

Digital Image Processing

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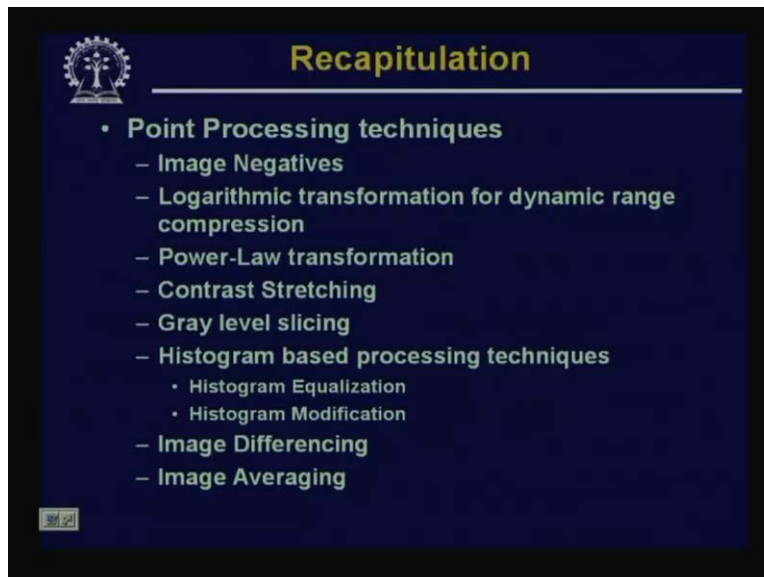
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Lecture - 20

Hello, welcome to the video lecture series on digital image processing. For last few classes, we were discussing about image enhancement techniques and we have completed our discussion on point processing techniques for image enhancement.

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So, what we have done till now is the point processing techniques for image enhancement and under this, the first operation that we have done is image negatives and there we have seen that image negative operation is useful in case the image contains information which are either contained in the gray level or in the white pixels that are embedded in dark image regions.

So, if you take the negative of such images, then the information content becomes dark where as background becomes white and visualization of that information in those negative images is much more easier.

The second operation that we have done is the logarithmic transformation for dynamic range compression. We have done this logarithmic transformation because we have seen that in some cases, the dynamic range that is the difference between the minimum intensity value and the maximum intensity value of an image is so high that a display device is normally not capable to deal with such a high dynamic range.

So, for such cases, we have to reduce the dynamic range of the image so that the image can be displayed properly on the given display device and this logarithmic transformation operation gives such a dynamic range compression operation.

Then the next technique we have talked about is the power-law transformation. In case of power-law transformation, we have seen that many of the devices like whether the image printing device or the image display device or the image acquisition device; these different devices, they themselves introduce some sort of power-law operation on the image that is to be displayed.

As a result, the image that we want to display or the image that we want to print, they become distorted. The appearance of the output image is not same as the image that is to be outputted that is intended to be outputted. So, this power-law which is introduced by those devices has to be corrected by some power-law operation.

So, we have seen that in case of this power-law compensation or power-law operation, we introduced a pre processed image having a power-law which is inverse of the power-law that is introduced by the device and as a result; the pre process image, when it goes to the device, then the output of the device will be almost similar to the image that is intended.

So, for this kind of operation, to nullify the effect of the device, we go for power-law compensation technique. Then the next operation that we have done is contrast stretching. So, in case of contrast stretching, we have seen that in many cases, we can get a very very dark image because of the fact that may be the scene when the image was taken was not properly illuminated or the scene was very very poorly illuminated.

The other reason why we can get such a dark image is that while taking the photograph, the camera lens was not properly set that is the aperture of the lens was not properly set or we can also get dark image because of the limitation of the sensor itself. The image sensor, if the dynamic range of the image sensor is a very narrow; in that case, such a kind of sensor also leads to an image which is a dark image.

So, to enhance such dark images so that the image can be visualized properly, we go for the contrast enhancement technique. The other kind of image enhancement we have talked about is the grey level slicing operation and this kind of grey level operation is useful in cases if the application needs to highlight certain range of grey levels in the image.

So, in such cases, in grey level slicing, we have seen 2 kind of techniques ((05:40))... region is highlighted whereas the grey levels outside that particular specified range is suppressed and the second kind of grey level slicing operation that we have seen is the grey levels within the specified range is highlighted whereas the grey levels outside the range remains as it is.

So, these are the 2 different types of grey level slicing operations we have talked about and as I said that if the application demands that the application needs enhancement of certain range of grey levels, the application is not interested in other grey level values or other intensity values; in that case, what we go for is the grey level slicing kind of operation.

Then we have talked about other enhancement techniques where these enhancement techniques are based on histogram based processing operations. In other point processing techniques, we define a transformation function where the transformation function simply works on a particular pixel of the input image to generate a processed pixel of the output image and those transformation functions, they do not take care of or they do not consider the overall appearance of the image and we have seen that the overall appearance of the image is actually reflected in what is called the histogram of the image.

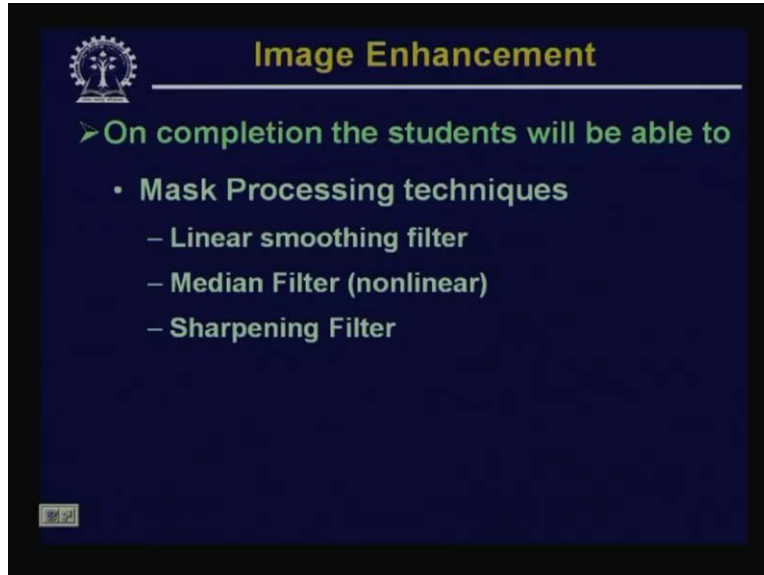
So, this histogram based processing techniques, they try to modify or they try to highlight the overall appearance of the image by modifying the histogram of that particular image and under this category, we have talked about 2 kinds of histogram based processing techniques; one was one of them was histogram equalization technique and the second one was histogram modification technique.

Then we have talked about 2 other kinds of image enhancement operations where these enhancement operations does not perform on a single image but it performs on multiple number of images. So, one of them we have talked about was image differencing operation. So, this image differencing operation, it highlights those regions in the image where there is a difference between the given 2 images. So, only those regions where the given 2 images are different, those regions will be highlighted and where the 2 images are similar, those regions will be suppressed.

The other kind of operation that we have done was image averaging operation and we have said that this kind of image averaging operation is very very useful where the object which is imaged that is of very very low intensity. So, for such kind of objects or while imaging such kind of objects, the image that you get is likely to be dominated by noise.

So, if I get multiple number of frames of such noisy images and if the noise that is added to the image is a 0 mean noise, then taking average of multiple number of frames of such noisy images is likely to cancel the noise part and ultimately what comes out after the averaging operation is the actual image that is desired. So, these are the different kind of operations, point processing techniques for image enhancement that we have done till our last class.

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The slide features a dark blue background with a white logo in the top left corner. The title 'Image Enhancement' is centered at the top in a yellow font. Below the title, a green arrow points to the text 'On completion the students will be able to'. Underneath, a white bullet point lists 'Mask Processing techniques', followed by three sub-points: 'Linear smoothing filter', 'Median Filter (nonlinear)', and 'Sharpening Filter'.

