**Digital Image Processing**

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**Lecture - 19**

**Image Enhancement Point Processing-III**

Hello, welcome to the video lecture series on digital image processing. For last few classes we have started our discussion on image enhancement techniques.

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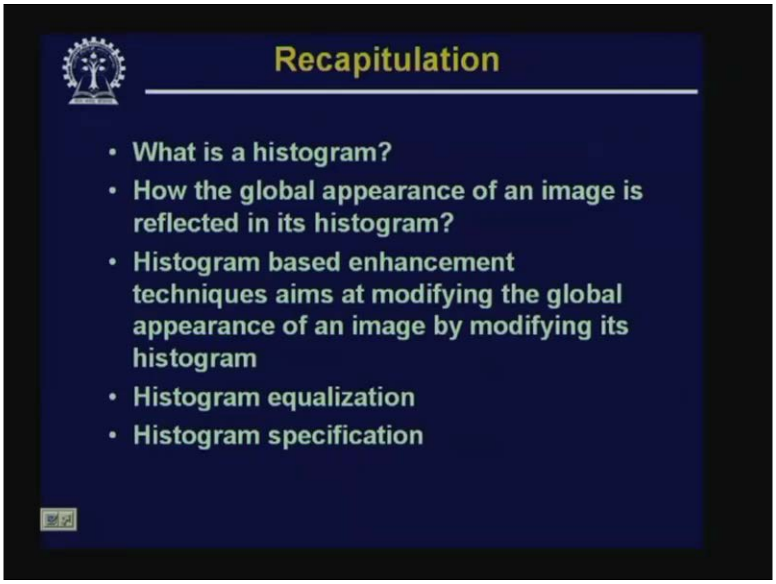


So, in the previous class, we have seen what is meant by histogram; we have seen how the global appearance of an image is reflected in its histogram, we have seen that histogram based enhancement techniques aims at modifying the global appearance of an image by modifying its histogram. Then we have started discussion on histogram equalization technique and histogram specification or histogram matching techniques.

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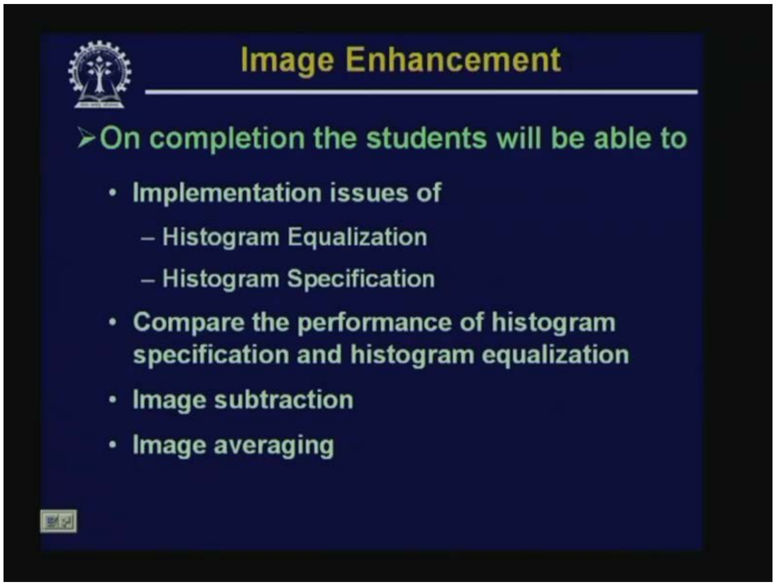


So today's class, we will talk about some implementation issues of histogram equalization and histogram specification techniques and we will talk about this implementation issues with respect to some examples. Then we will also compare the performance of histogram specification and histogram equalization techniques with the help of some results obtained on some images.

Then lastly, we will talk about two more point processing techniques for histogram equalization; one of them is histogram subtraction and other one is histogram averaging techniques. So now, let us briefly recapitulate what we have done in the last class.

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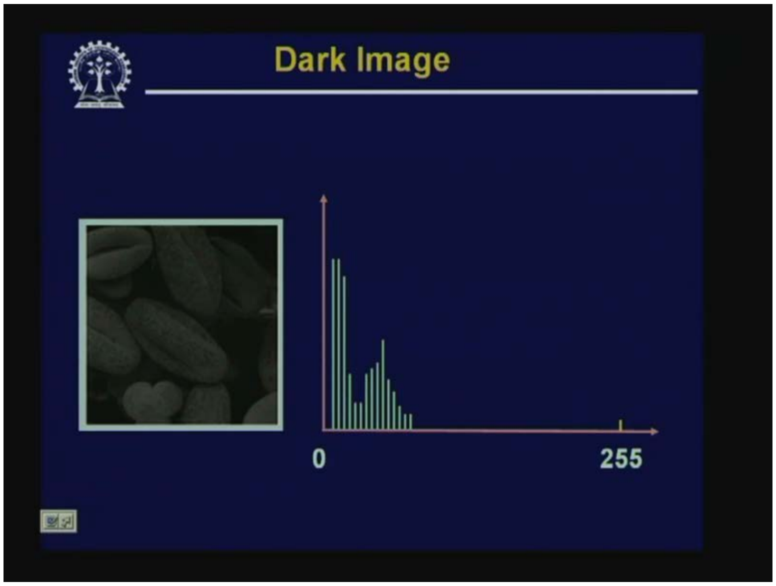


As we have said that histogram of an image that indicates what is the global appearance of an image. We have also seen these images in the last class but just for a quick recapitulation; you will find that on the left hand side, we have shown an image which is very dark and we call this as the dark image and on the right hand side, we have shown the corresponding histogram and you will find that this histogram shows that most of the pixels in this particular image, they are having an intensity value which is near about 0 and there is practically no pixel having higher intensity values and that is what gives this particular image a dark appearance.

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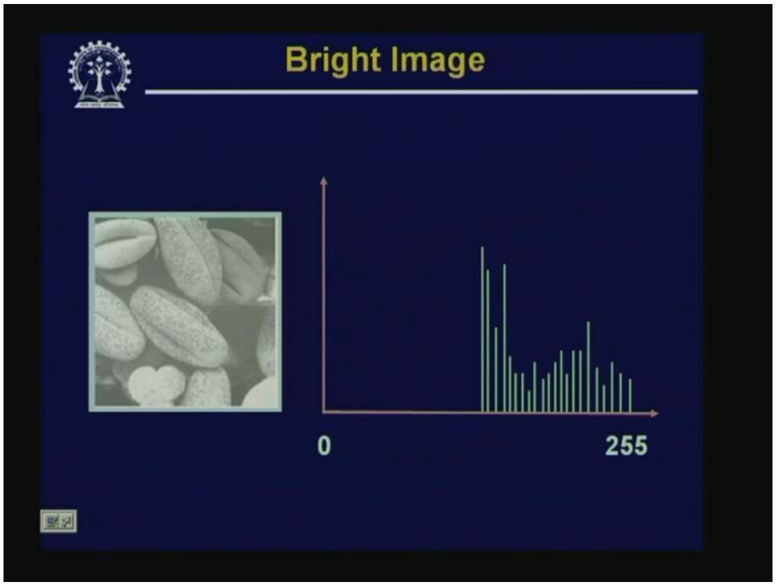

Then the second one that we have shown is a bright image or a light image and again from this particular histogram, you will find that most of the pixels in this particular image have intensity values which are near to maximum value that is 255 in this particular case and since we are talking about all the images in our application which are quantized where every pixel is quantized with 8 bits, so the intensity levels will vary from 0 to the 255.

So, in our case, the minimum intensity of a pixel will be 0 and the maximum intensity of a pixel will be 255. So, in this particular example, you will find that the intensity of the images as this histogram shows that most of the pixels have intensity which is near 255 that is maximum value.

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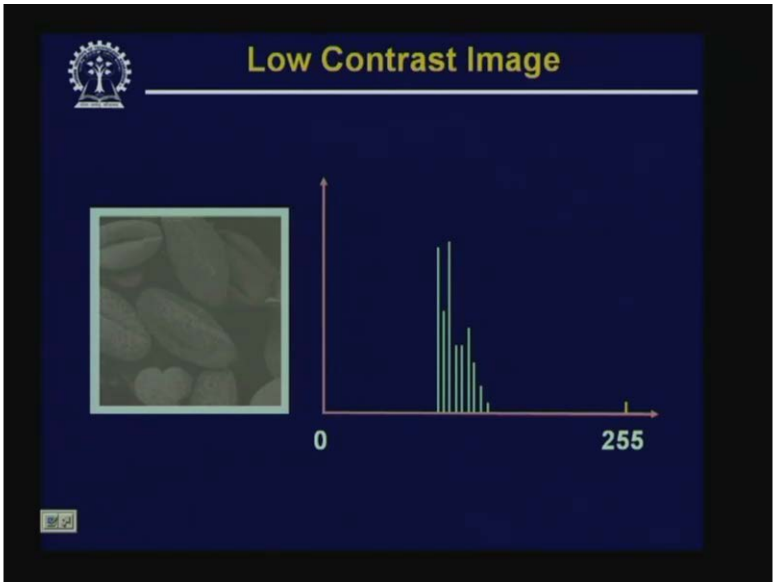


Then, the next image shows that here the image has got pixels having intensity value in the middle range but the range of intensity value is very is narrow. So, as a result, the image is neither very bright nor very dark but at the same time because dynamic range of the intensity values is very low, the image contrast is very poor.

