Digital Image Processing

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Lecture - 22

Image Restoration - I

Hello, welcome to the video lecture series on digital image processing. During our last few lectures, we have talked about various image enhancement techniques.

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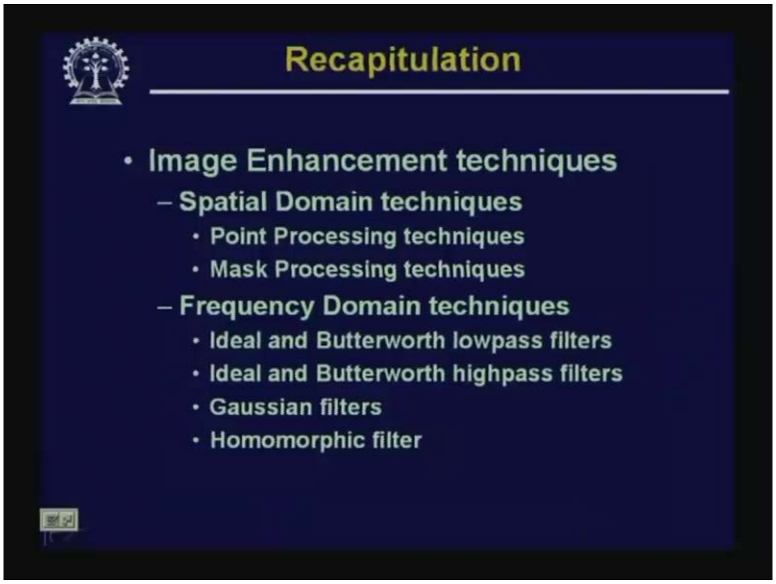

So, we have talked about image enhancement techniques both in the spatial domain as well as in the frequency domain. So, among spatial domain techniques, we have talked about the point processing techniques and we have also talked about the mask processing techniques and in frequency domain; we have talked about ideal and butter worth low pass filters, we have talked about ideal and butter worth high pass filters, we have talked about Gaussian filters and we have also talked about homomorphic filters and we have said that when we are filtering an image in the frequency domain using a low pass filter, if the low pass filter is an ideal low pass filter; in that case, there is a ringing effect in the output of the image.

The ringing effect is reduced by using the butter worth filter because of smooth transition which is given by the butter worth filter from low frequency region to the high frequency region. However, even in the butter worth filter if we use a butter worth filter of order more than 1 that is if I use a butter worth filter of order 2 or order 3 and so on; in such cases also, the butter worth filter leads to the ringing effect.

However, we have discussed that if we use Gaussian filters, then Gaussian filters do not lead to ringing effect at all. Same is the situation in case of the high pass filters where the high pass filters try to enhance the high frequency components or detailed contents of an image and it suppresses the low frequency components and that is the reason that the output of a high pass filter we have seen that if there is any smooth region in the image, the smooth region is almost appearing as black in the processed image.

Homomorphic filter as we have discussed is a very very interesting filter. It tries to enhance the reflectance component in an image and it tries to suppress the contribution of the intensity component of the image or the effect of the illumination of the same object and by using this, we have seen some interesting result that even in areas of very low illumination where the areas is not illuminated properly while taking the images, even in such areas, some details of the image, we have been able to extent.

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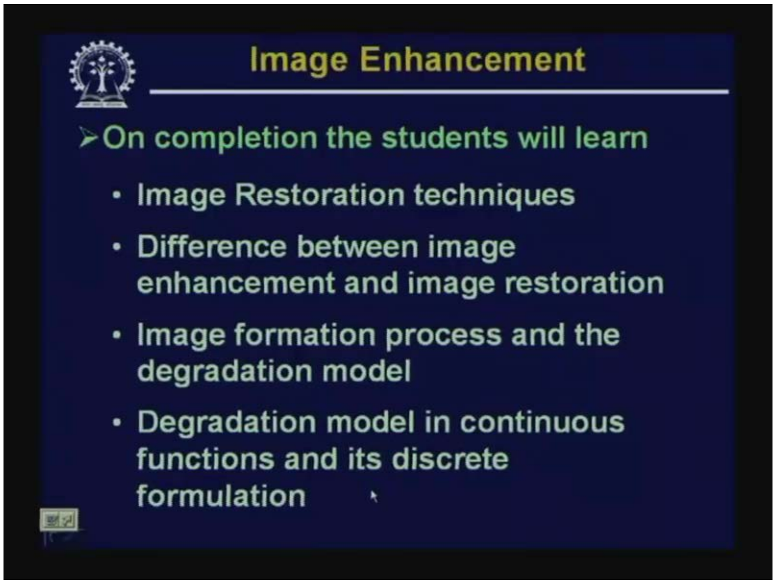

Now, in today's lecture or in a number of lectures starting from today, we will talk about image restoration techniques. So, we will talk about image restoration techniques and we will see what is the difference between image enhancement and image restoration. We will talk about image formation process and the degradation model involved in it and we will see the degradation model and the degradation operation in continuous functions and how it can be formulated in the discrete domain.

Now, when we have talked about the image enhancement, particularly using a low pass filter or using smoothing masks in the special domain; we have seen that one of the effect of using a low pass filter or the effect of using a smoothing mask in the special domain is that the noise content of the image gets reduced.

The simple reason is the noise content leads to high frequency components in the displayed image. So, I if can remove or reduce the high frequency components that also leads to reduction of the noise. Now, this type of reduction of the noise is also a sort of restoration. But these are not usually termed as restoration. Rather a process which tries to recover or which tries to restore an image which has been degraded by some knowledge of a degradation method which has degraded the image; this is an operation which is known as image restoration.

So, in case of image restoration, the image degradation model is very very important. So, we have to find out what is the phenomena or what is the model which has degraded the image and once that model, the degradation model is known; then we have to apply the inverse process to recover or restore the desired image.

So, this is the difference between an image enhancement or simple noise filtering in terms of image enhancement and image restoration. That is in case of image enhancement or simple noise filtering, we do not make use of any of the degradation model or we do not bother about what is the process which is degrading the image. Whereas in case of image restoration, we will talk about the degradation model, we will try to estimate the model that has degraded the image and using that model; we apply the inverse process and try to restore the image.

So, the degradation modeling is very very important in case of image restoration and when we try to restore an image, in most of the cases, we define some goodness criteria. So, using this goodness criteria, we can find out an optimally restored image which more or less which is almost same as the original image and we will see later that image restoration operations can be applied as in case of image enhancement both in the frequency domain as well as in the spatial domain.

So, first of all, let us see that what is the image degradation model that we will consider in our subsequent lectures. So, let us see the image degradation model first.

