Digital Image Processing

Prof. P. K. Biswas

Department of Electronics and Electrical Communication Engineering

Indian Institute of Technology, Kharagpur

Lecture - 2

Image Digitization- I

Hello, welcome to the video lecture series on digital image processing. In our earlier class that is during the introductory lecture on this digital image processing, we have seen the various applications of digital image processing technique.

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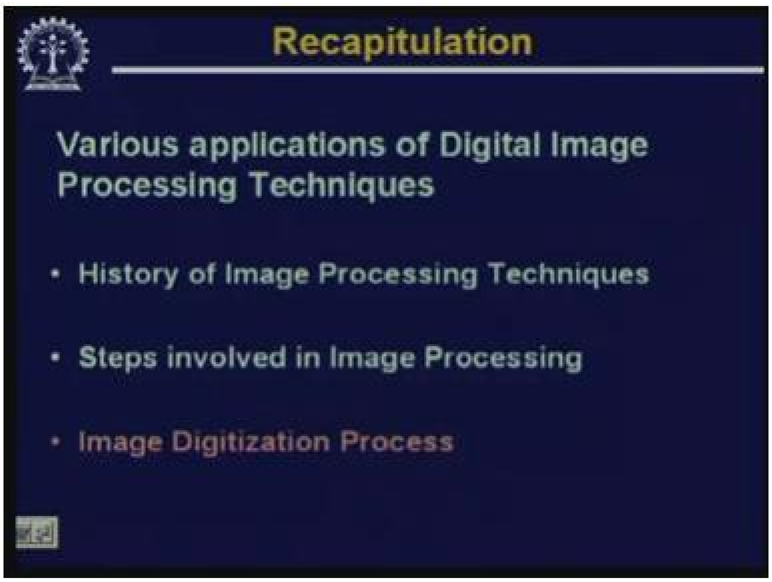

We have also talked about the history of image processing techniques and we have seen that though the digital image processing techniques are very popular and used in wide application areas these days but the digital image processing techniques is quite old.

In fact, we have seen that as early as in 1920's, the digital image processing techniques were been used to transmit the newspaper images from one place to another. After talking about the history, we have also seen the various steps that are involved in image processing techniques and while talking about the various steps, we have seen that the first step that has to be done before any processing can be done on the images is digitization of images.

So, in today's lecture and in the next lecture, we will talk about the digitization process through which an image taken from a camera can be digitized and that digital image can be finally processed by a digital computer. So, in today's lecture, we will talk about digital image digitization techniques.

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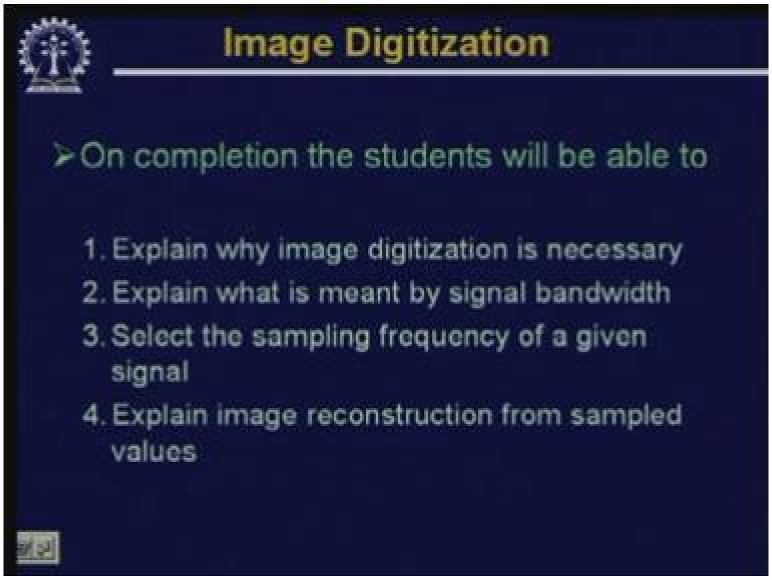

Now, during this course, we will talk about why image digitization is necessary, we will also talk about what is meant by signal bandwidth, we will talk about how to select the sampling frequency of a given signal and we will also see the image reconstruction process from the sampled values.

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So, in today's lecture, we will try to find out the answers to 3 basic questions. The first question is why do we need digitization? Then, we will try to find out the answer to what is meant by digitization and thirdly, we will go to how to digitize an image. So, let us talk about this one after another. Firstly, let us see that why image digitization is necessary.

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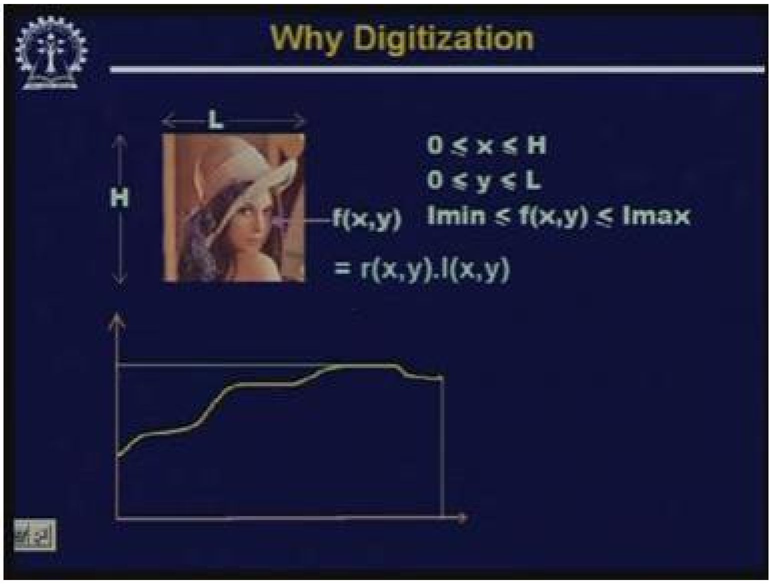

You find that in this slide, we have shown an image, this is the image of a girl and as we have just indicated in our introductory lecture that an image can be viewed as a 2 dimensional function given in the form of f (x, y).

Now, this image has certain length and certain height. The image that has been shown here has a length of L. This L will be in units of distance or units of length. Similarly, the image has a height of H which is also in units of distance or units of length. Any point in this 2 dimensional space will be identify the image coordinates X and Y.

Now, find that conventionally, we have said that X axis is taken as vertically downwards and Y axis is taken as horizontal. So, every coordinate in this 2 dimensional space will have a limit like this. That value of X will vary from 0 to H and value of L value of Y will vary from 0 to L.

Now, if I consider any point X Y in this image, the point X Y or the intensity or the colour value at the point X Y which can be represented as a function of X and Y where X Y identifies a point in the image space that will be actually a multiplication of 2 terms. One is r (x, y) and other one is i (x, y).

We have said during our introductory lecture that this r (x, y) represents the reflectance of the surface point of which these particular image points corresponds to and i (x, y) represents the intensity of the light that is falling on the object surface. Theoretically, this r (x, y) can vary from 0 to 1 and i (x, y) can vary from 0 to infinity.

So, a point f (x, y) in the image can have a value anything between 0 to infinity. But practically, the intensity at a particular point or the colour at a particular point given by X Y that varies from certain minimum which is given by I_{min} and certain maximum I_{max}. So, the intensity at this point X Y that is represented by X Y will vary from minimum intensity value to certain maximum intensity value.

Now, find the second figure in this particular slide. It shows that if I take a horizontal line on this image space and if I plot the intensity values along that line; the intensity profile will be something like this. It again shows that this is the minimum intensity value along that line and this is the maximum intensity value along the line. So, the intensity at any point in the image or intensity along a line; whether it is a horizontal or vertical, can assume any value between the maximum and minimum.

Now, here lies the problem. When we consider a continuous image which can assume any value, intensity can assume any value between certain minimum and certain maximum and the coordinate points X and Y, they can also some value between X can vary from 0 to H, Y can vary from 0 to L.

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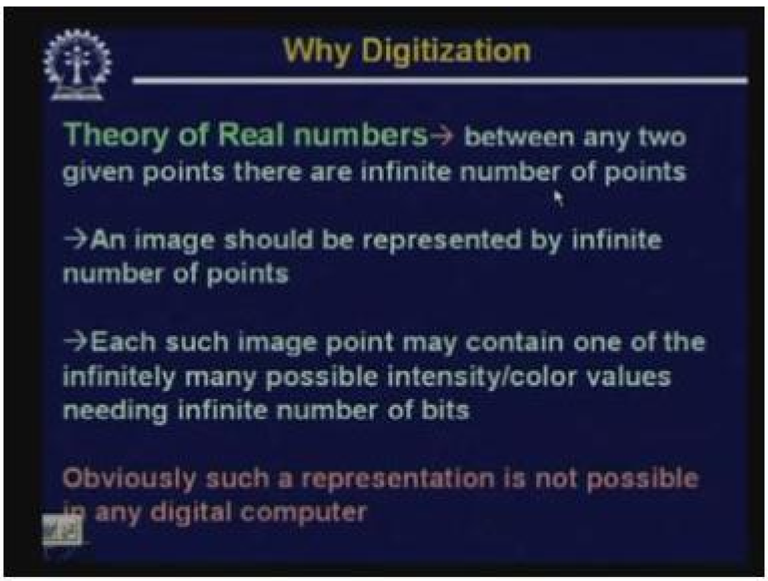

Now, from the theory of real numbers you know that given any 2 point that is between any 2 points, there are infinite numbers of points. So again, when I come to this image as X varies from 0 to H, there can be infinite possible values of X between 0 and H.

Similarly, there can be infinite values of Y between 0 and L. So effectively, that means that if I wants to represent this image in a computer, then this image has to be represented by infinite number of points and secondly when I consider the intensity value at a particular point, we have said that the intensity value f (x, y), it varies between certain minimum I_{min} and certain maximum I_{max}

Again, if I take these 2 - I_{min} and I_{max} to be minimum and maximum intensity values possible but here again the problem is the intensity values, the number of intensity values that can be between minimum and maximum is again infinite in number. So, which again means that if I want to represents an intensity value in a digital computer, then I have to have infinite number of bits to represent an intensity value and obviously such a representation is not possible in any digital computer.

So, naturally, we have to find out a way out. That is our requirement is we have to represent this image in a digital computer, in a digital form. So, what is the way out? In our introductory lecture, if you remember that we have said that instead of considering every possible point in the image space, we will take some discrete set of points and those discrete set of points are decided by grid.

So, if we have a uniform rectangular grid; then at each of the grid locations, we can take a particular point and we will consider the intensity at that particular point. So, this is the process which is known as sampling.

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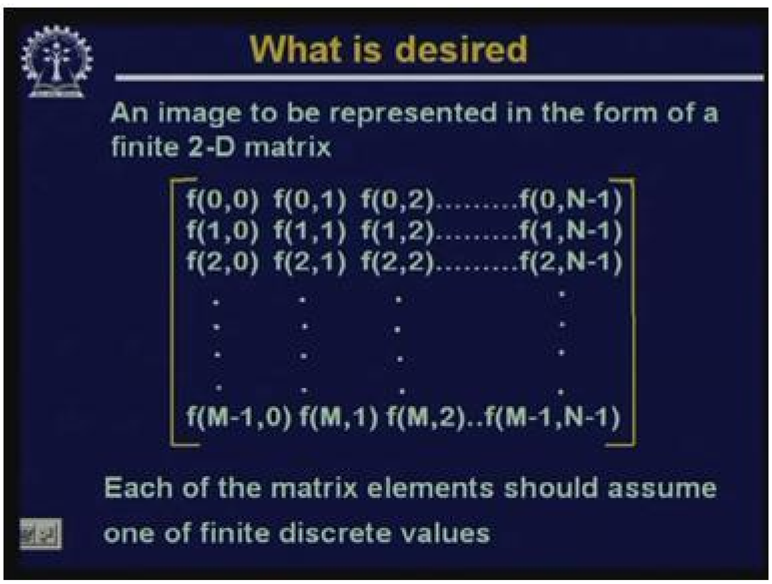

So, what is desired is we want that an image should be represented in the form of a finite 2 dimensional matrix like this. So, this is a matrix representation of an image and this matrix has got finite number of elements. So, if you look at this matrix, you find that this matrix has got M number of rows varying from 0 to M minus 1 and the matrix has got N number of columns varying from 0 to N minus 1.

