Digital Image Processing

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Lecture - 14

Fourier Transformation – II

Hello, welcome to the video lecture series on digital image processing.

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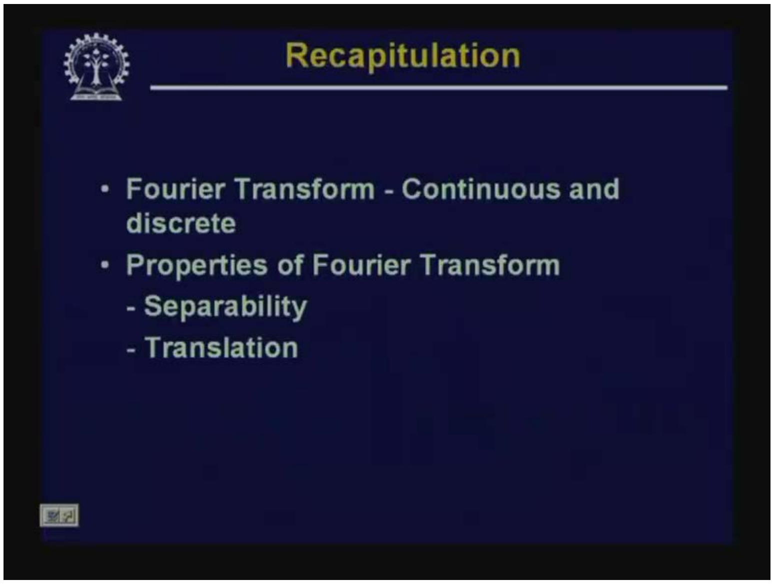

In our last lecture, we have started discussion on the Fourier transformation and towards the end we have seen some of the properties of the Fourier transformation. So, what we have done in the last class is we have talked about the Fourier transformation both in the continuous and in the discrete domain and we have talked about some of the properties of the Fourier transformation like the separability property and the translation property.

Today, we will continue with our lecture on the Fourier transformation and will see the other properties of the Fourier transformation and we will talk about how to implement Fourier transformation in a faster way. That is we will talk about the fast Fourier transformation algorithm.

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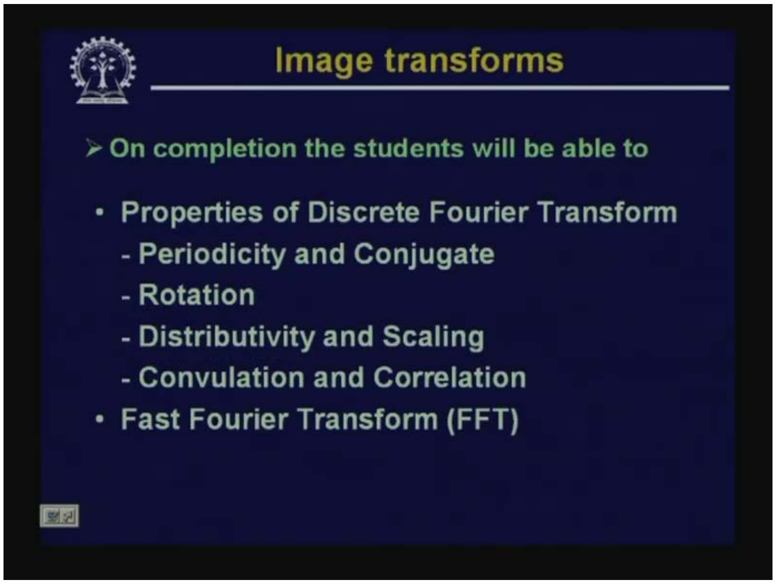

So, in today's lecture, we will see the properties of the discrete Fourier transformation, specifically the periodicity and conjugate property of the Fourier transformation. We will talk about the rotation property of the Fourier transformation, we will see the distributivity and the scaling property of the Fourier transformation followed by the convolution and correlation property of the Fourier transformation and then we will talk about an implementation, a fast implementation of the Fourier transformation which is called fast Fourier transform.

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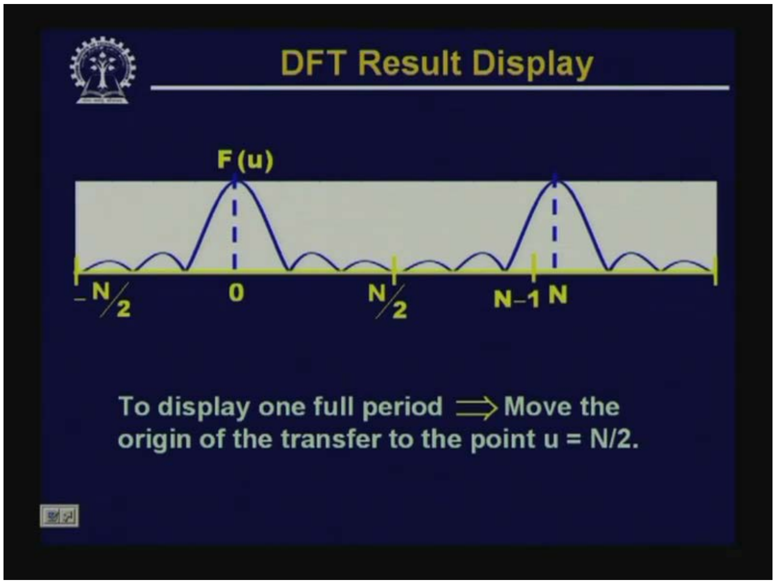

So first, let us see what just try to repeat, what we have done in the last class.

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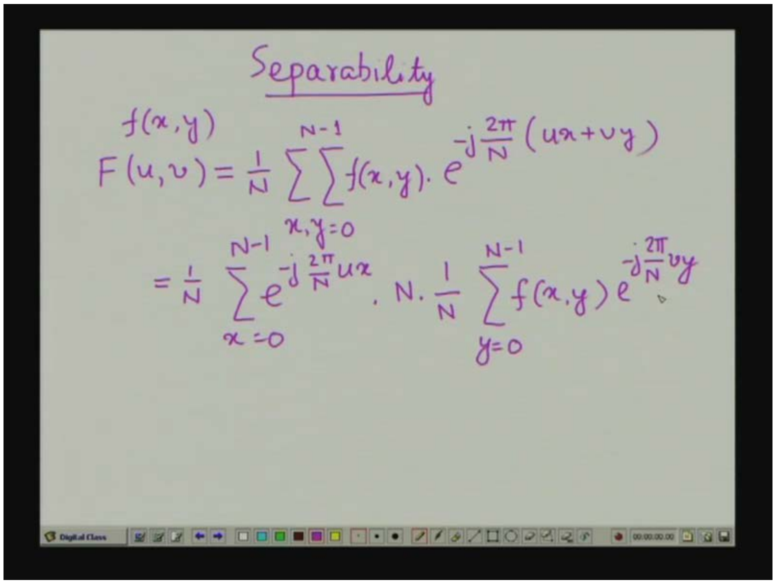

So in the last class, we have talked about the separability. We have talked about the separability of the Fourier transformation and here, we have seen that given a 2 dimensional signal f (x, y) in the discrete domain that is samples of this 2 dimensional signal f (x, y); we can compute the Fourier transformation of f (x, y) as F (u, v) which is given by the expression 1 upon capital N where our original signal f (x, y) is of dimension capital N by capital N.