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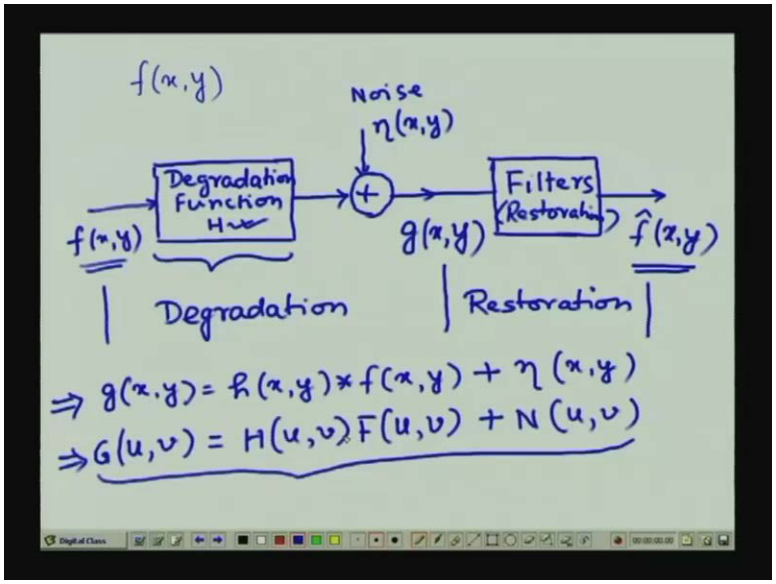

So here, we assume that our input image is image f (x, y). It is a 2 dimensional function as before and we assume that this f (x, y), the input image f (x, y) is degraded by a degradation function H. So, we will put it like this that we have a degradation function H which operates on the input image f (x, y).

Then, the output of this degradation function is added to an additive noise. So here, we add a noise term which we represent by say eta (x, y) which is added to the degradation output and this finally gives us the output image g (x, y). So, this g (x, y) is the degraded image which we want to recover. So, from this g (x, y), we want to recover the input image, the original input image f (x, y) using the image restoration techniques.

So, for recovering this f (x, y), what we have to do is we have to perform some filtering operation and we will see later that this filters, they are actually derived using the knowledge of the degradation function that is H and output of the filters is our restored image and let us put it as f hat (x, y) and we put it as f hat (x, y) because in most of the cases, we are unable to restore the image exactly. That means it is very difficult to get the exact image f (x, y) rather by using the goodness criteria that we have just mentioned; what we can do is we can get an approximation of the original image f (x, y). So, that is this reconstructed image f hat (x, y) which is an approximation of the original image f (x, y).

So, the blocks from here to here that is upto obtaining g (x, y), this is actually the process of degradation; so you will find that in the degradation, we first have a degradation function H which operates on the input image f a f (x, y), then the output of this degradation function block that is added with an additive noise which in this particular case we have represented as eta (x, y) and this degradation function output added to this additive noise that is what is the degraded image that we actually absorb and this degraded image is filtered by using the restoration filters. So, this filters that we use they are actually restoration filters.

So, this g (x, y) is passed through the restoration filters where we get the filter output as the reconstructed image f hat (x, y) and as we have just said that this f hat (x, y) is an approximation of the original image f (x, y). So, this particular block which represents an operation this is a restoration operation and as we have said that the process we call as image restoration in that, the knowledge of the degradation model is very very essential.

So, one of the fundamental task, one of the very important task in the restoration process is to estimate the degradation model of the degradation model which has degraded the input image and later on we will see various techniques of how to estimate the degradation model. That is how to estimate the degradation function H and we will see in a short while from now that this particular operation that is the conversion from f (x, y) to g (x, y), this can be represented in special domain as g (x, y) is equal to h (x, y) convolution with f (x, y) plus the noise eta (x, y).

So, this is the operation which is done in the spatial domain and the corresponding operation in frequency domain will be represented by G (u, v) is equal to H (u, v) into F (u, v) plus N (u, v) where H (u, v) is the Fourier transformation of H (x, y), F (u, v) is the Fourier transformation of the input image f (x, y), N (u, v) is the Fourier transform of the additive noise eta (x, y) and G (u, v) is the Fourier transform of the degraded image G (x, y).

And, this operation is the frequency domain operation and the equivalent operation in the spatial domain is the other one and here you see that in the special domain, we have represented this operation as the convolution operation and we had said earlier that a convolution in the special domain is equivalent to multiplication in the frequency domain. So, that is what the second term that is G (u, v) is equal to H (u, v) into F (u, v) plus N (u, v).

So here, the convolution in the spatial domain is replaced by the multiplication in the frequency domain. So, these 2 are very very important expressions and we will make use of these expressions subsequently more or less throughout our discussion on image restoration process.

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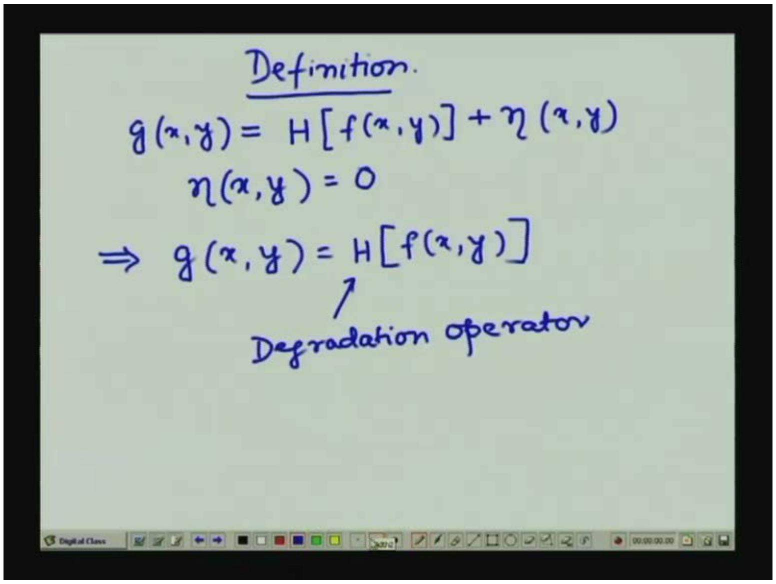

Now, before we proceed further, let us try to recapitulate some of the definitions. So, first we will look at some of the definitions that will be used throughout our discussion on image restoration. So here, what we have is we have a degraded image g (x, y) which now let us represent it is like this H of f (x, y) plus eta (x, y) where in this particular case, we assume that this H is the degradation operator which operates on the input image f (x, y) and that when added with the additive noise eta (x, y) gives us the degraded image g (x, y).

Now here, if we assume or for the time being if we neglect the term eta (x, y) or we said eta (x, y) equal to 0 for the time being for simplicity of our analysis, then what we get is g (x, y) is equal to H in f (x, y) and as we said that here this H, we assume that this is the degradation operator.

Now, the first term that we will define in our case is what is known as linearity. So, what do you mean by the linearity or we say that this degradation operator H is a linear operator.

