Digital Image Processing

Prof. P. K. Biswas

Department of Electronics & Electrical Communication Engineering

Indian Institute of Technology Kharagpur

Lecture - 13

Hello, welcome to the video lecture series on digital image processing. In the last 2 classes, we have seen the basic theories of unitary transformations and we have seen we have analyzed the computational complexity of the unitary transformation operations, particularly with respect to the image transformations.

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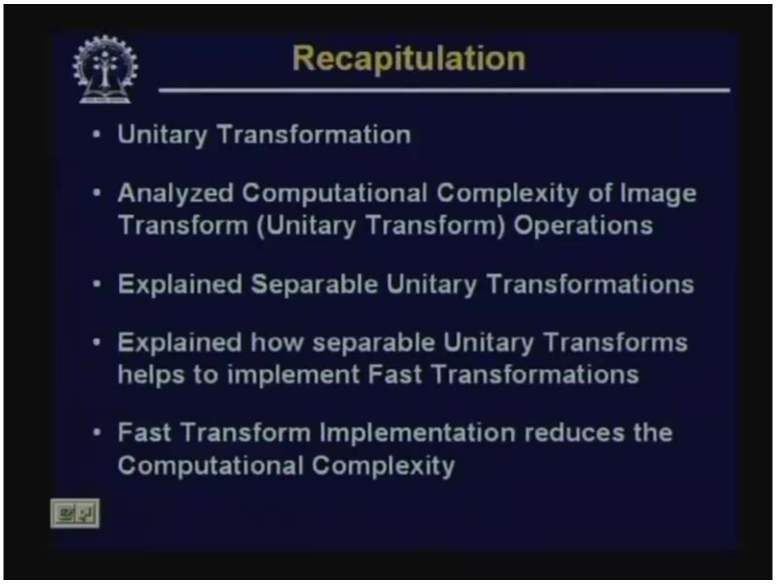

We have explained the separable unitary transformation where we have explained how separable unitary transformation helps to implement the fast transformations and fast transformation implementation as you have seen it during our last class; it reduces the computational complexity of the transformation operations.

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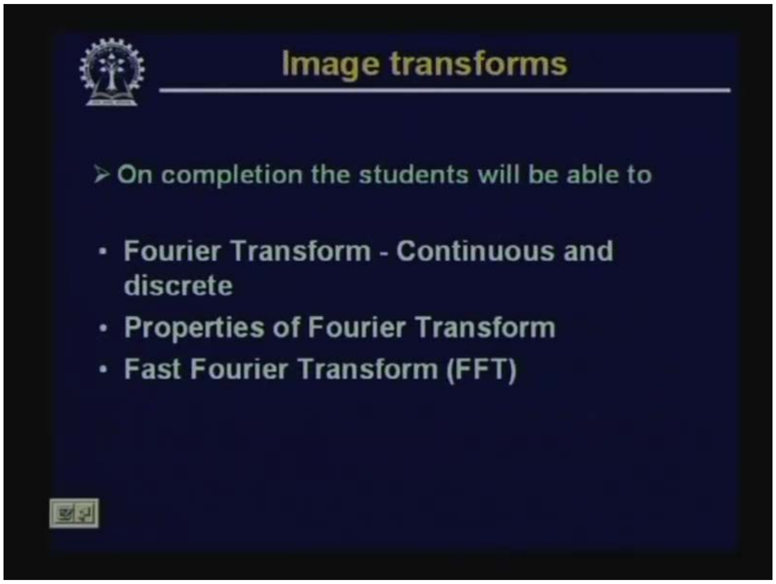

After giving the general unitary introduction to the general unitary transformations; in today's lecture we are going to discuss about the Fourier transformation which is a specific case of the unitary transformation. So, during today's lecture, we will talk about the Fourier transformation and we will talk about Fourier transformation both in the continuous domain as well as in discrete domain.

We will see what are the properties of the Fourier transformation operations and we will also see that what is meant by fast Fourier transform that is fast implementation of the Fourier transformation operation.

Now, this Fourier transformation operation, we have discussed in brief when we have discussed about the sampling theorem. That is given an analog image or continuous image while discretization, the first step of discretization was sampling the analog image. So, during our discussion on sampling, we have talked about the Fourier transformation and there we have said that Fourier transformation gives you the frequency components present in the image and for sampling, we must meet the condition that your sampling frequency must be greater than twice the maximum frequency present in the continuous image.

In today's lecture, we will discuss about the Fourier transformation in greater details. So, first let us see what is meant by the Fourier transformation.

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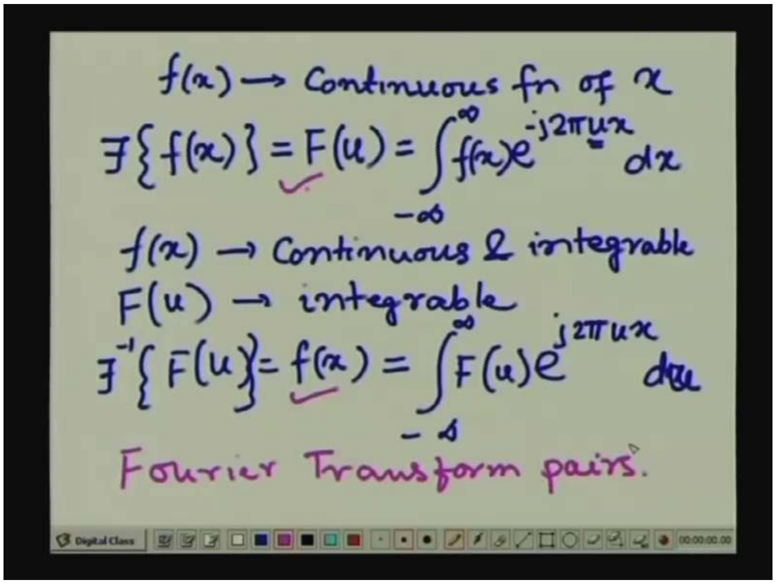

As we have seen earlier that if we assume a Function say f (x), so we will first talk about the Fourier transformation in the continuous domain and if we assume that f (x) is a continuous function, so this f (x) is a continuous Function of some variable say x; then the Fourier transformation of this Function f (x), we normally write it as the Fourier transformation of the Function f (x).

This is also written as capital F of u, this is given by the expression integral expression f (x) e to the power minus j 2 pi ux dx but the integration is carried over from minus infinity to infinity. Now, this variable ux, this is the frequency variable. So, given a Function f (x), a continuous Function f (x); by using this integration operation, we can find out the Fourier transformation of the Fourier transform of this continuous Function f (x) and the Fourier transform is given by F (u).

Now, for doing this continuous Fourier transformation, this Function f (x) has to meet some requirement. The requirement is the Function f (x) must be continuous, it must be continuous and it must be integrable. So, if f (x) meets these 2 requirements that is f (x) is continuous and integrable; then using this integral operation, we can find out the Fourier transformation of this continuous Function f (x).

Similarly, we can also have the inverse Fourier transformation. That is given the Fourier transform F (u) of a Function f (x) and if F (u) is integrable, F (u) must be integrable; then we can find out the inverse Fourier transform of F (u) which is nothing but the continuous Function f (x) and this is given by a similar integration operation and now it is F (u) integral e to the power j 2 pi ux dx and the sorry du and this integration again has to be carried out from minus infinity to infinity.

So, from f (x) using this integral operation, we can get the Fourier transformation which is the F (u) and if F (u) is integrable, then using the inverse Fourier transformation, we can get back the original continuous Function f (x) and these 2 expressions that is F (u) and f (x); the expressions for F (u) and expression for f (x), these 2 expressions are known as Fourier transform pairs. So, these 2 are known as Fourier transform pairs.

Now, from this expression, you find that because for doing the Fourier transformation; what we are doing is we are taking the Function f (x), multiplying it with an exponential e to the power minus j 2 pi ux dx and integrating this over the interval minus infinity to infinity. So naturally, this expression F (u) that you get is in general complex because e to the power minus j 2 pi ux this quantity is a complex quantity.