Typically, for image processing applications, we have mentioned that the dimension is usually taken either as 256 by 256 or 512 by 512 or 1 k by 1 k and so on. But still whatever be the size, the matrix is still finite; we have finite number of rows and we have finite number of columns. So, after sampling what we get is an image in the form of a matrix like this.

Now, the second requirement is if I do not do any other processing on this matrix elements; now what this matrix elements represent? Every matrix element represents an intensity value in the corresponding image location and we have said that these intensity values or the number of intensity values can again be infinite between certain minimum and maximum which is again not possible to be represented in a digital computer.

So, here what we want is each of the matrix elements should also assume one of finite discrete values. So, when I do both of this that is first operation is sampling to represent the image in the form of a finite 2 dimensional matrix and each of the matrix elements again has to be digitized so that the intensity value at a particular element or a particular element in the matrix can assume only values from a finite set of discrete values. These 2 together completes the image digitization process. Now, here is an example.

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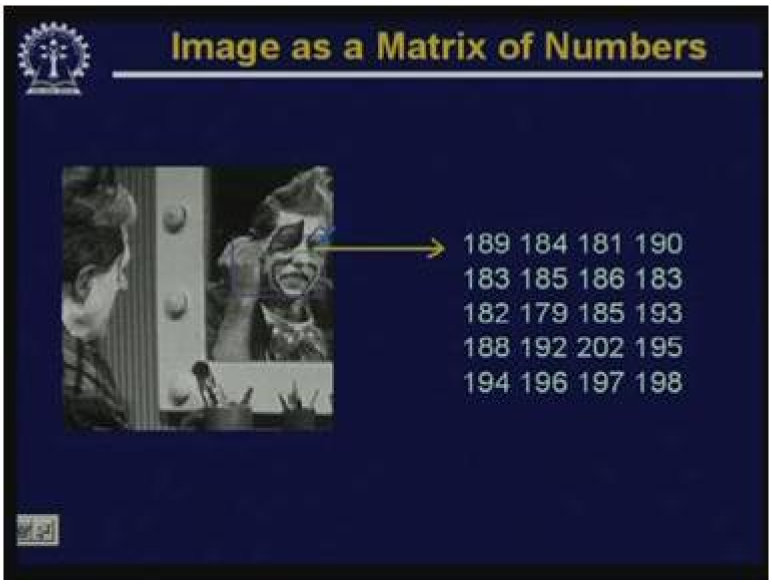

You find that we have shown an image on the left hand side and if I take a small rectangle in this image and try to find out what are the values in that small rectangle; you find that these values are in the form of a finite matrix and every element in this rectangular, in this small rectangle or in the small matrix assumes an integer value. So, an image when it is digitized will be represented in the form of a matrix like this.

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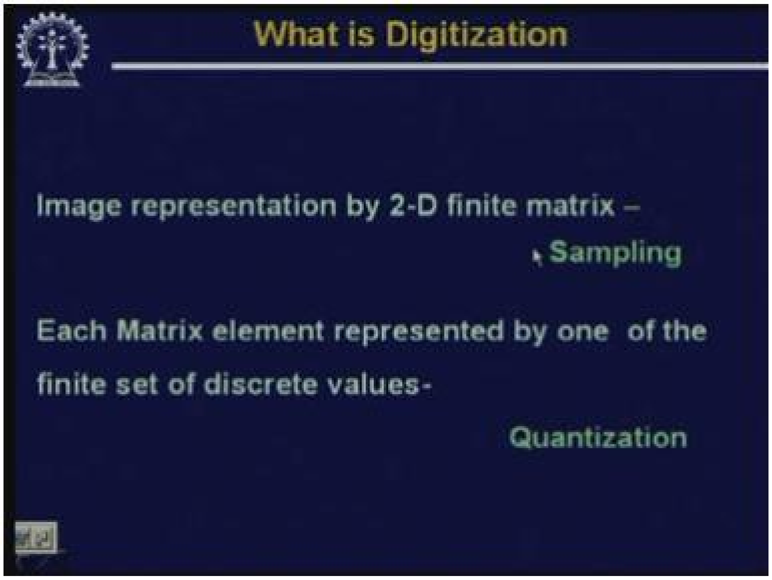

So typically, what we have said till now? It indicates that by digitization what we mean is an image representation by a 2D, 2 dimensional finite matrix; the process known as sampling. And, the second operation is each matrix element must be represented by one of the finite set of discrete values and this is an operation which is called quantization.

In today's lecture, we will mainly concentrate on the sampling and quantization we will talk about later.

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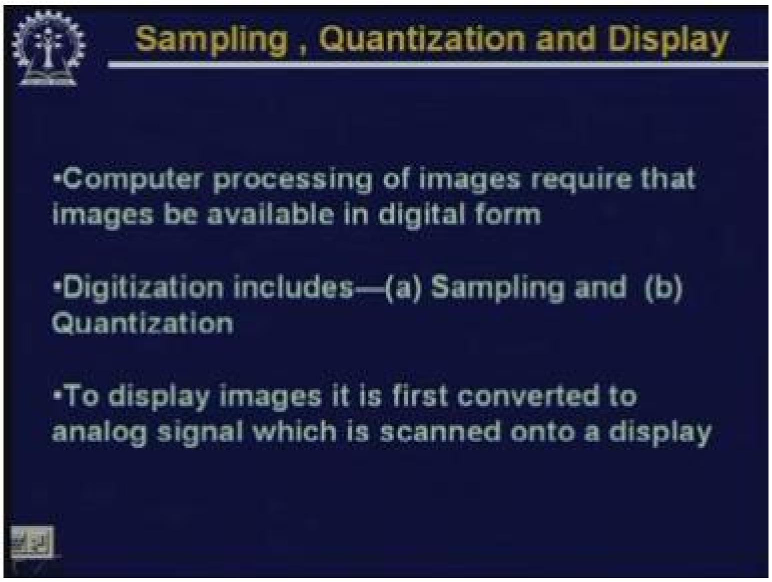

Now, let us see that how what should be the different blocks in an image processing system? Firstly, we have seen that computer processing of images require that images be available in digital form and so we have to digitize the image and the digitization process is a 2 step process.

The first step is sampling and the second step is quantization. Then finally, when we digitize an image processed by computer, then obviously our final aim will be that we want to see that what is the processed output.

So, we have to display the image on a display device. Now, when the image is being processed, the image is in the digital form. But when we want to have the display, we must have the display in the form of analog.

So, whatever process we have done during digitization; during visualization or during display, we must do the reverse process. So, for displaying the images, it has to be first converted into the analog signal which is then displayed on a normal display.

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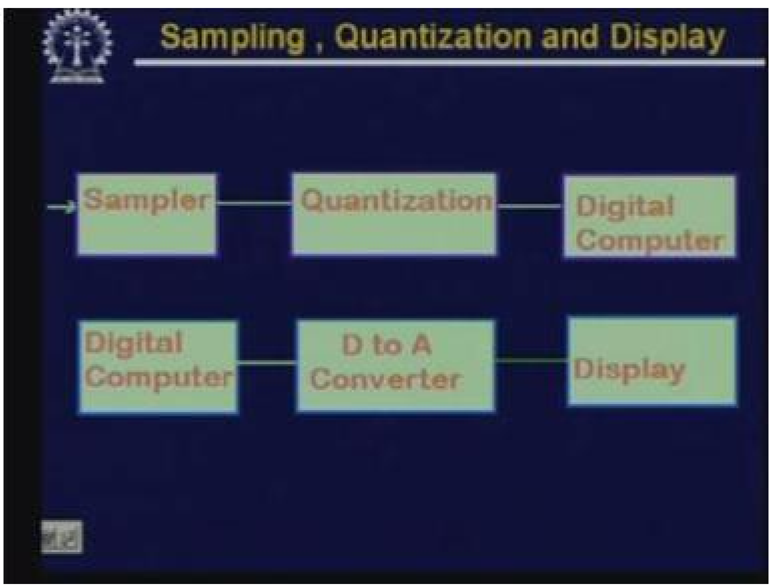

So, if you just look in the form of a block diagram, it appears something like this that while digitization; first we have to sample the image by a unit which is known as sampler, then every sample values we have to digitize - the process known as quantization and after quantization we get a digital image which is processed by the digital computer.

And, when we want to see the processed image, that is how does the image look like after the processing is complete; then for that operation, it is the digital computer which gives the digital output. This digital output goes to D to A converter and finally, the digital to analog converter output is fed to the display and on the display, we can see that how the processed image looks like.

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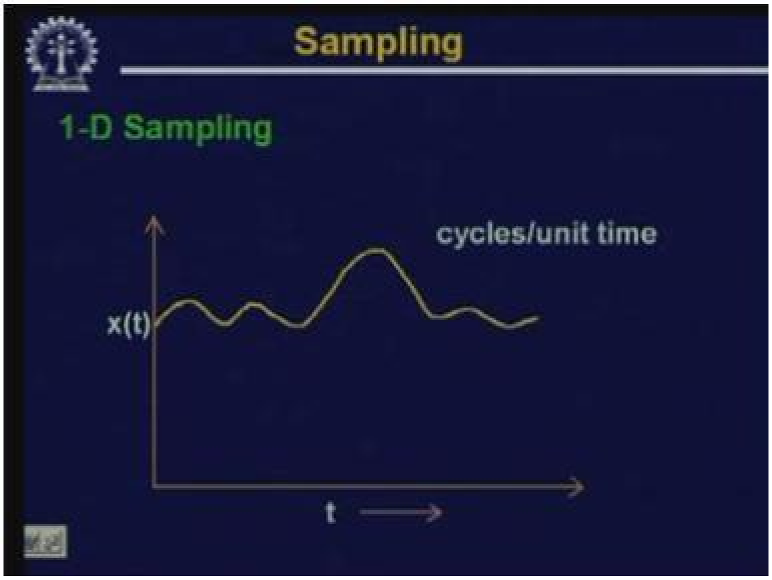

Now, let us come to the first step of the digitization process that is sampling. To understand sampling, before going to the 2 dimensional images, let us take an example from 1 dimension. That is let us assume that we have a 1 dimensional signal x (t) which is a function of t. Here, we assume this t to be time and you know that whenever some signal is represented as a function of time; whatever is the frequency content of the signal that is represented in the form of hertz and this hertz means it is cycles per unit time.

So here again, when you look at this particular signal X (t), you find that this is an analog signal. That is t can assume any value, t is not discretized. Similarly, the functional value X (t) can also assume any value between certain maximum and minimum. So obviously, this is an analog signal and we have seen that an analog signal cannot be represented in a computer.

So, what is the first step that we have to do? As we said that for digitization process, the first operation that you have to do is the sampling operation.

