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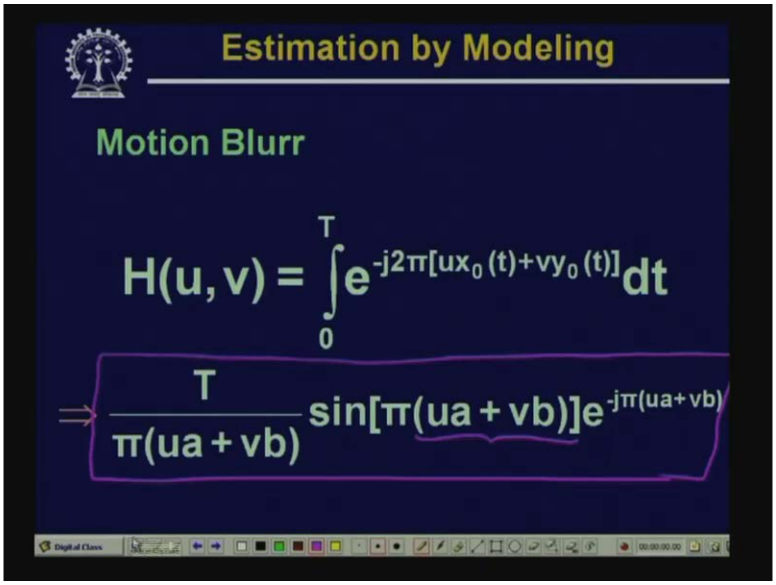

The reason is if you look at the degradation function, the motion degradation function, say for example in this particular case; you will find that the degradation function H (u, v) is given by this expression in the frequency domain. Now, this term will be equal to 0 whenever this component (ua plus vb), this is going to be integer.

So, for any integer value of (ua plus vb), the H (u, v), the corresponding component H (u, v) will be equal to 0 and for nearly integer values of this (ua plus vb), term H (u, v) is going to be very very low. So, for direct inverse filtering when we go for dividing G (u, v) by H (u, v), you have the Fourier transformation of the reconstructed image wherever H (u, v) is very low near about 0, the corresponding F (u, v) term will be abnormally high and when you take the inverse Fourier transform of this that very very high value is reflected in the reconstructed image and that is what gives to a reconstructed image as shown in this form.

So, what is the way out? Can't we use the inverse filtering for restoration of motion blurred image?

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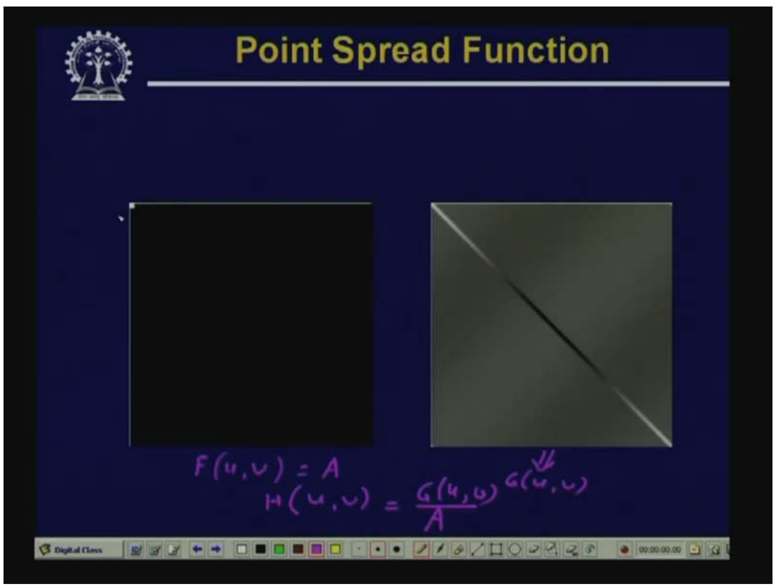

So, we have attempted round about approach. What we have done is again we have taken an impulse, try to find out what will be the point spread function of if I employ this kind of motion blur. So by using the same motion blur function or the motion blur model, you blur this impulse and what I get is an impulse response like this which is the point spread function in this particular case.

Now, once I have this point spread function; then as before, what I do is I take the Fourier transform of this and this Fourier transformation now gives me G (u, v) and because my input was an impulse, where so for this impulse F (u, v) is equal to constant which is say something like A, now from these two, I can compute recompute H (u, v) which is given by G (u, v) divided by this constant term A. Obviously, the value of the constant is same as what is the intensity of this impulse.

So, if it is an unit impulse, if I take an unit impulse, then the value of constant A will be equal to 1 and in that case, the Fourier transform of the point spread function directly gives me the degradation function H (u, v).

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Now, if I perform the inverse filtering using this recomputed degradation function; then we find that the reconstruction result is very very good. So, this was the blurred image, this is the reconstructed image during by using the inverse filtering, direct inverse filtering but here the degradation model was recomputed from the point spread from the Fourier transformation of the point spread function.

So, though using the direct inverse transform of the mathematical model of the motion blur does not give me good result but recomputation of that degradation function gives a satisfactory result. But again, with this inverse filtering approach the major problem is as we said that we have to consider the (u, v) values for reconstruction which is within a limited domain.

Now, how do you how do say that up to what extent of (u, v) value we should go? That is again image dependent. So, it is not very easy to decide that to what extent of frequency components we should consider for the reconstruction of the original image if i go for direct inverse filtering. So, there is another approach which is the minimum mean square error approach or it is also called the Wiener filtering approach. In case of Wiener filtering approach, the Wiener filtering tries to reconstruct the degraded image by minimizing an error function. So, it is something like this.