And, the Fourier transformation expression comes as f (x, y) into e to the power minus j 2 pi by N into ux plus vy where both x and y vary from 0 to capital N minus 1 and if I rearrange this particular expression, then this expression can be written in the form 1 upon capital N then summation e to the power minus j 2 pi by capital N ux and then multiply this quantity by capital N and then 1 upon N again a summation f (x, y) e to the power minus j 2 pi by capital N vy.

So, in the inner summation, it is taken from y equal to 0 to capital N minus 1 and the outer summation is taken from x equal to 0 to capital N minus 1 and here we have seen that this inner summation, this gives the Fourier transformation of different rows of the input image f (x, y) and the outer summation, this outer summation gives the Fourier transformation of different columns of the intermediate result that we have obtained.

So, the advantage of this separability property that we have seen in the last class is because of this separability property; we can do the Fourier transformation, 2 dimensional Fourier transformations in 2 steps. In the first step, we take the Fourier transformation of every individual row of the input image array and in the second step we can take the Fourier transformation of every column of the intermediate result that has been obtained in the first step. So now, the implementation of the 2 dimensional Fourier transformations becomes very easy.

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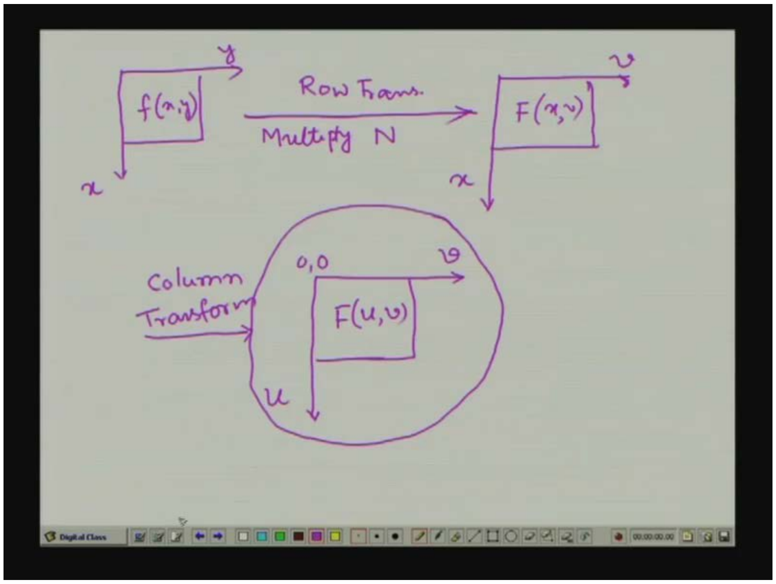

So, the scheme that we have said in the last class is like this. If I have an input array given by f (x, y) where this is the x dimension, this is the y dimension. So, first what we do is we do row transformation that is take Fourier transformation of every row of the input image, multiply the result by capital N. So, what I get is an intermediate result array and this intermediate result array gives Fourier transformation of different rows of the input image.

So, this is represented as F (x, v) and this is my x dimension and this becomes the v dimension. And after getting this intermediate result, I take the second step of the Fourier transformation and now the Fourier transformation is taken for every column. So, I do column transformation and that gives us the final result of the 2 dimensional Fourier transformations F (u, v). So, this becomes my u axis, the frequency axis u, this becomes frequency axis v and of course, this is the origin (0, 0).

So, it shows that because of the separability property, now the implementation of the 2 dimensional Fourier transformation has been simplified because the 2 dimensional Fourier transformation can now be implemented as 2 step of 1 dimensional Fourier transformation operations and that is how we get this final Fourier transformation F (u, v) in the form of the sequence of 1 dimensional Fourier transformations and we have seen in the last class that the same is also true for inverse Fourier transformation.

Inverse Fourier transformation is also separable. So, given an array F (u, v), we can do first inverse Fourier transformation of every row followed by inverse Fourier transformation of every column and that gives us the final output in the form of F (x, y) which is the image array. So, this is the advantage that we get because of separability property of the Fourier transformation.

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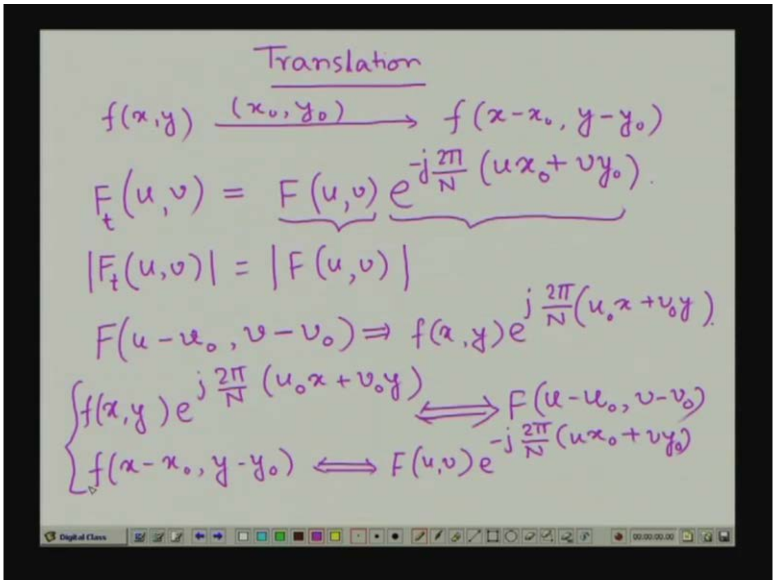

The second one, the second property that we have discussed in the last class is the translation property. So, this translation property says that if we have an input image f (x, y), input image array f (x, y); then translate this input image by (x_0, y_0). So, what we get is a translated image f (x minus x_0) and (y minus y_0). So, if we take the Fourier transformation of this, we have found that the Fourier transformation of this translated image which we had represented as F_t (u, v), this became equal to F (u, v) into e to the power minus j 2 pi by capital N ux_0 plus vy_0.

So, if you find, in this case, the Fourier transformation of the translated image is F (u, v) that is the Fourier transformer of the original image f (x, y) which is multiplied by e to the power of minus j 2 pi by N ux_0 plus vy_0. So, if we consider the Fourier spectrum of this particular signal, you will find that the Fourier spectrum that is F transpose F_t (u, v) will be same as F (u, v).

Now, this term e to the power of minus j 2 pi by N ux_0 plus vy_0, this simply introduces an additional phase shift. But the Fourier spectrum remains unchanged and in the same manner, if the Fourier spectrum F (u, v) is translated by u_0 v_0. So, instead of taking F (u, v), we take F(u minus u_0) (v minus v_0) which obviously is the translated version of u F (u, v) where F (u, v) has been translated by vector u_0 v_0 in the frequency domain. And if I take the in inverse Fourier transform of this, the inverse Fourier transform will be f (x, y) into e to the power j 2 pi by N into u_0 x plus v_0 y. So, this also can be derived in the same manner in which we have done the forward Fourier transformation.

So here, you find that if f (x, y) is multiplied by this exponential term e to the power j 2 pi by N u_0 x plus v_0 y; then the corresponding in the frequency domain, its Fourier transform is simply translated by the vector (u_0, v_0). So, what we get is F_u minus u_0 and v minus v_0. So, under this translation property, now the DFT pair becomes if we have F (x, y) e to the power j 2 pi by capital N u_0 x plus v_0 y. The corresponding Fourier transformation of this is F (u minus u_0, v minus v_0) and if we have the translated image f (x minus x_0, y minus y_0), the corresponding Fourier transformation will be F (u, v) e to the power minus j 2 pi by N ux_0 plus vy_0.