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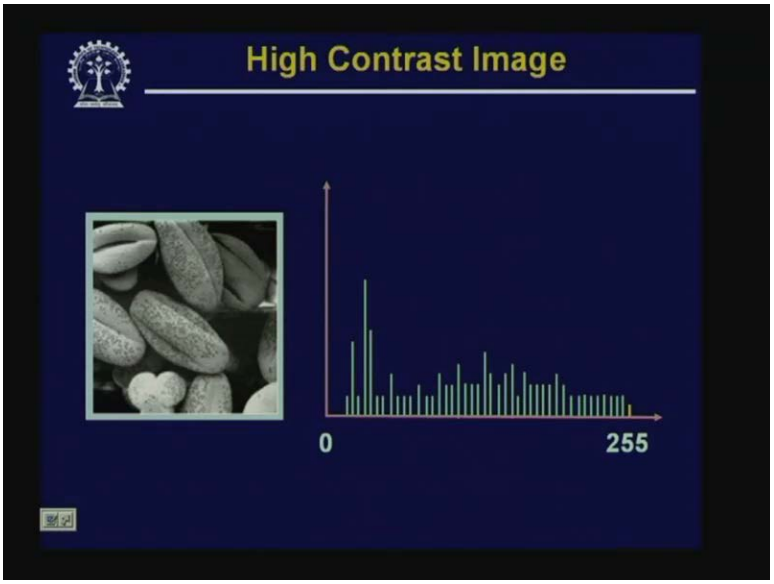


So, as the next slide shows which we call a high contrast image where you find that most of the details of the objects present in the image are visible and by looking at the corresponding histogram, we find that the pixels in this particular image have wide range of intensity values starting from very low value which is near about 0 to the maximum value which is near about 55.

So, this says that we will said that a particular image has a high contrast if its intensity values, the pixel intensity values have a wide range of values starting from a very low value to a very high value. So, all these 4 examples tell us that how the global appearance of an image is reflected in its corresponding histogram and that is why all the histogram based enhancement techniques, they try to adjust the global appearance of the image by modifying the histogram of the corresponding image.

So, the first technique of this histogram based enhancement that we have discussed in the last class is called histogram equalization. So, let us quickly review what we mean by histogram equalization.

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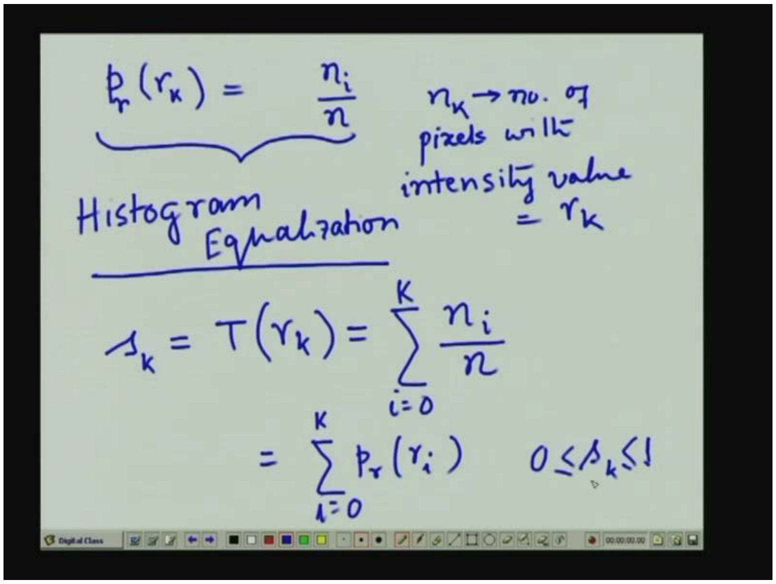


So here, in case of histogram equalization, if I consider a discrete case; in a discrete case we have seen that the histogram of an image is given in the form like this that is $p_r$ $(r_k)$ where $r_k$ is an intensity level present in the image and which is given by summation of $n_i$ by n, i varying from 0 to k where $n_k$ is the number of pixels with intensity value is equal to $r_k$ sorry this is not summation $p_r$ $(r_k)$ is given by $n_i$ by $n_k$.

So, as this expression suggests that this particular expression tells us what is the probability of a pixel having a value $r_k$ present in the image and plot of all these p $(r_k)$ values for different values of $r_k$ defines what is the histogram of this particular image.

Now, when you talk about the histogram equalization, the histogram equalization technique makes use of this histogram to find out the Transformation function between a intensity level in the original image to an intensity level in the processed image and that transformation function is given by $S_k$ is equal to, so transformation function we represent by T $(r_k)$ which is given by summation of $n_i$ by n where i varies from 0 to k and which is nothing but summation of $p_r$ $(r_i)$ where i varies from 0 to k.

So, this is the Transformation function that we get which is to be used for histogram equalization purpose. Now, find that in this particular case, because the histogram which is defined that is $p_r$ $(r_k)$ equal to $n_i$ by n, it is a normalized histogram. So, every value of $p_r$ $(r_k)$ will be within the range 0 to 1. And similarly, this transformation function that T $(r_k)$ when it gives us a value $S_k$ corresponding to an intensity level in the input image which is equal $r_k$ , the maximum value of $S_k$ also in this particular case will vary from 0 to 1.

So, the minimum value of the intensity as suggested by this particular expression will be 0 and the maximum value of the intensity will be equal to 1. But we know that when we are talking about the digital images, the minimum intensity of an image can be a value 0 and the maximum intensity can have a value $r_L$ minus 1, $s_k$ varies from 0 to L minus 1 and in our discrete case, this $r_L$ minus 1 is equal to 255 because in our case, the intensity values of different images are quantized with 8 bits.

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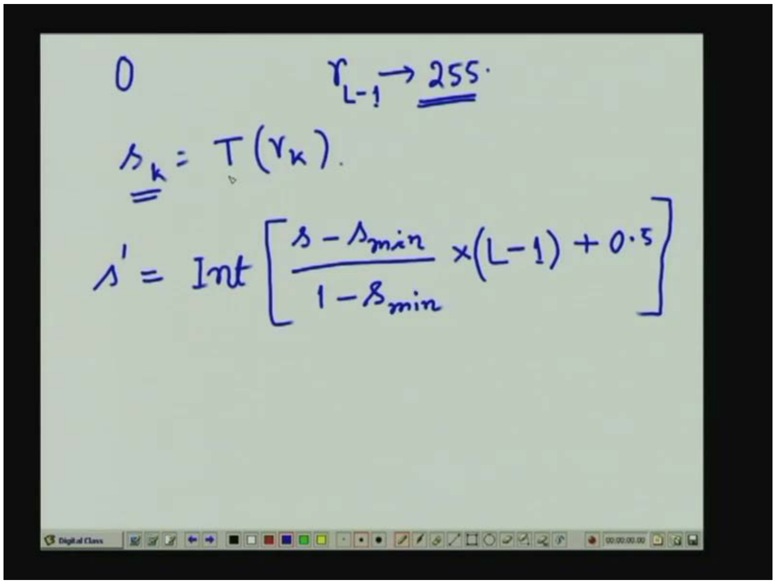


So, we can have an intensity varying from 0 to 255 whereas, this particular Transformation function that is $s_k$ equal to T ($r_k$), this gives us a maximum intensity value $s_k$ in the processed image which is equal to 1. So for practical implementation, we have to do some sort post processing so that all these $s_k$ values that you get in the range 0 to 1 can now be mapped to the maximum dynamic range of the image that is from 0 to 255 and the kind of mapping function that we have to use is given by say I can write it as s dash is equal to say integer value because we will getting only integer values into s minus s minimum divided by 1 minus s minimum into L minus 1 where L minus 1 is the maximum intensity level plus you give DC a shift of 0.5.

So, whatever value of s we get by this Transformation $s_k$ equal to T ($r_k$), that value of s has to be scaled by this function to give us an intensity level in the processed image which varies from 0 to maximum level that is 0 to capital L minus 1 and in our case, this capital L minus 1 will be equal to a value 255. Now, let us take an example to illustrate this.