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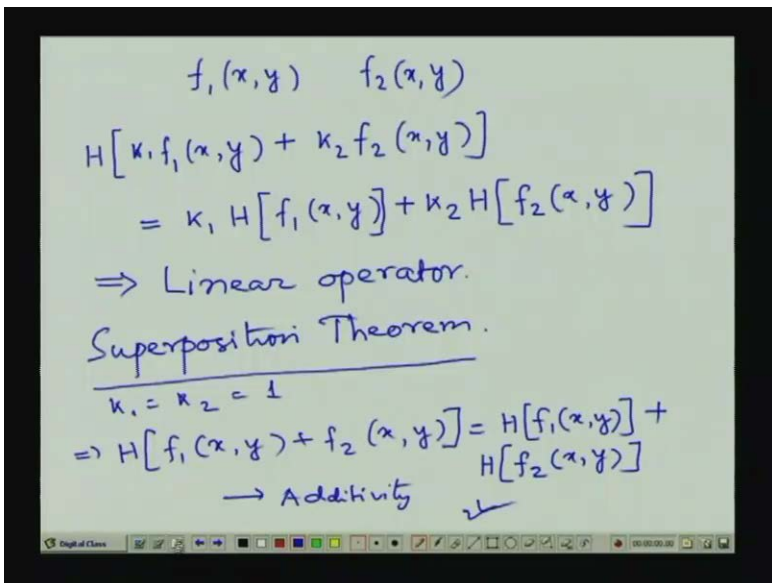

So, for defining linearity, we know that if we have 2 functions say f_1 (x, y) and f_2 (x, y); then we say that if H [k_1 f_1 (x, y) plus some constant k_2 f_2(x, y)], this is equal to k_1 H [f_1 (x, y)] plus k_2 H [f_2 (x, y)]. So, if for these 2 functions f_1 (x, y) and f_2 (x, y) and for these 2 constants k_1 and k_2, this particular relation is true that is H [k_1 f_1 (x, y) plus k_2 f_2 (x, y)] is equal to k_1 H [f_1 (x, y)] plus k_2 H [f_2 (x, y)] if this relation is true, then the operator H is said to be a linear operator.

And, we know very well from our linear system theory that this is nothing but the famous super position theorem. So, this is what is known as the super position theorem and as per our definition of a linear system, we know already that the super position theorem must hold true if the system is a linear system. Now, using this same equation if I said say k_1 is equal to k_2 is equal to 1, then the same equation leads to H [f_1 (x, y) plus f_2 (x, y)] this is nothing but H [f_1 (x, y) plus H f_2 (x, y)].

Simply, we have replaced $k_1$and k_2 by 1 and this is what is known as additivity property. So, the additivity property simply says that the response of the system to the sum of 2 inputs is same as the sum of their individual responses. So here, we have 2 inputs f_1 (x, y) and f_2 (x, y).

So, if I take the summation of f_1 (x, y) and f f_2 (x, y) and then allow H to operate on it, then whatever result we will get that will same as when H operates on f_1 and f_2 individually and we take the sum of those individual responses and this 2 must be equal to true for a linear system and this is what is known as the additivity property. So, this is what is the additivity property in this particular case.

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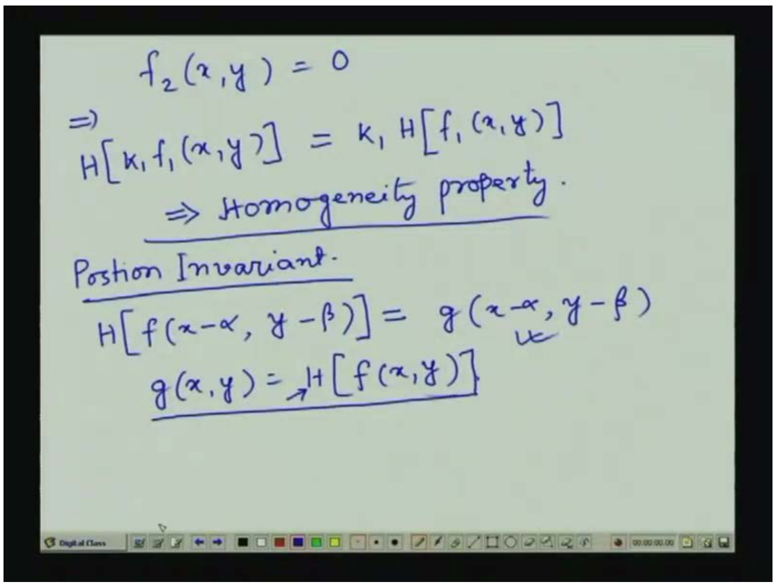

Now here, again if i assume that f $_2$ (x, y) is equal to 0. So, this gives H of k_1 f_1 (x, y) should be equal to k_1 H [f_1 (x, y)] and this is the property which is known as homogeneity property. So, these are the different properties of a linear system and the system is also called position invariant if certain properties hold.

So, the system will be position invariant or location invariant if H [f (x minus alpha, y minus beta)] is same as g of x minus alpha, y minus beta. So, in this case obviously, what we have assumed is g (x, y) is equal to H [f (x, y)]. So, when this is true that g (x, y) is equal to H [f (x, y)], then this particular operator H will be called to be position invariant if H (x minus alpha, y minus beta) is equal to g (x minus alpha, y minus beta) and that should be true for any function f (x, y) and any value of alpha, beta.

So, this position invariant property this simply says that the response at any point in the image, the response of H at any point in the image should solely depend upon the value of the pixel at that particular point and the response will not depend upon the position of the point in the image and that is what is given by this particular expression that is H [f (x minus alpha y, minus beta)] equal to g (x minus alpha, y minus beta).

Now given these definitions, let us see that what will be the degradation model for what will be the degradation model in case of continuous functions.

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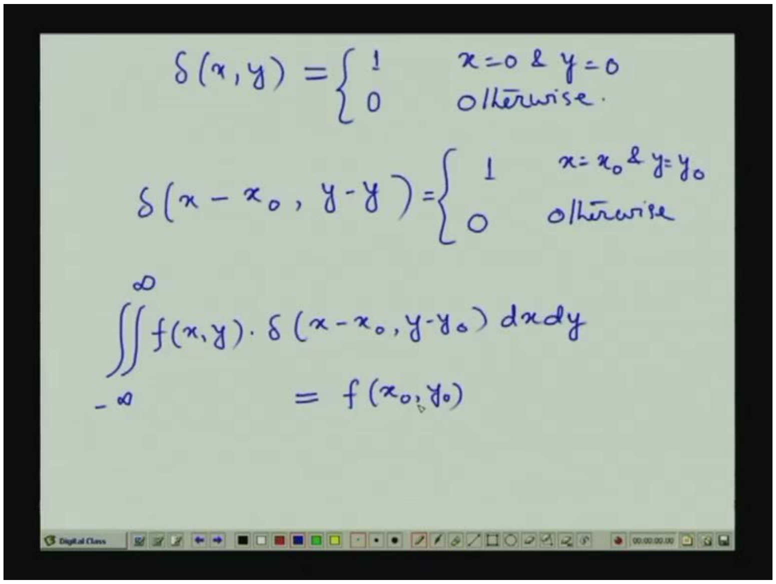

So, to look at the degradation model in case of continuous functions; we make use of an old mathematical expression where we have seen that if I take a delta function say delta (x, y) and the definition of delta (x, y) we have seen earlier that this is equal to 1 if x equal to 0 and y equal to 0 and this is equal to 0 otherwise.

So, this is the definition of a delta function that we have already used and we can use a shifted version of this delta function. That is delta x minus x_0 and y minus y_0 will be equal to 1 if x equal to x_0 and y equal to y_0 and it will be 0 otherwise. So, this is the definition of a delta function.

Now, earlier we have seen that if we have an image say f (x, y) or a 2 dimensional function f (x, y), then multiply this with delta x minus x_0, y minus y_0 and integrate this product over the interval minus infinity to infinity. Then the result of the integral will be simply equal to f (x_0, y_0).