So, these are the Fourier transform pairs under translation. So, this f (x, y) and f (x minus x_0, y minus y_0); so these 2 expressions gives you the Fourier transform pairs, the DFT pairs under translation. So, these are the 2 properties that we have discussed in the last lecture.

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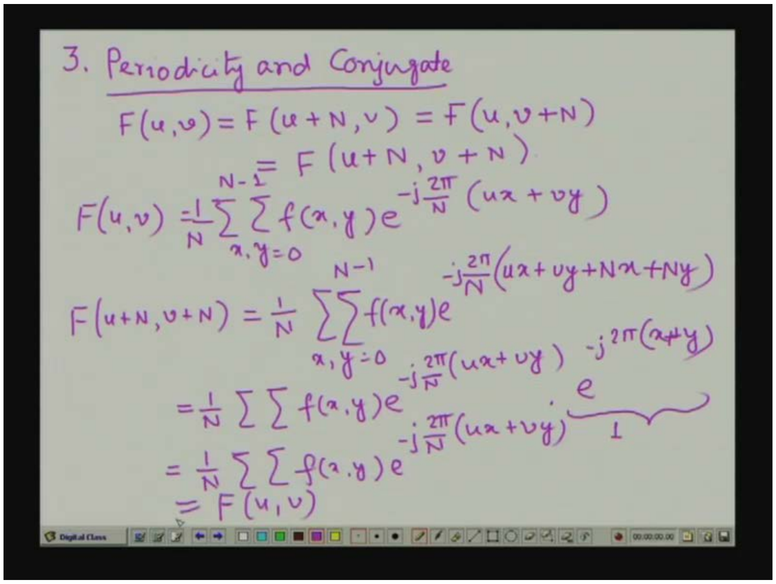

Today, let us talk about some other properties. So, the third property that we will talk about today is the periodicity and conjugate property. So, the first one that we will discuss is the periodicity and the conjugate property. The Periodicity property says that both the discrete Fourier transform and the inverse discrete Fourier transform that is DFT and IDFT are periodic with a period capital N. So, let us see how this periodicity can be proved.

So, this periodicity property says that F (u, v), this is the Fourier transform of our signal f (x, y). This is equal to F (u plus N, v) which is same as F (u, v plus N) which is same as F (u plus capital N, v plus capital N). So, this is what is meant by periodic. So, you will find that the Fourier transformation F (u, v) is periodic both in x direction and in y direction that gives rise to F (u, v) is equal to F (u plus N), F (v plus N) which is same as F (u plus N, v) and which is also same as F (u, v plus N).

Now, let us see how we can derive or we can prove this particular property. So, you have seen the Fourier transformation expression as we have discussed many times F (u, v) is equal to double summation f (x, y) e to the power minus j 2 pi by capital N ux plus vy. Of course, we have to have the scaling factor 1 upon capital N where both x and y vary from 0 to capital N minus 1.

Now, if we try to compute F (u plus capital N, v plus capital N) then what do we get? Following the same expression, this will be nothing but 1 upon capital N then double summation f (x, y) e to the power minus j 2 pi upon capital N and now we will have ux plus vy plus capital Nx because now u is replaced by u plus capital N, so we will have capital Nx plus capital Ny where both x and y will vary from 0 to capital N minus 1.

Now, this same expression, if we take out this capital Nx and capital Ny in a separate exponential, then this will take the form 1 upon capital N double summation f (x, y) e to the power minus j 2 pi upon capital N ux plus vy into e to the power minus j 2 pi into x plus y. Now, if you look at this second exponential term that is e to the power minus j 2 pi x plus y, you will find that x and y are the integer values. So, x plus y will always be integer. So, this will be the exponential e to the power minus j some k times 2 pi and because this is an exponentiation of some integer multiple of 2 pi; so the value of this second exponential will always be equal to 1.

So finally, what we get is 1 upon capital N double summation f (x, y) into e to the power minus j 2 pi upon capital N into ux plus vy and you will find that this is exactly the expression of F (u and v). So, as we said that the discrete Fourier transformation is periodic with period N, capital N both in the u direction as well as in the v direction and that is that can very easily we proved like this by this mathematical derivation; we have found that F (u plus capital N, v plus capital N) is same as F (u, v) and the same is true in case of inverse Fourier transformation.

So, if we derive the inverse Fourier transformation, then we will get the similar result showing that the inverse Fourier transformation is also periodic with period capital N. Now, the other property that we said is the conjugate property.

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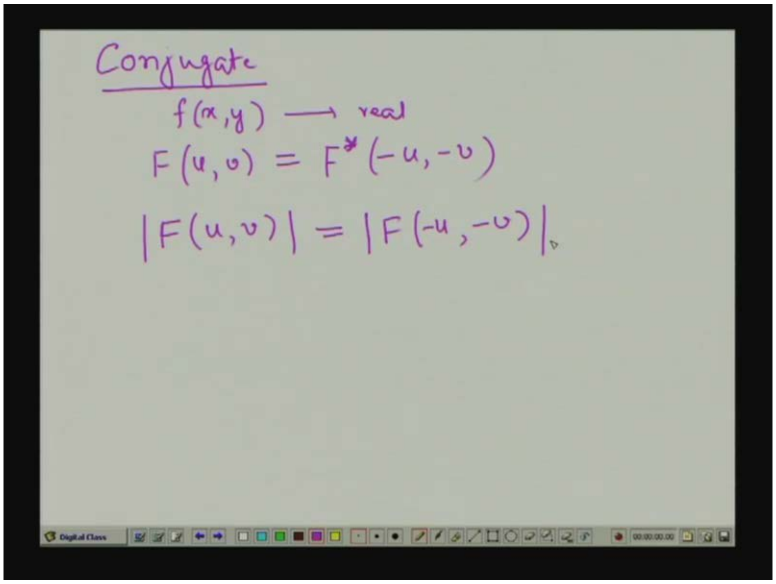

The conjugate property says that if f (x, y), this function, if this is a real value function; f (x, y) if it is a real value function, in that case the Fourier transformation F (u, v) will be F star (minus u, minus v) where this F star indicates that it is complex conjugate and obviously because of this, if I take the Fourier spectrum, F (u, v) will be same as F of (minus u, minus v). So, this is what is known as the conjugate property of the discrete Fourier transformation.

Now, find that using the periodicity property helps to visualize the Fourier spectrum of a given signal. So, let us see how this periodicity property helps us to properly visualize the Fourier spectrum. So for this, we will consider a 1 dimensional signal. Obviously, this can very easily be extended to a 2 dimensional signal.