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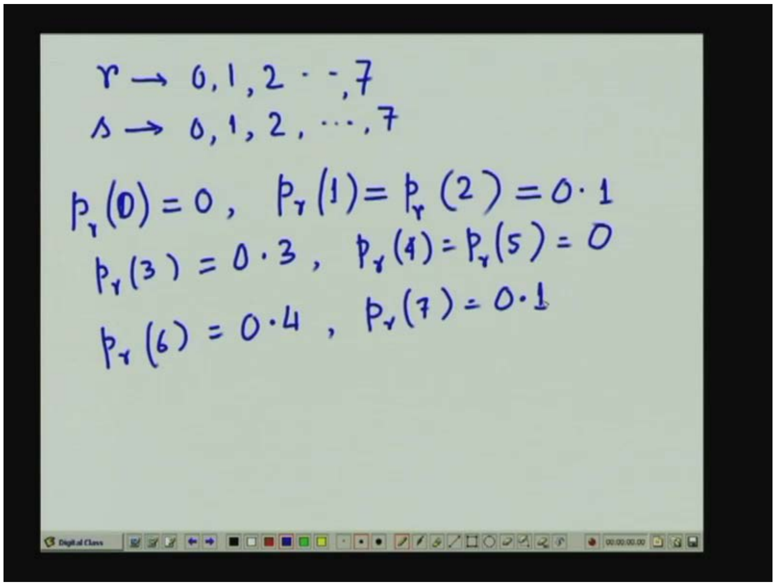


Suppose, we have an input image having 8 discrete intensity values that is r varies from 0, 1, 2 upto say 7. So, we have 8 discrete intensity values. Similarly, the processed image that you want to generate, that will also have 8 discrete intensity values varying from 0 to 7.

Now suppose, the probability density functions or the histogram of the input image is specified like this; so it is given that $p_r$ (1) that is the probability that an intensity value will be equal to 1 sorry $p_r$ (0) the probability that an intensity value will be equal to 0 is equal to 0, $p_r$ (1) that is probability that intensity value will be equal to 1 is same as $p_r$ (2) which is given as say 0.1, $p_r$ (3) that is given as 0.3, $p_r$ (4) is equal to $p_r$ (5) which is given equal to 0, $p_r$ (6) is given as 0.4 and $p_r$ (7) is given as 0.1.

Now, our aim is that given this histogram of the input image, we want to find out the transformation function T (r) which will map such an input image to the corresponding output image and the output image will be equalized. So to do this what we have to do is we have to find out the mapping function T (r).

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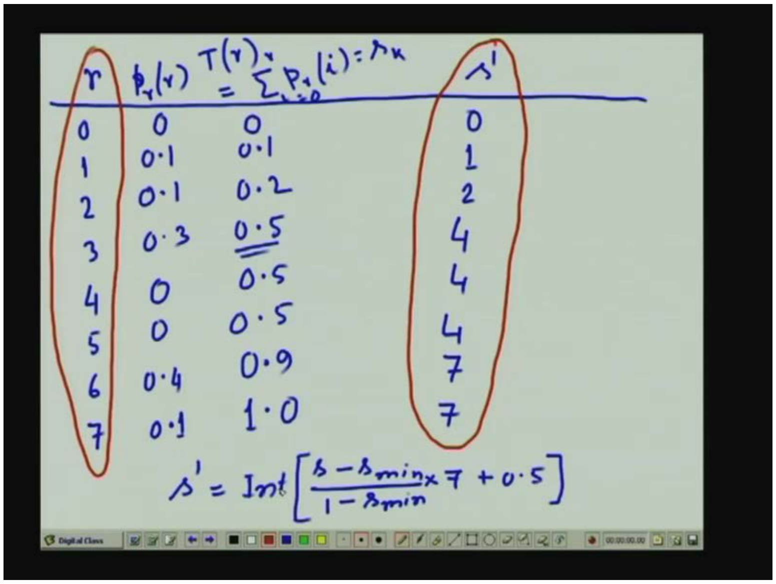


So, this mapping function, we can generate in this form: let us have all these values in the form of a table. So, I have this r, r varies from 0 to 7. The corresponding $p_r$, the probability values are given by 0, 0.1, 0.1, 0.3, 0, 0, 0.4 and 0.1. Then obviously, from this probability density function, we can compute the transformation function T (r) which is nothing summation of say $p_r$ (i) where i varies from 0 to r.

So, if we compute the transformation function, we will find that the transformation function comes out to be like this; this is 0, this is 0.1, here it is 0.2, here it is 0.5 because the next 2 probability density function values are 0, so it will remain as 0.5, this will also remain as 0.5 then this will be 0.9 and here I will get 1.0.

So, this is the Transformation function that we have. So, this means that if my input intensity is 0, this transformation function will give me a value s; this is nothing but the value say $s_k$ which will be equal to 0. If the intensity is 1, input intensity is 1, the output $s_k$ will be equal to 0.1. If the input intensity is 2, output $s_k$ will be equal to 0.2.

Similarly, if the input intensity is 6, the output intensity value will be 0.9 but naturally because the output intensities has to vary from 0 to the maximum value which is equal to 7; so we have to scale this particular function, this particular s values the cover this entire range of intensities and for that we use the same expression, the same mapping function as we have said that s dash equal to integer of s minus s min divided by 1 minus s min and in this particular case, L minus 1 equal to 7 plus 0.5.

So, doing this calculation and taking the nearest integer value whatever we get that will be my reconstructed intensity level. So, if I do this, then you will find that for all these different values of s, the reconstructed s dash will be for r equal to 0 the reconstructed s dash will be equal to 0. For r equal to 1, the reconstructed s dash will also be equal to 1;

for r equal to 2, the reconstructed s dash will also be equal to 2 but for r equal to 3, so in this case r equal to 3, I get s equal to 0.5 and minimum s is 0. So, this becomes 0.5 and denominator is also equal to 1; so 0.5 into 7, that gives you 3.5 plus 0.5 which is equal to 4. So, when my input intensity is 3, the corresponding output intensity will be equal to 4.

Similarly, for r equal to 4, the output intensity will also be equal to 4; for r equal to 5, the output intensity will also be equal to 4. For r equal to 6, now the output intensity if you calculate following the same relation, it will come out to be 7; for r equal 7, the output intensity will also be equal to 7.

So, this first column that is for different values of r and the last column that is the different values of s; so this first column and the last column, these gives us the corresponding mapping between the given intensity value to the corresponding output intensity value and this is the image which is a processed image or the enhanced image which is to be displayed.

So, this is how the histogram equalization operations have to be done and we have seen in the last class that using such histogram equalization operations, we have got the results which are given like this.

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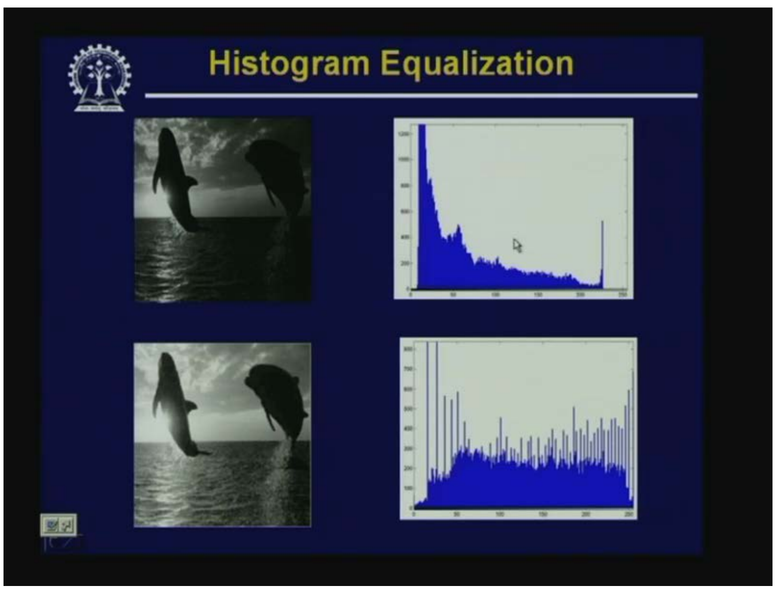


So here, we have shown on image which is a very very dark image and on the right hand side, we have the corresponding histogram and once we do histogram equalization, then what we get is an equalized image or the processed images and on the bottom row, you find that we have a brighter image which is the histogram equalized image and on the right side we have the corresponding histogram.

And, as we have mentioned in our last class that whenever we are going for histogram equalization; then the probability density function of the intensity values of the equalized image, they are ideally normal ideally uniform distribution.

In this particular case, you will find that this histogram of this equalized image that we have got, this is not absolutely uniform. However, this is near to uniform. So, that theoretical derivation which shows us that the distribution value, intensity distribution will be uniform that is the theoretical one.

In practical case, in discrete situations, in most of the cases we do not get a uniform probability distribution, a uniform intensity distribution. The reason being that in discrete cases, there may be situation that many of the allowed pixel values will not be present in the image and because of this the histogram that you get or the intensity distribution that you get in most of the cases, they will not be uniform.

So, this shows us the cases of histogram equalization. Now, let us come to the case of histogram specification or histogram modification as it is called.