So, this says that if I multiply a 2 dimensional function f (x, y) with the delta function delta x minus x_0, y minus y_0 and integrate the product over the interval minus infinity to infinity, then the result will be simply the value of the 2 dimensional function f (x, y) at location (x_0, y_0).

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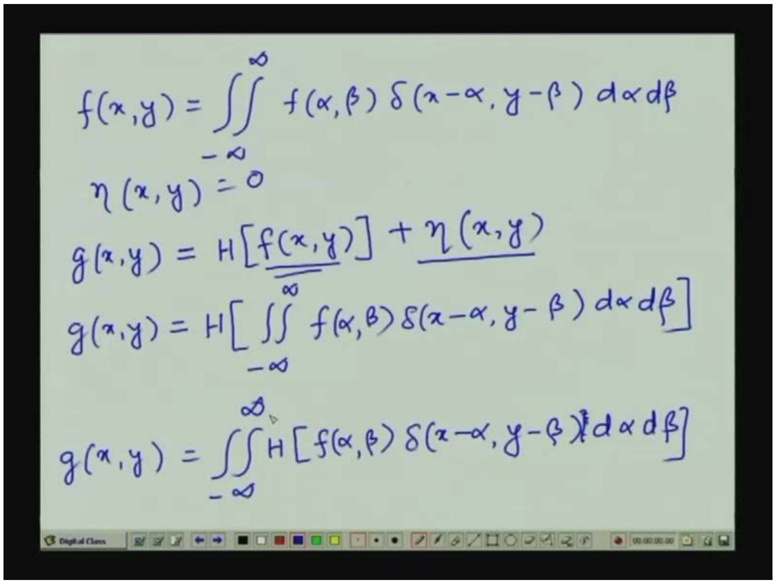

So, by slightly modifying this particular expression, we can have an equivalent expression which is given by I can formulate the 2 dimensional function f (x, y) as a similar integral operation and in this case, I will take f (alpha, beta) delta (x minus alpha, y minus beta) d alpha d beta and take the integral from minus infinity to infinity.

So, we find that we have an equivalent mathematical expression which is equivalent to just the earlier expression that we have said and in this case, we can formulate f (x, y) the 2 dimensional function f (x, y) in terms of the value of the function at a particular point alpha beta and in terms of the delta function delta (x minus alpha, y minus beta).

Now, for the time being if we consider say the noise term eta (x, y) is equal to 0 for simplicity, then we can write the degraded image g (x, y), we have seen earlier that g (x, y) we have written as H f (x, y) plus eta (x, y); so for the time being, we are assuming that this additive noise term eta (x, y) is 0 or it is negligible, then the degraded image g (x, y) can now be written in the form H of… I replace this f (x, y) by this integral term. So, this will be simply H of double integral f (alpha, beta) delta (x minus alpha, y minus beta) d alpha d beta where the integral has to be taken from minus infinity to infinity.

So, I can write, I can get an expression of the degraded image g (x, y) in terms of this integral definition of the function f (x, y) which is operated by the degradation operator H. Now, once I get this kind of expression, now if I apply the linearity and additivity property of the linear system; then this particular expression gets converted to g (x, y) is equal to… I can take this double summation outside, it becomes H of f (alpha, beta) delta (x minus alpha, y minus beta) d alpha d beta, take the integral from minus infinity to infinity and this is what we have obtained by applying the linearity and additivity property to this earlier expression of this degraded image.

Now, here you find that this term f (alpha, beta), this is independent of the variables x and y. So, because the term f (alpha, beta) is independent of the variables x and y, the same expression can now be rewritten in a slightly different form.

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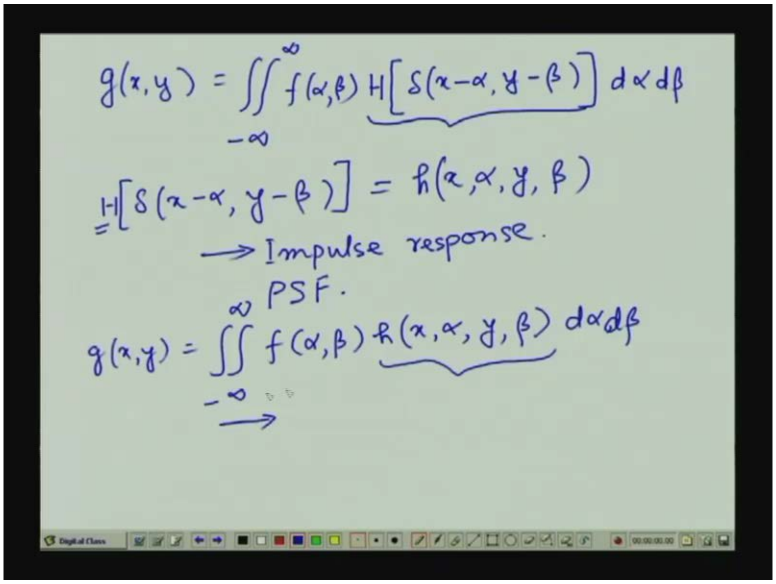

So, that form give us that g (x, y) can now be written as same double integral. We take f (alpha, beta) outside the scope of the operator H. So, this simply becomes f (alpha, beta), then H delta (x minus alpha, y minus beta) d alpha d beta. Take the integral over minus infinity to infinity.

Now, this particular term H of delta (x minus alpha, y minus beta), we can write this as h (x, alpha, y, beta) and this is nothing but what is known as the impulse response of H. So, this is what is known as the impulse response. That is the response of the operator H when the input is an impulse given in the form delta (x minus alpha, y minus beta) and in case of optics, this impulse response is popularly known as point spread function or PSF.

So, using this impulse response, now the same g (x, y), we can write as double integral again f of (alpha, beta) h (x, alpha, y, beta) d alpha d beta, integral from minus infinity to infinity and this is what is popularly known as super position integral of first kind. Now, this particular expression is very very important. It simply says that if the impulse response of the operator H is known, then it is possible to find out the response of this operator H to any arbitrary input f (alpha, beta).

So, that is what has been done here that using the knowledge of this impulse response h (x, alpha, y, beta), we have been able to find out the response of this system to an input f (alpha, beta) and this impulse response is the one which uniquely or completely characterizes a particular system. So, given any system, if we know what is the impulse response of the system, then we can find out what will be the response of that system to any other arbitrary function.