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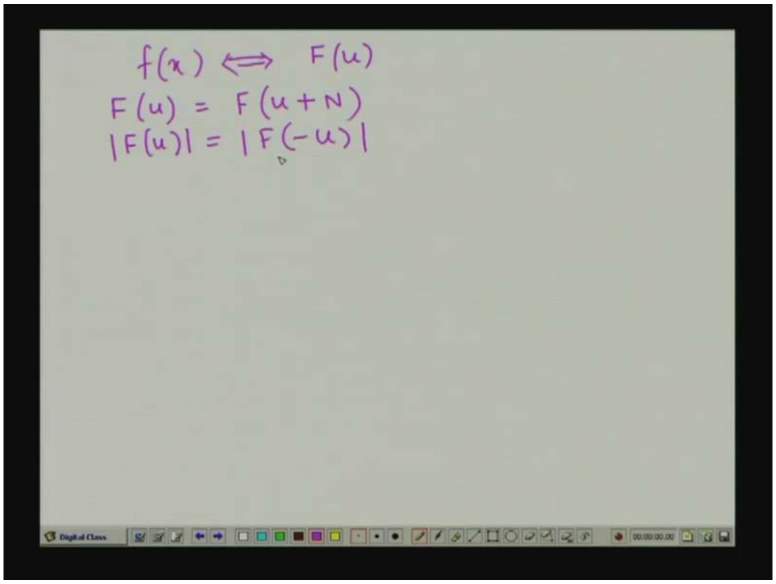

So, by this, what we mean is if we have a 1 dimensional signal say f (x) whose Fourier transform is given by capital F (u); then as we said, that the periodicity property says that F (u) is equal to F of u plus capital N and also the Fourier spectrum F of u is same as F of minus u. So, this says that F (u) has a period of length capital N and because the spectrum F (u) is same as F of minus u, so the magnitude of the Fourier spectrum of the Fourier transform is centered at the original. So by this, what we mean is, let us consider a figure like this.

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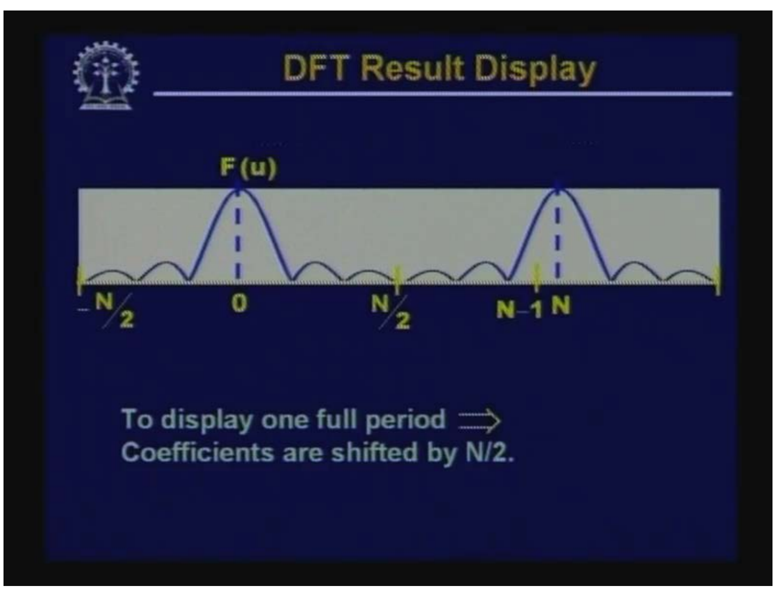

You will find that this is the typical this is a typical Fourier transform of a particular signal and here you find that this Fourier spectrum, the Fourier's transform is centered at the origin and if you look at the frequency access; so this is the u access, if you look at this frequency access, you will find that F of minus u, the magnitude of F of minus u is same as the magnitude of the F of plus u.

So, this figure shows that the transform values, if we look at the transform values from N by 2 plus 1; so that is somewhere here, this is N by 2 plus 1 to N minus 1, so that is somewhere here. So, find that the transform values in the range N by 2 plus 1 to N minus 1, this is nothing but the transform values in the left, transform values of the half period in the left half in the left of the origin.

So, just by looking at this, the transform values from N plus 1 N by 2 plus 1 to N minus 1, you will find that these values are nothing but the reflections of the half period to the left of the origin 0. But what we have done is we have computed the Fourier transformation in the range 0 to N minus 1. So, you will get all the Fourier coefficients in the range 0 to N minus 1. So, the Fourier coefficients ranging the values of u from 0 to N minus 1 and because of this conjugate property, you will find we find that in this range 0 to capital N minus 1, what we get is 2 back to back half periods of this interval. So, this is nothing but 2 back to back half periods. So, this is 1 half period, this is 1 half period and they are placed back to back.

So, to display these Fourier transformation coefficients in the proper manner, what we have to do is we have to displace the origin by a value capital N by 2.

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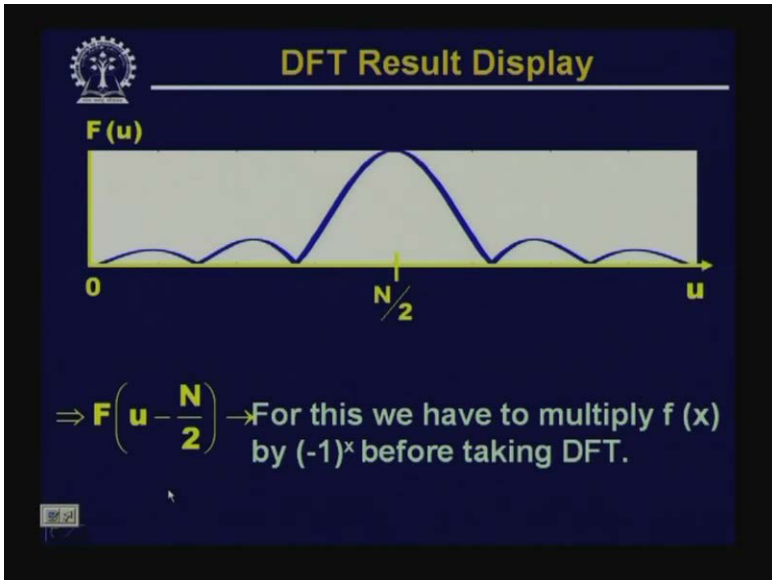

So, by displacement what we get is this. So, here you will find that in this particular case, the origin has been shifted to capital N by 2. So now, what we are doing is instead of considering the Fourier transformation F (u), we are considering the Fourier transformation F (u minus capital N by 2) and for this displacement, what we have to do is we have to multiply f (x) by minus 1 to the power x.

So, every f (x) has to be multiplied by minus 1 to the power x and this result, if you take the DFT of this; then what you get is the Fourier transformation coefficients in this particular form and this comes from the shifting property of the inverse Fourier transformation. So, this operation we have to do if we want to go for the proper display of the Fourier transformation coefficients.

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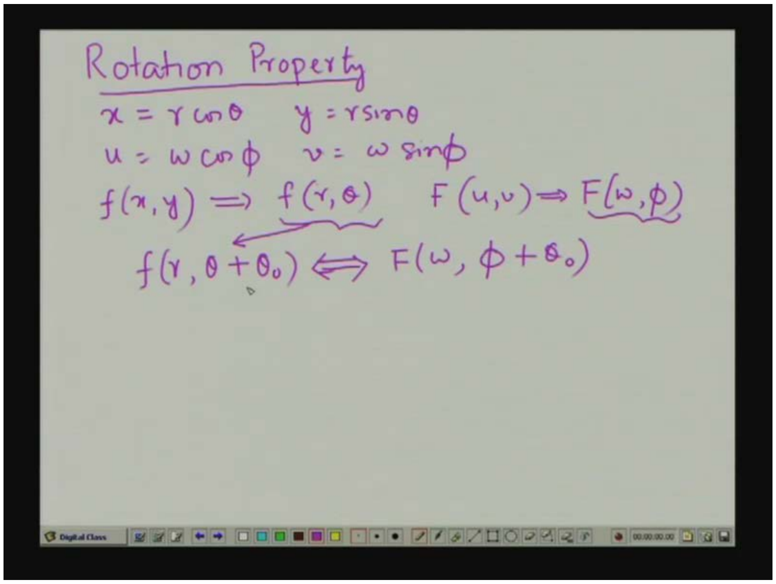

The next property that we will talk about is the rotation property, rotation property of the discrete Fourier transformation. So, to explain this rotation property, we will introduce the polar coordinated coordinate system that is we will now replace x by r cosine theta, y will be replaced by r sin theta, u will be replaced by omega cosine phi and v will be replaced by omega sin phi.