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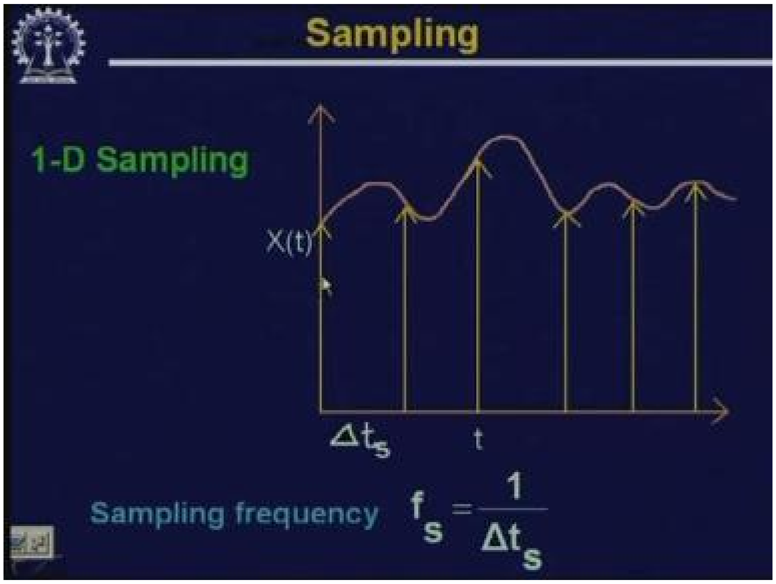

So, for sampling what we do is instead of taking considering the signal values at every possible value of t; what we do is we consider the signal values at certain discrete values of t. So here, in this figure it is shown that we assume the value of the signal X (t) at t equal to 0. We also consider the value of the signal X (t) at t equal to 2 delta t $_S$. Assume the value of signal X (t) at t equal to delta 2 t $_S$, at t equal to delta 3 t $_S$ and so on.

So instead of considering the signal values at every possible instant, we are considering the signal values at some discrete instants of time. So, this is a process known as sampling and here when we are considering the signal values at an interval of delta t $_S$, so we can find out what is the sampling frequency.

So, delta t $_S$ is the sampling interval and corresponding sampling frequency if I represent it by f $_S$, it becomes 1 upon delta t $_S$. Now, when you sample the signal like this, you find that there are many in formations which are being missed. So for example, here we have a local minimum, here we have a local maximum, here again we have a local minimum local maximum, here again we have a local maximum and when we sample at an interval of delta t $_S$, these are the information which cannot be captured by these samples. So, what is the alternative?

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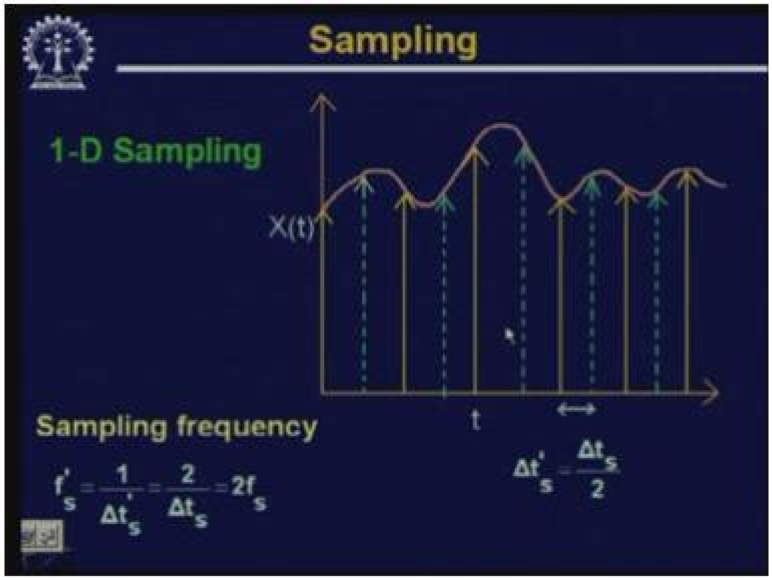

The alternative is; let us increase the sampling frequency or let us decrease the sampling interval. So, if I do that you find that these bold lines, bold golden lines, they represent the earlier samples that we had like this. Whereas, this dotted green lines, they represent the new samples that we want to take and when we take this new samples; what we do is we reduce the sampling interval by half. That is our earlier sampling interval was delta t_S, now I make the new sampling interval which are represented as delta t_S dash which is equal to delta t_S by 2.

And obviously, in this case, the sampling frequency which is f_S dash equal to 1 upon delta t_S dash, now it becomes twice of f_S. That is earlier we had the sampling frequency of f_S, now we have the sampling frequency of delta 2 f_S, twice f_S and when I increase the sampling frequency, you find that with the earlier samples represented by this solid lines, you find that this particular information that is steep in between these 2 solid lines were missed.

Now, when I introduce a new sample in between, then some information of this minimum or of this local maximum can be retained. Similarly here, some information of this local minimum can also to be retained. So obviously, it says that when I increase the sampling frequency or I reduce the sampling interval, then the information that I can maintain in the sampled signal will be more than when the sampling frequency is less.

Now, the question comes whether there is a theoretical background by which we can decide that what is the sampling frequency, proper sampling frequency for certain signals that we can decide. We will come to that a bit later.

Now, let us see that what does the sampling actually mean. We have seen that we have a continuous signal X (t) and for digitization; instead of considering the signal values at every possible value of t, we have consider the signal values at some discrete instants of time t.

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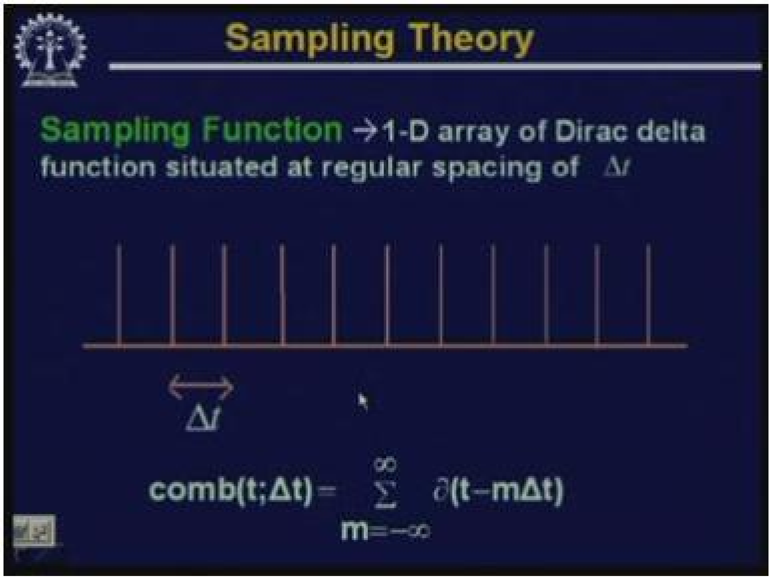

Now, this particular sampling process can be represented mathematically in the form that if I have if I consider that I have a sampling function and this sampling function is a 1 dimensional array of Dirac delta functions which are situated at a regular spacing of delta t. So, this sequence of Dirac delta functions can be represented in this form.

So, you find that each of these are sequences of Dirac delta functions and the spacing between 2 delta functions is delta t. In short, these kind of function is represented by comb function, a comb function t at an interval of delta t and mathematically, this comb function can be represented as delta t minus m into delta t, where m varies from minus infinity to infinity.

Now, this is the Dirac delta function. The Dirac delta function says that if I have a Dirac delta function delta t, then the functional value will be 1 whenever t equal to 0 and the functional value will be 0 for all other values of t. In this case, when I have delta t minus m of delta t, then this functional value will be 1 only when this quantity that is t minus m delta t within the parenthesis becomes equal to 0. That means this functional value will assume a value 1 whenever t is equal to m times delta t for different values of m varying from minus infinity to infinity.

So effectively, this mathematical expression gives rise to a series of Dirac delta functions in this form where at an interval of delta t, I get a value of 1. For all other values of t, I get values of 0.

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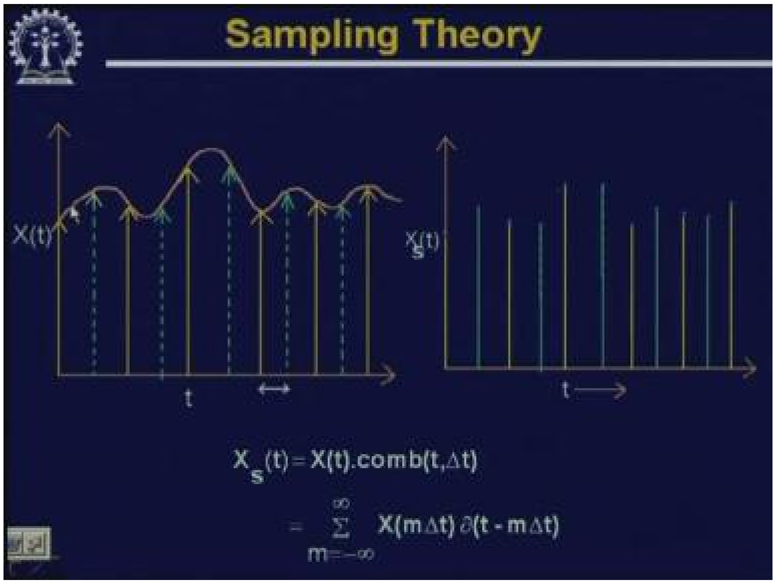

Now this sampling, as you find that we have represented the same figure here, we had this continuous signal X (t), original signal. After sampling, we get a number of samples like this. Now here, these samples can now be represented by multiplication of X (t) with the series of Dirac delta functions that we have seen that is comb of t delta t.

So if I multiply this, whenever this comb function gives me a value 1; only the corresponding value of t will be retained in the product and whenever this comb function gives you a value 0, the corresponding points, the corresponding values of X (t) will be said to 0.

So effectively, this particular sampling when from this analog signal, this continuous signal, we have gone to this discrete signal; this discretization process can be represented mathematically as x_S (t) is equal to X (t) into comb of t delta t and if I expand this comb function and consider only the values of t where this comb function has a value 1, then this mathematical expression is translated to x of m delta t into delta t minus m delta t where m varies from minus infinity to infinity.

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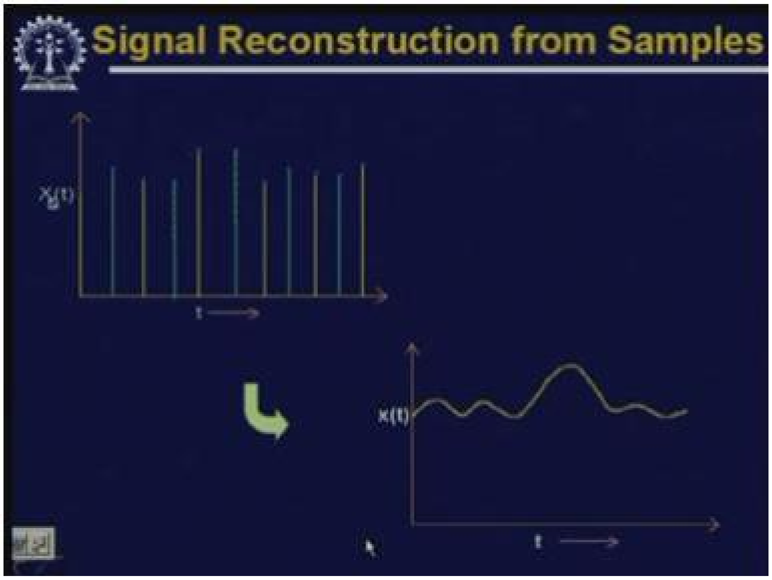

So, after sampling, what you have got is from a continuous signal we have got the sampled signal represented by $x_S(t)$ where the sample values exist at discrete instant of time. Sampling, what we get is a sequence of samples as shown in this figure where $x_S(t)$ has got the signal values at discrete time instants and during the other time intervals, the value of the signal is said to 0.

Now, this sampling will be proper if we are able to reconstruct the original continuous signal X (t) from these sampled values and we will find out that while sampling, we have to maintain certain conditions so that the reconstruction of the analog signal X (t) is possible.

Now, let us look at some mathematical back ground which will help us to find out the conditions which we have to impose for this kind of reconstruction.