Image Enhancement

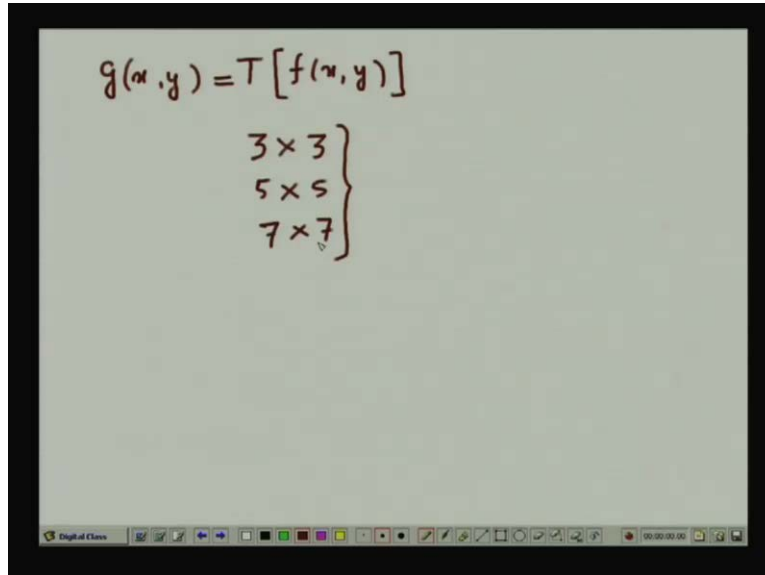
➤ On completion the students will be able to

- Mask Processing techniques
 - Linear smoothing filter
 - Median Filter (nonlinear)
 - Sharpening Filter

Now, in today's class we will talk about another special domain technique which is called mask processing technique. The previous lectures also we were dealing with the special domain techniques and we have said that image enhancement techniques can broadly be categories into special domain techniques and frequency domain techniques. The frequency domain techniques we will talk about later on.

So, in today's class, we will talk about another special domain techniques which are known as mask processing techniques and other under this, we will discuss 3 different types of operations. The first one is the linear smoothing operation; the second one is a nonlinear operation which is based on the statistical features of the image which is known as the median filtering operation and the third kind of mask processing technique that we will talk about is the sharpening filter. Now, let us see what this mask processing technique means.

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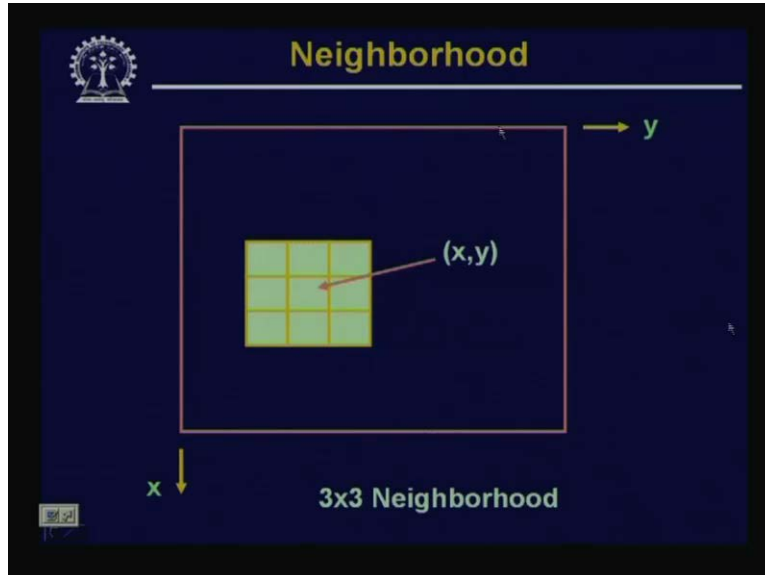
The image shows a whiteboard with a black border. At the top, the equation $g(x, y) = T[f(x, y)]$ is written in brown ink. Below it, three neighborhood sizes are listed vertically: 3×3 , 5×5 , and 7×7 . These three lines are enclosed in a large right-facing curly bracket. At the bottom of the whiteboard, there is a toolbar with various icons and a timestamp that reads "10:30:30".

Now, in our earlier discussions we have mentioned that while going for this contrast enhancement, what we basically do is given an input image say $f(x, y)$, we transform this input image by a transformation operator say T which gives us an output image $g(x, y)$ and what will be the nature of this output image $g(x, y)$ that depends upon what is this transformation operator T .

In point processing technique, we have said that this transformation operation T that operates on a single pixel in the image. That is it operates on a single pixel intensity value. But as we earlier said that T is an operator which operates on a neighborhood of the pixels at location (x, y) ; so for point processing operation, the neighborhood size was 1 by 1. So, if we consider a neighborhood of size more than 1 that is we can consider a neighborhood of size say 3 by 3, we may consider a neighborhood of size say 5 by 5, we may consider a neighborhood of size 7 by 7 and so on.

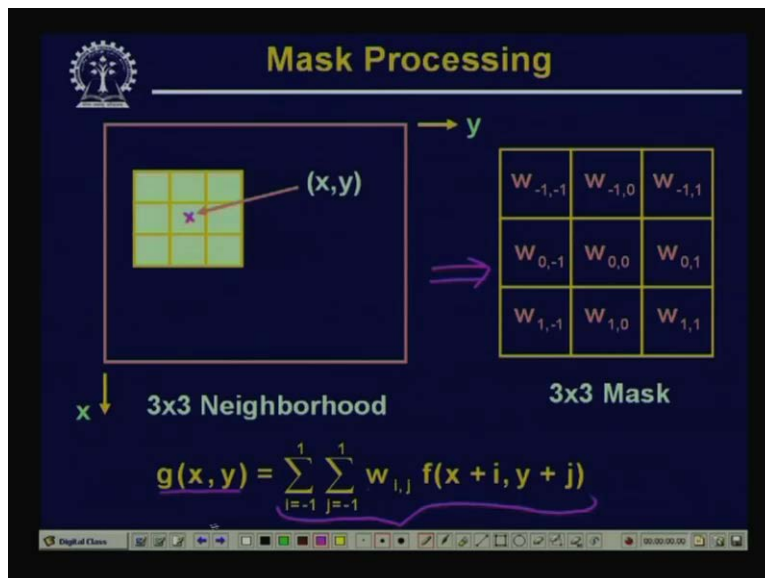
So, if we consider a neighborhood of size more than 1, then the kind of operation that we are going to have that is known as mask processing operation. So, let us see what does this mask processing operation actually mean.

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Here, we have shown a 3 by 3 neighborhood around a pixel location (x, y). So, this outer rectangle represents a particular image and in the middle of this, we have shown a 3 by 3 neighborhood and this 3 by 3 neighborhood is taken around a pixel at location (x, y).

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By mask processing what we mean is; so if I consider a neighborhood of size 3 by 3, I also consider a mask of size 3 by 3. So, we find that here on the right hand side, we have shown a mask. So, this is a given mask of size 3 by 3 and these different elements in the mask that is $W_{-1,-1}$, $W_{-1,0}$, $W_{-1,1}$, $W_{0,-1}$, $W_{0,0}$, $W_{0,1}$ and so on upto $W_{1,1}$; these elements represent the coefficients of this mask.

So, for all these mask processing techniques what we do is we place this mask on this image where the mask center coincides with the pixel location (x, y). Once you place this mask on this particular image, then you multiply every coefficient of this mask by the corresponding pixel on the image and then you take the sum of all these products.

So, the sum of all these products is given by this particular expression and whatever sum you get that is placed at location (x, y) in the image g(x, y). So, by mask processing operation, this is the mathematical expression we get that g(x, y) equal to $\sum_{i,j} W_{ij} f(x+i, y+j)$. You have to take the summation of this product over j varying from minus 1 to 1 and i varying from minus 1 to 1.