So by this, now our original 2 dimensional signal, 2 dimensional array in the plane f (x, y) gets transformed into f (r, theta) and the Fourier transformation F (u, v), the Fourier transform coefficients F (u, v) now gets transformed into F of omega phi. Now, using these polar coordinates if we find out, compute the Fourier transformation; then it will be found that f of r theta plus theta_0, the corresponding Fourier transformation will be given by capital F omega phi plus theta_0.

So, this will be the Fourier transformation pair in the polar coordinate system. So, this indicates our original signal was f (r, theta). If I rotate this f (r, theta) by an angle theta 0, then the rotated image becomes f (r, theta plus theta_0) and if I take the Fourier transform of f (r, theta plus theta_0) that is the rotated image which is now rotated by an angle theta_0, then the Fourier transform becomes F omega phi plus theta_0 where F omega phi was the Fourier transform of the original image f (r, theta).

So, this simply says that if I rotate image f (x, y) by an angle say $theta_0$, its Fourier transformation will also be rotated by the same angle $theta_0$ and that is what is obvious from this particular expression because f of r theta plus $theta_0$ gives rise to the Fourier transformation f of capital F omega plus omega phi theta phi plus $theta_0$ where f omega phi was the Fourier transformation of f (r, theta). So, by rotating an input image by an angle $theta_0$, the corresponding Fourier transform is also rotated by the same angle $theta_0$.

So, to illustrate this, let us come to this particular figure.

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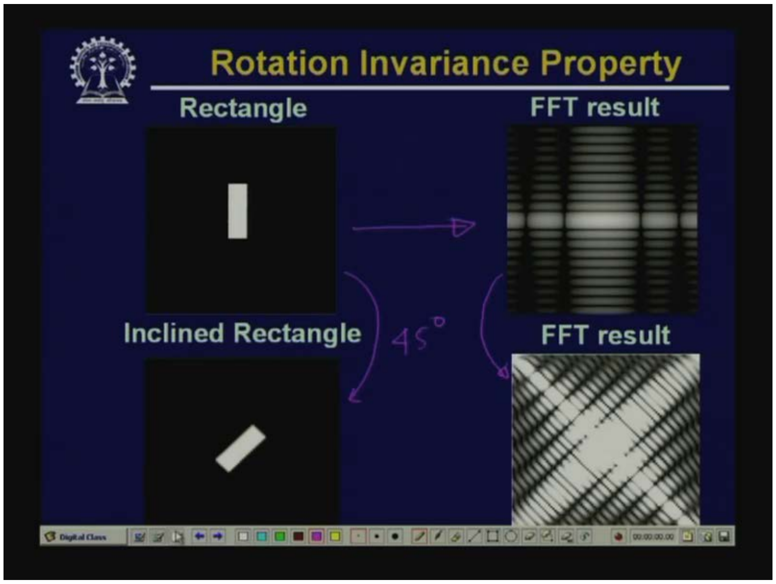

So here, you find that we had a rectangle, an image where we have all values we have pixel values equal to 1 within a rectangle and outside this, the pixel values are equal to 0 and the corresponding Fourier transformation is this. So here, the Fourier transformation coefficients or the Fourier spectrum is represented in the form of intensity values in an image. The second pair shows that the same rectangle is now rotated by an angle 45 degree.

So here, we have rotated this rectangle by angle 45 degree and here you find that if you compare the Fourier transformation of the original rectangle and the Fourier transformation of this rotated rectangle; here also you will find that the Fourier transform coefficients, they are also rotated by the same angle of 45 degree. So, this illustrates the rotation property of the discrete Fourier transformation.

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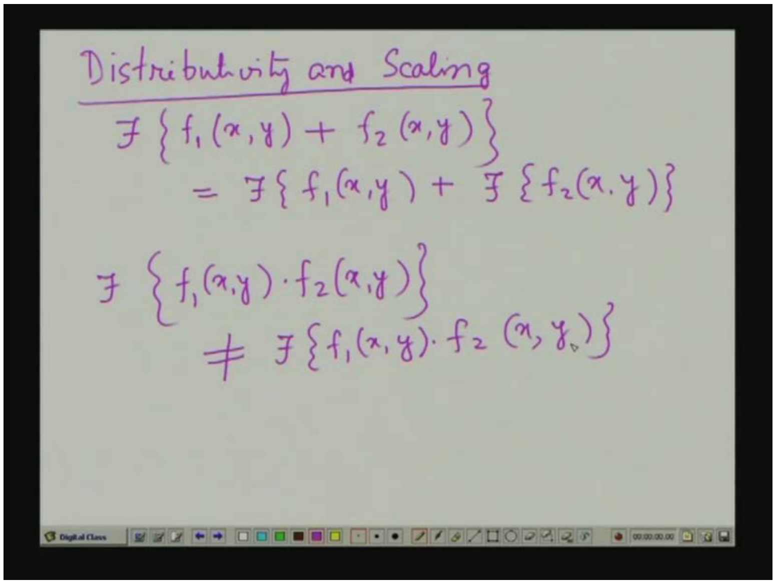

The next property that we will talk about is what is called distributivity and scaling property. The distributive property says that if I take 2 signals, 2 arrays f_1 (x, y) and f_2 (x, y); so these are 2 arrays, take the summation of these 2 arrays f_1 (x, y) and f_2 (x, y) and then you find out the Fourier transformation of this particular result. That is f_1 (x, y) plus f_2 (x, y) and take the Fourier transform of this.

Now, this Fourier transformation will be same as the Fourier transformation of f_1 (x, y) plus Fourier transformation of f_2 (x, y). So, this is true under addition. That is for these 2 signals f_1 (x, y) and f_2 (x, y) if I take the addition, if I take the summation and then take the Fourier transformation; the Fourier transformation of this will be the summation of the Fourier transformation of individual signals f_1 (x, y) and f_2 (x, y).

But if I take the multiplication that is if I take f_1 (x, y) into f_2 (x, y) and take the Fourier transformation of this product; this in general is not equal to the Fourier transform of f_1 (x, y) into the Fourier transform of f_2 (x, y). So, this shows that the discrete Fourier transformation and same is true for the inverse Fourier transformation.

So, this shows that the discrete Fourier transformation and its inverse is distributive over addition but the discrete Fourier transformation and its inverse is in general not distributive over multiplication. So, the distributivity property is valid for addition of the 2 signals but it is not in general valid for multiplication of 2 signals.