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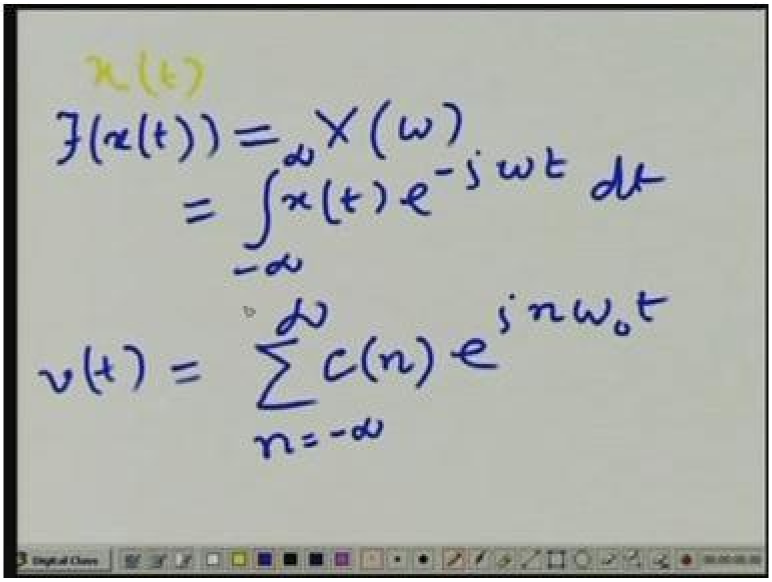

So, here you find that if we have a continuous signal in time which is represented by X (t), then we know that the frequency components of this signal X (t) can be obtained by taking the Fourier transform of this X (t).

So, if I take the Fourier transform of X (t) which is represented by f of X (t) which is also represented in the form of capital X of omega where omega is the frequency component and mathematically, this will be represented as X (t) e to the power minus j omega t dt and we have to take the integrate integration of this from minus infinity to infinity. So, this mathematical expression gives us the frequency components which is obtained by the Fourier transform of the signal X (t).

Now, this is possible if the signal X (t) is aperiodic. But when the signal X (t) is periodic, in that case; the instead of taking Fourier transform, we have to go for Fourier series expansion and the Fourier series expansion of a periodic signal say v (t) where we assume that v (t) is a periodic signal is given by this expression where omega 0 is the fundamental frequency of this signal v (t) and we have to take the summation from n equal to minus infinity to infinity.

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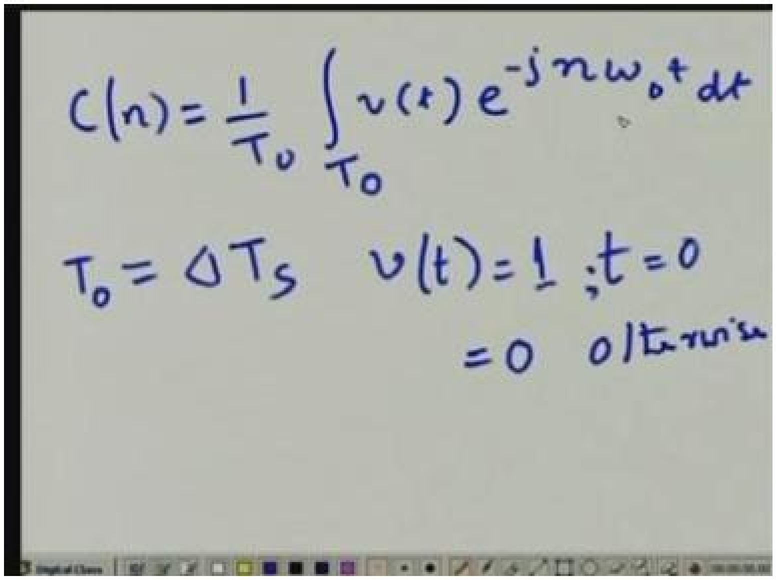

Now, in this case, the C (n) is known as Fourier coefficient. So, n'th Fourier coefficients and the value of C (n) is obtained as C (n) is equal to one upon T_0 v (t) e to the power minus j n omega naught t dt and this integration has to be taken over a period that is T_0.

Now, in our case, when we have v (t) in the form of series of Dirac delta functions, in that case we know that the value of v (t) will be equal to 1 when t equal to 0 and value of v (t) is equal to 0 for any other value of t within a single period. So, in our case T_0 that is the period of this periodic signal is equal to delta T_S because every delta function appears at an interval of delta $T_{S.}$

And, we have v (t) is equal to 1 for t is equal to 0 and v (t) is equal to 0 otherwise.

Now, if I impose this condition to calculate the value of C (n); in that case, we will find that the value of this integral will exist only at t equal to 0 and it will be 0 for any other value of t.

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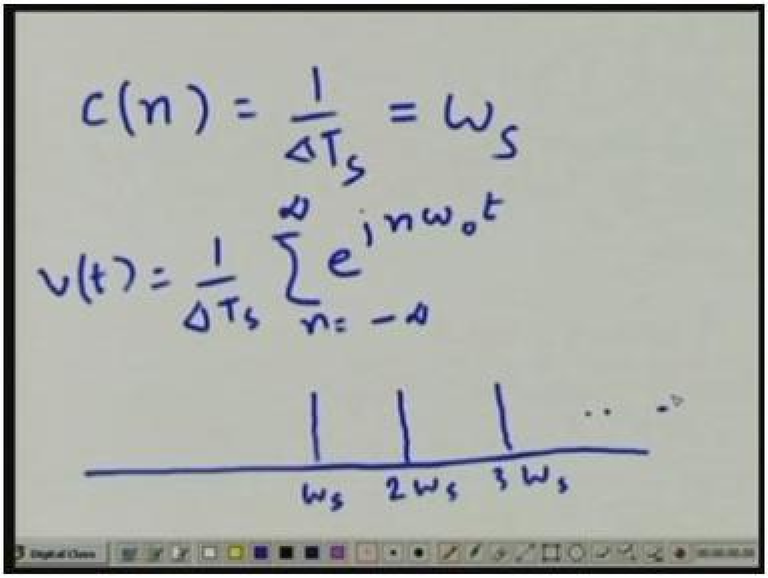

So by this, we find that C (n) now becomes equal to 1 upon delta t $_S$ and this 1 upon delta t $_S$ is nothing but the sampling frequency we will put as say omega s. So, this is the frequency of the sampling signal.

Now, with this value of C (n), now the periodic signal v (t) can be represented as 1 upon delta t $_s$ summation of e to the power j n omega naught t for n equal to minus infinity to infinity. So, what does it mean? This means that if I take the Fourier series expansion of our periodic signal which is in our case Dirac delta function; this will have frequency components, various frequency components where the fundamental components of the frequency is omega naught and it will have other frequency components of twice omega naught, thrice omega naught, 4 times omega naught and so on.

So, if I plot those frequencies or frequency spectrum, we find that we will have the fundamental frequency omega naught or in this case this omega naught is nothing but same as the sampling frequency that is omega s, we will also have a frequency component of twice omega s, we will also have a frequency component of thrice omega s, and this continues like this.

So, you find that the comb function as the sampling function that we have taken, the Fourier series expansion of that is again a comb function.

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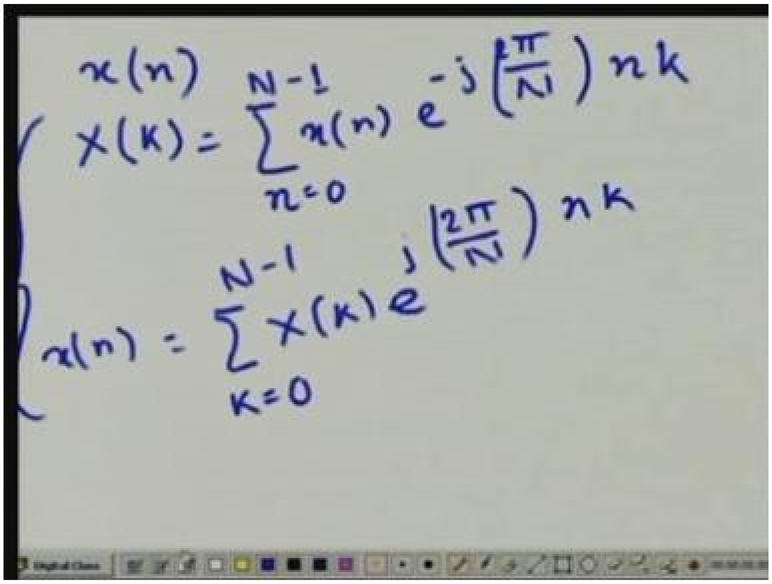

Now, this is about the continuous domain. When we go to discrete domain; in that case, for a discrete time signal say x (n) where n is the nth sample of the signal x, the Fourier transform of this is given by X (k) is equal to sum of x (n) e to the power minus j 2 pi by N into n k where value of n varies from 0 to N minus 1, where this capital N indicates that the number of samples that we have for which we are taking the Fourier transform.

And, given this Fourier transform, we can find out the original sampled signal by the inverse Fourier transformation which is obtained as x (n) is equal to sum of X (k) e to the power j 2 pi by N n k and this time the summation has to be taken over k for k equal to 0 to N minus 1.

So, you find that we get a Fourier transform pair. In one case, from the discrete time signal, we get the frequency components, discrete frequency components by the forward Fourier transform and in the second case, from the frequency components, we get the discrete time signal by the inverse Fourier transform and these 2 equations taken together forms a Fourier transform pair. Now, let us go to another concept, a concept called convolution.

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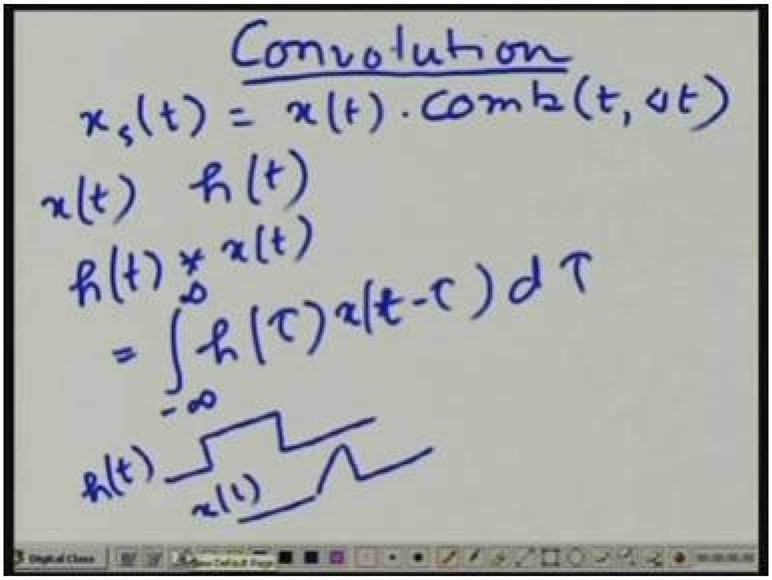

You find that we have represented our sampled signal as x $_S$ (t) is equal to X (t) multiplied by comb function t delta t. So, what we are doing is we are taking 2 signals in time domain and we are multiplying these 2 signals. Now, what will happen if we take the Fourier transform of these 2 signals?

Or let us put it like this, I have 2 signals X (t) and I have another signal say h (t). Both these signals are in the time domain. We define an operation called convolution which is defined as x h (t) convolution with X (t). This convolution operation is represented as h of tau x of t minus tau d tau. Integration is taken over tau from minus infinity to infinity. Now, what does it mean?

This means that whenever we want to take the convolution of 2 signals h (t) and X (t); then firstly what we are doing is we are time inverting the signal X (t). So, instead of taking x tau we are taking x of minus tau. So, if I have 2 signals of this form say h (t) is represented like this and we have a signal say X (t) which is represented like this; then what we have to do is as our expression says that the convolution of h (t) X (t) is nothing but h tau X (t) minus tau d tau integration over minus infinity to infinity and h (t) is like this and X (t) is like this. This is h (t) and this is X (t).