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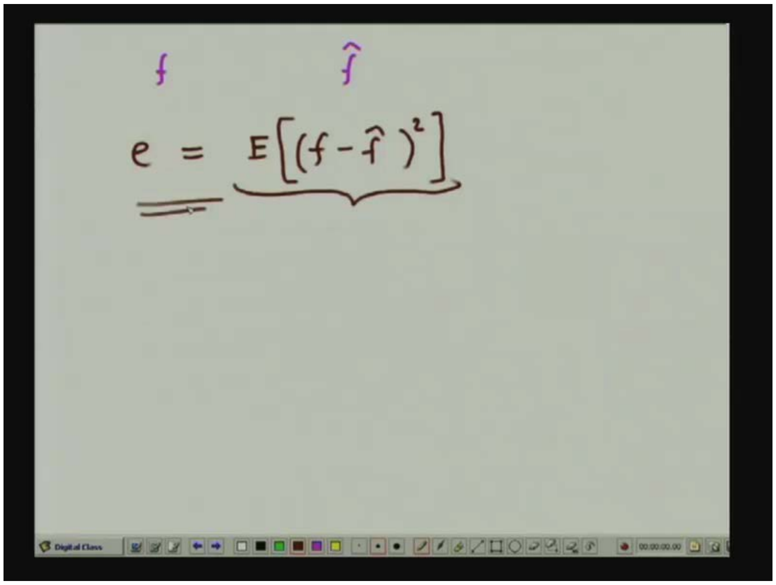

So, if my original image is f and my reconstructed image is f hat, then the Wiener filtering tries to minimize the error function which is given by expectation value of f minus f hat square. So, the error value e which is given by the expectation value of f minus f hat square where f is the original degraded image and f hat is the restored image from the degraded image; so f minus f hat square, this gives you the square error and this wiener filtering tries to minimize the expectation value of this error.

Now here, our assumption is that the image intensity and the noise intensity are uncorrelated and using that particular assumption, this Wiener filtering works.

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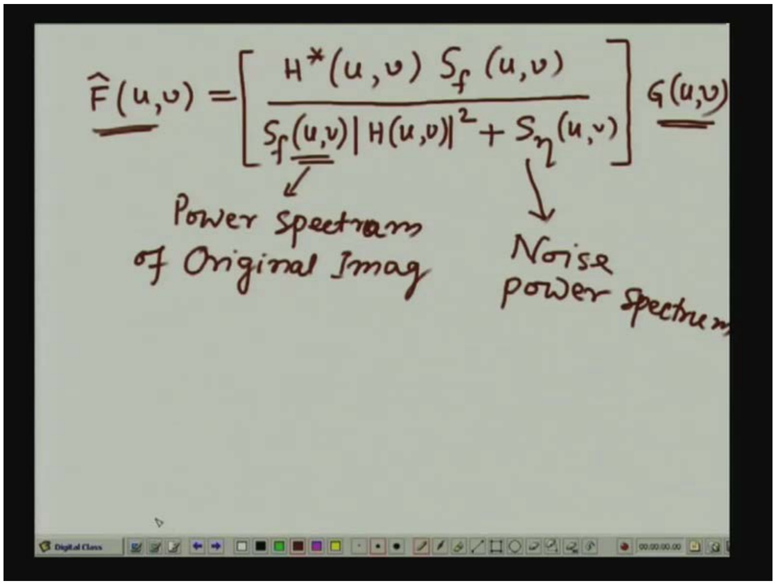

So here, we will not go into the mathematical details of the derivation but it can be shown that a frequency domain solution of this, I mean whenever this error function is minimum; the corresponding F (u, v) in frequency domain is given by F hat (u, v). This will be equal to [H star (u, v) into S_f (u, v) divided by S_f (u, v) into H (u, v) square plus S_{eta} (u, v)] this into G (u, v) where this H star indicates it is the complex conjugate of H (u, v) and G (u, v) as before, it is the Fourier transform of the degraded image and F hat (u, v) is the Fourier transform of the reconstructed image and in this particular case, this term S_f (u, v), this is the power spectrum power spectrum of original image undegraded image and S_{eta} (u, v) is the noise power spectrum.

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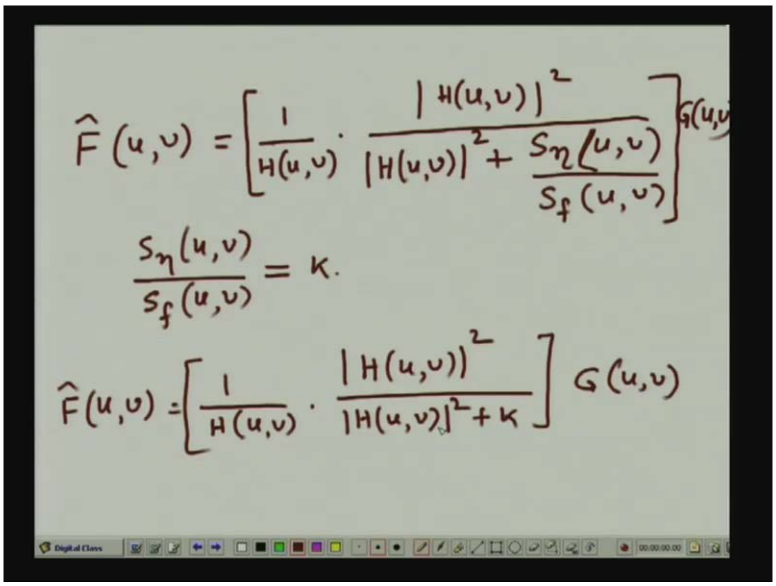

Now, if I simplify this particular expression, I get an expression of this form that F hat (u, v) is equal to [1 upon H (u, v) into H (u, v) square upon H (u, v) square plus S_{eta} (u, v) upon S_f (u, v)] into G (u, v). So, this is the expression of the Fourier transform of the reconstructed image when I use Wiener filtering.