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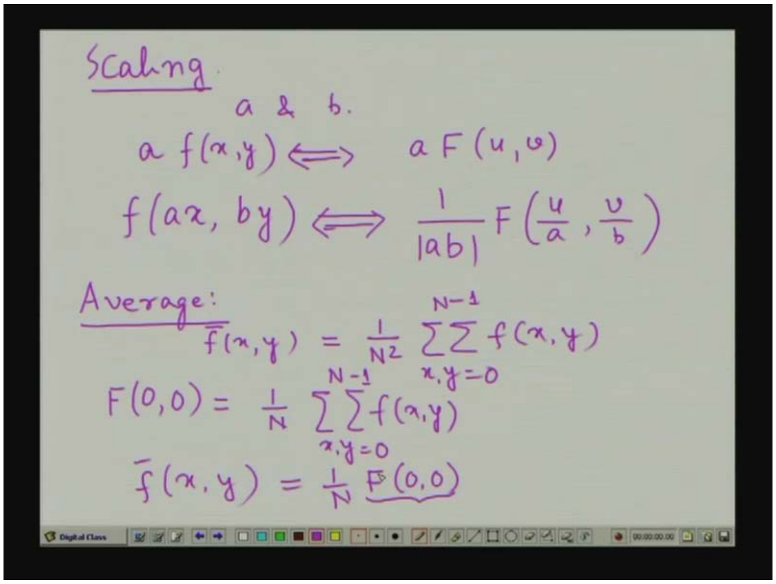

So, the next property of the same discrete Fourier transform that we will talk about is the scaling property. The Scaling property says that if we have 2 scalar quantities a and b; now given a signal f (x, y), multiply this by the scalar quantity a, it’s corresponding Fourier transformation will be F (u, v) multiplied by the same scalar quantity and the inverse is also true.

So, if I multiply a signal by a scalar quantity a and take its Fourier transformation; then we will find the Fourier transformation of this multiplied signal is nothing but the Fourier transformation of the original signal multiplied by the same scalar quantity and the true but the same is true for the reverse that is also for inverse Fourier transformation.

And, the second one is if I take f of ax, by that is now you scale the individual dimensions x is scaled by the scalar quantity a, the dimension y is scaled by the scalar quantity b; the corresponding Fourier transformation will be 1 upon a into b, then Fourier transformation u by a and v by b and this is the reverse. So, these are the scaling properties of the discrete Fourier transformation.

Now, we can also compute the average value of the signal f (x, y). Now, the average value for f (x, y) is given by if I represent it like this, this is nothing but 1 upon capital N square into summation of f (x, y) where the summation has to be taken for x and y varying from 0 to capital N minus 1. So, this is what is the average value of the signal f (x, y).

Now, you find that for the Fourier coefficient, the transform coefficient say f (0, 0); what is this coefficient? This is nothing but 1 upon capital N then double summation f (x, y) because all the exponential terms will lead to a value 1 and this summation has to be taken for x and y varying from 0 to capital N minus 1.

So, you will find that there is direct relation between the average of the 2 dimensional signal f (x, y) and its 0’th Fourier coefficient, DFT coefficient. So, this clearly shows that the average value f (x, y), the average value is nothing but 1 upon capital N into the 0'th coefficient 0'th discrete Fourier transformation coefficient and this is nothing but because here the frequency u equal to 0, frequency v equal to 0, so this is nothing but the DC component of the signal. So, the DC component divided by N gives you the average value of the particular signal.

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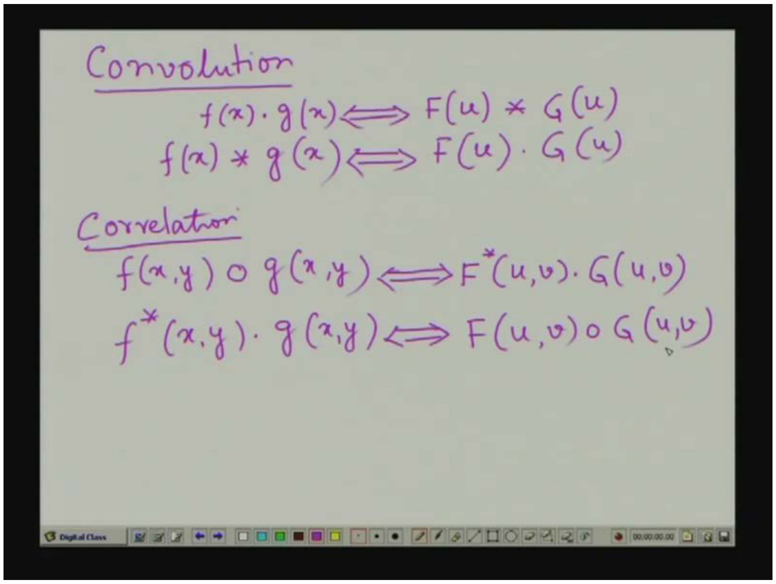

The next property, this we have already discussed in one of our earlier lectures when we have discussed about the sampling and quantization. That is the convolution property. In case of convolution property, we have said that if we have say 2 signals f (x), multiply this with the signal g (x); then the Fourier transform in the frequency domain, this is equivalent to F of u convolution with G of u.

Similarly, if I take the convolution of 2 signals f (x) and g (x); the corresponding Fourier transformation in the Fourier domain, it will be the multiplication of F (u) and G (u). So, the convolution of 2 signals in the special domain is equivalent to multiplication of the Fourier transformations of the same signals in the frequency domain. On the other hand, multiplication of 2 signals in the special domain is equivalent to convolution of the Fourier transforms of the same signals in the frequency domain. So, this is what is known as the convolution property.

The other one is called the correlation property. The correlation property says that if we have 2 signals say f (x, y) and g (x, y), so now we are taking 2 dimensional signals and if I take the correlation of these 2 signals say f (x, y) and g (x, y); in the frequency domain, this will be equivalent to the multiplication F star (u, v) where this star indicates the complex conjugate into G (u, v).

And similarly, if I take the multiplication in the special domain that is f star xy into g (x, y); in the frequency domain, this will be equivalent to F (u, v) correlation with G (u, v). So, these are the 2 properties which are known as the convolution property and the correlation property of the

Fourier transformations. So with this, we have discussed the various properties of the discrete Fourier transformation.

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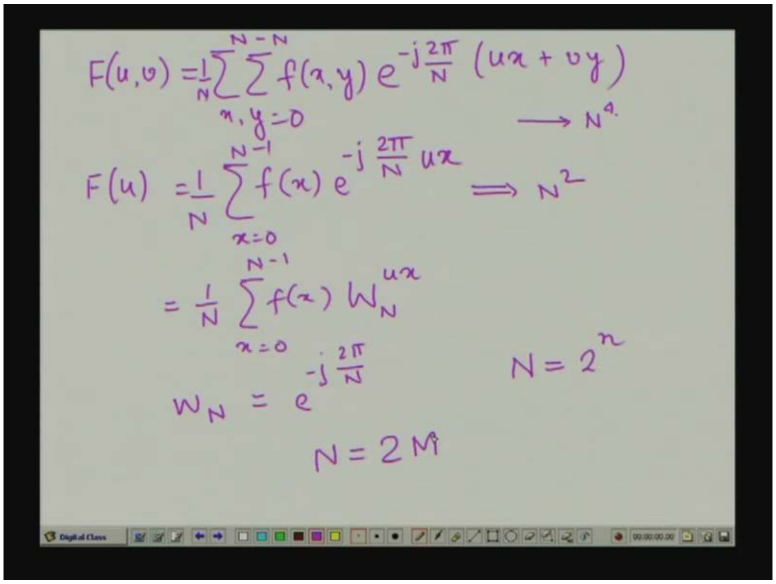

Now, let us see an implementation of the Fourier transformation because if you look at the expression of Fourier transformation, the expression we have told many times; this is F (u, v) which is same as f (x, y) e to the power minus j 2 pi by capital N ux plus vy where both x and y vary from 0 to capital N minus 1 and this divided by 1 upon N. So if I compute, if I analyze this particular expression which we have done earlier also in relation with unitary transformation, you will find that this text N to the power 4 number of computations.