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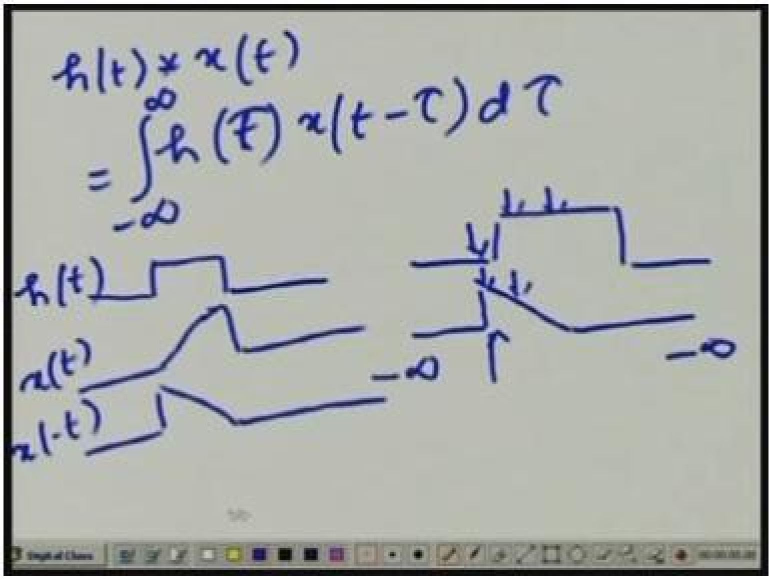

Then what we have to do is for convolution purpose, we are taking h of tau and x of minus tau. So, if I take x of minus t, this function will be like this. So, this is x of minus t and for this integration, we have to take h of tau for a value of tau and x of minus tau that has to be translated by this value t and then the corresponding values of h and x have to be multiplied and then we have to take the integration from minus infinity to infinity.

So, if I take an instance like this, so at this point I want to find out what is the convolution value. Then I have to multiply the corresponding values of h with these values of x. Each and every time instants, I have to do the multiplication, then I have to integrate from minus infinity to infinity. I will come to the application of this a bit later.

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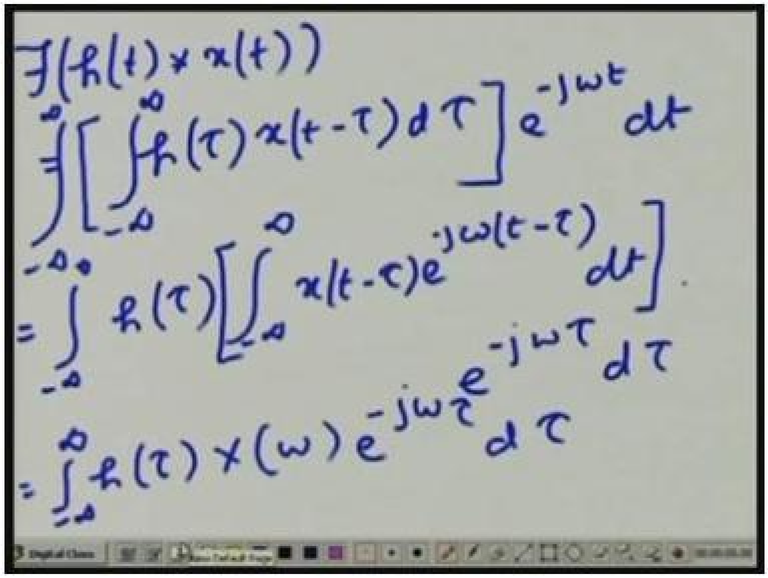

Now, let us see that if we have a convoluted signal, say we have h (t) which is convoluted with X (t) and if I want to take the Fourier transform of this signal, then what will get? The Fourier transform of this will be represented as h tau x of t minus tau d tau. So, this is the convolution. Integration over tau from minus infinity to infinity and then for the Fourier transform, I have to do e to the power minus j omega t dt and then again, I have to take the integral from minus infinity to infinity. So, this is the Fourier transform of the convolution of those 2 signals h (t) and X (t).

Now, if you do this integration, you find that this same integration can be written in this form. I can take out h tau out of the inner integral; the inner integral I can represent as x of t minus tau e to the power minus j omega t minus tau dt. So, I can put this as the inner integral, then I have to multiply this whole term by e to the power minus j omega tau d tau and then this integration will be from tau equal to minus infinity to infinity.

Now, find that what does this inner integral mean? From the definition of Fourier transform, this inner integral is nothing but the Fourier transform of X (t). So, this expression is equivalent to h of tau x of omega e to the power minus j omega tau d tau where this integration will be taken over tau from minus infinity to infinity.

Now, what I can do is because this x omega is independent of tau, so I can take out this x omega from this integral.

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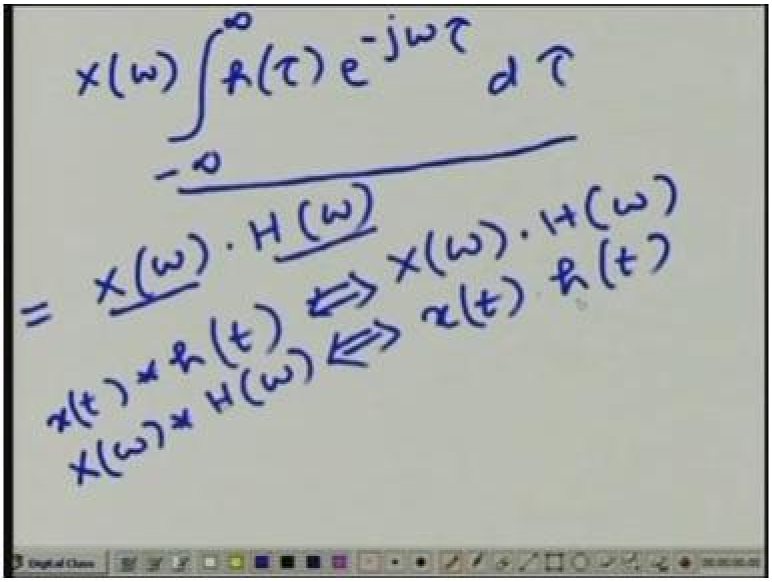

So, my expression will now be x omega, then within the integral, I have h of tau e to the power minus j omega tau d tau where the integration is taken over tau from minus infinity to infinity. Again, you find that from the definition of Fourier transformation, this is nothing but the Fourier transformation of the time signal h (t).

So effectively, this expression comes out to be x of omega into h of omega where x of omega is the Fourier transform of the signal X (t) and h of omega is the Fourier transform of the signal h (t). So effectively, this means that if I take the convolution of 2 signals X (t) and h (t) in time domain, this is equivalent to multiplication of the 2 signals in the frequency domain. So, convolution of the 2 signals X (t) and h (t) in the time domain is equivalent to multiplication of the same signals in the frequency domain. The reverse is also true.

That is if we take the convolution of x omega and h omega in the frequency domain, this will be equivalent to multiplication of X (t) and h (t) in the time domain. So, both these relations are true and we will apply these relations to find out that how the signal can be reconstructed from its sample values.

So, now let us come back to our original signal. So here, we have seen that we have been given this sample values and from the sample values our aim is to reconstruct this continuous signal X (t).

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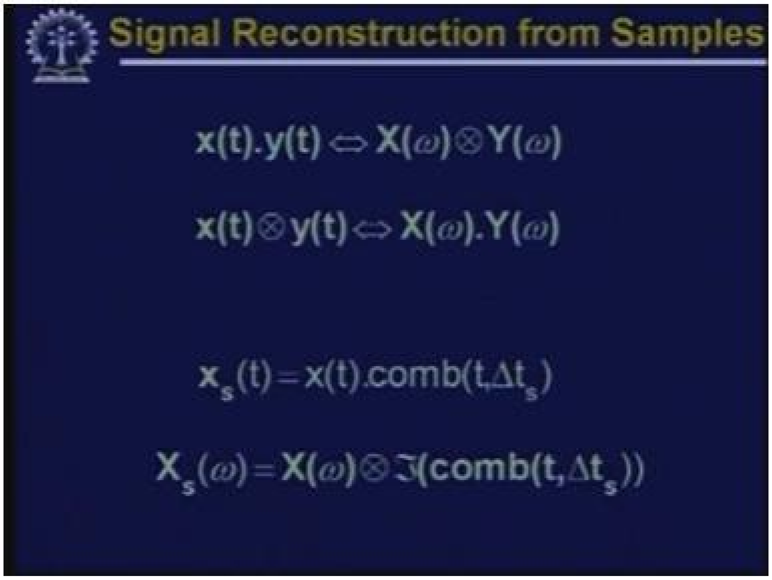

And, we have seen that this sampling is actually equivalent to multiplication of 2 signals in the time domain. One signal is X (t), the other signal is comb function, comb of t delta t. So, these relations, as we have said that these are true that if I multiply 2 signals X (t) and y (t) in time domain that is equivalents to convolution of the 2 signals x omega and y omega in the frequency domain.

Similarly, if I take the convolution of 2 signals in time domain, that is equivalent to multiplication of the same signals in frequency domain. So, for sampling when we have said that we have got x_S (t) that is the sampled values of the signal X (t) which is nothing but multiplication of X (t) with the series of Dirac delta functions represented by comb of t delta t. So, that will be equivalent to in frequency domain, I can find out x_S of omega which is equivalent to the frequency domain representation x omega of the signal X (t) convoluted with the frequency domain representation of the comb function, comb t delta t and we have seen that this comb function, the Fourier transform or the Fourier series expansion of this comb function is again a comb function.

So, what we have is we have a signal x omega, we have another comb function in the frequency domain and we have to take the convolution of these 2.

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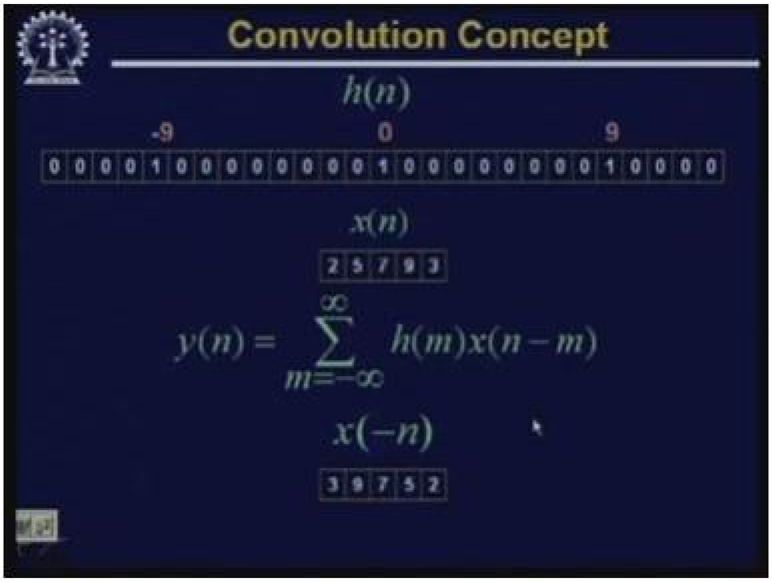

Now, let us see this convolution in details. What does this convolution actually mean? Here we have taken 2 signals h (n) and x (n). Both of them, for this purpose are in the sample domain. So, h (n) is represented by this and x (n) is represented by this.

You find that this h (n) is actually nothing but a comb function where the delta t_S in this case we have value of h (n) is equal to 1 at n equal to 0, we have a value of h (n) equal to 1 at n equal to minus 1, we have value of h (n) equal to 1 at n equal to minus 9, we have a value of h (n) equal to 1 at n equal to plus 9 and these things repeats.

So, this is nothing but representation of a comb function and if I assume that my x (n) is of this form that is at n equal to 0; value of x (n) is equal to 7, x minus 1 that is at n equal to minus 1 it is 5, n minus 1 minus 2 it is equal to 2.