Now, in this case, you might notice that if the image does not contain any noise; then obviously, S_{eta} (u, v) which is the power spectrum of the noise will be equal to 0 and in that case, this wiener filter becomes identical with the inverse filter. But if the noise contains additive if the degraded image also contains additive noise in addition to the blurring; in that case, the wiener filter and the inverse filter is different.

Now here, you find that this Wiener filter considers the ratio of the power spectrum of the noise power of the noise power and the power of the power spectrum of the original undegraded image. Now, even if I assume that the additive noise which is contained in the degraded image is a white noise for which the noise power spectrum will be constant; but it is not possible to find out what is the power spectrum of the original undegraded image. So, for that purpose, what is done is normally this ratio that is S_{eta} (u, v) upon S_f (u, v) that is the ratio of the power spectrum of the noise to the power spectrum of the original undegraded image is taken to be a constant k.

So, if I do this, in that case, the expression for F hat (u, v) comes out to be 1 upon H (u, v) into H (u, v) square upon H (u, v) square plus k into G (u, v) where this k, the term k is a constant which has to be adjusted manually for the optimum reconstruction or for the reconstructed image which appears to be visually best. So, using this expression, let us see what kind of image that or what kind of reconstructed image that we get.

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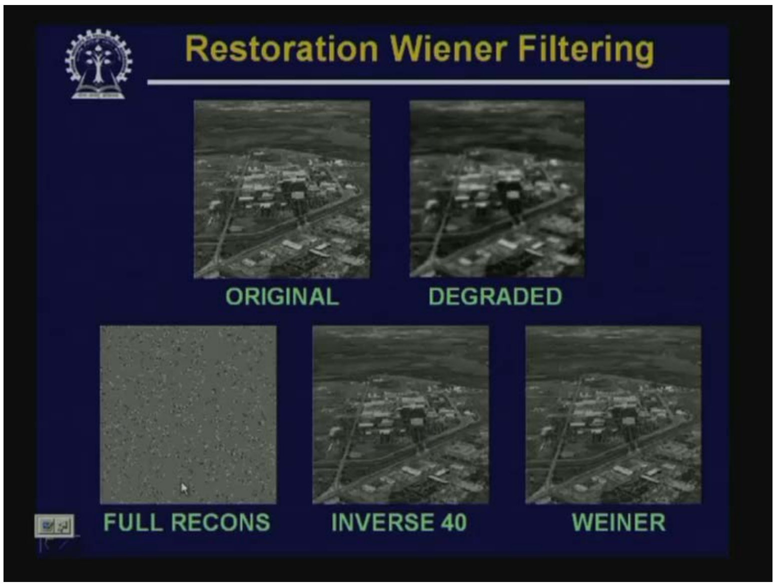

So here, we have shown the image restoration of the degraded image using Wiener filter. Again on the left hand side of the top row, it is the original image. Right hand side of the top row gives you the degraded image. Left hand, left most image on the bottom row that shows you the full reconstructed image or the reconstructed image using the inverse filtering where all the frequency components were considered.

The middle one shows the reconstructed image using inverse filtering where only the frequency components within a distance of 40 from the center of the frequency plane has been considered for reconstruction and the right most one is the one which is obtained using wiener filtering and for obtaining this particular reconstructed image, the value of k was manually adjusted for best appearance.

Now, if you compare these 2, that is the inverse filtered image with the distance equal to 40 with the Wiener filtered image; you will find that the reconstructed images are more or less same but if you look very closely, it may be found that the Wiener filtered image is slightly better than the inverse filtered image. However visually, they appear to be they appear to be almost same.

The advantage in case of Wiener filter is that I do not have to decide that what extent of frequency components I have to consider for reconstruction or for restoration of the undegraded image. But still, the wiener filter has got a disadvantage. That is the manual adjustment of the value of k and as we have said that the value of k has been used for simplification of the expression where this constant k is nothing but a ratio of the power spectrum of the noise to the power spectrum of the undegraded original image and in all the cases, taking this ratio to be a constant may not be justified approach.

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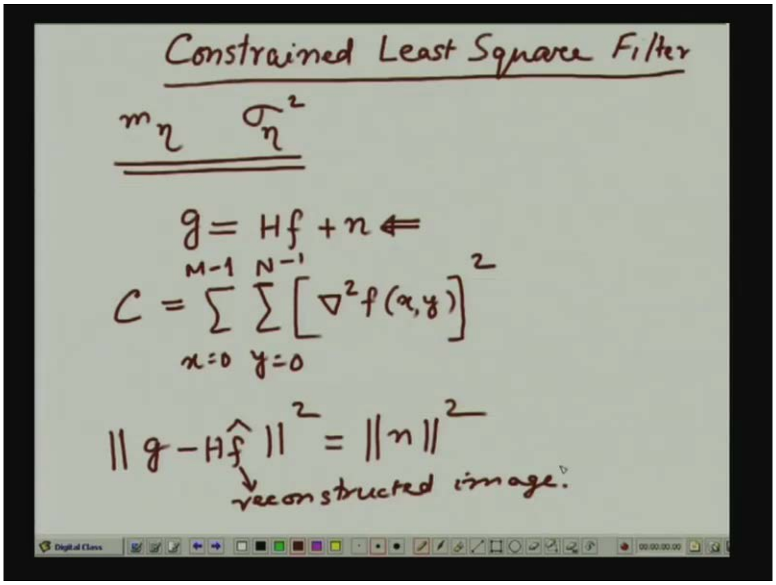

So, we have another operation, another kind of filtering operation which is called constant least square filtering. So now, we will consider a filtering operation which is called constant least square filter. Now, unlike in case of Wiener filtering where the performance of the Wiener filtering depends upon the correct estimation of the value of k that is the performance of Wiener filtering depends upon how correctly you can estimate what is the power spectrum of the original undegraded image.