In case of 1 dimensional signal, F (u) will be given by f (x) e to the power minus j 2 pi by capital N ux summation of this over x equal to 0 to capital N minus 1 and you have to scale it by 1 upon N. This particular expression takes N square number of computations. So obviously, the number of computations and each of these computations are complex addition and multiplication operations. So, you find that a computational complexity of N square for a data set of size capital N is quite high. So, for implementation, we have discussed earlier that if our transformations are separable; in that case, we can go for fast implementation of the transformations.

Let us see how that fast implementation can be done in case of this discrete Fourier transformation. So, because of this separability property, we can implement this discrete Fourier transformation in a faster way. So, for that what I do is let us represent this particular expression F (u) is equal to 1 upon capital N f (x) e to the power minus j 2 pi by N ux, take the summation from x equal to 0 to capital N minus 1; we represent this expression in the form 1 upon capital N F (x).

Now, I introduce a term W_N to the power ux where x varies from 0 to capital N minus 1. Now here, this W_N is nothing but e to the power minus j 2 pi by capital N. So, we have simply introduced this term for simplification of our expressions. Now, if I assume which generally is the case that the number of samples N is of the form say 2 to the power N; so if I assume that number of samples is of this form, then this capital N can be represented as 2 into capital M and let us see that how this particular assumption helps us.

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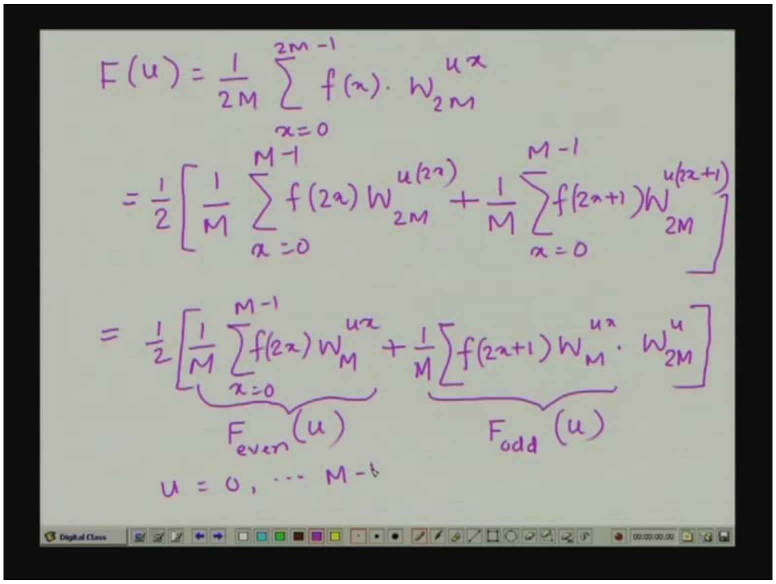

And with this assumption, now we can represent, rewrite F (u) as 1 upon 2 M because N is equal to 2 M. So now, I can write 1 upon 2 M, then take the summation f (x) into W $_{2M}$ to the power ux where x now varies from 0 to 2 m minus 1. The same expression I can rewrite as half 1 upon capital M summation F (2x) W $_{2M}$ to the power u into 2x plus 1 upon capital M summation F (2x plus 1) W $_{2M}$ to the power u into 2 x plus 1 where x varies from 0 to capital M minus 1, here also x varies from 0 to capital M minus 1.

Now by this, you see that what we have done. F (2x), as x varies from 0 to capital M minus 1, this gives us only the even samples of our input sequence. Similarly f (2x plus 1), as x varies from 0 to capital M minus 1, this gives us only the odd samples of the input sequence. So, we have simply separated out the even samples from the odd samples and if I further simplify this particular expression, this expression can now be written in the form half into 1 upon capital M summation f (2x) into W capital M to the power ux where x varies from 0 to capital M minus 1 plus 1 upon capital M summation f (2x) plus 1 W $_{M}$ to the power ux into W $_{2M}$ to the power u.

So after simplification, after some simplification, the same expression can be written in this particular form. Now, if you analyze this particular expression, you will find that the first summation, this one gives you the Fourier transform of all the even samples. So, this gives you F $_{even}$ u and this quantity in the second summation, this gives you the Fourier transformation of all the odd samples. So, I read it will write it as odd u and in this particular case, u varies from 0 to capital M minus 1.

So, by separating the even samples and odd samples, I can compute the Fourier transformation of the even samples to give me F $_{even}$ u; I can compute the Fourier transformation of the odd samples to give me F $_{odd}$ u and then I can combine these 2 to give me the Fourier DFT coefficients of values from 0 to capital M minus 1.

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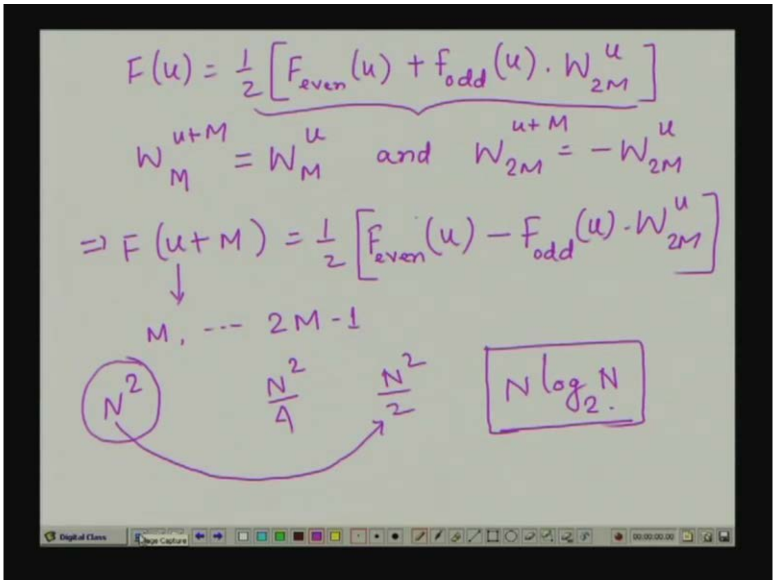

Now, following some more property, so effectively what we have got is F (u) is equal to half F $_{even}$ u plus F $_{odd}$ u into W $_{2M}$ to the power u. Now, we can also show that W_M to the power u plus M is same as W_M to the power u. This can be derived from the definition of W_M and also we can find out that W $_{2M}$ u plus M is same as minus W_{2m} to the power u. So, this tells us that capital F u plus capital M is nothing but half of F $_{even}$ u minus F $_{odd}$ u into W $_{2M}$ to the power u.

So here again, u varies from 0 to m minus 1. That means this gives us the coefficients from M to 2 M minus 1. So, I get back all the coefficients. The first part, this part gives us the coefficient from 0 to m minus 1 and this half gives us the coefficients from capital M to 2 M minus 1. Now, what is the advantage that we have got? In our original formulation, we have seen that the number of complex multiplications and additions were of the order of N square.

Now, we have divided the N number of samples into 2 half's. For each of them for each of the half's, when I compute the discrete Fourier transformation, the amount of computation will be N square by 4 for each of the half's and the total amount of computation will be of order N square by 2 taking 2 half's, considering 2 half's separately.