Similarly on this side, for n equal to 1, x (1) equal to 9 and x (2) equal to 3 and the convolution expression that we have said in the continuous domain, in discrete data domain, the convolution expression is translated to this form. That is y (n) equal to h (m) into x (n) minus m where m varies from minus infinity to infinity. So, let us see that how this convolution actually takes place.

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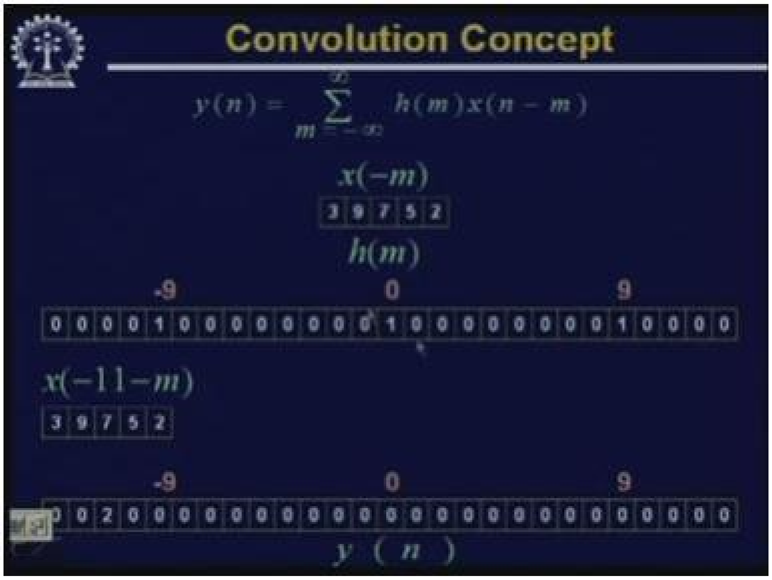

So, if I really understand this particular expression that h (m) x of n minus m, sum of this from m equal to minus infinity to infinity; we said that this actually means that we have to take the time inversion of the signal x (n).

So, if I take the time inversion, the signal will be something like this – 3, 9, 7, 5 and 2 and when I take the convolution that is I want to find the various value of y (n), that particular expression can be computed in this form.

So, if I want to take the value of y minus 11; so what I have to do is I have to give a translation of minus 11 to this particular signal x of minus m. So, it comes here, then I have to take the summation of this product from m equal to minus infinity to infinity. So, here what these do?

You find that I do point by point multiplication of these signals, so here 0 multiplied with 3 plus it will be 0 multiplied with 9 plus 0 multiplied with 7 plus 0 multiplied with 5 plus 1 multiplied with 2. So, the value that I can get is 2 and this 2 comes at this location y of minus 11.

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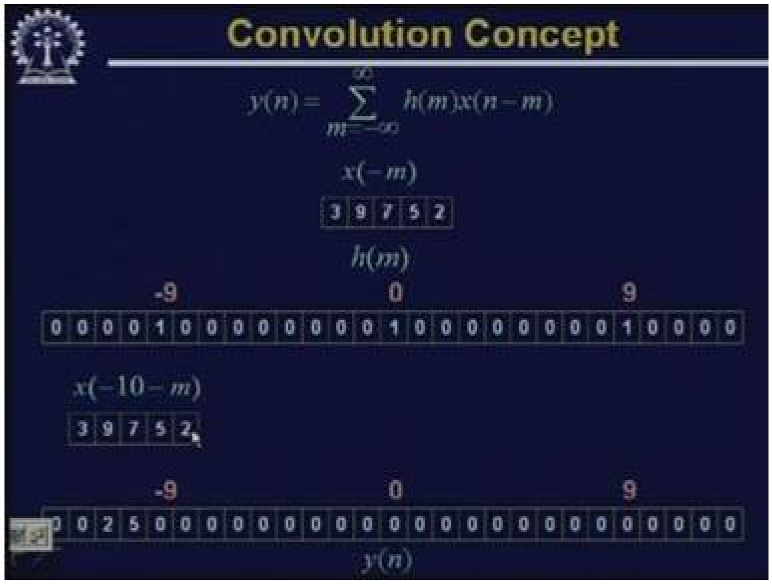

Now, for getting the value of y of minus 10, again I do the same computation and here you find that this 1 gets multiplied with 5 and all other values gets multiplied with 0 and when you take the summation of all of them, I get 5 here.

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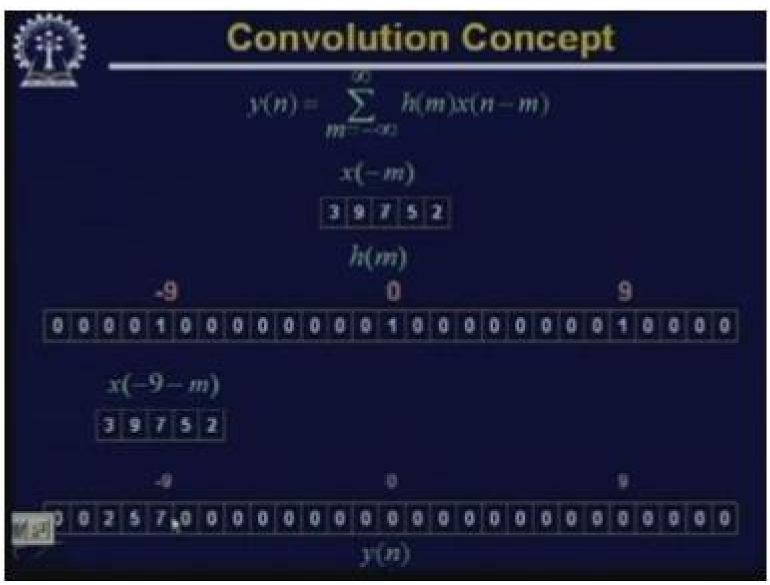

Then I get value at minus 10, I get 7 here following the same operation. Sorry, this is at minus 9.

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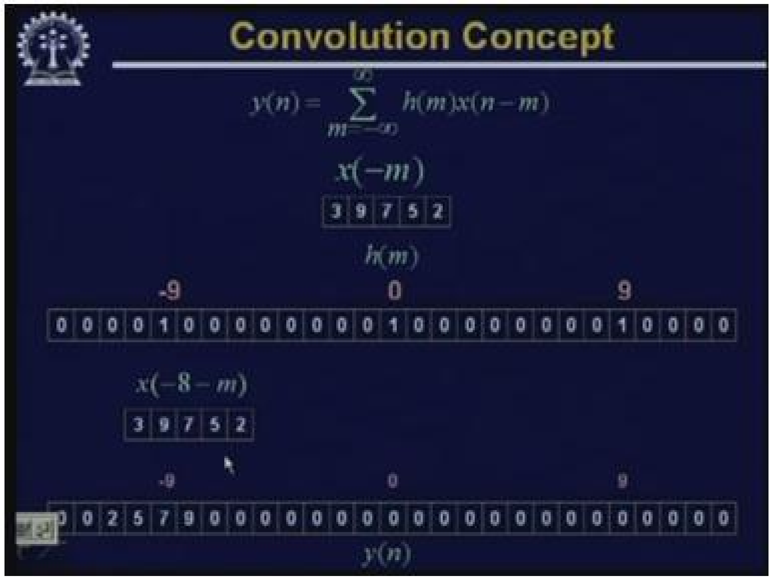

I get at minus 8.

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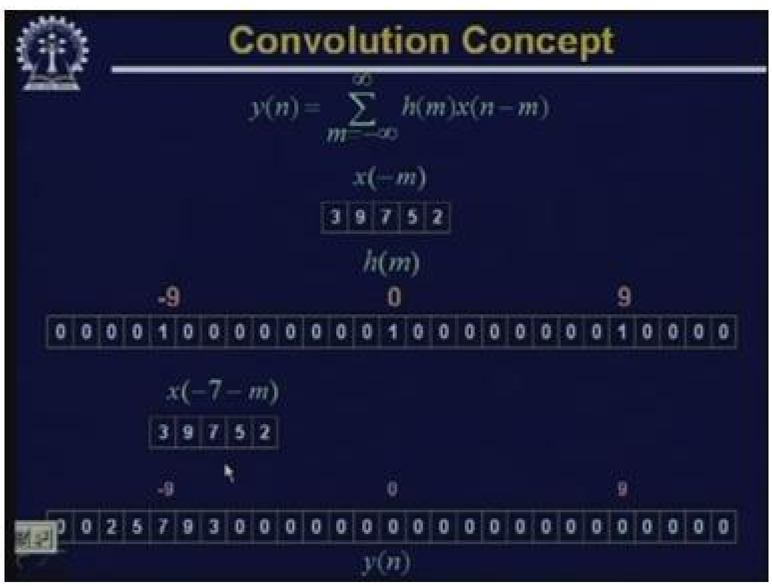

I get at minus 7.

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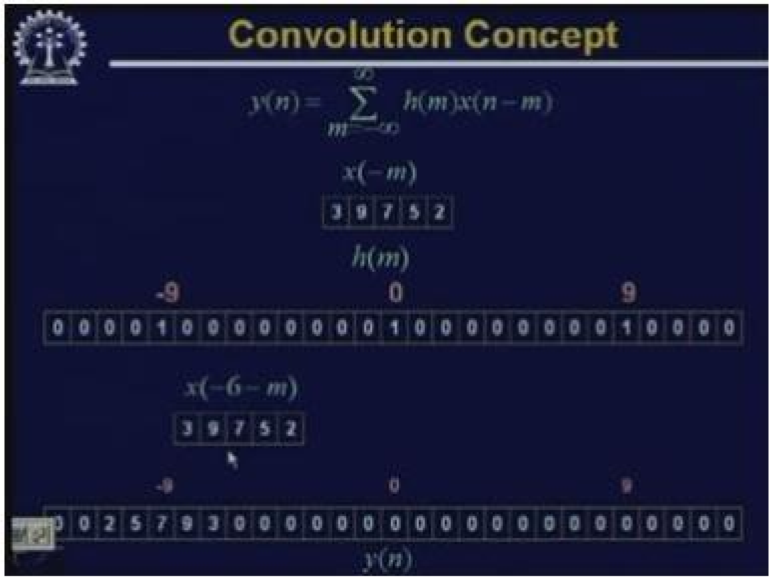

I get at minus 6. At minus 6, you find that the value is 0.

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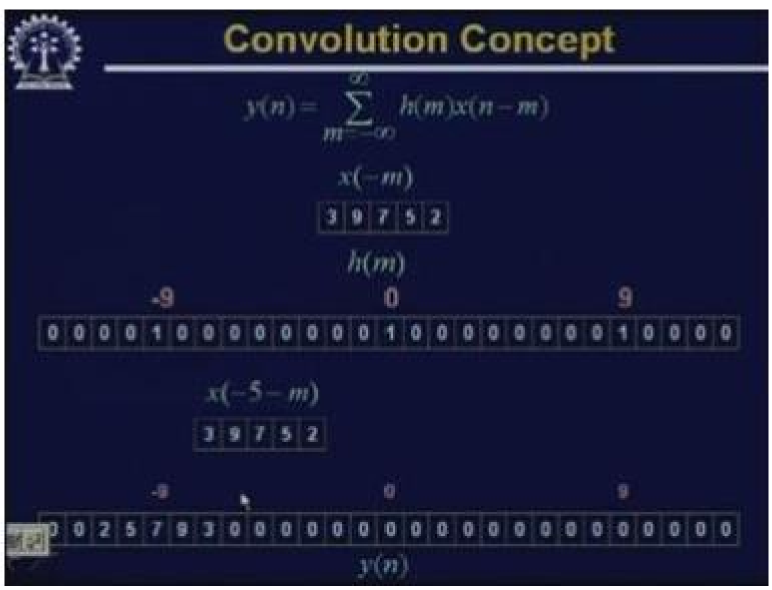

If I continue like this, here again at n equal to minus 2, I get value equal to 2.