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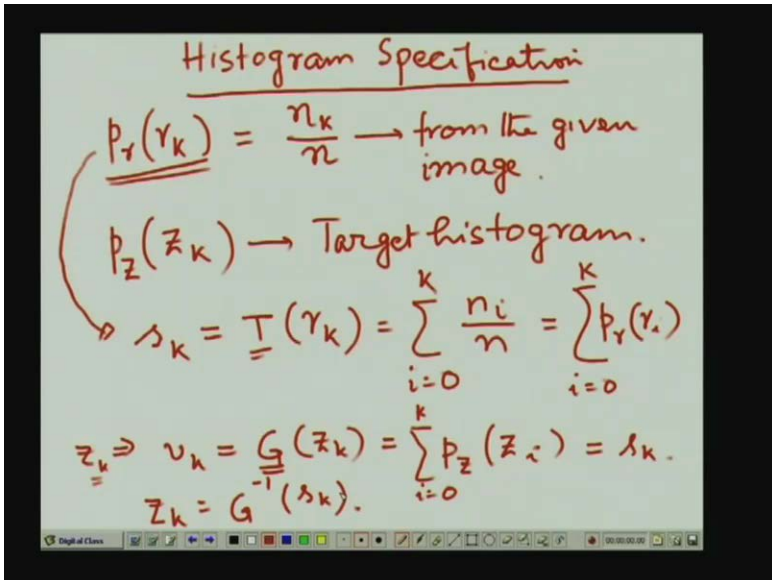


So, we will talk about histogram specification. So, as we have told in our last class that histogram equalization is an automated process; so whatever output you get, whatever process image you get by using the histogram equalization techniques that is fixed and histogram equalization techniques is not suitable for interactive image manipulation whereas, interactive image manipulation or interactive enhancement can be done by histogram specification techniques

So, in histogram specification techniques what we do is we have the input image, we can find out what is the histogram of that particular image, then a target histogram is specified and you have to process the input image in such a way that the histogram of the processed image will resemble, will be close to the target histogram which is specified.

So, here we have 2 different cases. Firstly, we have $p_r$ ($r_k$) which has we had seen is nothing but in $n_k$ divided by n where $n_k$ is the number of pixels in the given image with an intensity value equal to $r_k$ and this we compute from the give image which is to be processed. And, we have a target histogram which is specified which is to be so that or processed image will have a histogram which is almost close to the target histogram and the target histogram is specified in the form $p_z$ ($z_k$). So, this is the target histogram which is specified.

You note that we do not have the image corresponding to this particular histogram. So, it is the histogram which is specified and we use the subscript $p_r$ ($r_k$) and $p_z$ ($z_k$); so this subscripts r and z, they are used to indicate that these 2 probability distribution probability density functions $p_r$ and $p_z$, they are different.

So, in case of histogram specification, what we have to do is the process is done in this manner; firstly using this $p_r$ ($r_k$), you find out the transformation function corresponding to equalization and that transformation function as we have seen earlier is given by $s_k$ equal to T of $r_k$ which is nothing but sum of $n_i$ by n where i varies from 0 to k and which is obviously equal to $p_r$ ($r_i$), i varying from 0 to k.

So, this is a transformation function that is computed from the histogram $p_r$ ($r_k$) which is obtain from the given image and to obtain this histogram specification, the process is like this; you define a variable say $Z_k$ such that which will follow this property, we will have the transformation function say $V_k$ equal to G ($z_k$) and that will be equal to we can compute this from the specified histogram which is given in the form of $p_z$ say ($z_i$) where i varies from 0 to k and we define these to be equal to say $s_k$.

So, you find that this intermediate stage that is $V_k$ equal to G ($z_k$) where this transformation function G ($z_k$) is given by $p_z$ ($z_i$) summation i from 0 to k, this is a hypothetical case because we really do not have the image corresponding to the specified histogram $p_z$ $p_z$ ($z_k$). Now once I get this, to get the reconstructed image or the processed image; I have to take the inverse transformation.

So here, you find that as we have defined that for this particular Z $_k$, we have $V_k$ equal to G ($z_k$) which is equal $s_k$ and G ($z_k$) is computed in this form and from here to get the value $Z_k$, the intensity value $Z_k$ in the processed image; we have to take the inverse transformation but in this case the inverse transformation is not taken with respect to T but the inverse transformation has to be taken with respect to G. So, our $Z_k$ in this case will be equal to G inverse of $s_k$.

So, what we have to do is for the given image, we have to find out the transformation function T (r) corresponding to the histogram of the given image and using this transformation function, every intensity value of the input image which is $r_k$ has to be mapped to an equalized intensity value which is equal to $S_k$. So, that is the first step.

The second step is from the specified histogram $p_z$ we have to get a transformation function G and then the $s_k$ that we have obtained in the previous step that has to be inversed transformed using this transformation function G to give you the processed image intensity value in the processed image which is equal to $Z_k$.

Now, so far we have concerned, we have discussed that finding out T ($r_k$) is very simple that is this forward process is very simple but the difficulty comes for getting the inverse transformation that is G inverse. It may not always be possible to get analytical expressions for T and G and similarly, it may not always be possible to get an analytical expression for G inverse.

So, the best possible way to solve this inverse process is to go for an iterative approach. Now, let us see that what does this entire expression mean.

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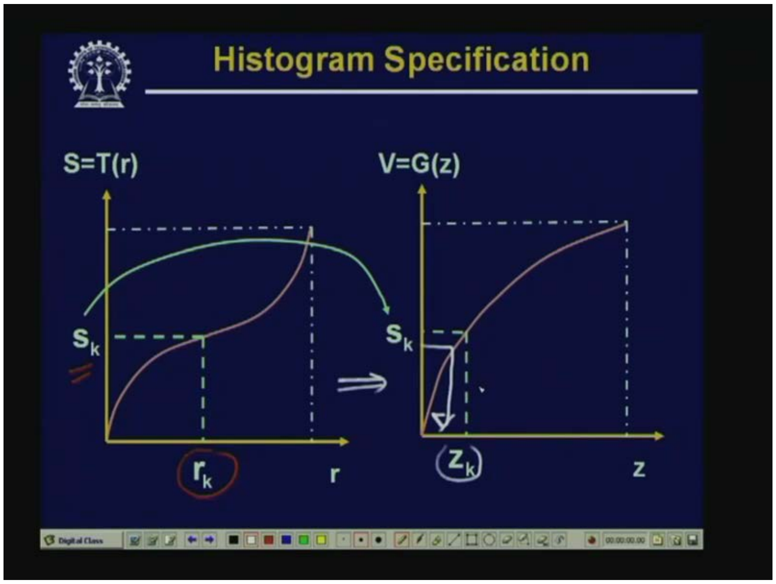


So here, we have shown the same formulations graphically. On the left hand side, we have the transformation function S equal to T (r) and this Transformation function is obtained from the histogram of the given image and on the right side, we have given the transformation function V equal to G (z) and this Transformation function has to be obtained from the target histogram that is specified.

Now, once we have these 2 transformation functions, you will find that these 2 transformation functions straightway tell you that given an $r_k$, I can find out what is the value of $s_k$; given $Z_k$, I can find out what is the corresponding value of $V_k$. But the problem is $Z_k$ is unknown, we do not know what is the value of $Z_k$; this is the one that we have to find out by using the inverse transform G inverse.

So, the process as per our definition; since we have seen that $V_k$ equal to $S_k$, so what we do is for the given image $r_k$, we find out the corresponding value $S_k$ by using this transformation S equal to T (r) and once we get this, then we set this $V_k$ equal to $S_k$ that is from the first transformation function, we come to the second transformation function.

So, we set $V_k$ equal to $s_k$ and then find out $Z_k$ in the reverse direction. So, now our direction of the transformation is reverse and we find out $Z_k$ from this value of $S_k$ using this transformation curve G (z) but as we have mentioned that it may not always be possible to find out the analytical expressions for r and G; so though the method appears to be very simple but its implementation is not that simple. But in the discrete domain, the matter, this particular case can be simplified. It can be simplified in the sense that both these transformation functions that is S equal to T (r) and V equal to G (z), they can actually be implemented in the formation of arrays. So, the arrays are like this.