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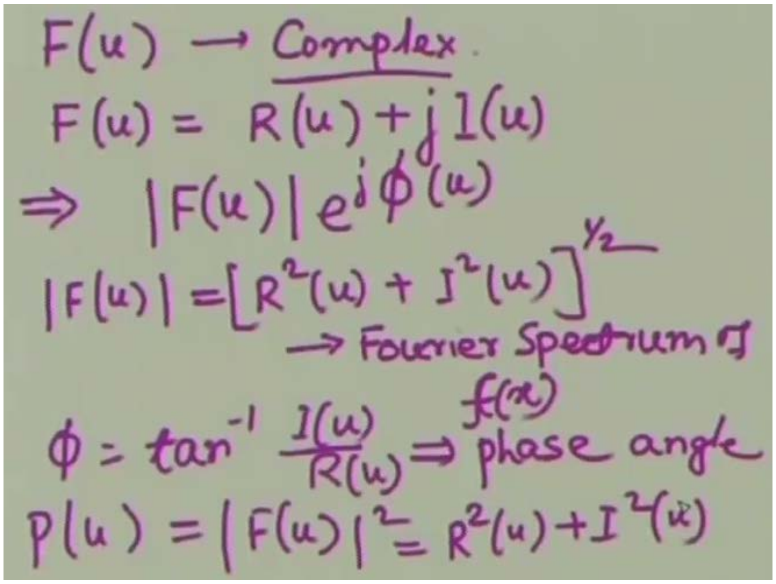

So, in general, the Function F (u), it is a complex function. In general, it is a complex Function and because this F (u) is a complex function; so we can write this F (u), we can break this F (u) in the real part - so the real part, we write as R (u) and the imaginary part - so it will be I (u). So, this F (u) which in general is a complex quantity is now broken into the real part and the imaginary part or the same F (u) can also be written in the form of modulus of F of u into e to the power j of phi u where this modulus of F of u which gives you the modulus of this complex quantity F (u) this is nothing but R (u) square plus I (u) square and square root of this and this is what is known as Fourier spectrum of f (x).

So, this we call as Fourier spectrum of the Function f (x) and this quantity - phi of u which is given by tan inverse I of u upon R of u, this is what is called the phase angle, this is the phase angle. So, from this we get what is known as the Fourier spectrum, Fourier spectrum of f (x) which is nothing but the modulus of the magnitude of the Fourier transformation F (u) and the tan inverse of the imaginary component I of u by the real component R of u. That is what is the phase angle for this particular, for a particular value of u.

Now, there is another term which is called the power spectrum. So, power spectrum of the Function f (x) which is also represented as p of u, this is nothing but F of u magnitude square and if you expand this, this will be simply R square u plus I square u. So, we get the power spectrum, we get the Fourier spectrum and we also get the phase angle from the Fourier transformation coefficients and this is what we have in case of 1 dimensional image because we have taken a Function f (x) which is a Function of a single variable x.

Now, because in our case, we were discussing about the image processing operations and we have already said that the image is nothing but a 2 dimensional Function which is a Function of 2 variables x and y; so we have to discuss about the Fourier transformation in 2 dimension rather than in single dimension.

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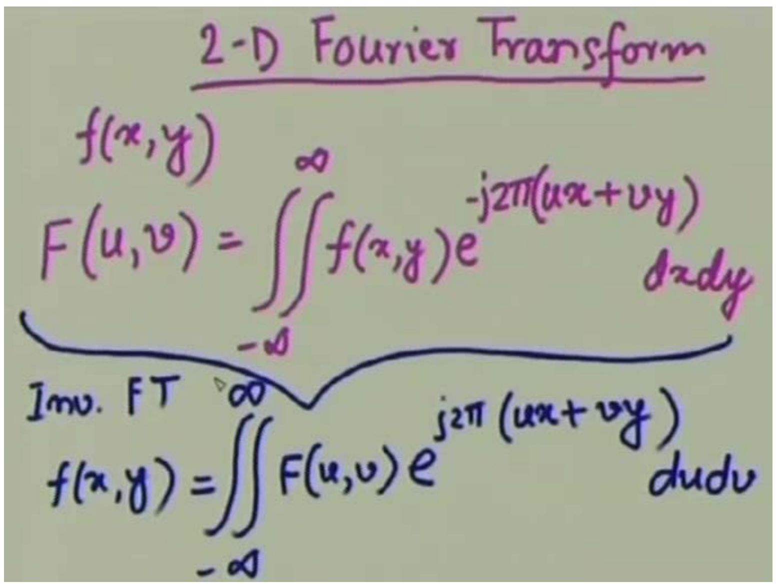

So, when we go for 2 dimensional Fourier transformation, so we will talk about 2D Fourier transform; the 1 dimensional Fourier transform that we have discussed just before can be easily extended to 2 dimension in the form that now in this case, a Function is a 2 dimensional Function f (x, y) which is a Function of 2 variables x and y and the Fourier transform of this f (x, y) is now given by F (u, v) which is equal to, now we have to have double integral f (x, y) e to the power minus j 2 pi ux plus vy dxdy and both these integrations have to be taken over the interval minus infinity to infinity.

So, we find that from a 1 dimensional Fourier transformation, we have easily extended that to 2 dimensional Fourier transformation and now this integration has to be taken over x and y because our image is a 2 dimensional image which is a Function of 2 variables x and y. So, the forward transformation is given by this expression F of (u, v) is equal to f (x, y) e to the power minus j 2 pi (ux plus vy) dxdy and integration has to be taken over from minus infinity to infinity.

In the same manner, the inverse Fourier transformation; so you can take the inverse Fourier transformation to get f (x, y) that is the image from its Fourier transform coefficients which are F (u and v) by taking the similar integral operation and in this case, it will be F (u, v) e to the power j 2 pi (ux plus vy) dudv and the integration has to be taken from minus infinity to infinity.

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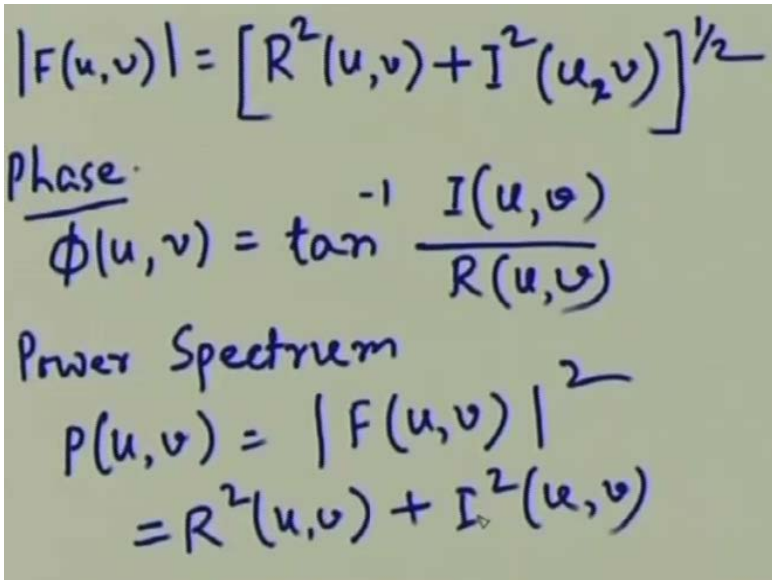

So, in this 2 dimensional signal, the Fourier spectrum F (u, v) is given by R square (u, v) so as before this R gives you the real component, plus I square (u, v) where I gives you the imaginary component and square root of this. So, this is what is the Fourier spectrum of the 2 dimensional signal f (x, y). We can get the phase angle in the same manner. The phase angle phi (u, y) (u, v) is given by tan inverse I (u, v) by R (u, v).

And, the power spectrum in the same manner, we get as P (u, v) is equal to F (u, v) square which is nothing but R square (u, v) plus I square (u, v). So, we find that all these quantities which we had defined in case of the single dimensional signal is also applicable in case of the 2 dimensional signals that is f (x, y). Now, to illustrate this Fourier transformation let us take an example.