In case of this constant least square filter, it does not make any assumption about the original undegraded image. It makes use of only the noise probability distribution function, probability density function noise pdf and mainly it uses the mean of the noise which we will write as say m_{eta} and the variance of the noise which we will write as sigma $_{eta}$ square. So, we will come, we will see that how the reconstruction using this constant least square filter approach makes use of this noise parameter like mean of the noise and the variance of the noise.

In case of this constant least square filter, to obtain this constant least square filter, we will start with the expression that we got in our first class that is g equal to Hf plus n. So, you remember that this is an expression which we had derived in the first class which tells us that what is the degradation model for degrading the image where H is the matrix which is derived from the impulse response H x and n is the noise vector.

Now here, you will notice that the value of H is very very sensitive to noise. So, to take care of that what we do is we define an optimality criteria and using that optimality criteria, the reconstruction has to be done and because this degradation function H or the degradation matrix

H is noise dependent, it is very very sensitive to noise; so for reconstruction, the optimality criteria that we will use is the image smoothness.

So, you know from our earlier discussion that the second derivative operation or the Laplacian operator, it tries to enhance the irregularities or discontinuities in the image. So, if we can minimize the Laplacian of the reconstructed image that will ensure that the image the reconstructed image will be smooth. So, our optimality criteria in this particular case is given by C is equal to double summation del square f (x, y) square where y varies from 0 to capital N minus 1 and x varies from 0 to capital M minus 1.

So, our assumption is the image that we are trying to reconstruct or the blurred image that we have obtained that image is of size capital M by capital M capital N. So, our optimality criteria are given by this where del square f (x, y) is nothing but the Laplacian operation. So, this optimality criteria is Laplacian operator based and our approach will be that we will try to minimize this criteria subject to the constraint that g minus H f hat square should be equal to n square where this f hat, this is the reconstructed image.

So, we will try to minimize these optimality criteria subject to the constraint that g minus Hf hat square is equal to n square and that is why it is called constant least square filtering. Again, without going into the details of mathematical derivation, we will simply give the frequency domain solution of this particular constant least square estimation where the frequency domain solution now is given by F hat (u, v) is equal to H star (u, v) upon H (u, v) square plus a constant gamma times P (u, v) square, this times G (u, v).

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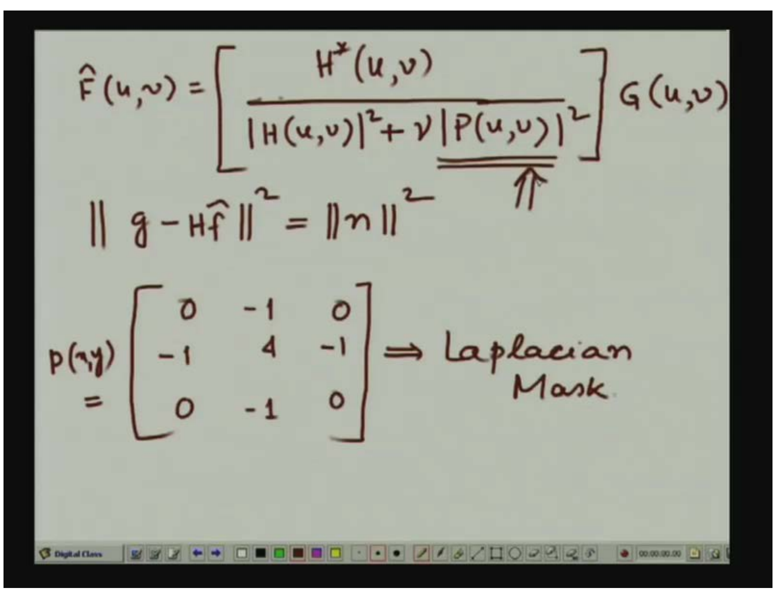

Again as before, this H star indicates that it is the complex conjugate of H. Here again, we have a constant term given as gamma where the gamma is to be a adjusted so that the specified constant that is g minus Hf hat square is equal to n square this constant is met.

So here, this gamma is a scalar quantity, scalar constant whose value is to be adjusted to for so that this particular constant is maintained and this quantity P (u, v), the term P (u, v), it is the Fourier spectrum or the Fourier transform of the mask given by (1 0, minus 1, 0) (minus 1, 4, minus 1) (0, minus 1, 0). So, this is my P (x, y) and this P (u, v) is nothing but the Fourier spectrum of or the Fourier transformation of this P (x, y) and you can easily identify that this is nothing but the Laplacian operator mask or the Laplacian mask that we have already discussed in our earlier discussion.

Now here, for implementation of this, for computation of this, you have to keep in mind that our image is of size capital M by capital N. So, before we compute the Fourier transformation of P (x, y) which is given in the form of a 3 by 3 mask, we have to paired appropriate number of zeros so that this P (x, y) also becomes a function of dimension capital M by capital N or an array of dimension capital M by capital N and after only converting these 2 an array of dimension capital M by capital N; we can compute P (u, v) and that P (u, v) has to be used in this particular expression.

So, as we said that this gamma has to be adjusted manually for obtaining the optimum result and the purpose is that this adjusted value of gamma, the gamma is adjusted so that the specified constant is maintained. However, it is also possible to automatically estimate the value of gamma by an iterative approach.