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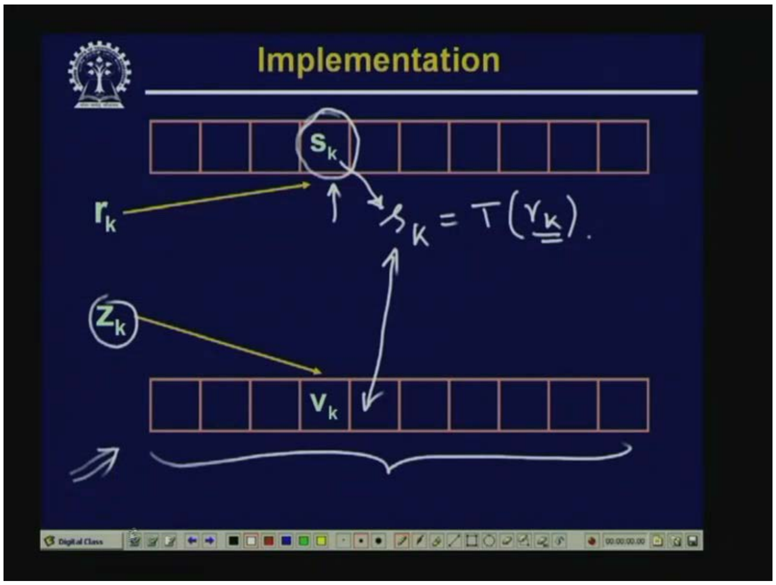


So, I have an array r as shown in the top where the intensity value of the input image $r_k$ is to be taken as an index to this particular array and the content of this array element that is S is the value the corresponding value $S_k$. Similarly, the second Transformation function $G_z$ that can also be implemented with the help of an array where $Z_k$ is an index is an index to this array and the corresponding element in the array gives us the value of $V_k$.

Now, you find that using these arrays the forward Transformation is very simple. When we want to find out $S_k$ is equal to Transformation function T ($r_k$); what we do is using this $r_k$, you simply come to the corresponding element in this particular array, find out what is the value stored in that particular location and that value gives you the value of $S_k$.

But the matter is not so simple when we go for the inverse transformation. So, for inverse Transformation what you have to do is we have to find out a location in this second array, we have to find out a location in the second array that is we have to find out the value of $Z_k$ where the element is equal to $S_k$. So, this is what we have to do.

So, you find that as the forward transformation that is $S_k$ equal to T ($r_k$) was very simple but the reverse transformation is not that simple. So, to do this reverse transformation, what we have to do is we have to go for an iterative procedure. The iterative is something like this. So, we do the iterative procedure following this.

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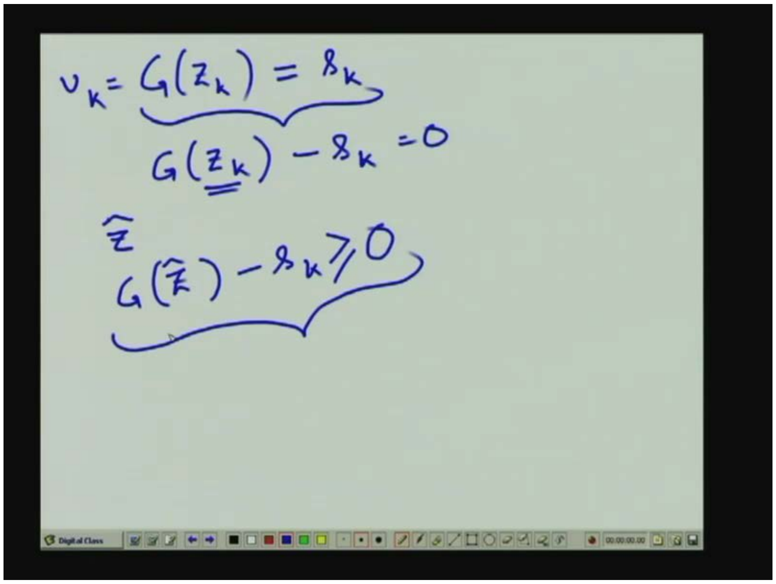

You find that as per our definition we have said that G ($z_k$), this is $V_k$, $V_k$ equal to G ($z_k$) is equal to $s_k$. So, if this is equal to 2, then we must have G ($z_k$) minus $S_k$ which is equal to 0. The solution would have been very simple if $Z_k$ was known but here we are trying to find out the value of $Z_k$.

So, to find out the value of $Z_k$, we take help of an iterative procedure. So, what we do is we initialize the value of $Z_k$ to some value say z hat and try to iterate on this z hat until and unless you come to a condition, until and unless a condition like G(z) hat minus $S_k$ is greater than or equal to 0.

So, until and unless this condition is satisfied, you go on iterating the value of z hat incrementing the value of z hat by 1 at every iteration and for this, what we have to do is we have to start with a minimum value of z hat, then increment the value of z hat by steps of 1 in every iteration until and unless we come to a condition like this that is G (z) hat minus S equal to 0 and the minimum value for z hat for which this condition is satisfied that gives you the corresponding value of $Z_k$. So, this is a simple iterative procedure. So, again as before we can illustrate this with the help of an example.

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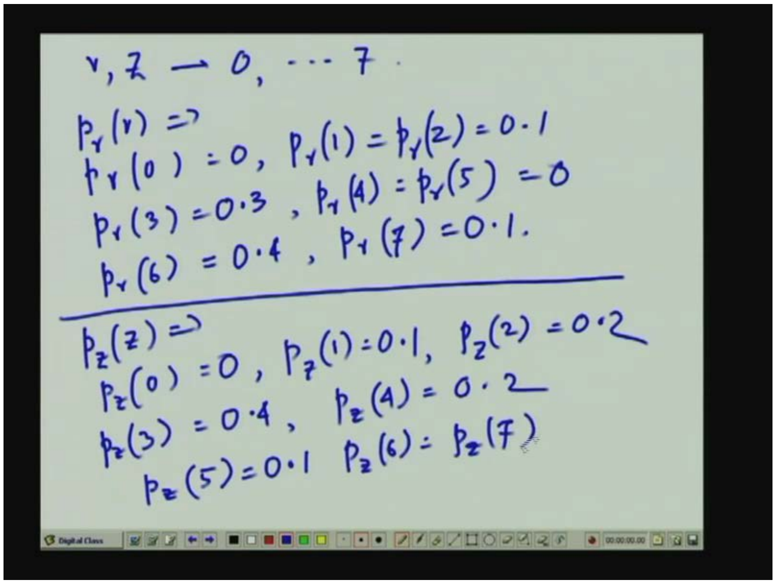


Here again, we take $P_r$. We assume that both r and z, there varies from they assumes value from 0 to 7 and we take the probability density functions of $P_r$ (r) like this that is $p_r$ (0) is equal to 0, $P_r$ (1) is equal to $P_r$ (2) is equal to 0.1, $P_r$ (3) is equal to 0.3, $P_r$ (4) is equal to $P_r$ (5) equal to 0, $P_r$ (6) is equal to 0.4 and $P_r$ (7) is equal 0.1.

So, this is what we assumed that it is obtained from the given image and similarly, the target histogram is given in the form $P_z$ (z) but the values are $P_z$ (0) is equal to 0, $P_z$ (1) is equal to 0.1, $P_z$ (2) is equal to 0.2, $P_z$ (3) is equal to 0.4, $P_z$ (4) is equal to 0.2, $P_z$ (5) is equal to 0.1, $P_z$ (6) is equal to $P_z$ (7) this is equal to 0. So, this is the target histogram that has been specified.

Now, our aim is to find out the transformation function or the mapping function from r to z. So, for doing this, we follow the similar procedure as we have done in case of histogram equalization.

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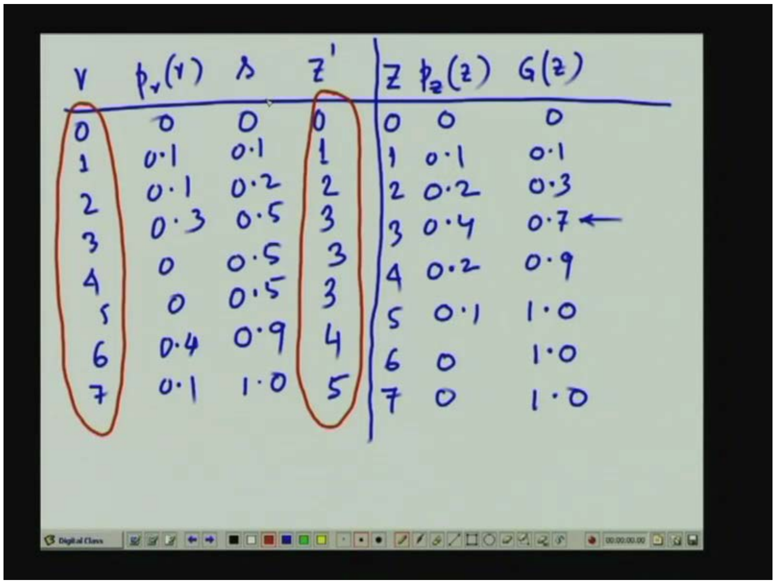


And in case of histogram equalization, we have found that for different values of r, we get $P_r$ (r) like this: for r equal to 0, we have $P_r$ (r) equal to 0; for r equal to 1, $P_r$ (r) equal to 0.1; for r equal to 2, this is also 0.1; 3, this is 0.3; for 4, this is 0; for 5, this is also 0; for 6, this is 0.4; for 7, this 0.1 and from this, we can find out what is the corresponding value of s and the corresponding values of s will be given by 0.1, 0.2, 0.5, 0.5, 0.5 then here you get 0.9 and here you get 1.0.