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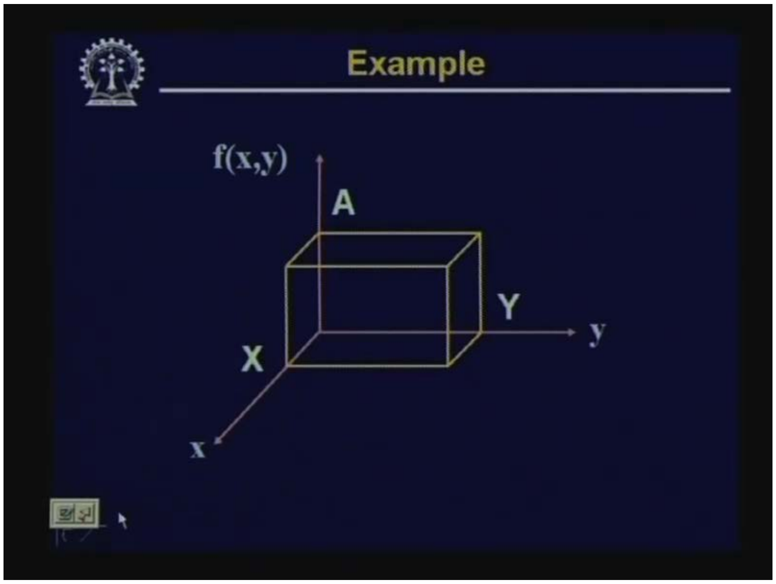

Suppose, we have a continuous Function like this, the Function f (x, y) which is again a Function of 2 variables x and y and the Function in our case is like this that f (x, y) assumes a value, a constant value say capital A for all values of x lying between 0 to capital X and all values of y lying between 0 to capital Y.

So, what we get is rectangular Function like this where all values of x greater than capital X, the Function value is 0 and all values of y greater than capital Y, the Function values also 0 and between 0 to capital X and 0 to capital Y, the value of the Function is equal to capital A. Let us see, how we can find out the Fourier transformation of these particular 2 dimensional signals.

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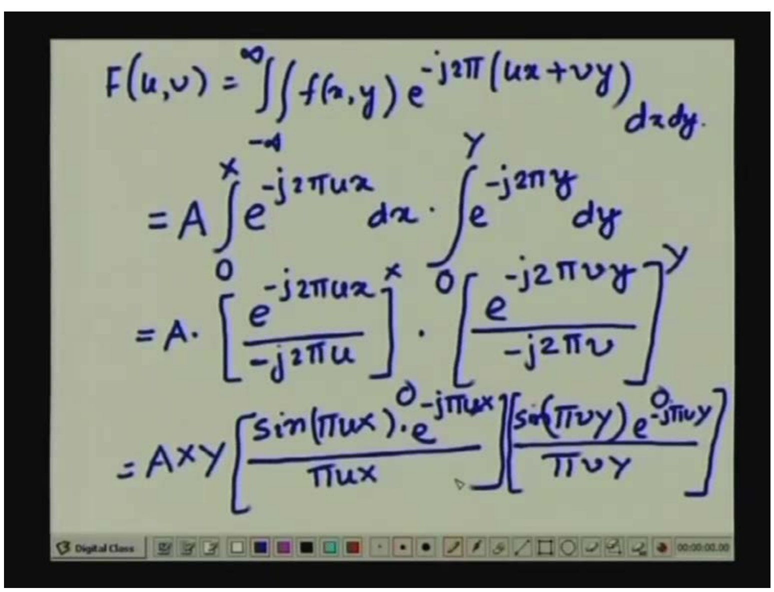

So, to compute the Fourier transformation, we follow the same expression. We have said that F (u, v) is nothing but double integration from minus infinity to infinity f (x, y) e to the power minus j 2 pi ux plus vy dxdy. Now, in our case, this f (x, y) is equal to constant which is equal to A as long as x lies between 0 to capital X and y is in between 0 to capital Y and outside this region, the value of f (x, y) is equal to 0.

So, you can break this particular integral in this form. This will be same as capital A, then take the integration over x which will be in this particular case e to the power minus j 2 pi ux dx. Now, this integration over x has to be from 0 to capital X multiplied by e to the power minus j 2 pi y dy where this integration will be in the range 0 to capital Y.

So, if I compute this these 2 integrations, these 2 integrals; you will find that it will take the form something like this and if you compute these 2 limits, you will find that it will take the value A capital X into capital Y into sin (pi ux) into e to the power minus j pi ux upon pi ux into sin (pi vy) into e to the power minus j pi vy upon pi vy.

So, after doing all these integral operations, I get an expression like this. So, from this expression, if you compute the Fourier spectrum; the Fourier X spectrum will be something like this.

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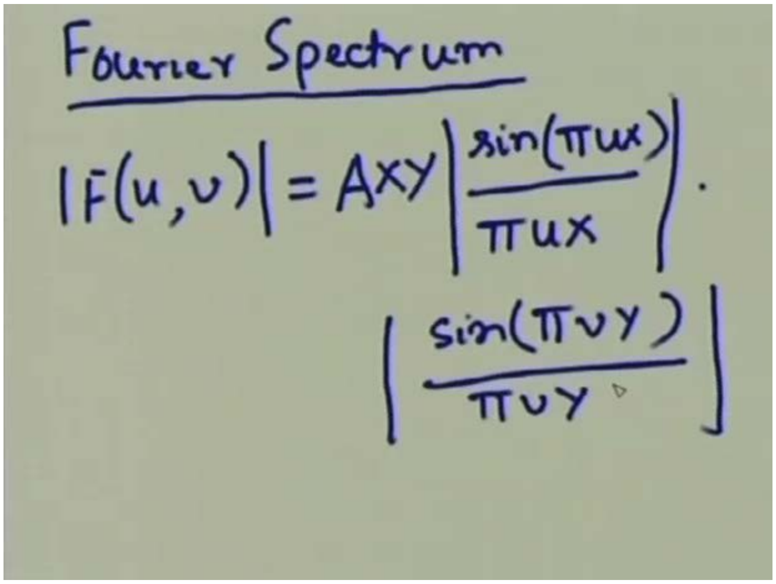

So, what we are interested in is the Fourier spectrum. So, the Fourier spectrum that is modulus of F (u, v) will be given by A capital X capital Y into sin (pi ux) upon pi ux into sin (pi vy) upon pi vy. So, this is what is the Fourier spectrum of the Fourier transformation that we have got. Now, if we plot to the Fourier spectrum, the plot will be something like this.

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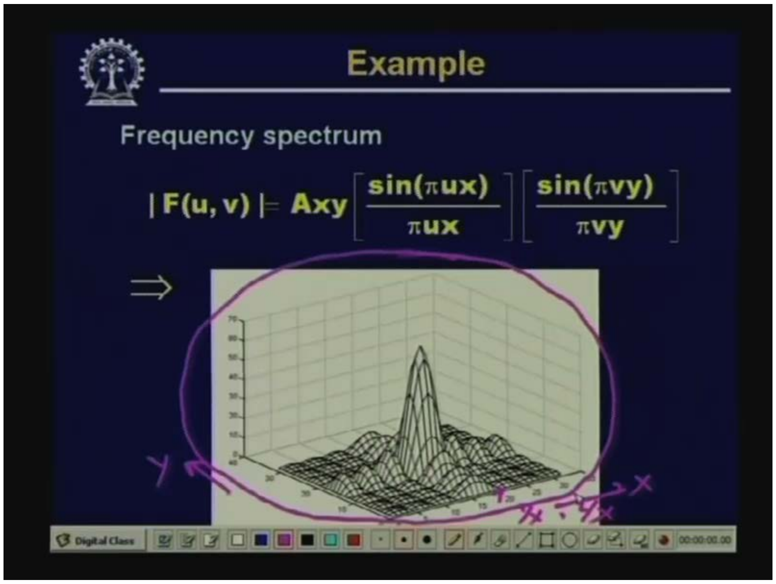

So, this is what is the plot of this Fourier spectrum. So, the Fourier spectrum plot is this one. So, you will find that this is again a 2 dimensional Function; of course in this case, the spectrum that has been shown is shifted so that the spectrum comes within the range for its complete feasibility.

So, for a rectangular Function, rectangular 2 dimensional Function; you will find that the Fourier spectrum will be something like this and we can find out that if I say that this is the x axis and this is the y axis and assuming the centre to be at the origin, you will find that along the x axis at point 1 upon capital X, similarly 2 upon capital X, the value of this Fourier spectrum will be equal to 0. Similarly, along the Y axis at values 1 upon capital Y, 2 upon capital Y; the values of this spectrum will also be equal to 0. So, what we get is the Fourier spectrum and the nature of the Fourier spectrum of the particular 2 dimensional signal.