So, this is the operation that has to be done for a 3 by 3 neighborhood in which case we get a mask again of size 3 by 3. Of course, as we said that we can have masks of higher dimension; we can have a mask of 5 by 5, if I consider a 5 by 5 neighborhood. I have to consider a mask of size 7 by 7, if I consider a 7 by 7 neighborhood and so on.

So, if this particular operation is done at every pixel location (x, y) in the image, then the output image g(x, y) for various values of x and y that we get is the processed image g. So, this is what we mean by mask processing operation.

Now, the first of the mask processing operation that we will consider is the image averaging or image smoothing operation. So, image smoothing is a special filtering operation where the value at a particular location (x, y) in the processed image is the average of all the pixel values in the neighborhood of x and y. So, because it is the average, this is also known as averaging filter and later on we will see that this averaging filter is nothing but a low pass filter. So, when we have such averaging filter, the corresponding mask can be represented in this form.

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The slide is titled "Smoothing Spatial Filter" and features a logo in the top left corner. Below the title, it says "Averaging filter (Lowpass filter)". In the center, there is a 3x3 grid of ones, with a multiplier of $\frac{1}{9}$ to its left. An arrow points from this grid to the text "Box filter". At the bottom, the mathematical expression for the filtered image is given as $g(x, y) = \frac{1}{9} \sum_{i=-1}^1 \sum_{j=-1}^1 f(x+i, y+j)$.

So, again here we are showing a mask, a 3 by 3 mask and here we find that all the coefficients in this 3 by 3 mask are equal to 1 and by going back to our mathematical expression, I get an expression of this form that is $g(x, y)$ equal to 1 upon 9 into $f(x + i, y + j)$. Take the summation over j equal to minus 1 to 1 and i equal to minus 1 to 1.

So naturally, as this expression says you find that what we are doing; we are taking the summation of all the pixels in the 3 by 3 neighborhood of the pixel location (x, y) and then dividing these summation by 9. So, which is nothing but average of all the pixel values in the neighborhood of (x, y) in the 3 by 3 neighborhood of (x, y) including the pixel at location (x, y) and this average is placed at location (x, y) in the processed image g .

So, this is what is known as averaging filter and also this is called a smoothing filter **and the filter could** and the particular mask for which all the filter coefficients or mask coefficients are same or equal to 1 in this particular case, this is known as a box filter. So, this particular filtering operation that we are getting, this particular mask is known as a box filter.

Now, when we perform this kind of operation, then naturally because we are going for averaging of all the pixels in the neighborhood; so the output image is likely to be a smoothed image that means it will have a blurring effect, all the sharp transitions in the images will be removed and they will be replaced by a blurred image.

As a result, if there is any sharp edge in the image; the sharp images, the sharp edges will also be blurred. So, to avoid the effect of blurring, there is another kind of mask; averaging mask or smoothing mask which performs weighted average.

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Smoothing Spatial Filter

Weighted average

$\frac{1}{16} \times$	1	2	1
	2	4	2
	1	2	1

$$g(x, y) = \frac{1}{16} \sum_{i=-1}^1 \sum_{j=-1}^1 w_{i,j} f(x+i, y+j)$$

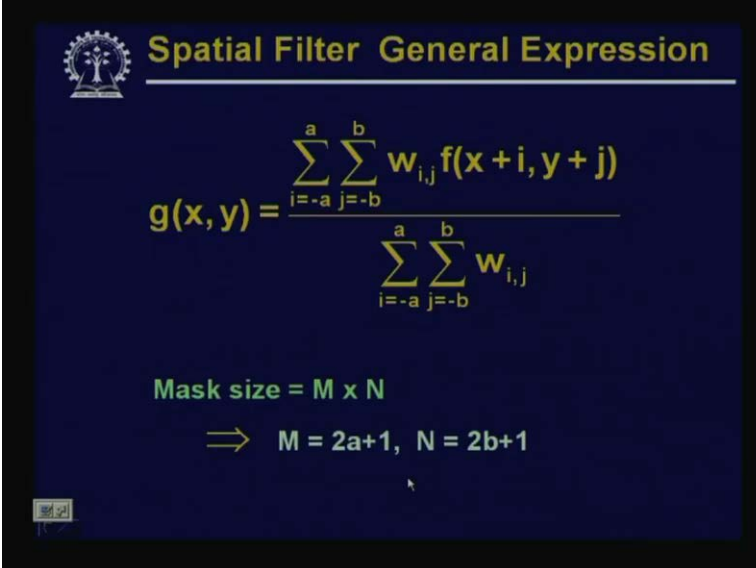
So, such a kind of mask is given by this particular mask. So, here you find that in this mask, the center coefficient is equal to 4. The coefficients vertically up, vertically down or horizontally left, horizontally right; they are equal to 2 and all the diagonal elements of the center elements in

this mask are equal to 1. So effectively, what we are doing is when we are taking the average, we are weighting every pixel in the neighborhood by the corresponding coefficients and what we get is a weighted average.

So, the center pixel that is the pixel at location (x, y) gets the maximum weightage and as you move away from the pixel locations, from the center location, the weightage of the pixels are reduced. So, when we apply this kind of mask, then our general expression of this mask operation that is valid which becomes $W_{ij} f(x \text{ minus } i, y \text{ minus } j)$. Take the summation from j equal to minus 1 to 1 and i equal to minus 1 to 1 and take 1 upon 16 of this and that will give the value which is to be placed at location (x, y) in the processed image g . So, this becomes the expression of $g(x, y)$.

Now, the purpose of going for this kind of weighed averaging is that because here we are weighting the different pixels in the image for taking the average, the blurring effect will be reduced in this particular case. So in case of box filter, the image will be very very blurred and of course the blurring will be more if I go for bigger and bigger neighborhood size or bigger and bigger mask size. When we go for averaging, weighted averaging; in such cases, the blurring effect will be reduced. Now, let us say what kind of result we get.

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Spatial Filter General Expression

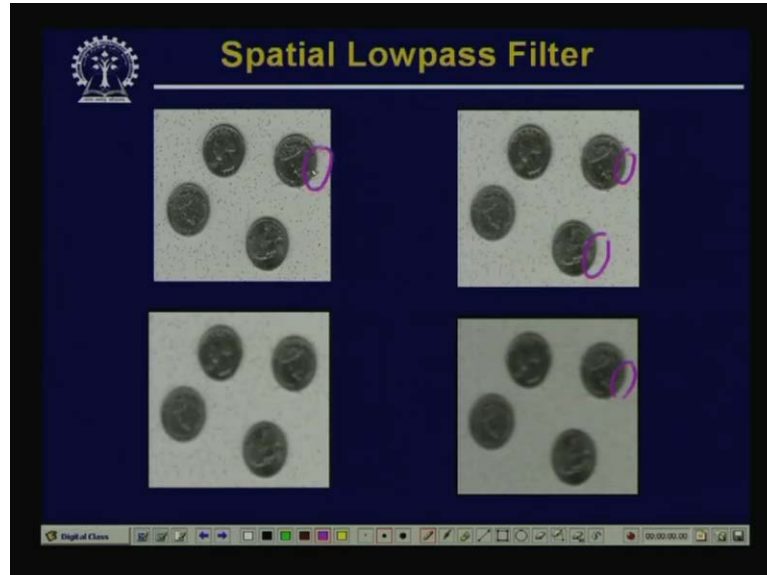
$$g(x, y) = \frac{\sum_{i=-a}^a \sum_{j=-b}^b w_{i,j} f(x+i, y+j)}{\sum_{i=-a}^a \sum_{j=-b}^b w_{i,j}}$$

Mask size = $M \times N$

$\Rightarrow M = 2a+1, N = 2b+1$

So, this gives the general expression that when we will consider W_{ij} , we have to have a normalization factor that is this summation has to be divided by sum of the coefficients and as we said that 3 by 3 neighborhood is only a special case, I can have neighborhoods of other sizes; so here it shows that we can have a neighborhood of size M by N where M equal to $2a$ plus 1 and N equal to $2b$ plus 1 where a and b are some positive integers and obviously here, **you show the here** it is shown that the mask is usually of odd dimension, it is not even dimension and that is normally the mask of odd dimension which is normally used in case of image processing.