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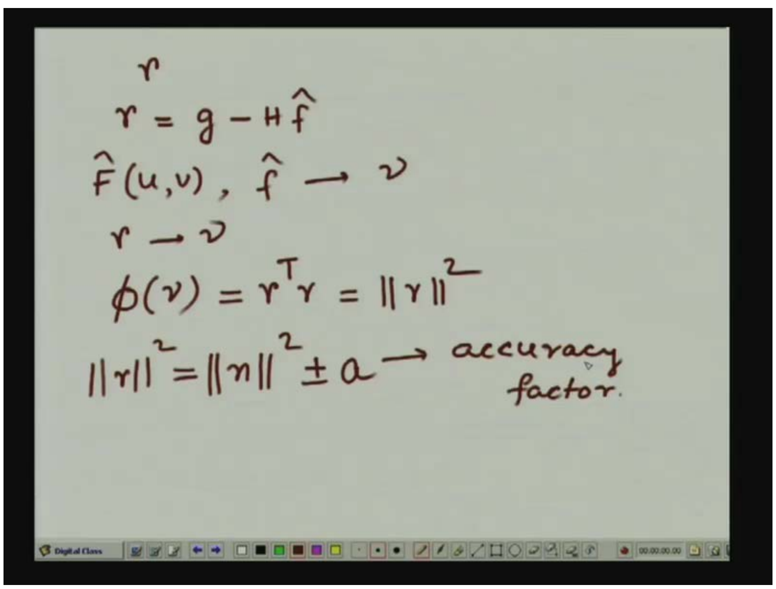

So, for that iterative approach, what we do is we use define a residual vector say r where this residual vector is nothing but r equal to g minus H f hat. So, you remember that this g is obtained from the degraded image. The matrix, degradation matrix H is obtained from the degradation function H x and f hat is actually the estimated restored image.

Now here, since f hat we have seen earlier that f hat sorry we have seen earlier that f hat (u, v) and so this f hat in the special domain, they are functions of gamma. So obviously, r which is the function of f hat; so, this r will also be a function of gamma. Now, if I define a function say phi of gamma which is nothing but r transpose r or which is nothing but the Euclidean norm of r, it can be shown that this function is a monotonically increasing function of gamma.

That means whenever gamma increases, this Euclidean norm of r also increases; if gamma decreases, the Euclidean norm of r also decreases. And by making use of this property, it is possible to find out what is the optimum value of gamma within some specified accuracy. So, our approach in this case, our aim is that we want to estimate the value of gamma such that the Euclidean norm of r that is r square will be equal to n square plus minus some constant A where this A is nothing but what is the specified accuracy factor or this gives you the tolerance of reconstruction.

Now obviously, here you find that if r square is equal to n square, then the specified constant is exactly met. However it is very very difficult, so you specify some tolerance by giving the accuracy factor A and we want that the value of gamma should be such that r will be the Euclidean norm of r will be within this range. Now, given this background; an iterative approach, an iterative algorithm for estimation of the value of gamma can be put like this.

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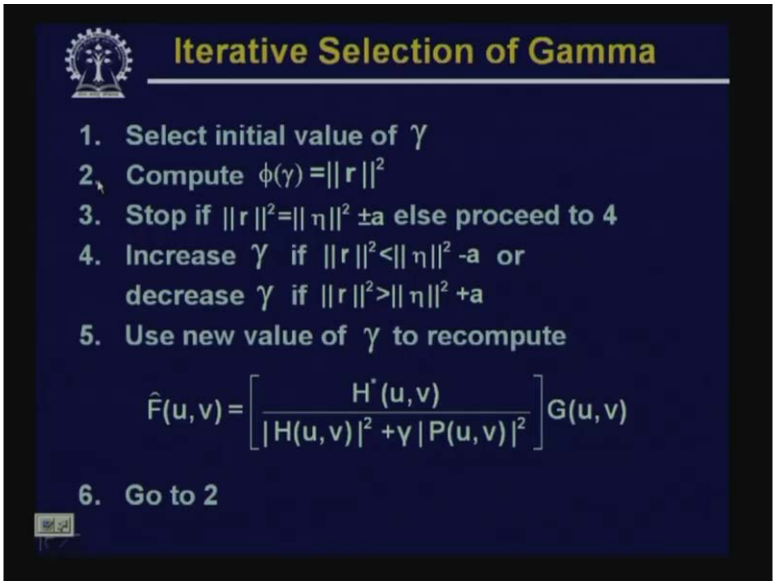

So, you select an initial value of gamma, then compute y phi gamma which is nothing but Euclidean norm of r. Then you terminate the algorithm if r square is equal to n square, here it has been written as eta square plus minus a. If this is not the case, then you proceed to step number 4 where you increase the value of gamma and if r square is less than eta increase the value of gamma if r square is less than eta square minus a or you decrease the value of gamma if r square is greater than eta square plus a.

Now, using whatever new value of gamma that you get, you re-compute the image and for that the image reconstruction function as we have said in frequency domain is given by this particular expression and with this reconstructed value of F, you go back to step number 2 and you do this iteration until and unless this termination condition that is r square is equal to n square plus minus a, this condition is met.

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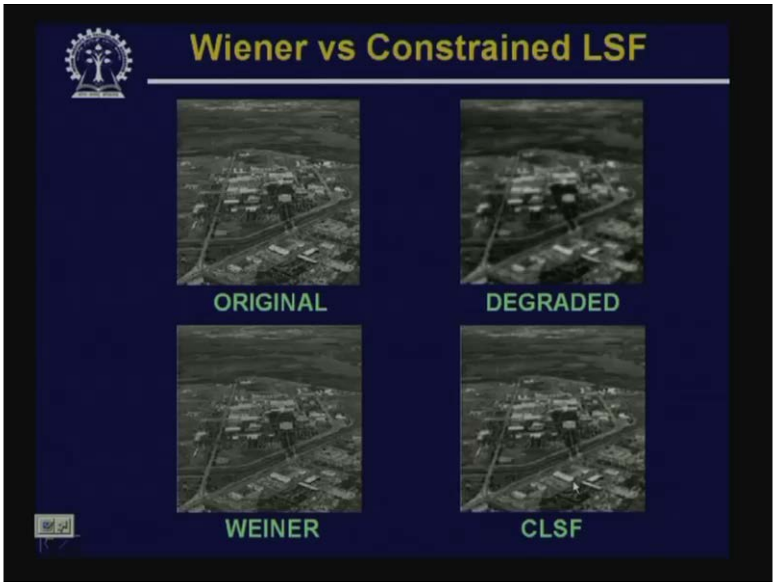

Now, using this kind of approach what we have obtained is we have got some reconstructed image. So here, you find that it is the same original image; this is the degraded version of that image. On the bottom row, the left hand side gives you the reconstructed image using the Wiener filtering and again on the bottom row on the right hand side, this gives you the reconstructed image which is obtained using the constant least square filter.