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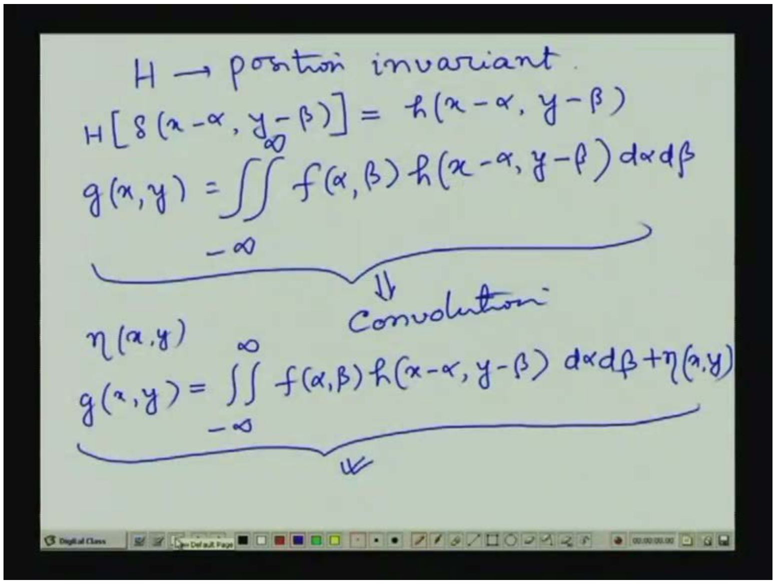

Now, in addition to this, if the function H, this operator H is position invariant; so we use H to be position invariant, so if H is position invariant, then obviously H [delta x minus alpha, y minus beta)] as per of our definition of position invariance will be same as h (x minus alpha, y minus beta). This is as per the definition of position invariance of a system.

Now, using this position invariance property, now we can write g (x, y) that is the degraded image as simply double integral f (alpha, beta) into h (x minus alpha, y minus beta) d alpha d beta, take the integral from minus infinity to infinity. And, if you look at this particular expression, you will find that this expression is not is nothing but the convolution operation. This is nothing but the convolution operation of the 2 functions f (x, y) and h (x, y) and that is what we said that when we have drawn our degradation model, we have said that input image f (x, y) is actually convolved by the degradation process that is H (x, y). So, this is nothing but that convolution operation.

And now, if I take, you will find that earlier we have considered this noise term eta (x, y) to be equal to 0. So now, if I consider this noise term eta (x, y), then our degradation function or the degradation model becomes simply g (x, y) is equal to f (alpha, beta) h (x minus alpha, y minus beta) d alpha d beta, take the integral from minus infinity to infinity plus the noise term eta (x, y).

So, this is the general image degradation model and you will find that here we have assumed that the degradation function H is linear and position invariant and it is very important to note that many of the degradation operations which we encounter in reality can be approximated by such linear space invariant or linear position invariant models.

The advantage is once a degradation model can be approximated by a linear position invariant model, then the inter mathematical tool of linear system theory can be used to find out the solution for such image restoration process. That means we can use all those tools of linear system theory to estimate what will be the restored image f (x, y) from a given degraded image g (x, y) provided, we know we have some knowledge of the degradation function that is H (x, y) and we have some knowledge of what is the noise function eta (x, y).

Now, this formulation that we have done till now, this formulation is for the continuous case and as we have said many times that in order to use this mathematical operation for our digital image processing techniques, we have to find out a discrete formulation of this mathematical model. So, let us see that how we can have an equivalent discrete formulation of this particular degradation model.

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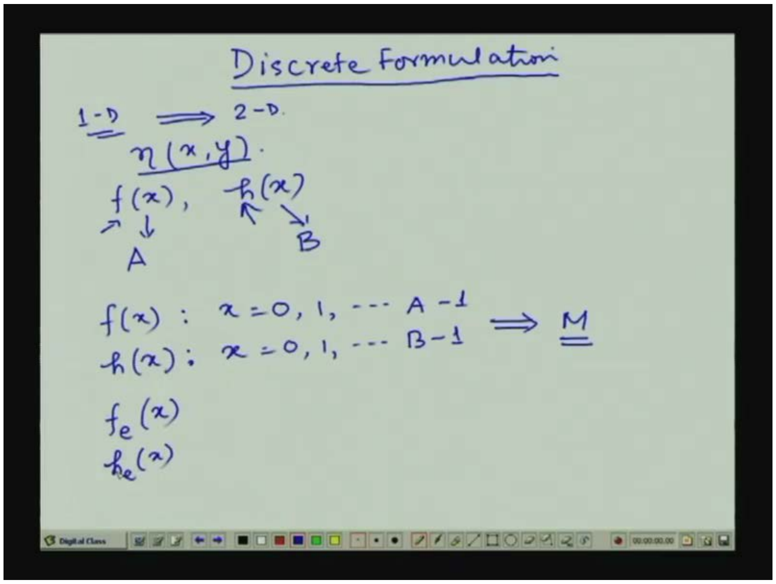

So, to get a discrete formulation, firstly we will consider; so we have to get a discrete formulation. So, to obtain this discrete formulation, for simplicity, initially we will assume the cases in 1 dimension and later on this we will extend to 2 dimensional cases for digital image processing operations. Again for simplicity, initially, we will neglect the contribution of the noise term that is eta (x, y).

So, in case of 1 dimension as we have done in case of in the continuous signal; we have 2 signals f (x) and another one is h (x). So, we have said that f (x) is the input signal and h (x) tells us that what is the degradation function. So, f (x) is the input function and h is the h (x) is the degradation function. For discretization of the same formulation, what we have to do is we have to uniformly sample these 2 functions f (x) and h (x) and we assume that f (x) is uniformly sampled to give an array of dimension A and h (x) is uniformly sampled to give an array of dimension B.

That means for f (x) in the discrete case, x varies from 0, 1 to A minus 1 and h (x) for h (x), x varies from 0, 1 to b minus 1. Then what we will do, we will add additional 0s to this f (x) and b (x) to make both of them of the same dimension and dimension equal to say capital M.

So, we make both of them to be of dimension capital M by adding additional number of 0s and we assume that both f (x) and h (x) after addition of this 0 terms and making both of them to be of dimension M, they become periodic with a periodicity capital M. So, once we have done this, now the same convolution operation that we have done in case of our continuous case, now can also be written in case of discrete case.

So, in discrete case, the convolution operation, we will write in this manner. So, after converting both f (x) and h (x) into arrays of dimension M, this new arrays that we will get, we represent it by f_e (x) that is f extended x as we have extended it and h we represent by h_e (x) that is the extended version of h (x).

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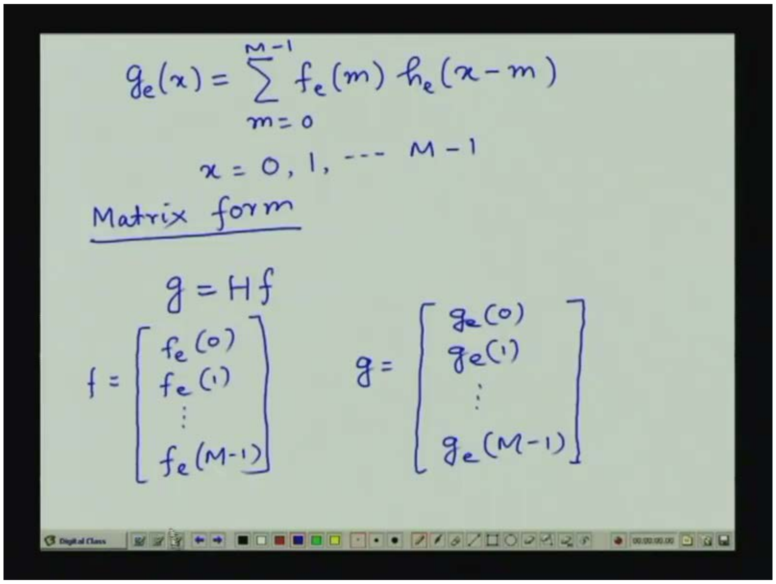

And now, in discrete domain, the convolution function can be written as g_e (x) is equal to summation f_e (m) h_e (x minus m) where this m will be varying from 0 to capital M minus 1 and x we will assume values from 0 to capital M minus 1. So, this is the discrete formulation of the convolution equation that we have obtained in case of continuous signal cases.