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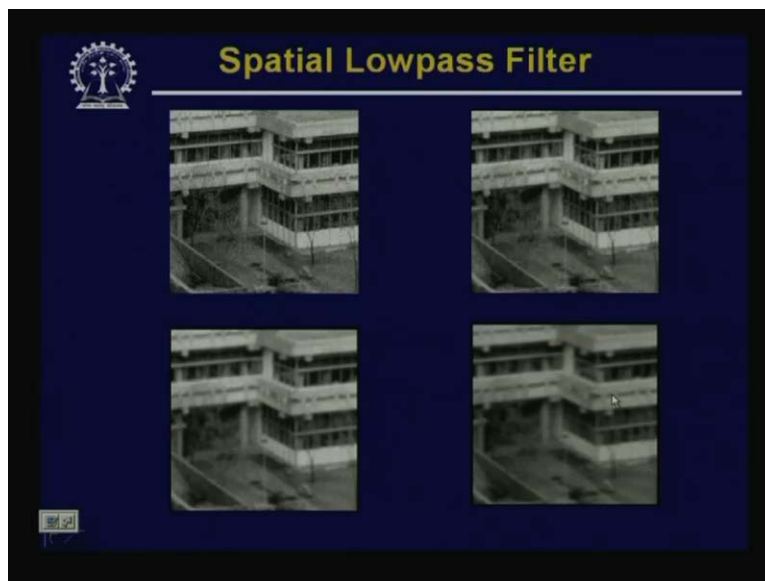


Now, using this kind of mask operation, here we have shown some results. You find that the top left image is noisy image. When you do the masking operation or averaging operation on this noisy image, the right top image shows the averaging with a mask of size 3 by 3, the left bottom image is obtained using a mask of size 5 by 5 and the right bottom image is obtained using a mask of size 7 by 7.

So, from these images, it is quite obvious that as I increase the size of the mask, the blurring effect becomes more and more. So, we find that the right bottom image which is obtained by a mask of size 7 by 7 is much more blurred compared to the other 2 images and this effect is more prominent if you look at the edge regions of this images.

Say, if I compare this particular region with the similar region in the upper image or the similar regions in the original image. You find that here, in the original image that is very very sharp whereas when I do the smoothing using a 7 by 7 mask, it becomes very blurred whereas the blurring effect when I use the 3 by 3 mask is much less. Similar such result is obtained with other images also.

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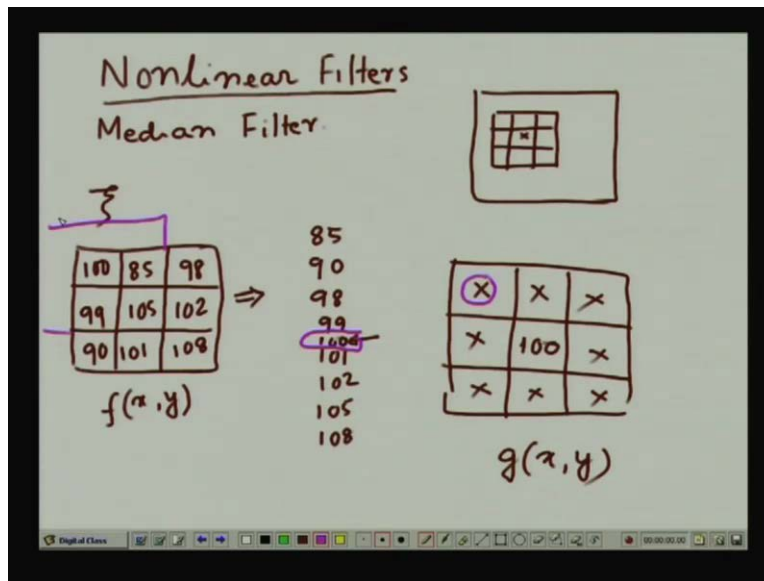


So, here is another image. Again, we do the masking operation or the smoothing operation with different mask sizes. On the top left, we have an original noisy image and the other images are the smoothed images using various mask sizes. So, on the right top, this is obtained using a mask of size 3 by 3, the left bottom is an image obtained using a mask of size 5 by 5 and the right bottom is an image obtained using a mask of size 7 by 7.

So, we find that as we increase the mask size, the reduction is in noise or the noise is reduced to a greater extent but at the cost of addition of blurring effect. So, though the noise is reduced but image becomes very blurred. So, that is the effect of using the box filters or the smoothing filters that though the noise will be removed but the images will be blurred or the sharp contrast in that image will be reduced.

So, there is a second kind of image, second kind of masking operations which are based on order statistics which will reduce this kind of blurring effects. So, let us consider one such filter based on order statistics.

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So, these kind of filters unlike in case of the earlier filters; these filters are nonlinear filters. So, here in case of this order statistic filters; the response is based on the ordering of the intensities, ordering of the pixel values in the neighborhood of the point under consideration. So, what we do is we take the set of intensity values which is in the neighborhood which are in the neighborhood of the point (x, y) , then order all those intensity values in a particular order and based on this ordering, you select a value which will be put at location (x, y) in the processed image g and that is how the output image that you get is a processed image.

But here the processing is done using the order statistics filter. A widely used filter under this order statistics is what is known as a median filter. So in case of a median filter, what we have to do is I have an image and what I do is around point (x, y) , I take a 3 by 3 neighborhood and consider all the 9 pixels, intensity values of all the 9 pixels in this 3 by 3 neighborhood.

Then, I arrange this pixel values, the pixel intensity values in a certain order and take the median of this pixel intensity values. Now, how do you define the median? We define the median say $zeta$ of a set of values such that half of the values in the set will be less than or equal to $zeta$ and the remaining half of the values of the set will be greater than or equal to $zeta$.

So, let us take a particular example. Suppose, I take a 3 by 3 neighborhood around a pixel location (x, y) and the intensity values in this 3 by 3 neighborhood, let us assume that this is 100, this is a 85, this is a 98, this may have a value 99, this may have a value say 105, this may have a value say 102, this may have a value say 90, this may have a value say 101, this may have a value say 108 and suppose this represents a part of my image say $f(x, y)$.

Now, what I do is I take all these pixel values, all this intensity values and put them in ascending order of magnitude. So, if I put them in ascending order of magnitude, you find that the

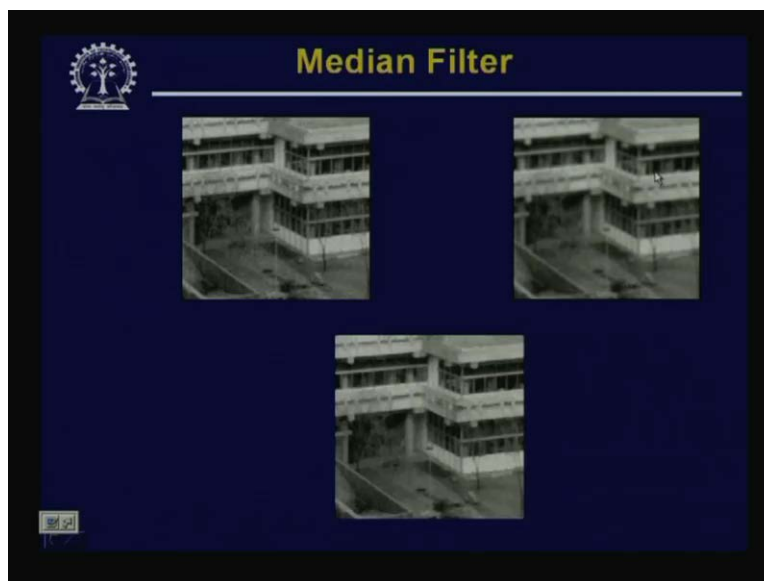
minimum of these values is 85, the next value say 90, and next one is 98, the next one is 99, the next one is 101, the next one is 102, the next one is 105 and the next one is 108. So, all these 9 intensity values, I have put in ascending order of magnitude. Here there will be one more, so there is one more value - 100. So, these are the 9 intensity values which are put in the ascending order of magnitude. So once I put them into ascending order of magnitude, from this I take the fifth maximum value which is equal to 100.