Similarly, from the target histogram, we get for different values of Z, 0, 1, 2, 3, 4, 5, 6, 7. The corresponding histogram is given by $P_z$ (z) which is 0, 0.1, 0.2, 0.4, 0.2, 0.1, 0, 0 and the corresponding G (z) is given by 0, 0.1, 0.3, 0.7, 0.9, 1.0, 1.0 and 1.0.

Now, if I follow the same procedure that to map from r to r to z; first I have to map from r to s, then I have to map from s to z and for that I have to find out the minimum value of z for which G (z) minus s is greater than or equal to 0.

So, for this, when I come to value of s is equal to 0; so here I put the corresponding values for z, let me put it as z prime. So, when s equal to 0, the minimum value of z for which G (z) minus s is greater tan or equal to 0 is 0. For s is equal to 0.1, again I start with z equal to 0. I find that the minimum value of z for which G (z) minus s will be greater than or equal to 0 is equal to 1.

Here, the minimum value of z for which G (z) minus s will be greater than or equal to 0 is equal to 2. When I come here that is r equal to 3, the corresponding value of s is equal to

0.5. Again, I do the same thing. You find that the minimum value of z for which the condition will be equal to 2 that is equal to 3. When I come to r equal to 4, you find that here the value of s equal to 0.5 and when I compute G (z) minus s, the minimum value of z for which G (z) minus s will be equal to greater than or equal to 0 that is equal to 3 because for 3, G (z) equal to 0.7. So, this will also be equal to 3 and if I follow the similar procedure, I will find that the corresponding functions will be like this.

So, for r equal to 1, the corresponding processed image will have an intensity value. So, for r equal to 0, the corresponding processed image will have intensity value equal to 0; for r equal to 1, it will equal to 1; r equal to 2, processed image will be equal to 2; r equal to 3, processed will also be equal to 3 but r equal to 4 and 5, the processed image will have intensity values which is a which are equal to 3. For r equal to 6, the processed image will have an intensity value which is equal to 4; r equal to 7, the processed imaged will have an intensity value which is equal to 5.

So, these 2 columns; the first column of r and the column of z prime, these 2 columns gives us the mapping between an intensity level and the corresponding processed image intensity level when I go for this histogram equalization sorry histogram matching.

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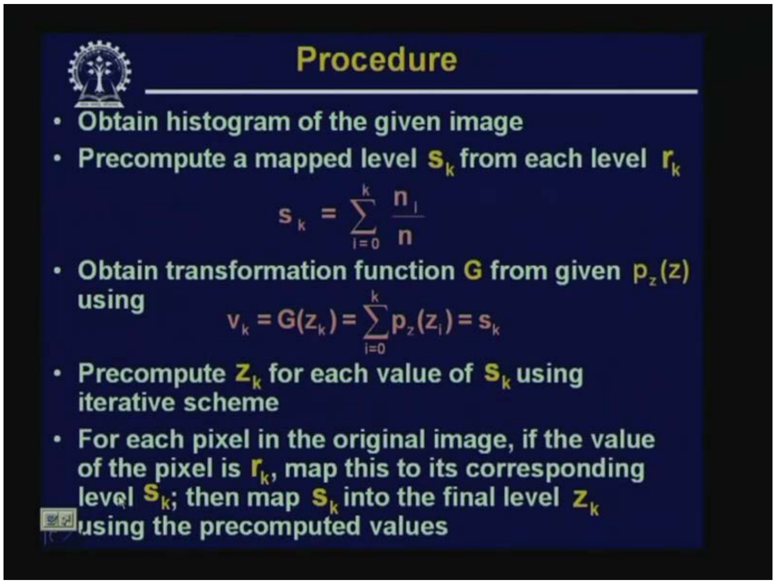


So, you find that our iterative procedure will be something like this that first you obtain the histogram of the given image, then pre compute a mapped level $S_k$ from for each level $r_k$ using this particular relation. Then from the target histogram, you obtain the mapping function G and for that this is the corresponding expression, then pre compute values of $Z_k$ for each values of $S_k$ using the iterative scheme.

And once these 2 are over, I have a pre computed transformation function which is in the form of a table which maps an input intensity value to the corresponding output intensity value and once that is done then for final enhancement of the images, I take the input intensity value and map to the corresponding output intensity value using the mapping function.

So, that will be our final step and if this is done for each and every pixel location in the input image, the final output image that will be an enhanced image where the enhanced image whose intensity levels will have a distribution which is close to the distribution that is specified.

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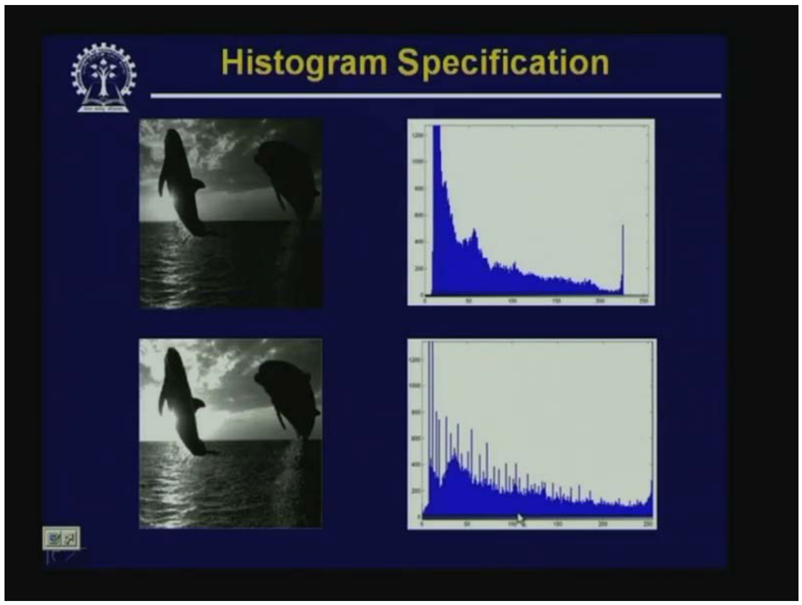


So, using these histograms equalization techniques, you find that what are the results that we get. Again, I take the same image that is dolphin image. On the top is the original image and on the right hand side, I have the corresponding histogram. On the bottom, I have ah an equal histogram matched image and on the right hand side, I have the corresponding histogram.

So, you find that these 2 histograms are quite different and I will come a bit later that what is the corresponding target histogram that we have taken.

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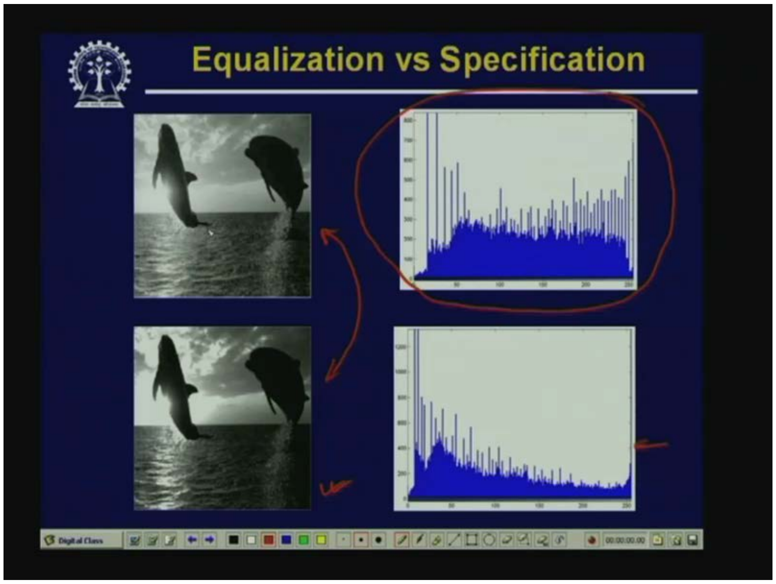


Now, to compare the result of histogram equalization with histogram specification, you find that on the top I have shown the same histogram equalized image as we have done that we have shown earlier and on the bottom row, we have shown this histogram matched image and at this point; this histogram specification, the target histogram which was specified was the histogram which is obtained using this equalization process.

So, this particular histogram was our target histogram. So, this histogram was the target histogram and using this target histogram when we did this histogram specification operation, then this is the processed image that we get and this is the corresponding histogram.

Now, if you compare these 2, the histogram equalized image and the histogram matched image; you can note a number of differences. For example, you find that this background, contrast of the background is much more than the contrast of the histogram equalized image and also the details on this waterfront, the water surface is more prominent in the histogram specified image than in the case of histogram equalized image and similar such differences can be obtained by specifying other histograms. So, this is our histogram specification operation.