Now, if you analyze this convolution expression, you will find that this convolution expression can be written in the form of a matrix, matrix operation. So we can have the matrix form. In matrix form, these equations will be like this - g equal to some matrix H times f where the function f or array f will be simply f_e (0), f_e (1), this way upto f_e (capital M minus 1) and function g similarly will be g_e (0), g_e (1), so like this it will be g_e (M capital M minus 1).

So, you recollect, you just recollect that f_e and g_e, these are the names which are given to the sample versions of the functions f (x) and g (x) after extending the functions by addition of addition by adding additional number of 0's to make them of dimension capital M.

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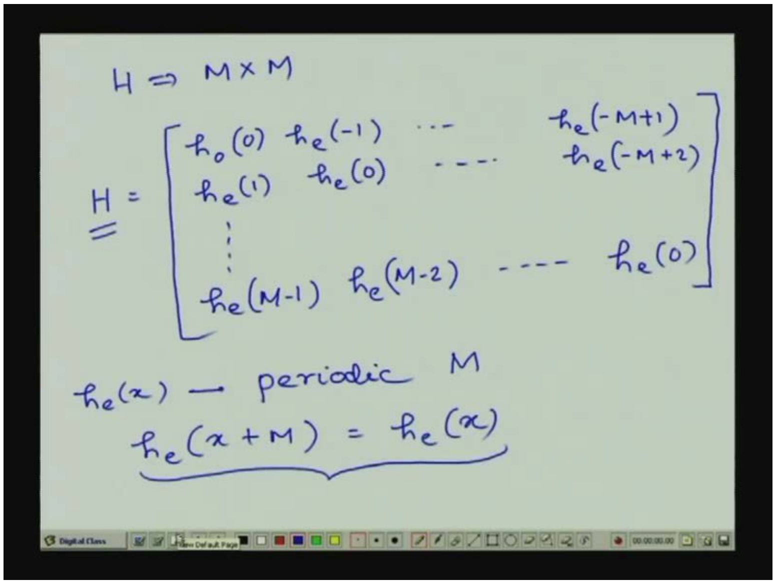

And, in this particular case, the matrix h will have the matrix h will be of dimension capital M by capital M. But the elements of H will be like this - h_e (0), h_e (minus 1), continue like this, it will be h_e (minus M plus 1), here it will be h_e (1), h_e (0), it will be h_e (minus capital M plus 2) and if we continue like this, it will be h_e (capital M minus 1), h_e (capital M minus 2), like this it will be h_e (0). So, this is the form of the matrix capital H which is the degradation matrix in this particular case.

And here, you find that that elements of this degradation matrix capital H are actually generated from the degradation function h_e (x). Now, remember that we have assumed that our h_e (x), this function is actually periodic. This is which we have assumed with periodicity of capital M. So, if this function is periodic with periodicity capital M that means he x plus capital M that will be same as h_e of x.

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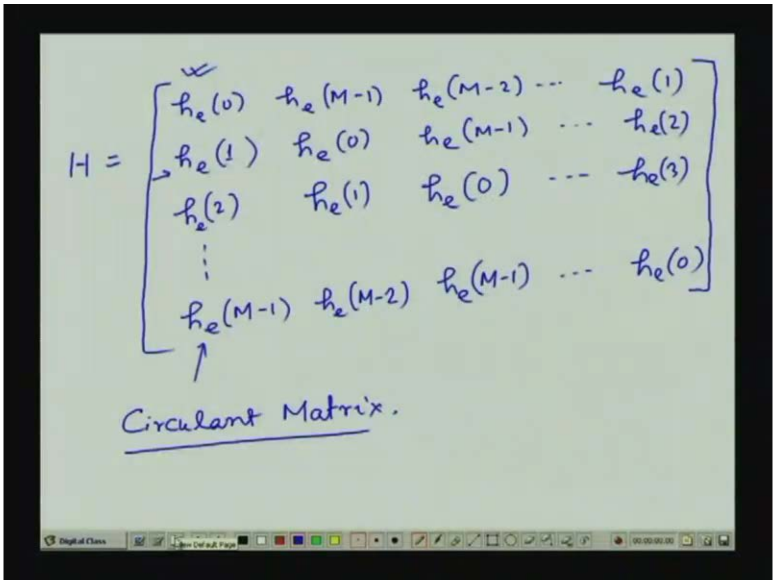

So, by using this periodicity assumption, now this particular degradation matrix H can be written in a different form where this matrix H will now be represented as h_e (0), h_e (capital M minus 1), h_e (capital M minus 2) upto h_e (1). The second row will be h_e (1), h_e (0), h_e (capital M minus 1) and this will be h_e (2). Third row will be h_e (2), h_e (1), h_e (0) like this it will be h_e (3) and the last row continue in the same manner will be h_e (capital M minus 1), h_e (capital M minus 2), h_e (capital M minus 3) and the last term will be equal to h_e (0).

Now, if you analyze this particular matrix, you will find that this degradation matrix capital H has a very very interesting property. That means the first property is different rows of this matrix are actually generated by rotation to the right of the previous term. So, here if you look at the second row; you will find that this second row is actually generated by rotating the first row to the right. Similarly, third row is generated by rotating the second row to right by 1.

So, this is so in this particular matrix, the different rows are actually generated by rotating the previous row to the right. So, this is called circulant matrix because different rows are generated by a circular rotation and the circularity in this particular matrix is also complete in the sense that if I rotate this last row to right, what I get is the first row of the matrix. So, this kind of matrix is known as a circulant matrix.

So here, I find we find that in case of discrete formulation, the discrete formulation is also a convolution operation and here in the matrix equation of the degradation model, the degradation matrix H that we obtain that is actually a circulant matrix. Now, let us extend the concept of this discrete formulation from 1 dimension to 2 dimensions.

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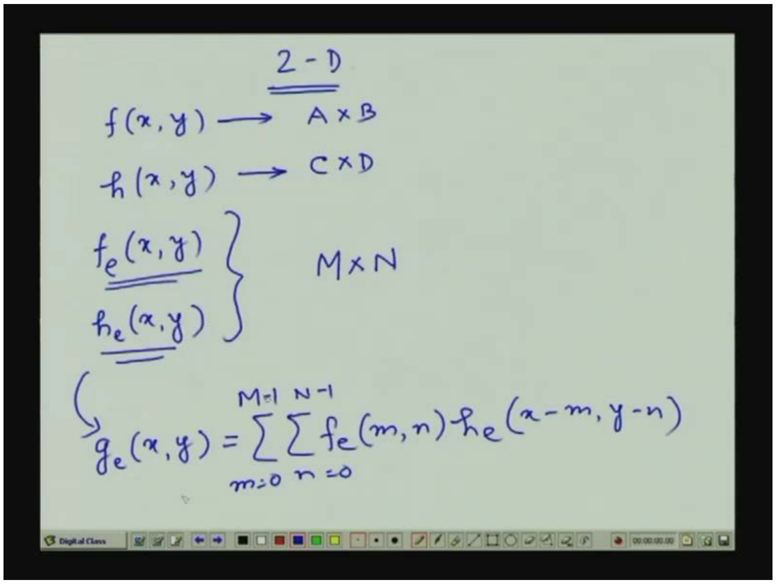

So, let us see what we get in case of 2 dimensional functions that is in case of 2 dimensional images. So, in case of 2 dimension, we have the input function or the image function which is given by f (x, y) and we have the degradation function which is given by h (x, y) and we assume that this if f (x, y) is sampled to an array of dimension capital A by capital B and say h (x, y) is sampled to an array of dimension say capital C by capital D.