Now, so far what we have discussed is the case of the continuous functions or analog functions but in our case, we have to be interested in the case for discrete images or digital images where the functions are not continuous but the Functions are discrete.

So, all these integration operations that we are doing in case of the continuous functions; they will be replaced by the corresponding summation operations.

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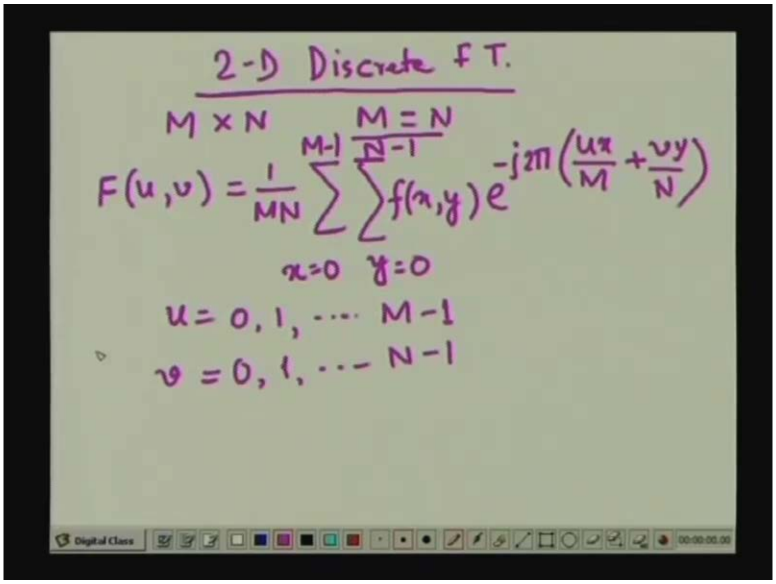

So, when you go for the 2 dimensional signal; so, in case of this discrete signals, the discrete Fourier transformation will be of this form F (u, v). Now, these integrations will be replaced by summations. So, this will take the form of 1 upon M into N. Then double summation f (x, y), the expression remains almost the same minus j 2 pi ux by capital M plus vy by capital N and now the summation will be for y equal to 0 to N minus 1, capital N minus 1 and x equal to 0 to capital M minus 1 because our images are of size M by N and the frequency variables u because our images are discrete, the frequency variables are also going to be discrete.

So, the frequency variables u will vary from 0, 1 upto M minus 1 and the frequency variable v will similarly vary from 0, 1 upto capital N minus 1. So, this is what is the forward discrete Fourier transforms, forward 2 dimensional discrete Fourier transformations. In the same manner, we can also obtain the inverse Fourier transformation for this 2 dimensional signal.

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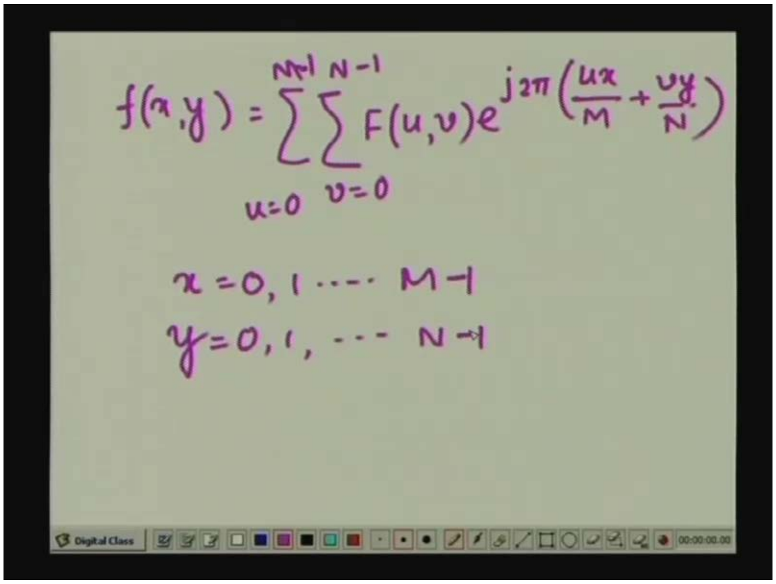

So, the inverse Fourier transformation will be given by f (x, y) will be double summation F (u, v) which is the Fourier transformation of f (x, y) e to the power j 2 pi ux by M plus vy by N and now the integration will be from v equal to 0 to capital N minus 1 and u equal to 0 capital N minus 1.

So, the frequency variables v varies from 0 to capital N minus 1 and u varies from 0 to capital M minus 1 and obviously, this will give you back the digital image f (x, y), the discrete image where x will now vary from 0 to capital M minus 1 and y will now vary from 0 to capital N minus 1.

So, we have formulated these equations in a general case where the discrete image is represented by a 2 dimensional array of size capital M by capital N. Now, as we said that in most of the cases the image is mostly represented in the form of square array where M is equal to N so if the image is represented in the form of square array; in that case, this transformation equations will be represented as

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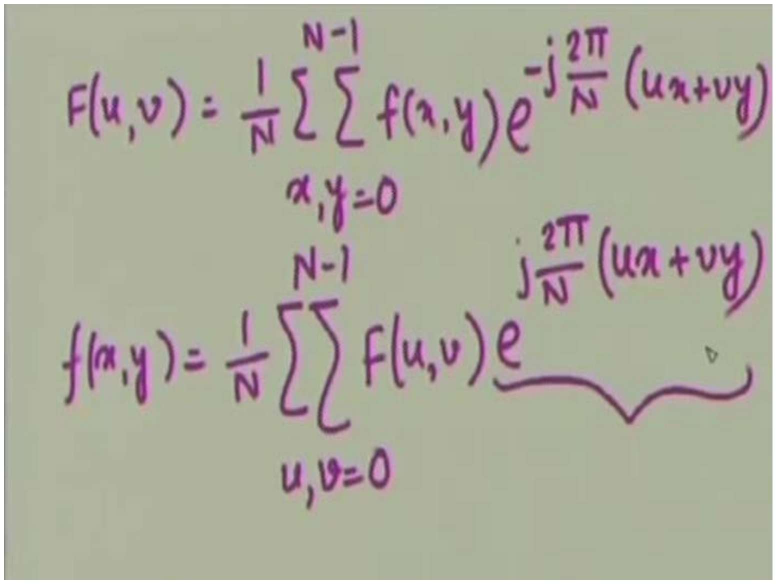

F (u, v) will be equal to 1 upon capital N double summation f (x, y) and now because M is equal to N, so the expression becomes e to the power minus j 2 pi by N ux plus vy where both x and y will now vary from 0 to capital N minus 1and similarly the inverse Fourier transform f (x, y) will be given by 1 upon capital N summation double summation F (u, v) e to the power j 2 pi by N ux plus vy but the variables u and v will now vary from 0 to capital N minus 1.

So, this is the Fourier transformation pair that we get in discrete case for a square image where the number of rows and the number of columns are same and we have discussed earlier that e to the power j 2 pi by N ux plus vy, this is what we have called as the basis images. This we have discussed when we have discussed about the unitary transformation and we have said, we have shown the time that these basis images will be like this.

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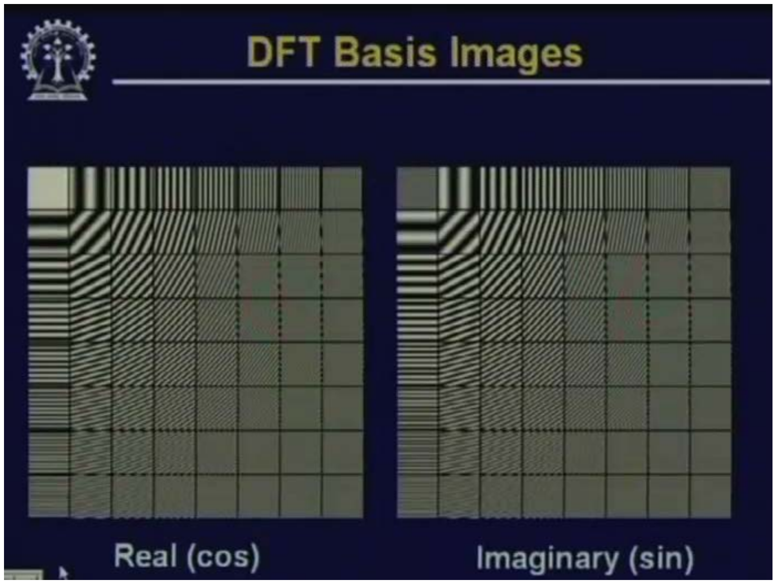

So, as the Fourier transformation, as we have seen that it is a complex quantity; so, for the Fourier transformation, we will have 2 basis images. One basis image corresponds to the real part, the other basis image corresponds to the imaginary part and these are the 2 basis images, one for the real part and the other one for the imaginary.