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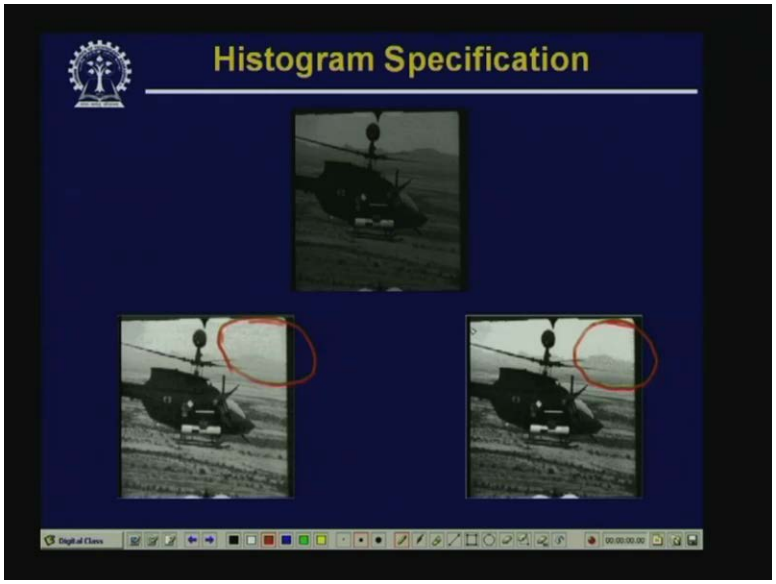


So, this shows another result with histogram specification. On the top we have the dark image, on the left bottom we have the histogram equalized image, on the right bottom we have the histogram specified image. Here again, you find that the background in case of histogram equalized image is almost washed out but the background is highlighted in case of histogram specified image.

So, this is the kind of difference that we can get between a histogram equalized image and a histogram specified image. So, this is what is meant by histogram specification and histogram equalization. Now, as we have said that I will discuss about 2 more techniques; one is image differencing techniques, the other one is image averaging techniques for image enhancement.

Now, as the name suggests, whenever we go for image differencing that means we have to take the difference of pixel values between 2 images.

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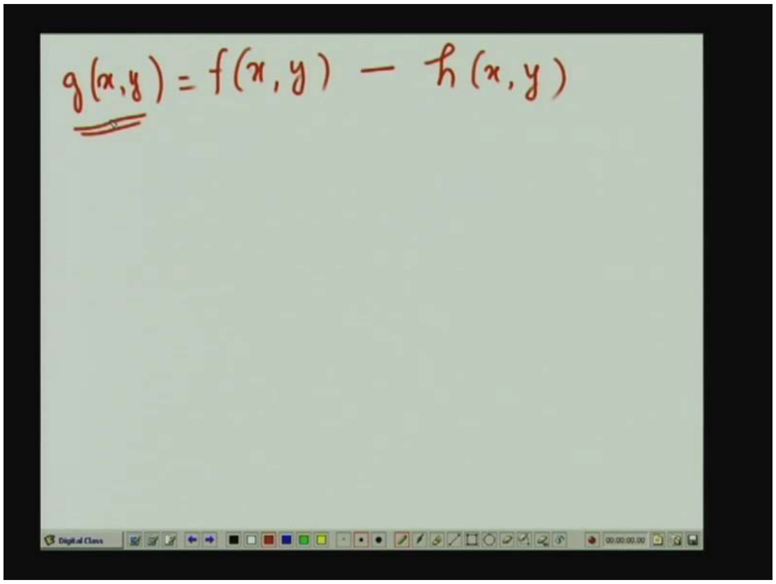


So, given 2 images; say one image is say f (x, y) and the other image say h (x, y). So, the difference of these 2 images is given by g (x, y) is equal to f (x, y) minus h (x, y). Now, as this operation suggests that g (x, y) in g (x, y), all those pixel locations will be highlighted wherever there is a difference between the corresponding locations in f (x, y) and the corresponding location in h (x, y). Wherever f (x, y) and g (x, y) are same the correspond f (x, y) and h (x, y) are same, the corresponding pixel in g (x, y) will have a value which is near to 0.

So, this kind of operation image, differencing operation mainly highlights the difference between 2 images or the locations where the 2 image contents are different. Such a kind of image difference operations is very very useful particularly in medical image processing.

So in case of medical image processing, there is operation which is called say mask mode radiography. In mask mode radiography, what is done is you take the image of certain body part of a patient, an x-ray image which is captured with the help of a tv camera where the camera is normally placed opposite to a x-ray shots and then what is done is you inject a contrast media into the blood stream of the patient and after injecting this contrast media, again you take a series of images using the same tv camera of the same anatomical portion of the patient body.

Now, once you do this; the first one, the image which is taken before injection of this contrast media that is called a mask and that is why the name is mask mode radiography. So, if you take the difference of all the frames that you obtain after the injection of the contrast media, take the difference of those images from the mask image; then you will find that all the regions where the contrast media that flows through the artery, those will be highlighted in the difference image and this kind of application and this kind of processing is very very useful to find out how the contrast media flows through artery of the patient and that is very very helpful to find out if there is any arterial disease of the patient. Say for example, if there is any blockade in the artery or similar such diseases.

So, this mask mode radiography makes use of this difference image operation to highlight the regions in the patient body or the arterial regions in the patient body and which is useful to detect any arterial disease or arterial disorder.

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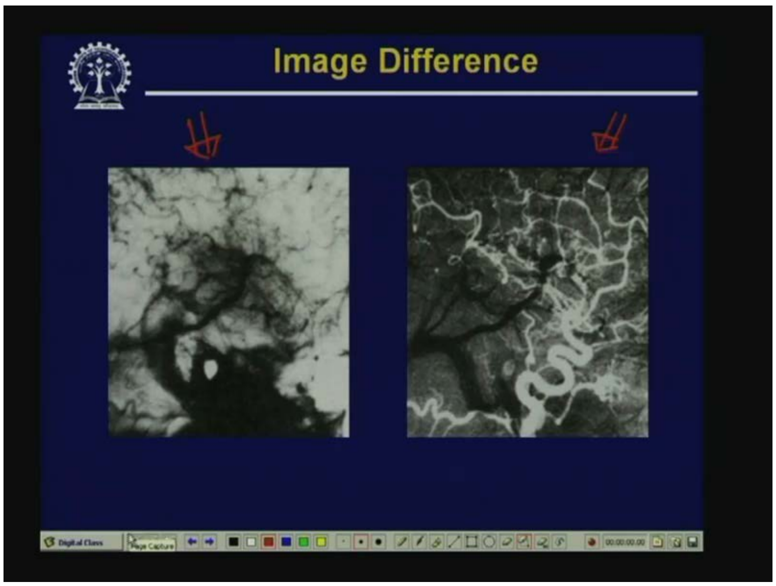


Now, to show a result, here this particular case, you find that on the left hand side whatever shown that is the mask which is obtained and on the right hand side, this is the difference image. This difference image is the difference of the images taken after injection of the contrast media with the mask and here you find that all these arteries through which the contrast media is flowing, those are clearly visible and because of this, it is very easy to find out if there is any disorder in the artery of the patient.

Now, the other kind of image processing applications as we said; so this is our difference image processing. So, as difference image processing can be used to enhance the contents of certain regions within the image wherever there is a difference between 2 images; similarly, if we take the average of a number of images of the same scene, then it is possible to reduce the noise of the image.

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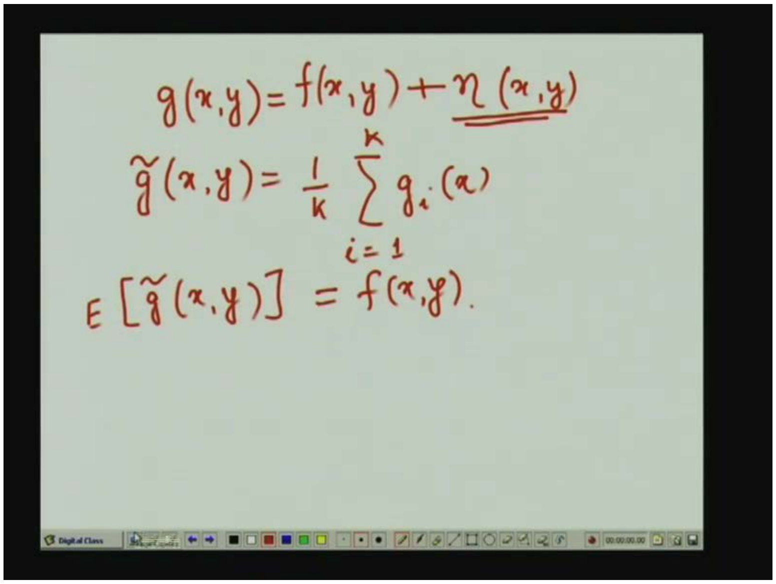


And, that noise deduction is possible because of the fact that normally if I have a pure image say f (x, y), then the image that we capture if I call it as g (x, y) that is the captured image, this captured image is normally the pure is image f (x, y) and on that we have a contaminated noise say eta (x, y).