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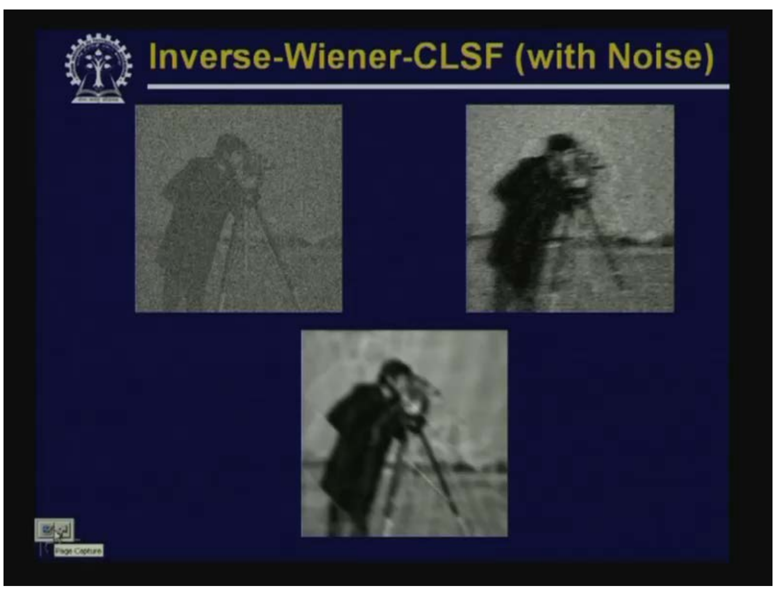

The same for the motion degraded image. Here, we have also considered some additive noise; so again on the top row on the left, this is the image which is obtained by direct inverse filtering and you find the prominence of noise in this particular case. The right one is the one which has been obtained by Wiener filtering. Here also you find that the amount of noise has been reduced but still the image is noisy and the bottom one is the one that has been obtained by using the constant least square filtering.

Now, if you look at these 3 images; you will find that the amount of noise is greatly reduced in the bottom one. That is this particular image which has been obtained by this restored image which has been obtained by constant least square filtering approach and as we said that the constant least square filtering approach makes use of the estimation of mean of the noise and the standard deviation of the noise; so it is quite expected that the noise performance of the least square constant least square error filter will be quite satisfactory and that is what is observed here that this image which is obtained using this constant least square filter, the image has been the noise has been removed to a great extent whereas, the other reconstructed image cannot remove the noise component to that extent.

However, if you look at these reconstructed images, the reconstruction quality of this image is not that good. So, that clearly says that using the optimality criteria the reconstructed image that you get, the optimum reconstructed image may not always be visually the best. So, to obtain a visually best image, the best approach is you manually adjust that particular constant gamma.

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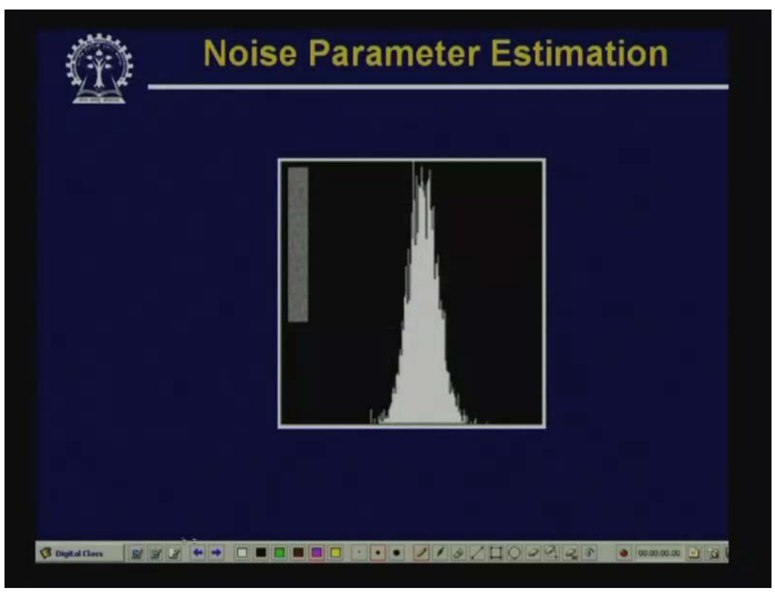

Now, as I said that this constant least square filtering approach makes use of the noise parameters that is mean of the noise and the variance of the noise; now how can we estimate the mean and variance of the noise from the degraded image itself? It is possible that if you look at a more or less uniform intensity region in the image; if you take a sub image of the degraded image where the intensity is more or less uniform and if you take the histogram of that, the nature of the histogram is same as the probability density function - pdf of the noise which is contaminated with that image.