Now as we have defined, the Fourier transform, the Fourier spectrum, the phase, the power spectrum in case of analog image; all these quantities can also be defined or also defined in case of discrete image in the same manner.

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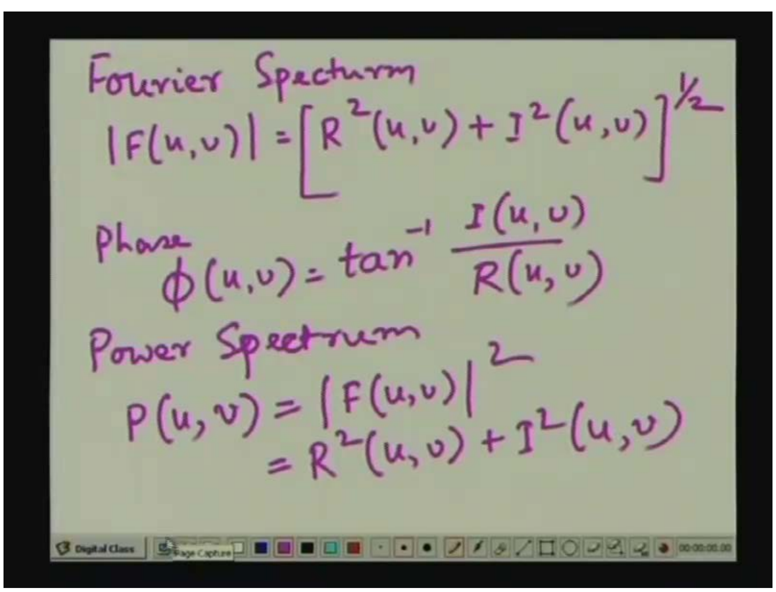

So, in case of this discrete image, the Fourier spectrum is given by similar expression. That is F (u, v) is nothing but R square (u, v) plus I square (u, v) square root of this, phase is given by phi (u, v) is equal to tan inverse I (u, v) upon R (u, v) and the power spectrum P(u, v) is given by the similar expression which is nothing but F (u, v) modulus square which is nothing but R square (u, v) plus I square (u, v) where R is the real part of the Fourier coefficient and I (u, v) is the imaginary part of the Fourier coefficient.

So, after discussing about this Fourier transformation both in the forward direction and also in the reverse direction; let us look at how this Fourier transform coefficients look like.

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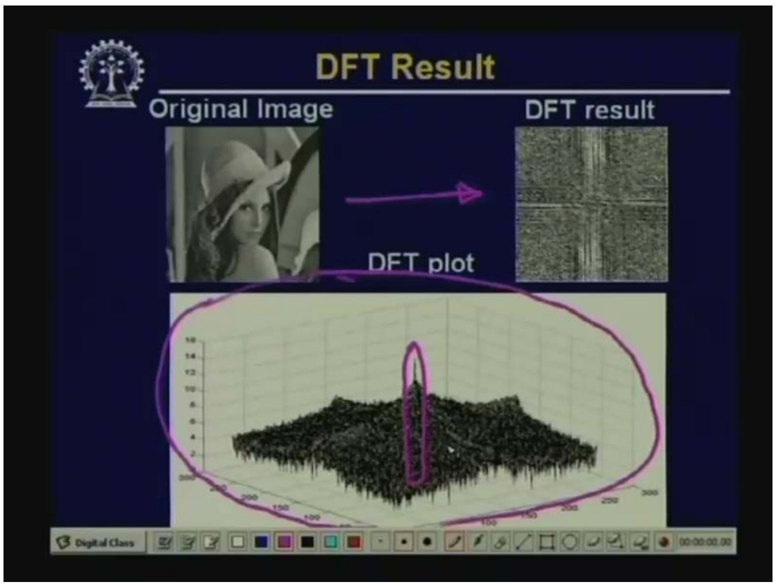

So, here we have the result of one of the images and you will find that this is a popular image very popular image which is sighted in most of the image processing text books that is the image of Lena. So, if you take the discrete Fourier transformation of this particular image, the right hand side, this one shows that DFT which is given in the form of an intensity plot and the bottom one that is this particular plot is the 3 dimensional plot of the DFT coefficients.

Here again, when these coefficients are plotted, it is shifted so that the origin is shifted at the centre of the plane so that you can have a better view of all these coefficients here. Here you find that at the origin, the intensity of the coefficient or the value of the coefficient is quite high compared to the values of the coefficients as you move away from the origin. So, this indicates that the Fourier coefficient is maximum at least for this particular image at origin that is when u equal to 0 or v equal to 0 and later on, we will see that u equal to 0, v equal to 0 gives you what is the DC component of this particular image.

And in most of the images, the DC component is maximum and as you move towards the higher frequency components, the energy of the higher frequency signals are less compared to the DC component. So, after discussing about all these Fourier transformation, the entire inverse Fourier transformation and looking at how the Fourier coefficients look like; let us see some of the properties of these Fourier transformation operations.

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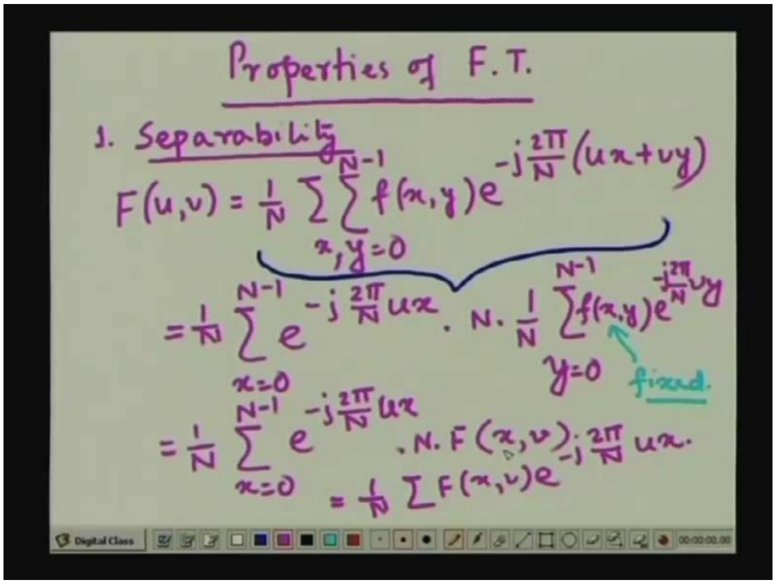

So now, we will see some of the properties, important properties of Fourier transformation. So, the first property that we will talk about is the separability. Now, if you analyze the expression of the Fourier transformation where we have said that the Fourier transformation F (u, v) is given by 1 upon N double summation f (x, y) e to the power minus j 2 pi by N; we are assuming a square image of size N by N into ux plus vy where both x and y varies from 0 to capital N minus 1.

Now, find that this particular expression, expression of the Fourier transformation, this particular expression can be rewritten in the form 1 upon N into e to the power minus j 2 pi by capital N ux where x varies from 0 to capital N minus 1 into N capital N into 1 upon capital N summation y varying from 0 to capital N minus 1 f (x, y) e to the power minus j 2 pi by capital N into vy.

So, it is the same Fourier transformation expression but now we have separated the variables x and y into 2 different summation operations. So, the first summation operation, you will find that it involves the variable x and the second summation operation involves the variable y. Now, if you look at this Function f (x, y) for which we are trying to find out the Fourier transformation; now this second summation operation where the summation is taken over y where y varies from 0 to capital N minus 1, you find that in this Function f (x, y) if we keep the value of x to be fixed that is for a particular value of x, the different values of f (x, y) that represents nothing but a particular row of the image.