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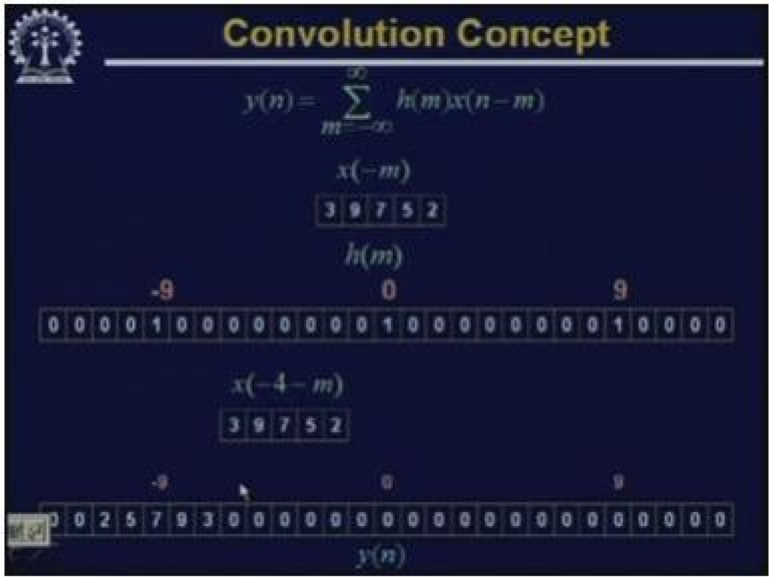

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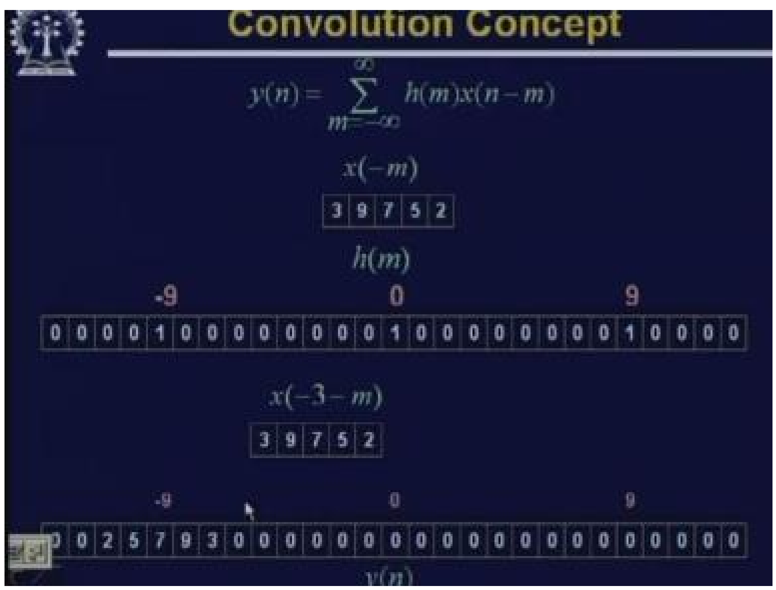

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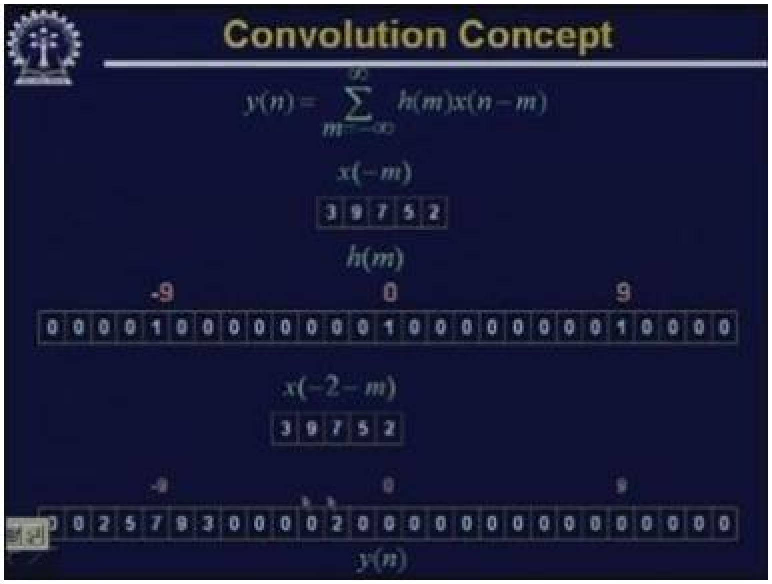

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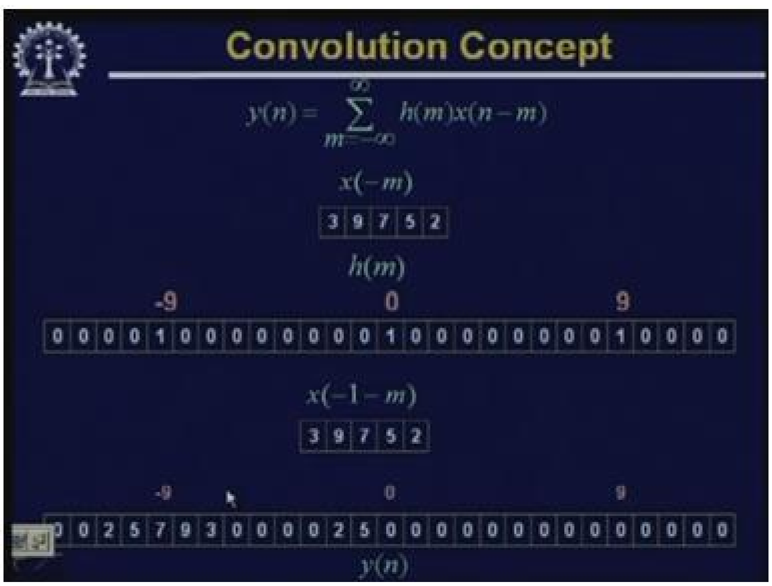

At n equal to minus 1, I get value equal to 5.

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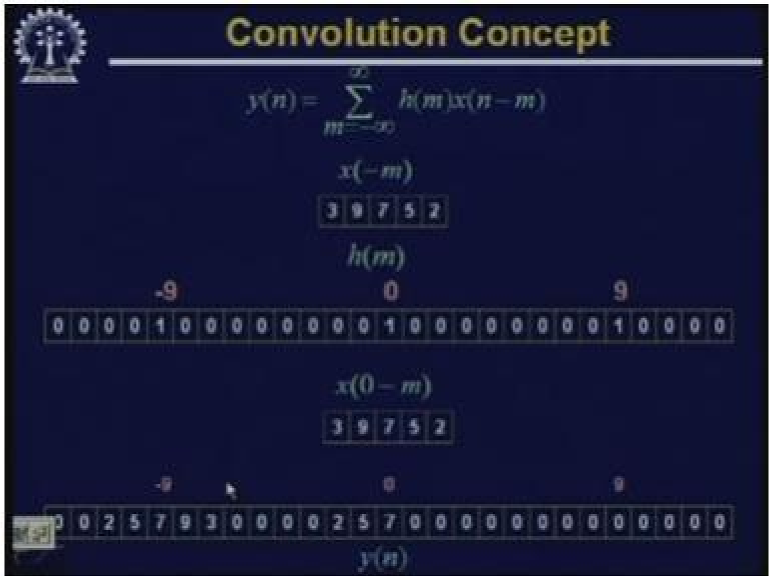

At n equal to 0, I get value of 7.

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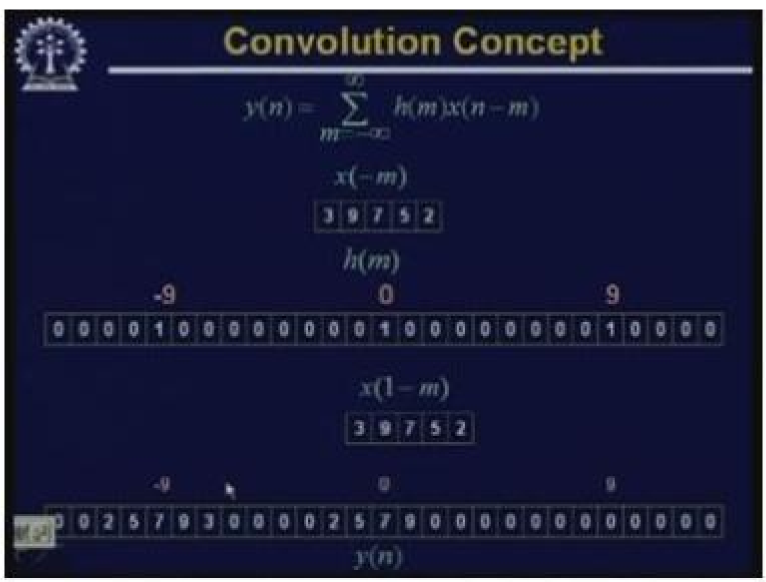

At n equal to plus 1, I get value of 9.

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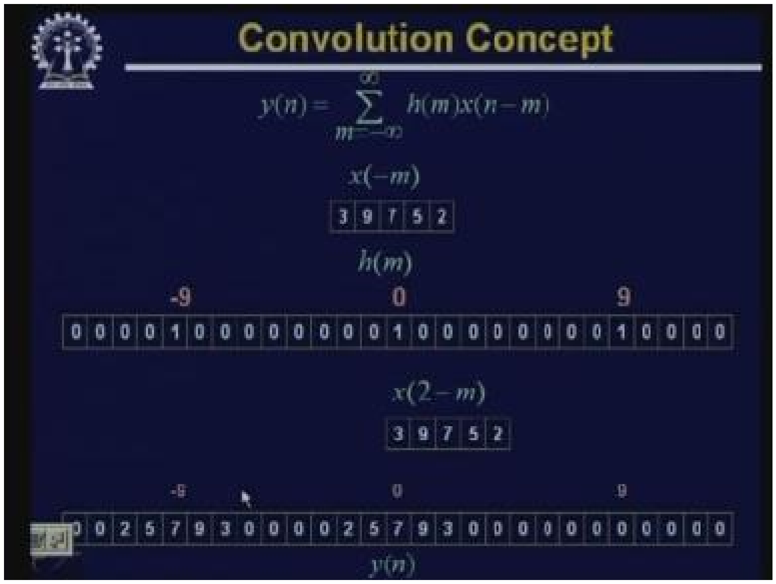

At n equal to plus 2, I get value of 3.

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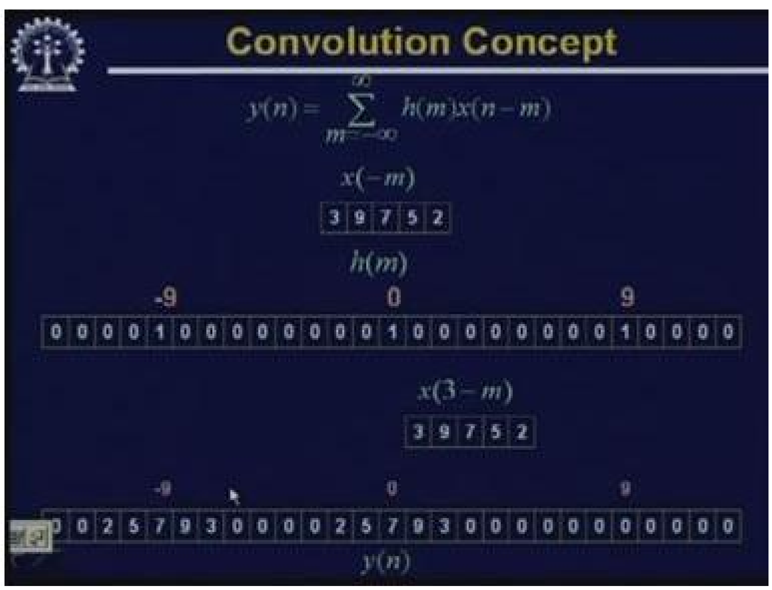

At n equal to plus 3, again I get the value of 0.