Now, as we have done in 1 dimensional case that is the functions f (x) and h (x) are actually extended by using by putting additional number of 0's to make both of them of same size say capital N; in the same manner, here we add additional number of 0's to both this f (x, y) and h (x, y) to get the extended functions f_e (x, y) and h_e (x, y) to make both of them of dimension say capital M by capital N and we also assume that this f_e (x, y) and h_e (x, y), they are periodic and in x dimension, the periodicity will be of period capital M and in y dimension, the periodicity will be of period capital N.

Now, following similar procedure, we can obtain a convolution expression in 2 dimensions which is given by g_e (x, y) which is nothing but f_e (m, n) h_e (x minus m, y minus n) where n varies from 0 to capital N minus 1 and m varies from 0 to capital M minus 1.

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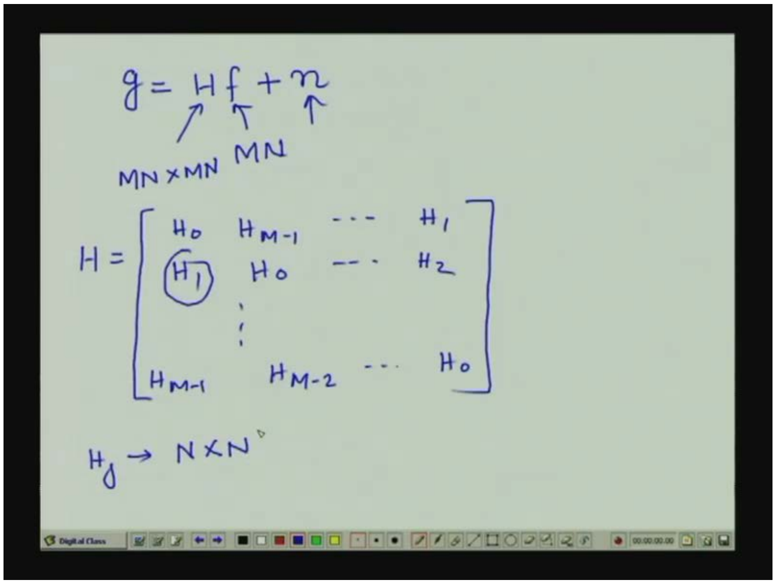

And, if I write this convolution expression in the form of a matrix and incorporating the noise term eta (x, y), I will get a matrix equation which is of the form g equal to Hf plus n where this matrix where this vector f is a vector of dimension capital M into N which is obtained by concatenating different rows of the 2 dimensional function f (x, y) that is the first N number of elements of this vector f will be the elements of the first row of matrix f (x, y).

Similarly, we also obtain this particular vector n by concatenation of the rows of the matrix eta (x, y) and this particular degradation matrix h (x) in this case will be of dimension M into N by M into N and this matrix H will have a very very interesting form. This matrix H can now be represented as H_0 $H_{M\ minus\ 1}$ like this upto H_1. The second row can be H_1 H_0 upto H_2 and the last row is $H_{M\ minus\ 1}$ $H_{M\ minus\ 2}$ like this we have H_0 where each of these terms H_j is a matrix, so each of this Hj is actually a matrix of dimension N by N where this H_j is generated from the j'th row of the degradation function H (x, y).

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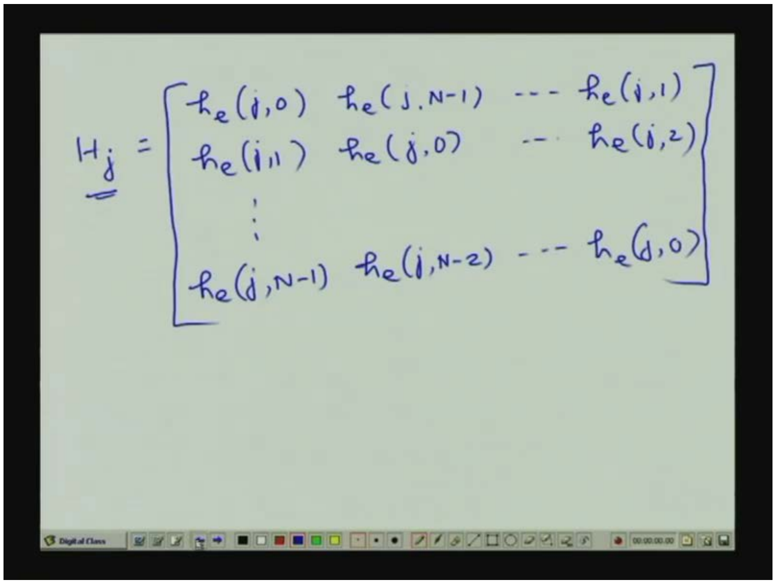

That is this H_j we can write this matrix H_j in the form h_e (j, 0) h_e (j, N minus 1) like this upto h_e (j, 1). Second row will be h_e (j, 1) h_e (j, 0) this way h_e (j, 2) and if I continue like this the last row will be h_e (j, N minus 1) h_e (j, N minus 2) like this if I continue, the last element will be h_e (j, 0). So, you find that this matrix H_j which is actually a component of the degradation matrix capital H is a circulant matrix that we have defined earlier and using this block matrix, the degradation matrix H is also have been subscripted in the form of a circulant matrix. So, this matrix H in this particular case is what is known as a block circulant matrix. So, this is what is called a block circulant matrix.

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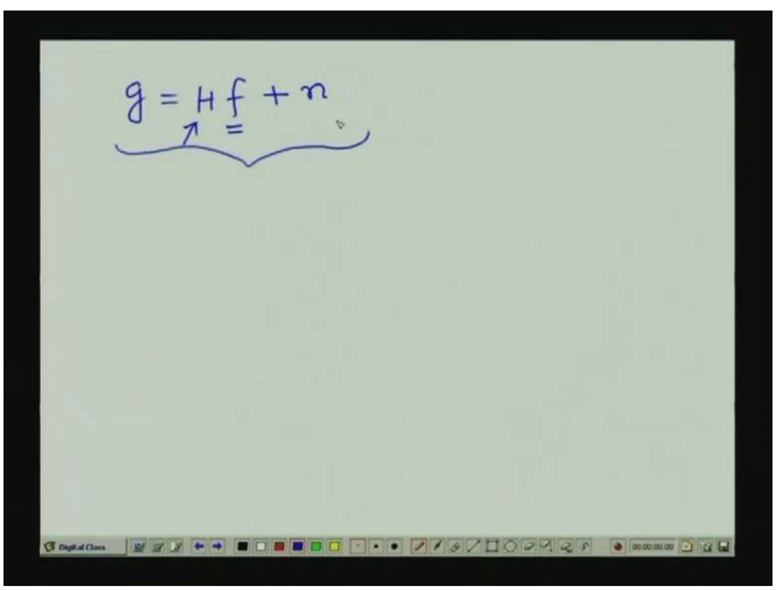

So, in case of 2 dimensional function that is in case of a digital image, we have seen that the degradation model can simply be represented by this expression g equal to H into f plus n where this vector f is a vector of dimension m into n and the degradation matrix H which is of dimension m into n by m into n is actually a block circulant matrix where for each block, the matrix is obtained from the j'th row of the degradation function H (x, y).