So, in this particular case, for a particular value of x, if I keep x to be fixed; so for a fixed value of x, this f (x, y) represents a particular row of the image which is nothing but an 1 dimensional signal. So, by looking at that what we are doing is we are transforming the rows of the image and different rows of the image for different values of x.

So, after expansion or elaboration of this particular expression, the same expression now gets converted to 1 upon capital N x equal to 0 to capital N minus 1 e to the power minus j 2 pi by capital N ux into F of (x, v). I represent these as F of (x, v) and of course, there is a multiplication term which is capital N and this is nothing but 1 upon capital N summation F of (x, v) e to the power minus j 2 pi by capital N ux.

So, once if you look at these expressions, you will find that the second summation, the second summation operations gives you the Fourier transformation of the different rows of the image and that Fourier transformation of the different rows which now we represent by F (x, v), this x represents the x is an index of a particular row and the second summation, what it does is it takes this intermediate Fourier coefficients and on this Fourier coefficients, now it performs the Fourier transformation over the columns to give us the complete Fourier transformation operation or F of (u, v).

So, the first operation that we are performing is the Fourier transformation over of different rows of the image multiplying this intermediate result by the factor of capital N and then this intermediate result or intermediate Fourier transformation matrix that we get, we further take the Fourier transformation of different columns of this intermediate result to get the final Fourier transformation.

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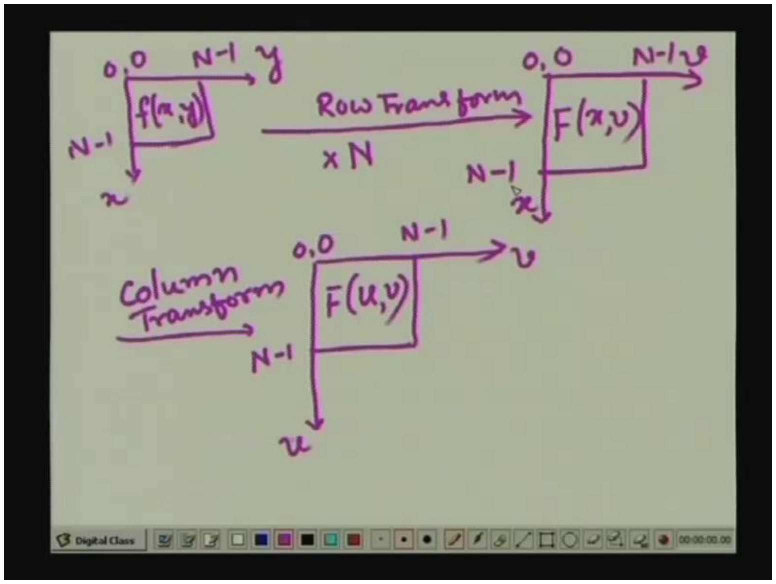

So graphically, we can represent this inter operation like this that this is our x axis, this is our y axis, I have an image f of (x, y). So, first of all what we are doing is we are taking the Fourier transformation along the row. So, we are doing row transformation and after doing row transformation, we are multiplying all these intermediate values by a factor N. So, you multiply by the capital by the factor capital N and this gives us the intermediate Fourier transformation coefficients which now we represent as capital F (x, v).

So, you get one of the frequency components which is v and then what we do is we take this intermediate result and initially we had done row transformation and then now we will do column transformation and after doing this column transformation, what we get is so here it will be x and it will be axis v and we get the final result as u, v and our final transformation coefficients will be capital F (u and v). Of course, this is the origin (0, 0), all these values are N minus 1, N minus 1. Here also, it is (0, 0), this is N minus 1, N minus 1. Here also it is (0, 0), here it is capital N minus 1, here it is capital N minus 1.

So, you will find that by using this separability property what we have done is this 2 dimensional Fourier transformation operation is now converted into 2 1 dimensional Fourier transformation operations.

So, in the first case what we are doing is we are doing the 1 dimensional Fourier transformation operation over different rows of the image and the intermediate result that you get, that you multiply with the dimension of the image which is n and this intermediate result, you take and now you do again 1 dimensional Fourier transformation across the different columns of this intermediate result and then you finally get the 2 dimensional Fourier transformation coefficient.

So because of separability, this 2 dimensional Fourier transformation has been converted to 2 1 dimensional Fourier transformation operations and obviously by using this, your operation will be much more simpler.

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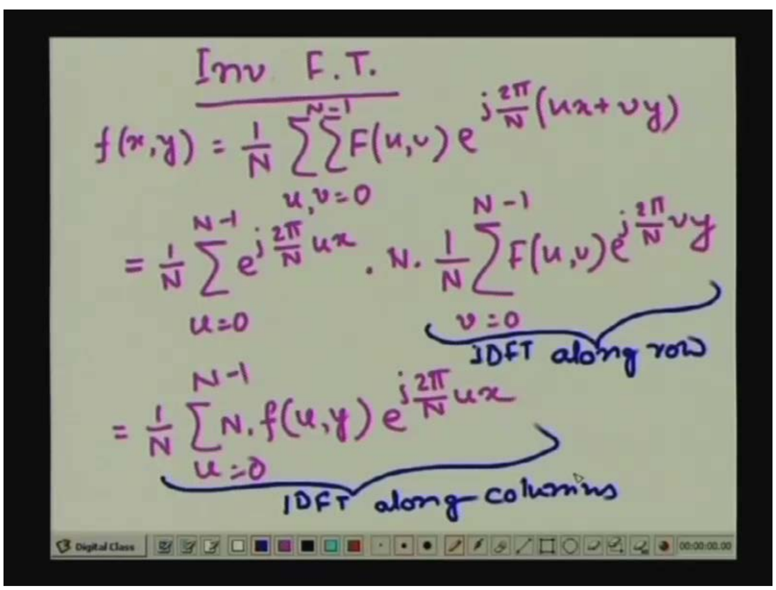

So, in the same manner as you have done in case of forward Fourier transformation, we can also have the inverse Fourier transformation, we can also have the Inverse Fourier transformation. So, in case of inverse Fourier transformation, our expression was f (x, y) is equal to 1 up on capital N double summation F (u, v) e to the power j 2 pi upon N ux plus vy where both u and v varies from 0 to capital N minus 1. So in the same manner, I can also break this expression into 2 summations.

So, the first summation will be e to the power j 2 pi by capital N ux. Here, u will vary from 0 to capital N minus 1 multiplied by N into 1 upon capital N F (u, v) e to the power j 2 pi upon capital N vy and now v will vary from 0 to capital N minus 1. So, again as before, you will find that this second operation, this is nothing but inverse discrete Fourier transformation along a row.
So, this second expression, this gives you the inverse Fourier transformation along the row and when you finally convert this and get the final expression, this will be 1 upon capital N summation N times F (u, y) into e to the power j 2 pi upon capital N ux and now u varies from 0 to capital N minus 1. This particular expression is inverse dispute Fourier transformation along columns.

So, as we have done in case of forward Fourier transformation that is for a given image, you first take the Fourier transformation of the different rows of the image to get the intermediate Fourier transformation coefficient and then take the Fourier transformation of different columns of that set of intermediate Fourier coefficients to get the final Fourier transformation.

In the same manner, in the inverse Fourier transformation; we can also take the Fourier coefficient array, do the inverse Fourier transformation along the rows and all those intermediate results that you get, for that, second step you do the inverse dispute Fourier transformation along the columns and these 2 operations completes the inverse Fourier transformation operation of the 2 dimensional array to give you the 2 dimensional signal f (x) of y, x and y.

So, because of this separability property, we have been able to convert the 2 dimensional Fourier transformation operation into 2 1 dimensional Fourier transformation operations and because now it has to be implemented as 1 dimensional Fourier transformation operation; so the operation is much more simple than in case of 2 dimensional Fourier transform transformation operation. Now, let us look at the second property of this Fourier transformation.