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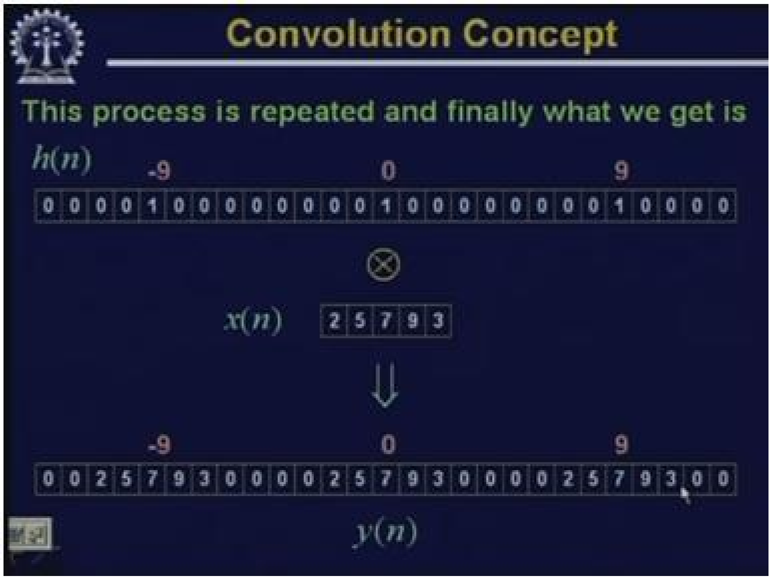

So, if I continue like this, you find that after completion of this convolution process, this h (n) convoluted with x (n) gives me this kind of pattern and here you notice one thing that when I have convoluted this x (n) with this h (n); the convolution output y (n), this is you just notice this that it is the repetition of the pattern of x (n) and it is repeated at those locations where the value of h (n) was equal to 1. So, by this convolution what I get is I get the repetition of the pattern x (n) at the locations of delta functions in the function h (n).

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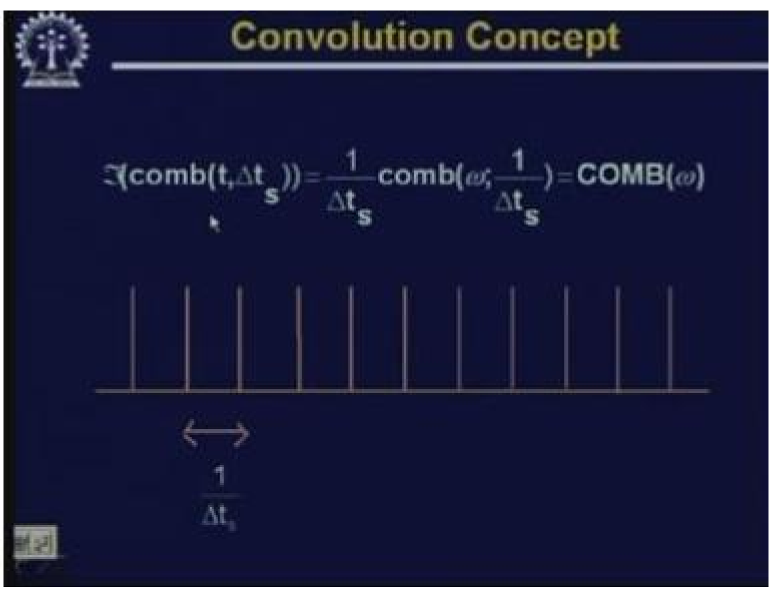

So, by applying this, when I convolute 2 signals X (t) and the Fourier transform of this comb function that is comb omega in the frequency domain; what I get is something like this.

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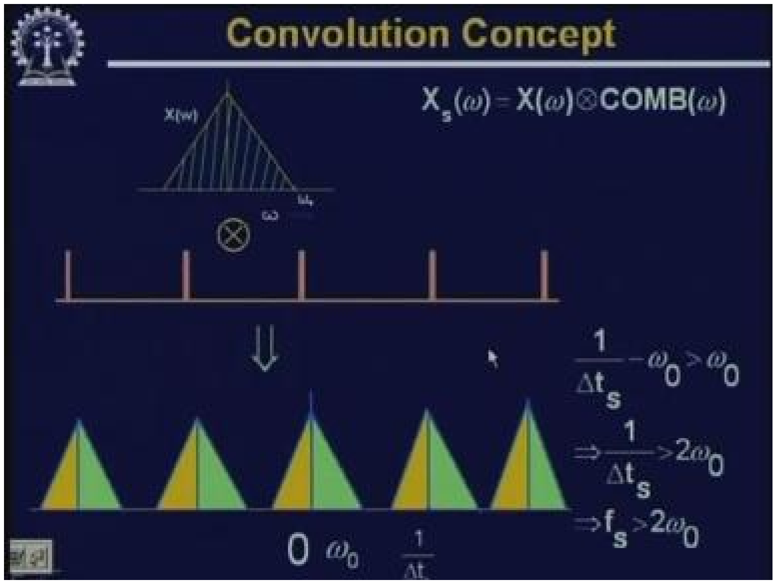

When X (t) is band limited, that means the maximum frequency component in the signal X (t) is omega naught; then the frequency spectrum of the signal X (t) which is represented by x omega will be like this.

Now, when I convolve this with this comb function comb of omega, then as we have done in the previous example; what I get is at those locations where the comb function had a value 1, I will get just a replica of the frequency spectrum x omega. So, this x omega gets replicated at all these locations.

So, what we find here? You find that the same frequency spectrum x omega, when it gets translated like this, when X (t) is actually sampled that means the frequency s spectrum of x_S or x_S omega is like this. Now, this helps us in reconstruction of the original signal X (t). So, here what I do is you find that around omega equal to 0, I get a copy of the original frequency spectrum.

So, what I can do is if I have a low pass filter whose cut off frequency is just beyond omega naught and this frequency signal, these spectrum, the signal with this spectrum; I pass through that low pass filter. In that case, the low pass filter will just take out this particular frequency band and it will cut out all other frequency bands.

So, since I am getting the original frequency spectrum of X (t), so signal reconstruction is possible. Now, here you notice one thing as we said that we will just try to find out that what is the condition that original signal can be reconstructed.

Here you find that we have a frequency gap between this frequency band and this translated frequency band. Now, the difference of between centre of this frequency band and the centre of this frequency band is nothing but 1 upon t $_S$ which is equal to omega s that is the sampling frequency.

Now, as long as this condition that is 1 upon t $_S$ minus omega naught is greater than omega naught, that is the lowest frequency of this translated frequency band is greater than the highest frequency of the original frequency band; then only these 2 frequency bands are disjoint and when these 2 frequency bands are disjoint, then only by use of a low pass filter, I can take out this original frequency band.

And from this relation, you get the condition that 1 upon delta t $_S$ or the sampling frequency omega s in this case, it is represented as f $_S$ must be greater than twice of omega naught where omega naught is the highest frequency component in the original signal X (t) and this is what is known as Nyquist product. That is we can reconstruct, perfectly reconstruct the continuous signal only when the sampling frequency is greater than, more than twice the maximum frequency component of the original continuous signal.

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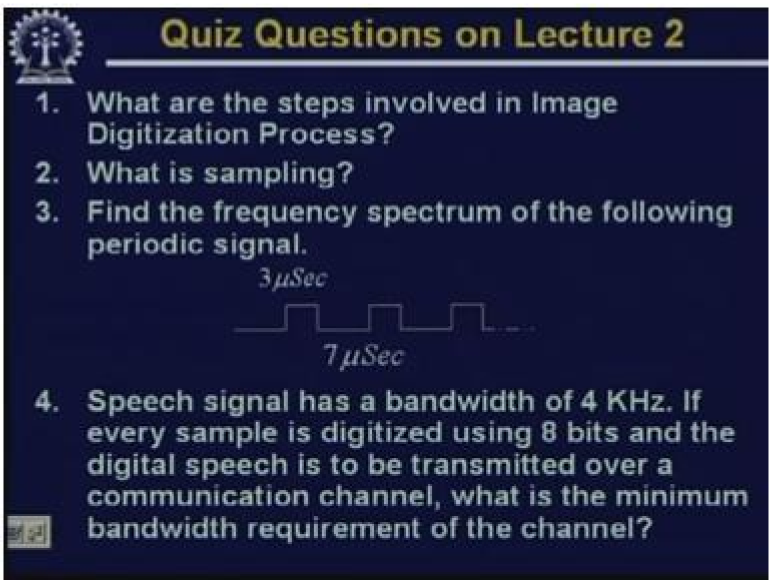

Now, let us have some quiz questions on today's lecture. The first question - what are the steps involved in image digitization process? I repeat, what are the steps involved in image digitization process? The second question - what is sampling? What is sampling?

The third question - here you find that we have given a periodic signal in time which is at periodic square wave, in this square wave the on time is 3 micro second and the off time is one micro 7 micro second. So, you have to find out the frequency spectrum of this periodic signal. So, for this periodic signal; on time is 3 micro second, off time is second micro second 7 micro second. So obviously, the time period of this periodic signal is 10 micro second. You can assume the amplitude of this signal to be 1 and you have to find out the frequency spectrum of this periodic signal.

The forth question - if a speech signal has a bandwidth of 4 kilo hertz, a speech signal has a bandwidth of 4 kilo hertz, then if every sample is digitized using 8 bits and the digital speech is to be transmitted over a communication channel; then what is the minimum bandwidth requirement of the channel?

So, speech signal is has a bandwidth of 4 kilohertz, every sample is digitized using 8 bits and the digital speech is to be transmitted over a communication channel; then you have to find out that what will be the minimum bandwidth requirement of the channel.

Obviously, because the signal is digital; so by band width requirement, I mean that what is the bit rate requirement of the channel.

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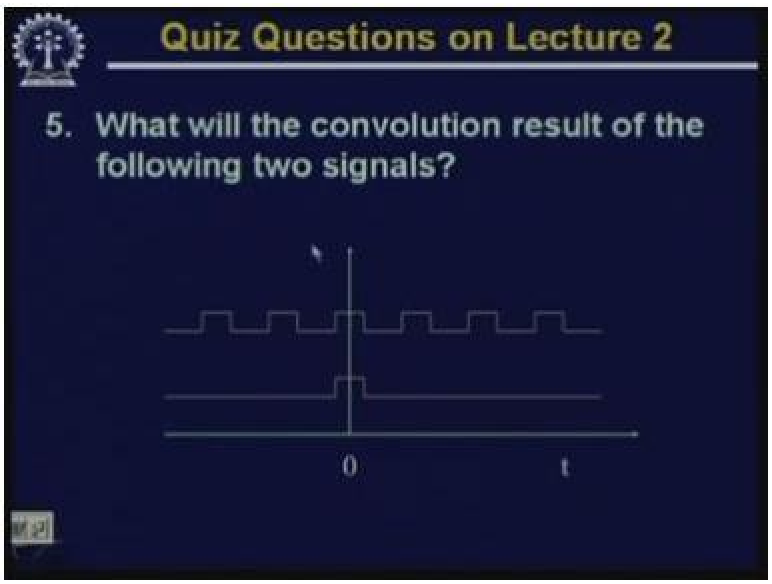

The next question - here again we have given 2 signals in time. One is a periodic square wave, the second signal is an aperiodic, it is just a square pulse. We can assume that on time of this square wave and on time of this square pulse is same. Then we have to find out that what will be the convolution result if you convolve these 2 signals in the time domain. So, you have to find out the convolution output when these 2 signals are convolved in the time domain.

Thank you.