Now, if this noise eta (x, y) is additive and 0 mean; then by averaging a large number of such noisy frames, it is possible to reduce the noise because simply because if I take the average of say k number of frames of this, then g tilde (x, y) which is the average of k number of frames is given by 1 over k then summation of $g_i$ (x, y) where i varies from 1 to k and if I take the acceptation value or the average value of this g tilde (x, y), then this average value is nothing but f (x, y) and our condition is the noise must be 0 mean additive noise and because it is 0 mean, I assume that at every pixel location, the noise is un correlated and the mean is 0.

So, that is why if you take the average of a large number of frames, the image the noise is going to get canceled out and this kind of operation is very very useful for astronomical cases because in case of astronomy, normally the objects which are imaged the intensity of the images are very very low. So, the image that you capture is likely to be dominated by the presence of noise.

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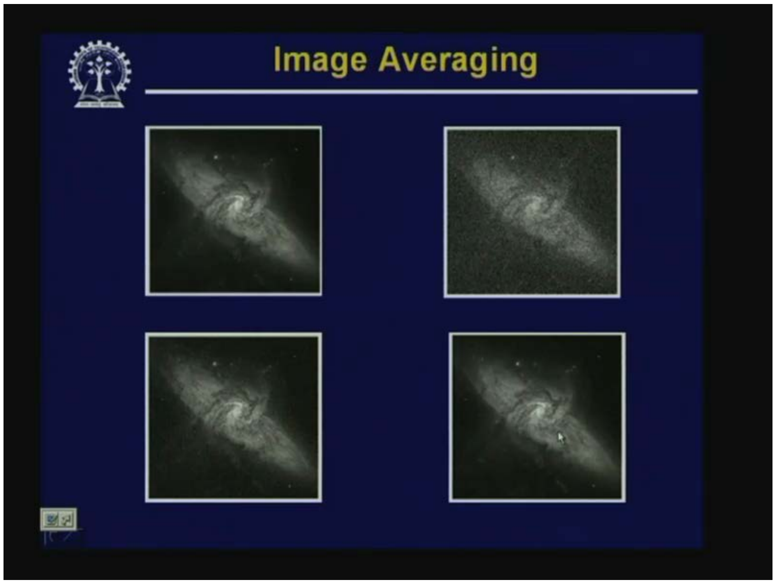


So, here this is the image of a galaxy. On the top left, on the top right; we have the corresponding noisy image and on the bottom, we have the images which are averaged over a number of frames. So, the last one is an average image where the average is taken over 128 number of frames and here the number of frames is less and as it is quite obvious from this that as you increase the number of frames, the amount of noise that you have in the processed image is less and less.

So, with this, we come to an end to our discussion on point processing techniques for image enhancement operations. Now, let us discuss the questions that we have placed in the last class. The first one is what is an image histogram? So, you find that few of the questions are very obvious. So, we are not going to discuss about them.

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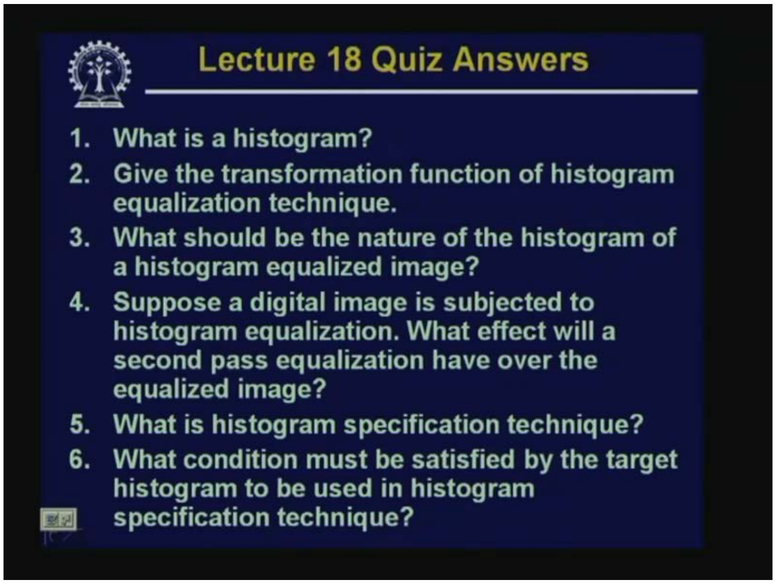


Now, the fourth one is very interesting that suppose a digital images is subjected to histogram equalization; what is the effect, what effect will a second pass equalization have over the equalized image? So, as we have already mentioned that once a image is histogram equalized, the histogram of the processed image will be a uniform histogram that means it will have an uniform probability density function and if I want to equalize this equalized image, then you find that the corresponding transformation function will be a linear one where the straight line will be inclined at an angle of 45 degree with the x axis.

So, that clearly indicates that whatever kind of equalization we do over an already equalized image that is not going to have any further effect on the processed image. So, this is ideal case but practically we have seen that after equalization, the histogram that you get is not really uniform. So, there will be some effect in the second pass the effect but the effect may be negligible. Sixth one is again a tricky one; what condition must be satisfied by the target histogram to be used in histogram specification technique?

You find that in case of histogram specification techniques, the target histogram is used for inverse transformation that is G inverse. So, it must be that the it must be true that the transformation function G has to be monotonically increasing and that is only possible if you have the value of $P_z$ (z) non 0 for every possible value of z. So, that is the condition that must be satisfied by the target histogram.

Now, coming to today's questions; first one is explain why the discrete histogram equalization technique does not in general yield a flat histogram.

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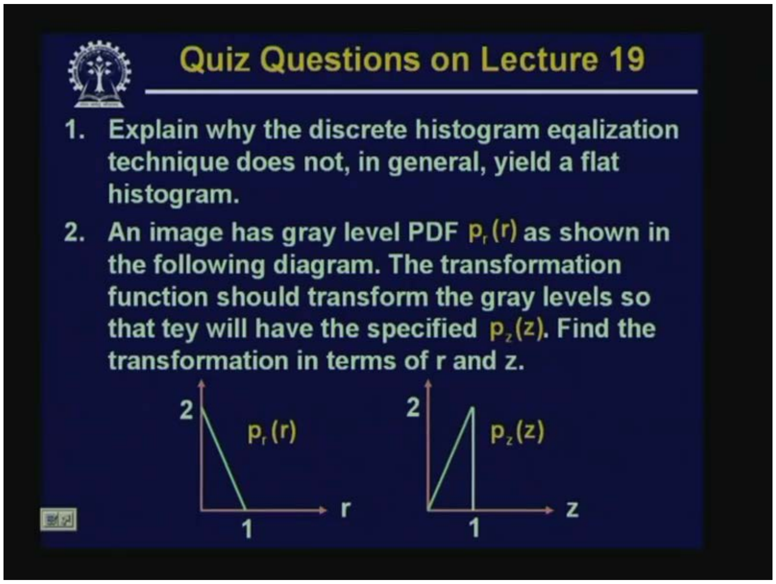


The second, an image has gray level PDF $P_r$ (r) as shown here and the target histogram as shown on the right. We have to find out the transformation in terms of r and z that is what is the mapping from r to z.

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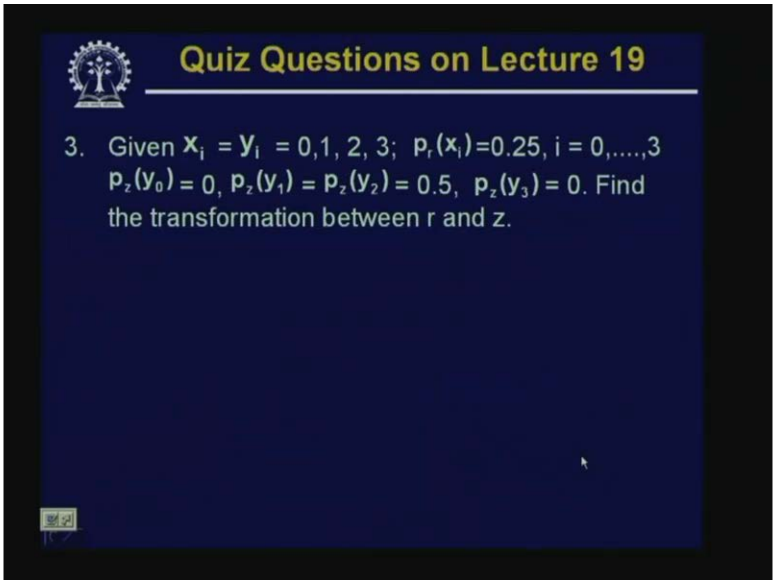


The third question, we have given the probability density functions to probability density functions. Again, you have to find out the Transformation between r and z.

Thank you.