So, if I take the fifth maximum value, you find that there will be equal number of values which is greater than this fifth value; greater than or equal to this fifth number and there will be same number of values which will be less than or equal to this fifth number. So, I consider this particular pixel value 100 and when I generate the image $g(x, y)$, in $g(x, y)$ at location (x, y) , I put this value 100 which is the median of the pixel values within this neighborhood. So, this gives my processed image $g(x, y)$.

Of course, the intensities in other locations in other pixel regions will be decided by the median value of the neighborhood of the corresponding pixels. That is if I want to find out what will be the pixel value at this location, then the neighborhood that I have to consider will be this particular neighborhood.

So, this is how I can get the median filtered output and as you can see that this kind for filtering operation is based on statistics. Now, let us see that what kind of result that we can have using this median filter.

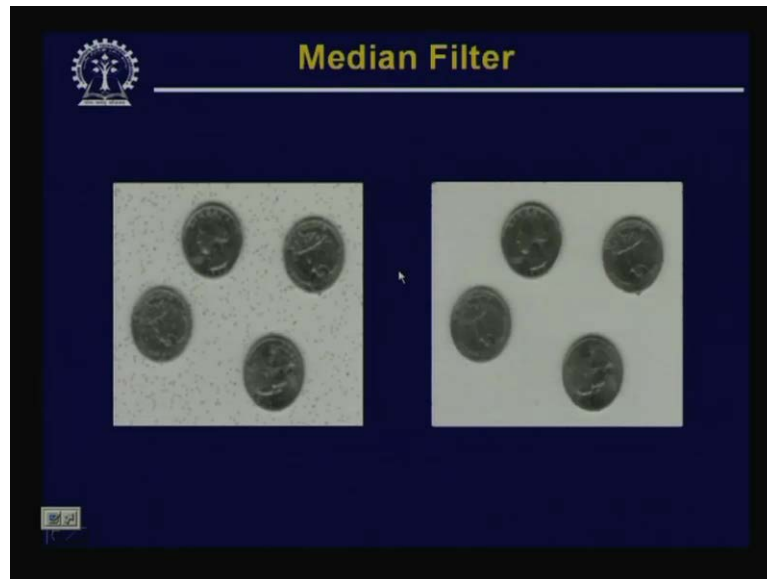
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So here, you find that it is again on the same building image. The left top is our original noised image; on the right hand side, it is the smoothed image using box filter and on the bottom, we have the image using this median filter.

So here again, as you see that **the image obtained using** the processed image obtained using median filter operation, maintains the sharpness of the image to a greater extent than that obtained using the smoothing images.

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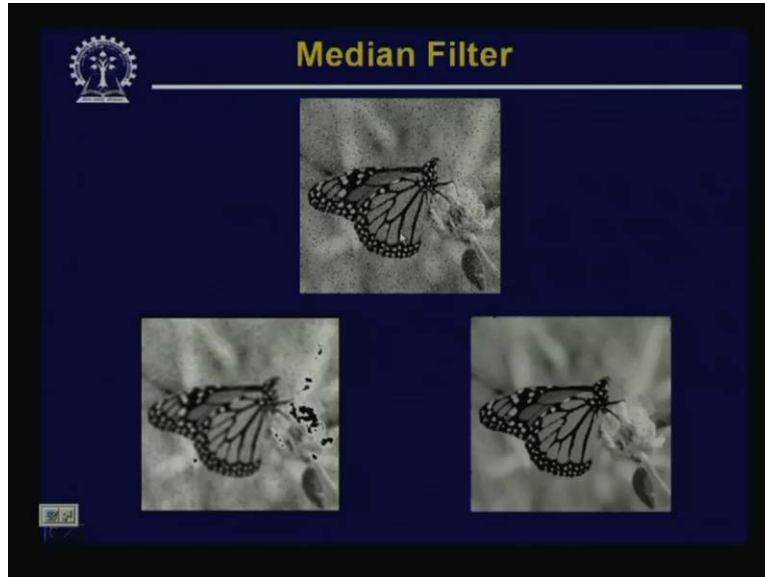


Coming to the second one, again this is one of the images that we have shown earlier a noise image having 4 coins. Here again, you find that after doing the smoothing operation, the edges become blurred and at the same time, the noises are not reduced to a great extent. Still this particular image is noisy.

So if I want to remove all these noise, what I have to do is I have to smooth this image using a higher neighborhood size and the moment I will go for the larger neighborhood size, the blurring effect will be more and more. On the right hand side, the image that we have; so this particular image is also the processed image but here the filtering operation which is done is median filter.

So here, we find that because of the median filtering operation; the output image that we get, the noise in this output image is almost vanished but at the same time the contrast of the image or the sharpness of the image remains more or less intact. So, this is an advantage that you get if we go for median filtering rather than **smooth** smoothing filtering or averaging filtering. To show the advantage of this median filtering, we will take another example.

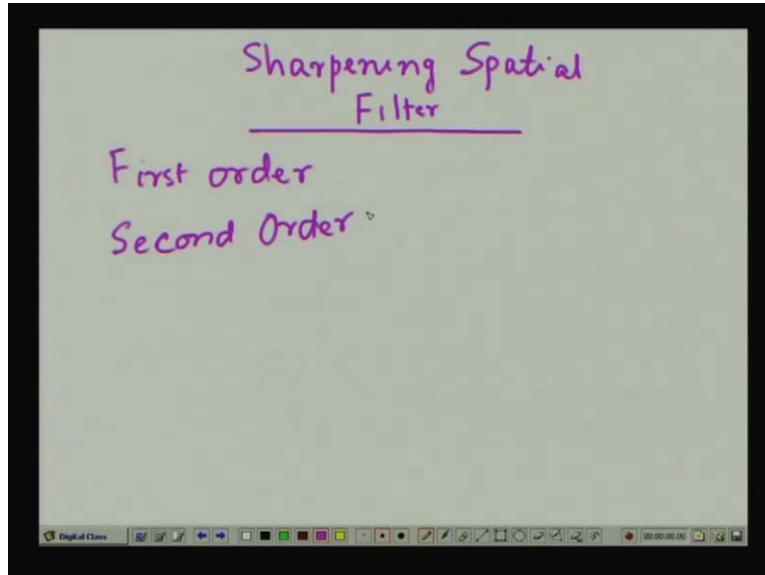
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So, this is the image of the butterfly, a noisy image of a butterfly. On bottom left, the image that is shown, this is an averaged image where the averaging is done over a neighborhood of size 5 by 5. On the bottom right is the image which is filtered by using median filter. So, this particular image clearly shows, this result clearly shows the superiority of the median filtering over the smoothing operation or averaging operation and such median filtering is very very useful for a particular kind of noise where the noise is a random noise which are known as salt and pepper noise because of the appearance of the noise in the image.

So, these are the different filtering operations which reduces the noise in the particular image or the filtering operations which introduce blurring or smoothing over the image. We will now consider another kind of spatial filters which increases the sharpness of the image. So, the spatial filter that we will consider now is called sharpening spatial filter.