So, straight way we have got a reduction in the computation by a factor of 2. So, it is further possible that this odd half of the samples and the even half of the samples that we have got, we can further sub divide it. So, from N by 2, we can go to N by 4; from N by 4, we can go to N by 8 number of samples; from N by 8, we can go to N by 16 number of samples and so on until we are left with only 2 samples.

So, if I go further breaking this sequence of samples into smaller sizes, compute the DFT's of each of those smaller size samples and then combine them together, then you will find that we can gain enormously in terms of amount of computation and it can be shown that for this first Fourier transform implementation, the total number of computation is given by N log N and log is taken with this 2.

So, this gives enormous amount of computation, as enormous gain in computation as against N square number of computations that is needed for direct implementation of discrete Fourier transformation. So with this, we have come to the end of our discussion on Fourier transformation.

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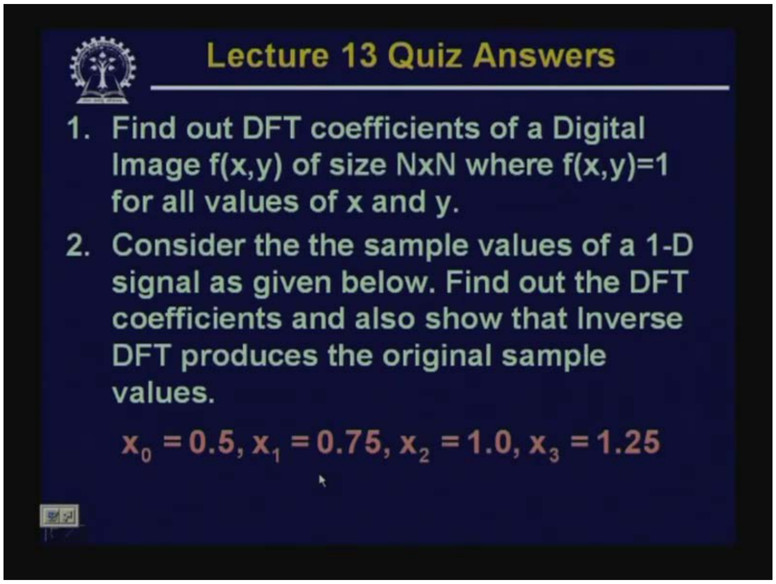

Now, let us discuss about these questions that we have given in our last class. The first question we said that find out DFT coefficients of a digital image f (x, y) of size capital N by capital N where f (x, y) equal to 1 for all values of x and y. Now, this computation is very simple.

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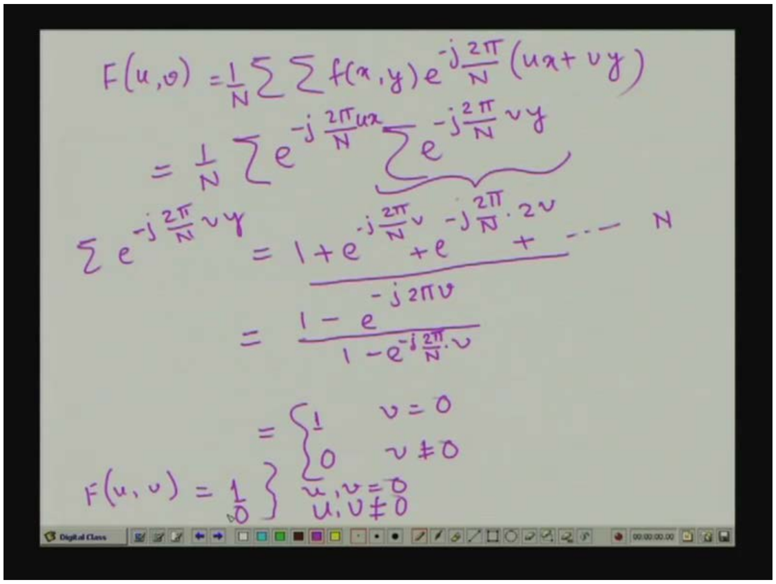

Here, you find that F (u, v) will be simply summation 1 upon N f (x, y) e to the power minus j 2 pi by capital N ux plus vy. Now, for simplicity, let me break it into 2 summations. So, I will write it as 1 upon capital N into e to the power minus j 2 pi by capital N summation f (x, y); no, f (x, y) is equal to 1, so I can simply forget this term f (x, y). So, this will be simply e to the power minus j sorry this is ux e from minus j 2 pi by N vy.

Now, let us take 1 of these terms. If I expand this, so summation e to the power minus j 2 pi by capital N vy, this will be simply 1 plus e to the power minus j 2 pi by capital N plus e to the power minus j 2 pi by capital N into 2 v, this is into v plus it continues like this and there will be total capital N number of terms and if you look at this particular series, it is nothing but a GP series having capital N number of terms.

So, this summation will simply be 1 minus e to the power minus j 2 pi into v divided by 1 minus e to the power minus j 2 pi by capital N into v and this particular term will be equal to 1 only when v equal to 0 and it will be equal to 0 when v is non 0. So, by substituting this and same is the case for the other summation. So, by substituting this result in this expression what we get is F (u, v) is equal to 1 in this particular expression when u and v is equal to 0 and this is equal to 0 when u and v are non 0. So, this is the final result that we will get for the first problem.

Now coming to the second problem, consider the sample values of a 1 dimensional signal as given below; find out the DFT coefficients and also show that the inverse DFT produces the original sample values. This is very simple. You simply replace these values in our DFT expressions. So, you get the DFT coefficients f_0, f_1, f_2 and f 3 and whatever value you get as the coefficients, you replace those values in our inverse DFT expression and you will see that you can get back the same sample values.

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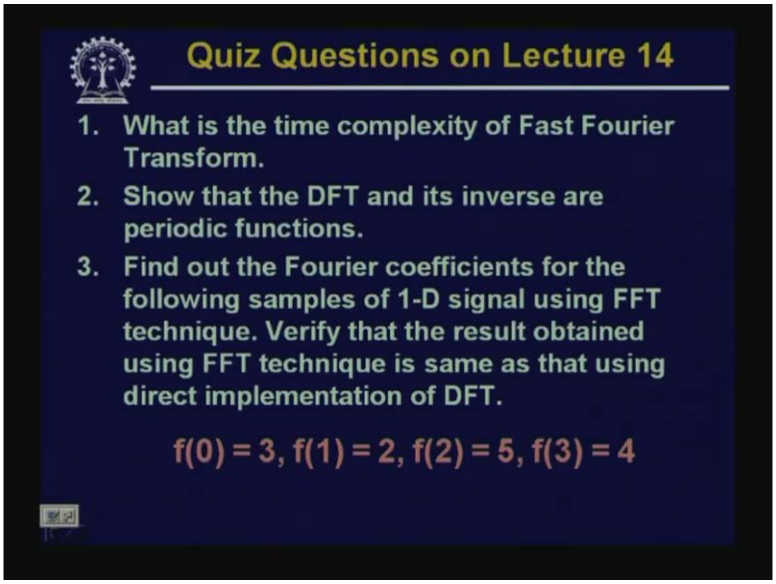

Now, coming to the today's questions, the today's questions are: what is the time complexity of fast Fourier transformation, the second question is show that the discrete Fourier transformation and its inverse are periodic functions, third question is find out the Fourier coefficients for the following set of 1 dimensional signal using the fast Fourier transformation technique and verify that the result obtained using the fast Fourier transformation technique is same as that using direct implementation of discrete Fourier transformation technique.

Thank you.