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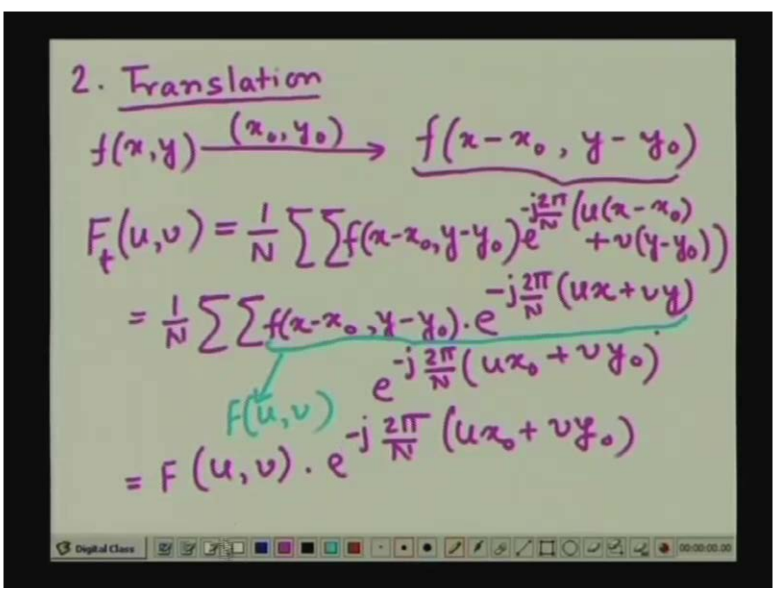

The second property that we will talk about is the translation property. Translation property says that if we have a 2 dimensional signal say f (x, y) and translate this by a vector x_0, y_0. So, along x direction, you translate it by x_0 and along y direction, you translate it by y_0. So, the Function that you get is f (x minus x_0, y minus y_0).

So, if I take the Fourier transformation of this translated signal f (x minus x_0, y minus y_0); how the Fourier transformation will look like? So, you can find out the Fourier transformation of this translated signal and let us call this Fourier transformation as F_t (u, v). So, I represent this as F_t (u, v). So, going by the similar expression, this will be nothing but 1 upon capital N f of x minus x_0, y minus y_0 into e to the power minus j 2 pi by capital N into u (x minus x_0) plus v (y minus y_0).

So, if I expand this, what I will get is 1 upon capital N into double summation f of (x minus x_0, y minus y_0) into e to the power minus j 2 pi by N (ux plus vy) into e to the power minus j 2 pi by N ux_0 plus vy_0; by simply expanding this particular expression.

Now here, if you consider the first expression that is f (x minus y_0 y minus y_0) e to the power minus j 2 pi by N ux plus vy summation from x equal to 0 to N minus 1, y equal to 0 to N minus 1; this particular term is nothing but a Fourier transformation f of (u, v). So, by doing this translation what we get is the final expression F_t of (u, v) will come in the form F of (u, v) into e to the power minus j 2 pi by capital N into ux_0 plus vy_0. So, this is the final expression of this translated signal that we get.

So, if I compare, if you compare these 2 expressions F (u, v) and F_t (u, v), you will find that the Fourier spectrum of the signal after translation does not change because the magnitude of this F_t (u, v) and the magnitude of F (u, v) will be the same. So because of this translation, what you get is only it will produces some additional phase difference.

So, whenever f (x, y) is translated by x_0 y_0, the additional phase difference which is introduced by the e to the power minus j 2 pi by capital N ux_0 plus vy_0 but otherwise, the magnitudes of the Fourier spectrum or the magnitude of the Fourier transformation that is the Fourier spectrum that remains unaltered.

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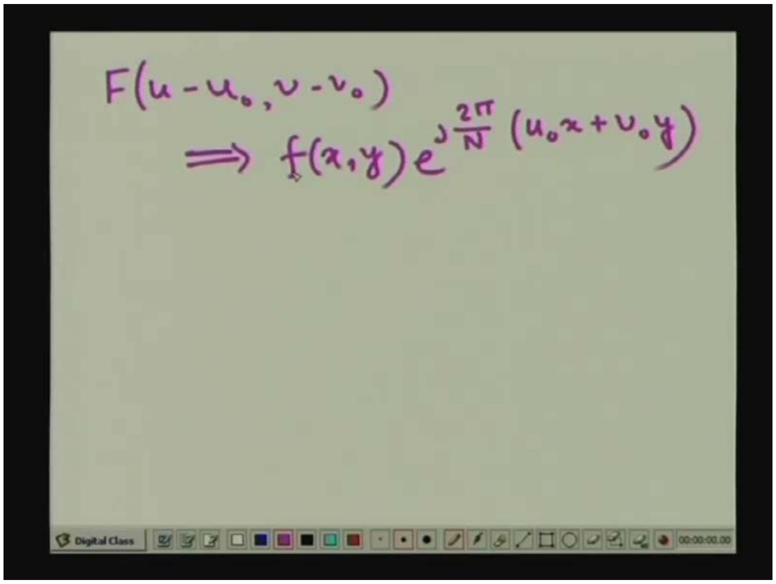

In the same manner, if we talk about the inverse Fourier transformation; the inverse Fourier transformation F of (u minus u_0, v minus v_0), this will give raise to f of xy e to the power j 2 pi by capital N (u_0 x plus v_0 y). So, this says that if f (x, y) is multiplied by this exponential term, then its Fourier transformation is going to be replaced is going to be displaced by the vector u_0 v_0 and this is the property which will we will use later on to find out that how the Fourier transformation coefficients can be better visualized.

So here, in this case, we get the Fourier transformation, the forward Fourier transformation and the inverse Fourier transformation with translation and you will find that and we have found that the shift in f (x, y) by say x_0 y_0 does not change the Fourier spectrum of the signal. What we get is just an additional phase term gets introduced in the Fourier spectrum.

So, with this let us conclude today's lecture on the Fourier transformation. We will talk about the other Fourier transformation, other properties of the Fourier transformation in our subsequent lectures.

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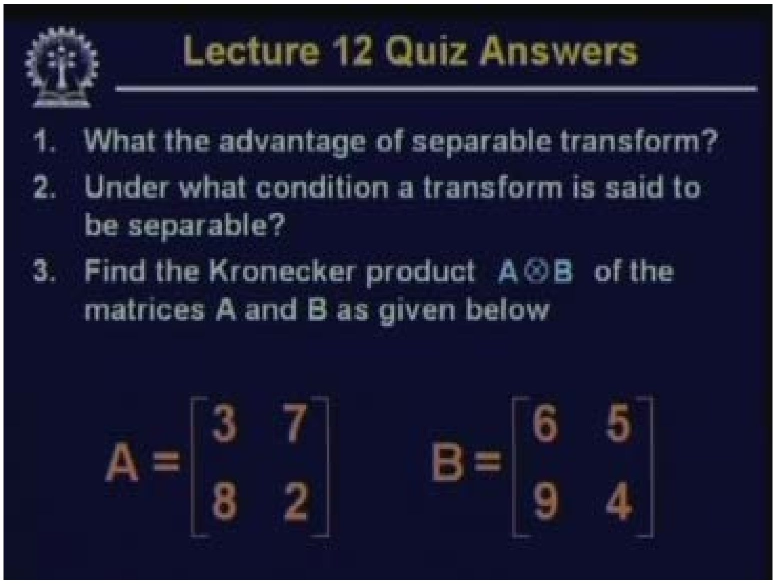

Now, let us see some of the answers to some of the questions that we had presented during our last lecture. So, the first 2 questions that we had that what is the advantage of separable transform? This we have already discussed during our lecture that if the transformation is separable, then we can go for the first implementation of the transformation that is computational complexity of the transformation implementation will be much less if the transformation is separable.

The second question: under what condition a transform is said to be separable? This also we have discussed during the our previous discussion that we have said that a transformation is separable if the transformation matrix can be represented as a product of 2 matrices for the transformation was unitary which is this transformation unitary matrix is now represented as product of 2 matrices say a_1 and a_2 and if both these matrices a_1 and a_2 are also unitary; in that case, we will say that the transformation is separable.

We have also said that this can also be discussed; this can also be explained in terms of Kronecker products. That is if the transformation, original transformation at matrix A can be represented as Kronecker product of 2 other matrices say A and B both of which are unitary matrices; then also the matrix is separable and the advantage is obviously, for a separable transformation, we can go for faster implementation of that transformation.

Now, the third question: find the Kronecker product of A and B.