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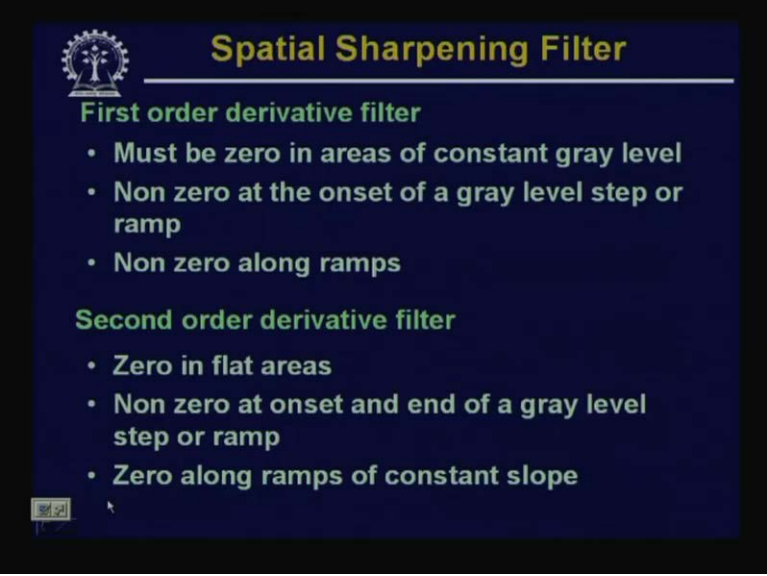


So, we will consider sharpening spatial filter. So, the objective of this sharpening spatial filter is to highlight the details, the intensity details or variation details in an image. Now, through our earlier discussion, we have seen that if I do averaging over an image or smoothing over an image, then the image becomes blurred or the details in the image are removed. Now, this averaging operation is equivalent to integration operation.

So, if I integrate the image, then **what I do is** what I am going to get is a blurring effect or a smoothing effect of the image. So, if integration gives a smoothing effect, so it is quite logical to think that if I do the opposite operation that is instead of integration, if I do differentiation operation, then the sharpness of the image is likely to be increased. So, it is the derivative operations or the differentiations which are used to increase the sharpness of an image.

Now, when I go for the derivative operations, I can use 2 types to derivatives. I can use the first order derivative or I can also use the second order derivative. So, I can either use the first order derivative operation or I can use the second order derivative operation to obtain or to enhance the sharpness of the image. Now, let us see what are the desirable effects that these derivative operations are going to give.

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Spatial Sharpening Filter

First order derivative filter

- Must be zero in areas of constant gray level
- Non zero at the onset of a gray level step or ramp
- Non zero along ramps

Second order derivative filter

- Zero in flat areas
- Non zero at onset and end of a gray level step or ramp
- Zero along ramps of constant slope

If I use a first order derivative operation or a first order derivative filter, then the desirable effect of this first order derivative filter is it must be 0, the response must be 0 in areas of constant grey level in the image and the response must be non zero at the onset of the grey level step or or at the onset of a grey level ramp and it should be non zero along ramps. Whereas, if I use a second order derivative filter; then the second order derivative filter response should be 0 in the flat areas, it should be non zero at the onset and end of a grey level step or grey level ramp and it should be 0 along ramps of constant slope. So, these are the desirable features or the desirable responses of a first order derivative filter and the desirable response of a second order derivative filter.

Now, whichever derivative filter I use; whether it is a first order derivative filter or a second order derivative filter, I have to look for discrete domain formulation of those derivative operations. So, let us see how we can formulate the derivative operations; the first order derivative or the second order derivative in discrete domain.

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The image shows handwritten mathematical derivations on a whiteboard. At the top, the first derivative is defined as $\frac{df(x)}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$. Below this, the first derivative in discrete domain is given as $\frac{\partial f}{\partial x} = f(x+1) - f(x) \Rightarrow 1^{st}$. A red arrow points to the text "2nd Order." which is underlined. Below that, the second derivative is given as $\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$. The whiteboard also shows a toolbar at the bottom with various drawing tools and a timestamp of 00:00:00.

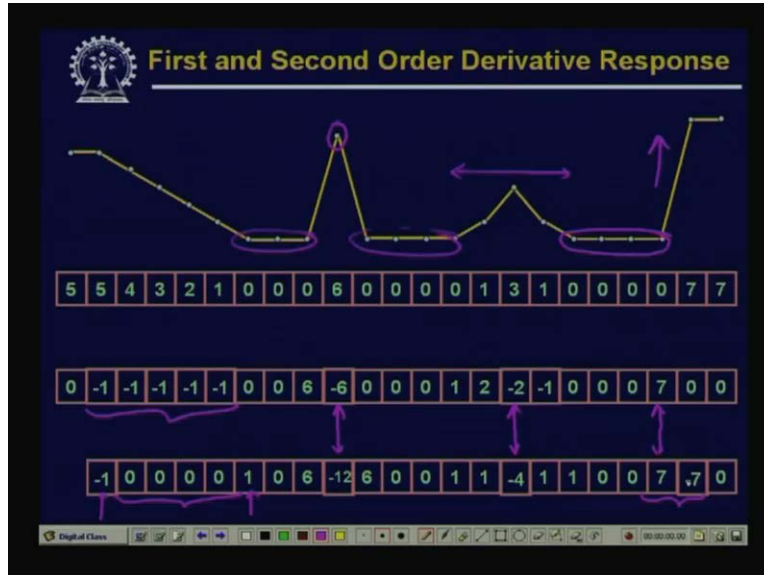
Now, we know that in continuous domain the derivative is given by let us consider a 1 dimensional case that is if I have a function $f(x)$ which is a function of variable x , then I can have the derivative of this which is given by $df(x)/dx$ which is given by limit Δx tends to zero $f(x + \Delta x) - f(x)$ upon Δx . So, this is the definition of derivative in continuous domain.

Now, when I come to discrete domain; in case of our digital images, the digital images are represented by a discrete set of points or pixels which are represented at different grid locations and the minimum distance between 2 pixels is equal to 1.

So, in our case, we will consider the value of Δx equal to 1 and this derivative operation in case of 1 dimension, now reduces to $\frac{\partial f}{\partial x}$ is equal to $f(x+1) - f(x)$. Now, here I use the partial derivative notation because our image is a 2 dimensional image. So, when I take the derivative in 2 dimensions, we will have partial derivatives along x and we will have partial derivatives along y . So, the first derivative, the first order derivative for 1 dimensional discrete signal is given by this particular expression.

Similarly, the second order derivative of a discrete signal in 1 dimension can be approximated by $\frac{\partial^2 f}{\partial x^2}$ which is given by $f(x+1) + f(x-1) - 2f(x)$. So, this is the first order derivative and this is the second order derivative and you find that these 2 derivations, these 2 definitions of the derivative operations, they satisfy the desirable properties that we have discussed earlier. Now, to illustrate the response of these derivative operations, let us take an example.

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So, this is a 1 dimensional signal where the values of the 1 dimensional signals for various values of x are given in the form of an array like this and the plot of these functional values, these discrete values are given on the top. Now, if you take the first order derivative of this as we have just defined, the first order derivative is given in the second array and the second order derivative is given in the third array.

So, if you look at this functional value, the plot of this functional value, this represents various regions, Say for example; here, this part is a flat region, this particular portion is a flat region, this is also a flat region, this is also a flat region. This is a ramp region, this represents an isolated point, this area represents a very thin line and here we have a step kind of discontinuity.

So now, if you compare the response of the first order derivative and the second order derivative of this particular discrete function; you find that the first order derivative is non zero during ramp, whereas the first order derivative is 0 along a ramp. The second order derivative is 0 along a ramp; the second order derivative is non zero at the onset and end of the ramp.