So, in our next lecture, we will see what will be the applications of this particular degradation model to restore an image from its degraded version.

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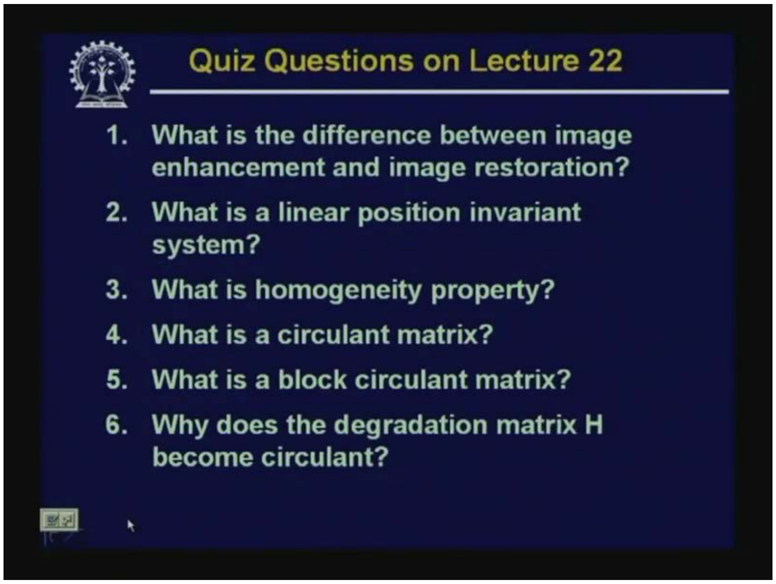

So now, let us see some of the questions of this particular lecture. So, the first question is what is the difference between image enhancement and image restoration? Second question is what is a linear position invariant system? Third question, what is homogeneity property? Fourth, what is a circulant matrix? What is a block circulant matrix? Why does the degradation matrix H become circulant?

Thank you.

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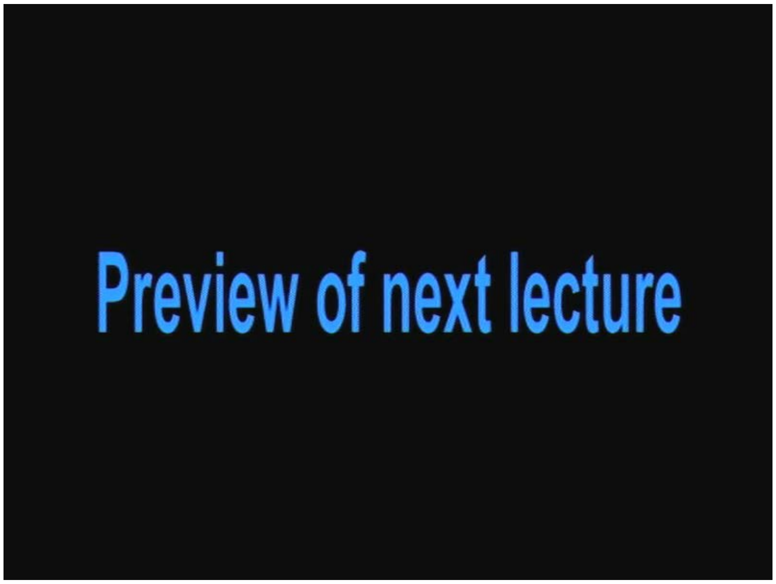

Hello, welcome to the video lecture series on digital image processing. In the last class, we have started discussion on image restoration.

We have said that there are certain cases where image restoration is necessary in the sense that in many cases, while capturing the image or while acquiring the image, some distortions appear in the image. For example, if you want to capture a moving object with a camera; in that case, because of the movement of the camera, it is possible that the image that is captured will be blurred which is known as motion blurring.

There are many other situations, say for example if the camera in not properly focused, then also the image that you get is a distorted image. So, in such situations, what we have to go for is restoration of the image or recovery of the original image from the distorted image.

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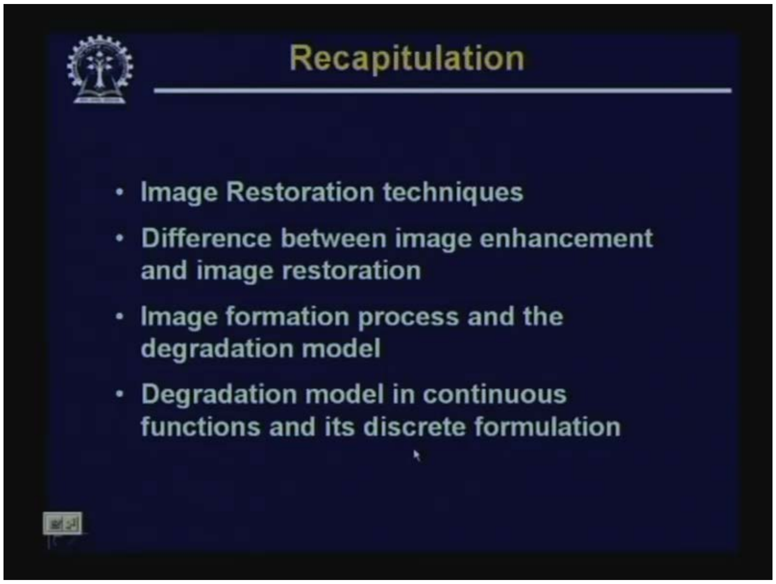

Now, regarding this in the last class, we had talked about what is image restoration technique. In previous classes, we have talked about image filtering that is if the image is contaminated with noise. Then, we have talked about various types of filters both in spatial domain as well as in frequency domain to remove that noise and we just mentioned in our last class that this kind of noise removal is also a sort of restoration because there also we are trying to recover the original image from a noisy image.

But conventionally, this kind of simple filtering is not known as restoration. But what is by restoration what I what we mean is that if we know a degradation model by which the image has been degraded and on that degradation model on the degraded image some noise has been added. So, recovery or restoration of the original image from a degraded image using the acquired knowledge of the degradation function of the model using which the image has been degraded; so, that kind of recovery is normally known as restoration process. So, this is the basic difference between restoration and image filtering or image enhancement.

Then, we have seen an image formation process where the degradation is involved and we have talked about the degradation model in continuous functions as well as its discrete formulation. So, in today's lecture, we will talk about the …..((59:19))

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