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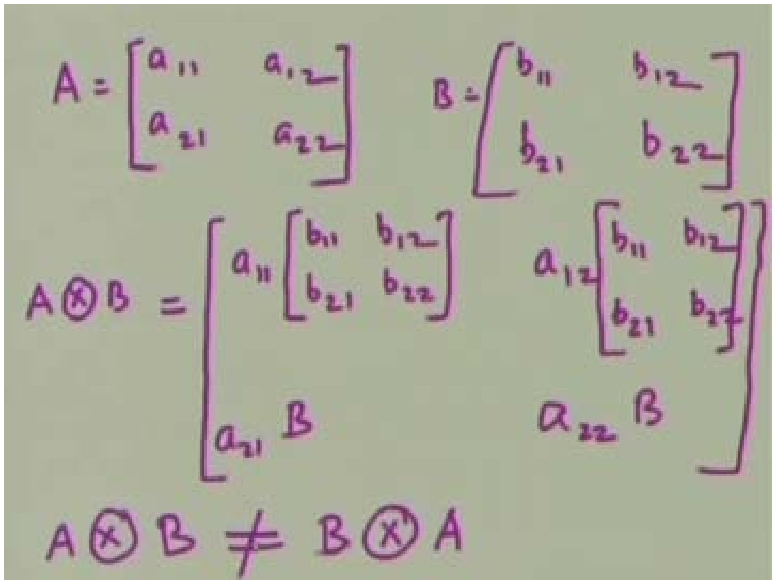

Now, the Kronecker product of 2 matrices A and B in this particular case will be represented as suppose the matrix A is given by a_{11} a_{12} a_{21} a_{22}, so this is on matrix A and the matrix B is suppose represented as b_{11} b_{12} b_{21} b_{22}, this is matrix B. Then the Kronecker product of these 2 matrices A and B will be represented as a_{11} into b_{11} b_{12} b_{21} b_{22} then a_{12} the same b_{11} b_{12} b_{21} b_{22} a_{21} again the same matrix B and a_{22} again the same matrix B. So, this entire matrix is the Kronecker product of the 2 matrices A and B.

Now, from this definition if I replace the values of a_{11} a_{12} a_{21} a_{22} b_{11} b_{12} b_{21} b_{22} all these different values from the given matrices A and B, what I get is the Kronecker product of the 2 matrices A and B. Now here, let me mention that from this definition it is quite obvious that A Kronecker product with B is not equal to B Kronecker product with A and this is in general true for matrix multiplication.

In general, A into B is not equal B into A. In the same manner, A Kronecker product with B is in general not equal to B Kronecker product with A.

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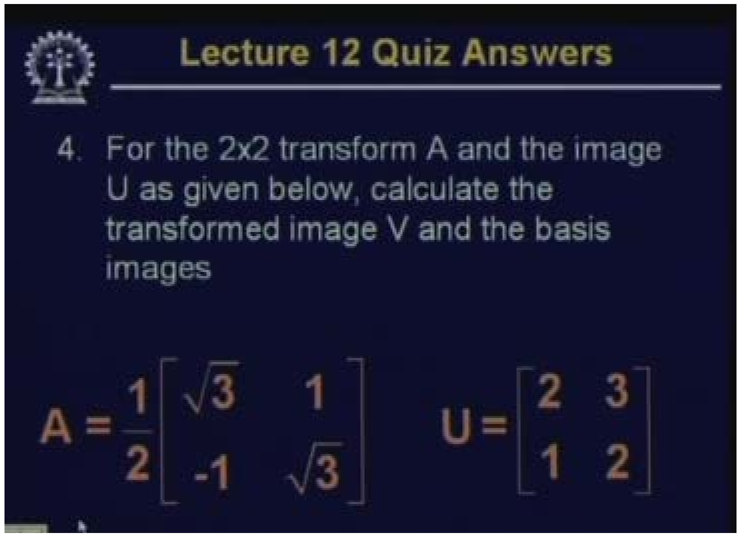

So, coming to our next question, the next question was for the 2 by 2 transform A and the image U as given below, calculate the transformed image v and the corresponding basis images. We had taken in during our lecture an example of exactly similar nature and there also we have said that if that you have the transformation matrix and the image; then the transformation coefficients V can be easily obtained as AUA transpose where A is the transformation matrix, e is the image U is the image.

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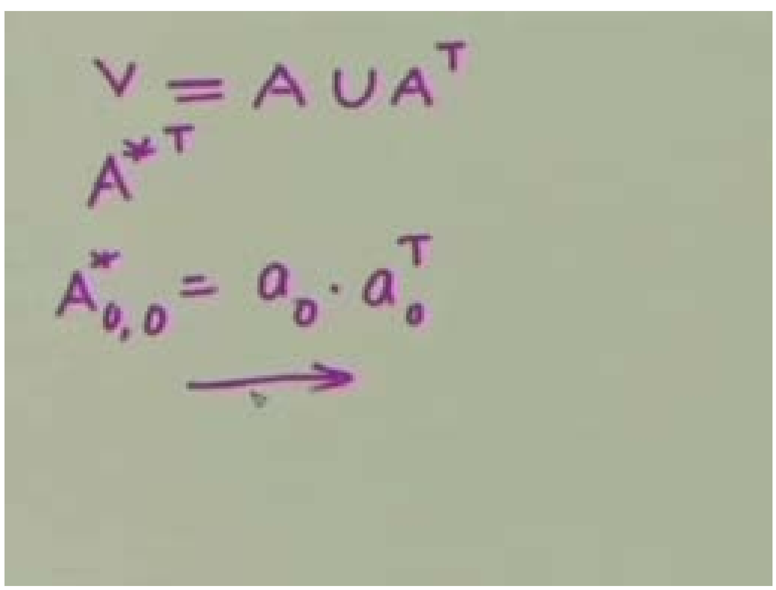

So, just by replacing the matrices A and U in this expression, we can get what it is the coefficient matrix V. To get the basis images, what you have to do is we have to take the outer products of the columns of matrix A conjugate transpose.

See, in this particular case, because it is the real matrix, so A conjugate will be same as A. We can find out take the transpose of it and after taking the transpose, what we will do is we will take different columns and take the outer products of the columns to find out the basis images. So in this particular case, the basis image $A_{0,0}$ conjugate will be nothing but a 0'th column outer product of this with a 0'th column transpose and following the similar approach, we can find out all other basis images for this given transformation.

So, we have discussed all the problems that we had given at the end of our last lecture. Now, coming to the today's problems, we are giving 2 problems. First one is find out the discrete Fourier transformation coefficients of a digital image f (x, y) of size capital N by capital N where f (x, y) equal to 1 for all values of capital X and capital Y.

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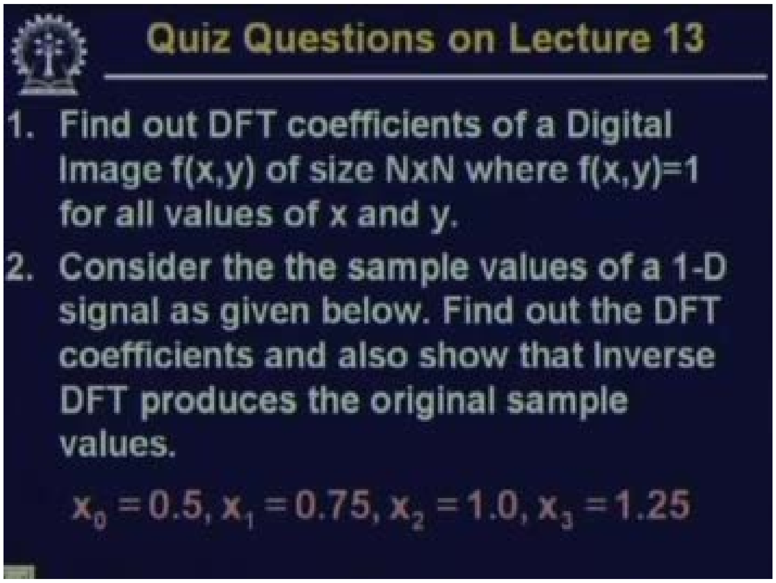

The second problem is; consider the sample values of one dimensional signal as given below and find out the DFT coefficients of this sample values.

Thank you.