Similarly coming to this isolated point, if I compare the response of the first order derivative and the response of the second order derivative, you find that the response of the second order derivative for an isolated point is much stronger than the response of the first order derivative.

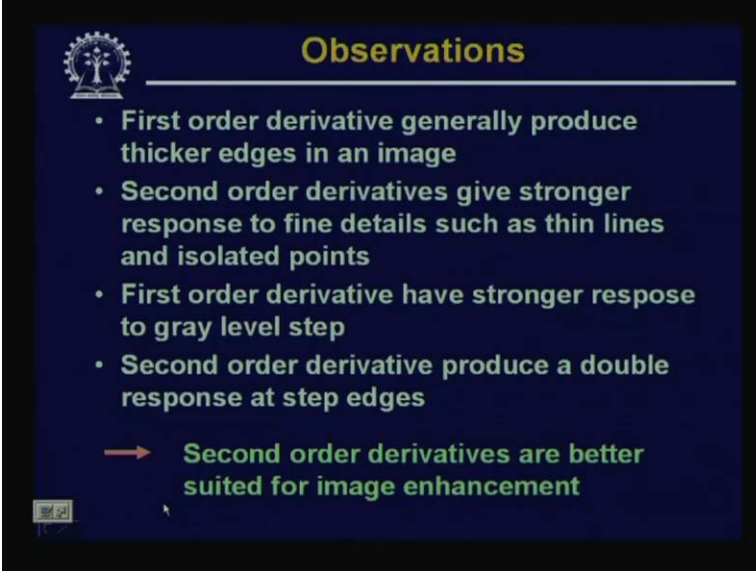
Similar is the case for a thin line. The response of the second order derivative is greater than the response of the first order derivative. Coming to this step edge, the response of the first order derivative and response of the second order derivative is almost same but the difference is in case of second order derivative, I have a transition from a positive polarity to a negative polarity.

Now, because of this transition from positive polarity to negative polarity, the second order derivatives normally leads to double lines **the moment** in case of a step discontinuity in a image

whereas, the first order derivative that leads to a single line. Of course, this double line, getting this double line; usefulness of this, we will discuss later.

Now, but as we have seen, **the first order** the second order derivative gives a stronger response to isolated points or to thin lines and because the details in an image normally has the property that it will be either isolated points or thin lines to which **the second order gives** second order derivative gives a stronger response; so, it is quite natural to think that the second order derivative based operator will be most suitable for image enhancement operations.

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Observations

- First order derivative generally produce thicker edges in an image
- Second order derivatives give stronger response to fine details such as thin lines and isolated points
- First order derivative have stronger response to gray level step
- Second order derivative produce a double response at step edges

→ Second order derivatives are better suited for image enhancement

So, our observation is as we have discussed previously that first order derivative generally produce a thicker edge because we have seen that during a ramp or along a ramp, the first order derivative is non zero whereas, the second order derivative along a ramp is 0 but it gives non zero values at the starting of the ramp and the end of the ramp.

So, that is why the first order derivatives generally produce a thicker edge in an image. The second order derivatives gives stronger response to find it is such as thin lines and isolated points. The first order derivative have stronger response to gray level step and the second order derivative produce a double response at step edges and as we have already said that as the details in the image are either in the form of isolated points or thin lines; so the second order derivatives are better suited for image enhancement operations.

So, we will mainly discuss about the second order derivatives for image enhancement. But to use this for image enhancement operation; obviously, because our images are digital and as we have said in many times that we have to have a discrete formulation of this second order derivative operations and the filter that will design that should be isotropic. That means the response of the second order derivative filter should be independent of the orientation of the discontinuity in the image and the most widely used or the popularly known second order derivative operator of isotropic nature is what is known as a Laplacian operator.

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Laplacian Operator

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$
$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x) \leftarrow$$
$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$
$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

So, we will discuss about the Laplacian operator and as we know that the Laplacian of a function is given by $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$. So, this is the Laplacian operator in continuous domain but what we have to have is the Laplacian operator in the discrete domain. And, as we have already seen that $\frac{\partial^2 f}{\partial x^2}$ in case of discrete domain is approximated as $f(x-1) + f(x+1) - 2f(x)$.

So, this is in case of a 1 dimensional signal. In our case, our function is a 2 dimensional function that is the function of variables x and y . So for this, we can write for 2 dimensional signal $\frac{\partial^2 f}{\partial x^2}$ which will be a simply $f(x+1, y) + f(x-1, y) - 2f(x, y)$. Similarly, $\frac{\partial^2 f}{\partial y^2}$ will be given by $f(x, y+1) + f(x, y-1) - 2f(x, y)$.

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The image shows a handwritten derivation of the discrete Laplacian operator. The first line is $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$. The second line is $= [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] - 4f(x, y)$. The background is a whiteboard with a black border. At the bottom, there is a software interface with various icons and a timestamp of 00:00:00:00.

And, if I add these 2, I get the Laplacian operator in discrete domain which is given by $\text{del}^2 f$ is equal to $\text{del}^2 f \text{ del } x^2$ plus $\text{del}^2 f \text{ del } y^2$ and we will find that this will be given as $f(x \text{ plus } 1, y)$ plus $f(x \text{ minus } 1, y)$ plus $f(x, y \text{ plus } 1)$ plus $f(x, y \text{ minus } 1)$ minus $4f(x, y)$ and this particular operation can be represented again in the form of a 2 dimensional mask. That is for this Laplacian operator, we can have a 2 dimensional mask and the 2 dimensional mask in this particular case will be given like this.

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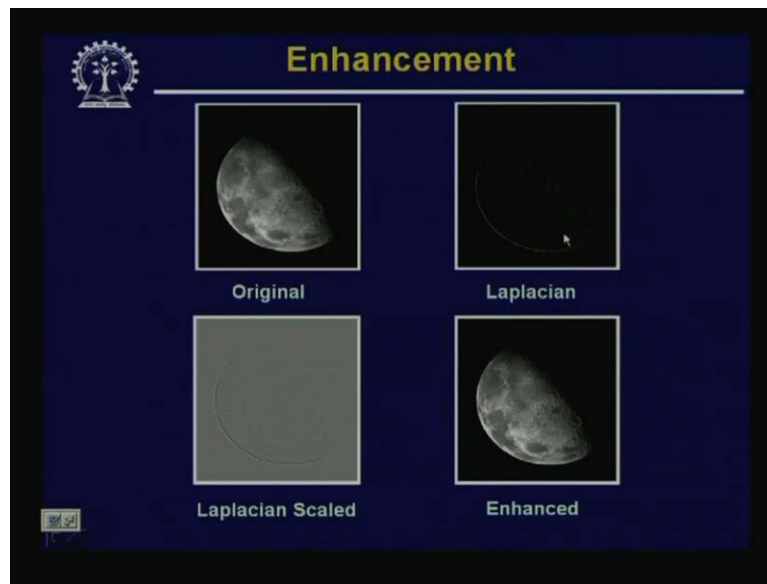
So, on the left hand side, the mask that is shown, this mask considers the Laplacian operation only in the vertical direction and in the horizontal direction and if we also include the diagonal

directions, then the Laplacian mask is given on the right hand side. So, we find that using this particular mask which is shown on the left hand side, I can always derive the expression that we have just shown.

Now, here I can have 2 different types of mask. Depending upon polarity of the coefficient at the center pixel, I can have the center pixel to have a polarity either negative or positive. So, if the polarity is positive, same of the center coefficient; then I can have a mask of this form where the center pixel will have a positive polarity but otherwise the nature of the mask remains the same.