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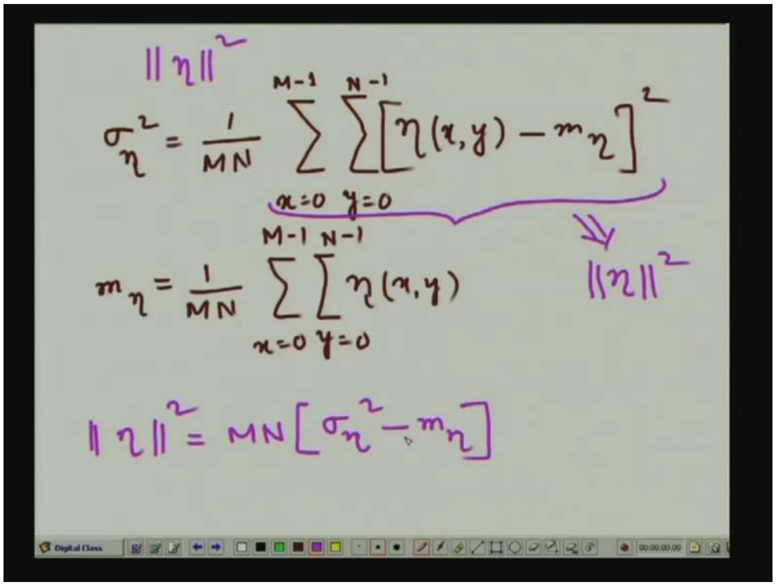

So, we can obtain the noise estimate or we can compute the noise term that is eta square in our expression which is used for this constant least square filtering in this way. We have the noise variance which is given by say sigma n square, sigma eta square which is nothing but 1 upon capital M into capital N into double summation eta (x, y) minus m eta which is the mean of the noise square where y varies from 0 to capital N minus 1 and x varies from 0 to capital M minus 1.

And, the noise mean that is m eta is given by the expression 1 upon capital M into capital N; then double summation eta (x, y) where again Y varies from 0 to capital N minus 1 and x varies from 0 to capital M minus 1.

Now, from this you find that this particular term, this particular term; this is nothing but what is our eta square. So, by making use of this and making use of the mean of the noise, we get that eta square this noise term is nothing but capital M into N where M into N is M and N the dimension of the image into sigma N square sigma eta square minus m eta and as in our constant that we have specified, it is the eta square which is used in the constant term and which is only dependent upon sigma eta and m eta; so, this clearly says that this optimum reconstruction is possible if I have the information of the noise standard deviation of the noise variance and the noise mean.

Now, the estimation of noise variance and noise mean is very very important. If I have only the degraded image what I will do is I will look at some uniform gray level region within the degraded image, find out the find out the histogram of that particular region and the nature of the histogram is same as the probability density function of the noise.

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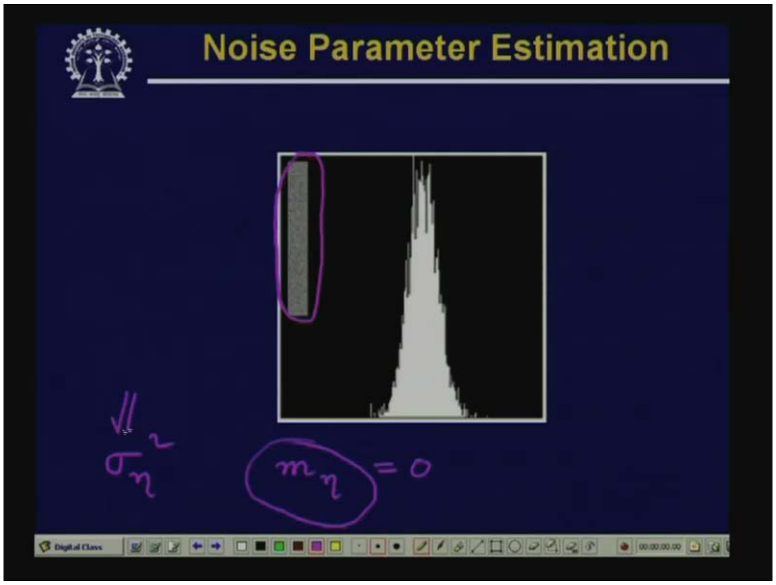

So, as has been shown here in this particular diagram, here you find that this bar that has been shown, this is taken from one such noisy image and this is the histogram of this particular region and this histogram tells you that what is the pdf of the noise which is contaminated with this image.

So, once I have this probability density function; from this, I can compute what is the standard or the variance sigma eta square and I can also compute what is the mean that is m eta and in most of the cases for the noise term, the noise is assumed to be 0 mean. So, this m eta is equal to 0. So, what is important for us is only this sigma eta square and using this sigma eta square, we can go for optimum reconstruction of the degraded image.

Now, in some situation, in many cases, it is also possible that the image is contaminated with periodic noise. So, how do we remove the periodic noise present in the image? You will find that if you take the Fourier transformation of the periodic noise and display the Fourier transformation; in that case, because the noise is periodic, the corresponding dots the corresponding at the corresponding (u, v) location in the Fourier transformation plane, you will get very very bright dots and that dot indicates that what is the frequency of the periodic noise present in the image.

Then, we can go for a very simple approach that once I know the frequency components, I can go for band pass filtering just to remove that part of the coefficients from the Fourier transform and whatever is the reminder Fourier coefficients we have, if we go for the inverse Fourier transformation of that, we will get the reconstructed image.