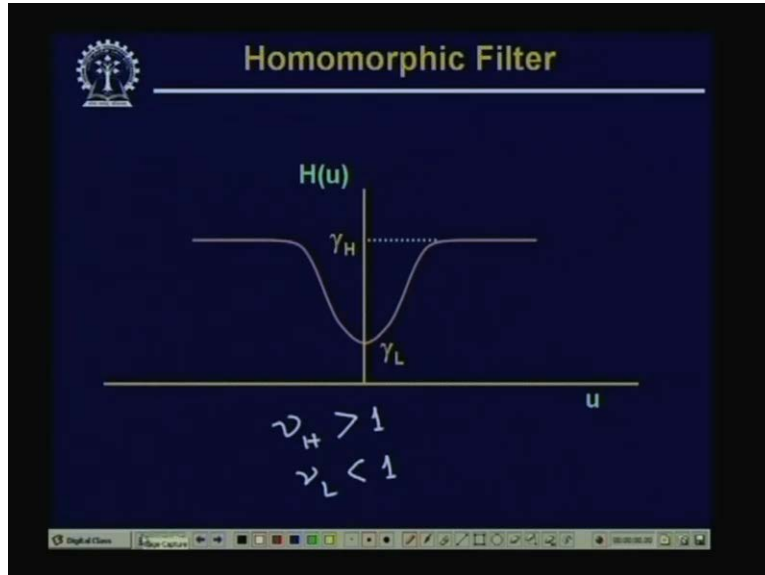
Now, if I have these kinds of operation, then you find that the image that you get **that will have** that will just highlight the discontinuous regions in the image whereas, all the smooth regions in the image will be suppressed. So, this shows an original image.

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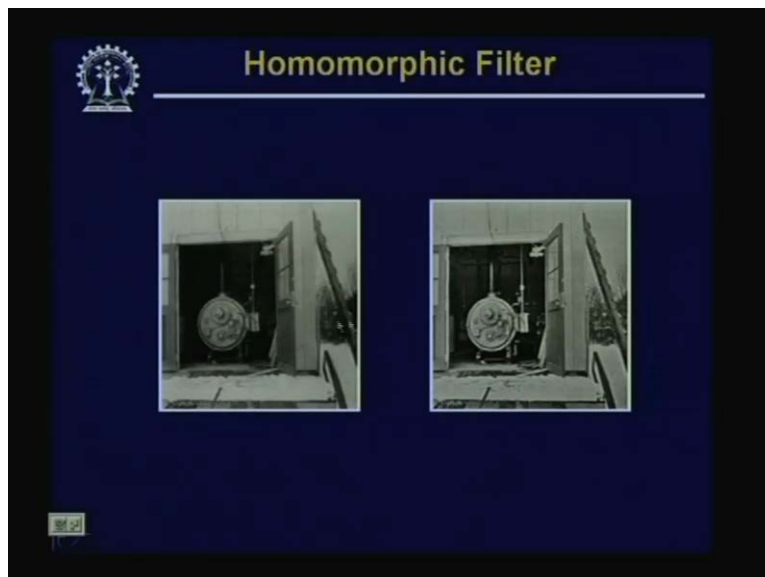
On the right hand side, we have the output of the laplacian and if you closely look at this particular image, you will find that all the discontinuous regions will have some value. However this particular image cannot be displayed properly.

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So, we have to have some scaling operation because I will have say gamma H greater than 1 and gamma L less than 1. This will amplify all the high frequency components that is the contribution of the reflectance and it will attenuate the low frequency components that is contribution due to the illumination. Now, using this type of filtering, the kind of result that we get is something like this.

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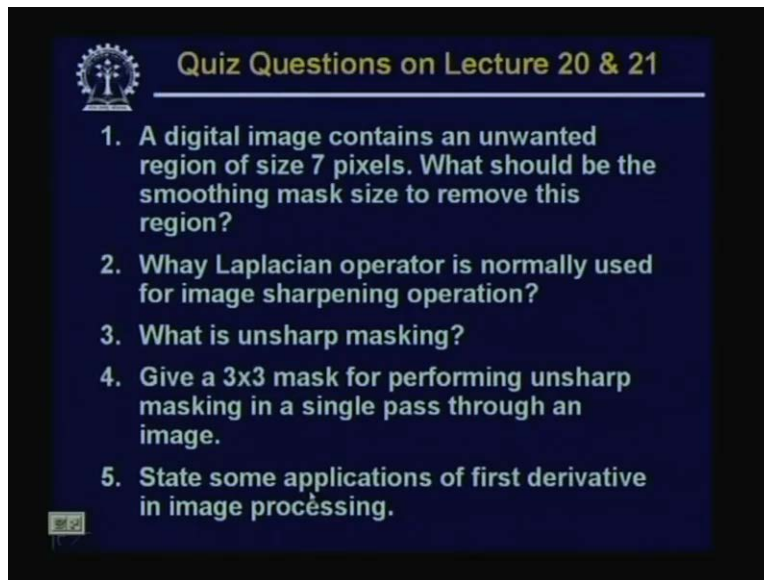


Here, on the left hand side is the original image and on the right hand side is the enhanced image and if you look in the boxes, you find that many of the details in the boxes which are not available in the original image is now available in the enhanced image. So, using such

homomorphic filtering, we can even go for this kind of enhancement or the illumination, the contribution due to illumination will be reduced; so even in the dark areas, we can take out the details.

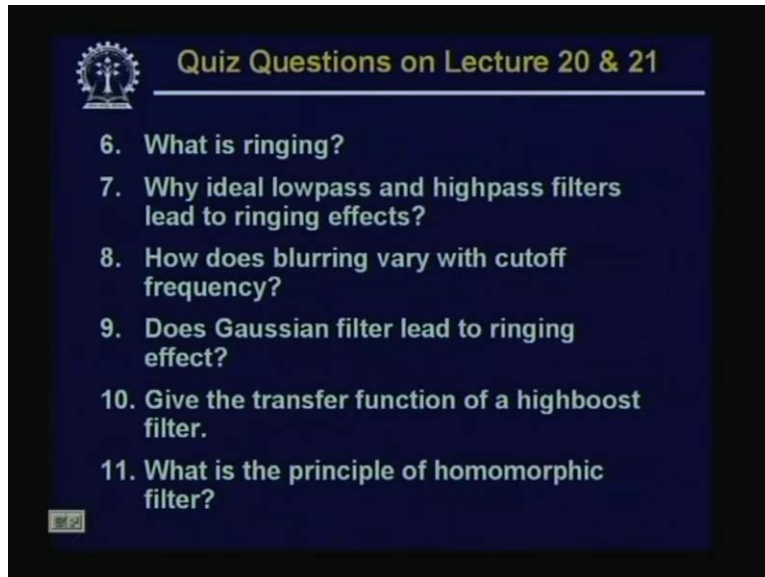
So with this, we come to an end to our discussion on image enhancements. Now, let us go to some questions of our today's lecture.

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The first question is a digital image contains an unwanted region of size 7 pixels. What should be the smoothing mask size to remove this region? Why Laplacian operator is normally used for image sharpening operation? Third question - what is unsharp masking? Fourth question - give a 3 by 3 mask for performing unsharp masking in a single pass through an image. Fifth, state some applications of first derivative in image processing.

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The image shows a slide titled "Quiz Questions on Lecture 20 & 21" with a list of 11 questions. The slide has a dark blue background with white text. A small logo is visible in the top left corner of the slide area.

Quiz Questions on Lecture 20 & 21

6. What is ringing?
7. Why ideal lowpass and highpass filters lead to ringing effects?
8. How does blurring vary with cutoff frequency?
9. Does Gaussian filter lead to ringing effect?
10. Give the transfer function of a highboost filter.
11. What is the principle of homomorphic filter?

Then, what is ringing? Why ideal low pass and high pass filters lead to ringing effects? How does blurring vary with cut off frequency? Does Gaussian filter lead to ringing effect? Give the transfer function filter and what is the principle of homomorphic filter?

Thank you.