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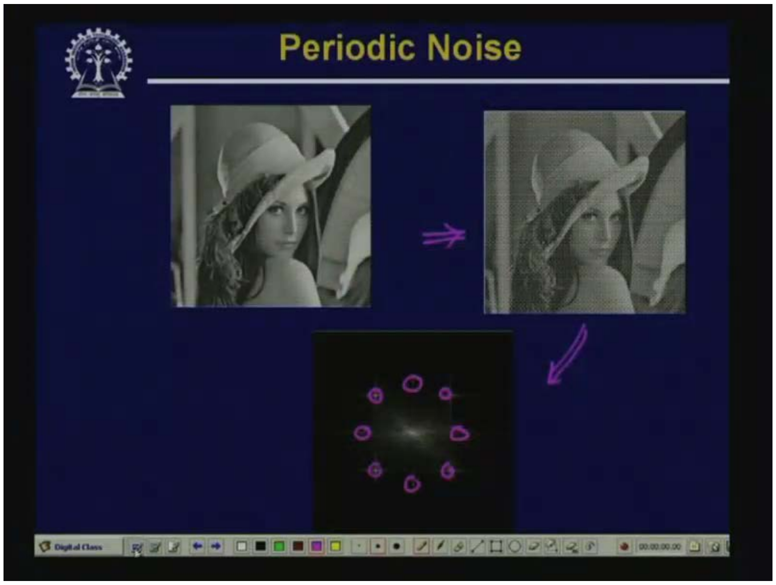

So, as has been shown here, see, here you find that we are taking the same image which we have taken a number of times earlier and if you look at the right most image and if you look closely to this right most image, you will find that this is contaminated with periodic noise and if I take the Fourier transform of this, here you find that in the Fourier transform, there are few bright dots; one bright dot here, one bright dot here, one bright dot here, one bright dot here, one bright dot here, one bright dot here, one is at this location and one is at this location.

So, all this bright dots tell us that what is the frequency of the periodic noise which contaminates the image. So, once I have this information, I can go for an appropriate band pass filtering to filter out that region from the Fourier transform or that part of the Fourier coefficients.

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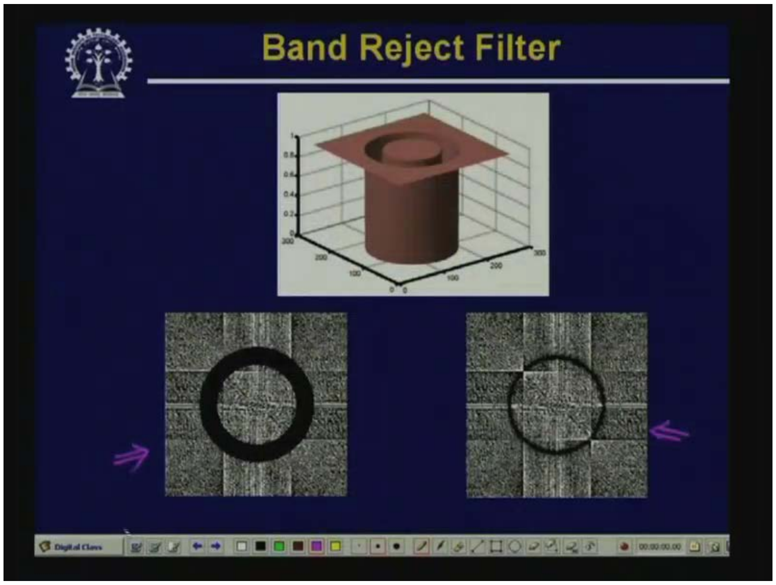

So, that is what has been shown next. So, this is what is a band reject filter. So, this is the perspective plot of an ideal band reject filter and here the band reject filters are shown super imposed on the frequency plane. So, on the left, what I have is an ideal band reject filter and on the right, what I have is the corresponding butter worth band reject filter.

So, by using this band reject filter, we are removing a band of frequencies from the Fourier coefficients corresponding to the frequency of the noise. So, after removal of these frequency components, if I go for inverse Fourier transform, then I am going to get back my reconstructed image and that is what we get in this particular case.

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So here, you find that on the left top, this is again the original image. On the right top, it is the noisy image contaminated with periodic noise. If I go for ideal band reject filter and then reconstruct, then this is the image I get which is on the bottom left and if I go for the butter worth band filter, then the image reconstructed image that I get is in the bottom right.

So, we have talked about the reconstruction or the restoration of images using variation operations and the last one that we have discussed is if we have an image contaminated with periodic noise; then we can make use of band reject filter, do frequency domain operation, employ a band reject filter to remove those frequency components and then go for inverse filtering to reconstruct the image.

And, here you find that the qualities of the reconstructed images are quite good where we have used this band reject filter in the frequency domain. So, with this, we complete our discussion on image restoration. Now, let us have some questions on today's lecture.

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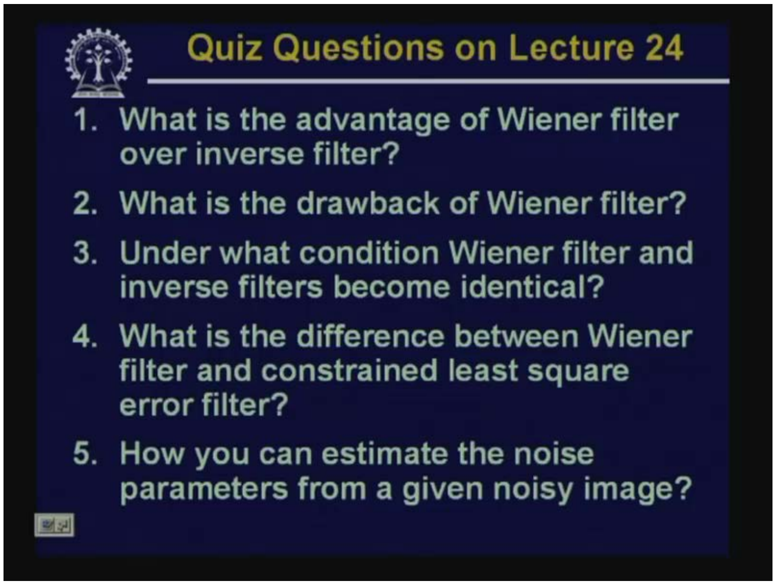

So, the first question is what is the advantage of Wiener filter over inverse filter? The second question - what is the drawback of Wiener filter? Third one, under what condition Wiener filter and inverse filters become identical? Fourth one, what is the difference between Wiener filters and constrained least square error filter? And the last question, how you can estimate the noise parameters from a given noisy image or from a given blurred image?

Thank you.