Digital Image Processing

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Lecture - 10

Image Interpolation – II

Hello, welcome to lecture series on digital image processing. In the last class, we started our discussion on image interpolation and resampling. Today we will continue with the discussion on the same topic that is image interpolation and resampling and today we will try to explain the concepts with the help of a number of examples.

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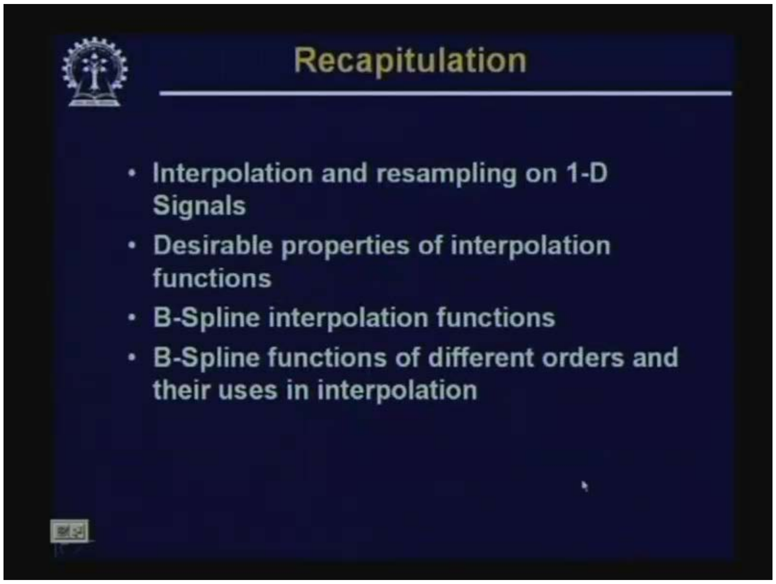

So in the last class, we have talked about interpolation and resampling and mostly we have discussed about the interpolation problems in case of 1 dimensional signal. We have also seen that what are the desirable properties of the interpolation functions and we have seen that it is the B Spline function which satisfies all the desired properties of interpolation. And, we have seen the B Spline functions; how different orders and we have seen, how this B Spline functions can be used for interpolation operations.

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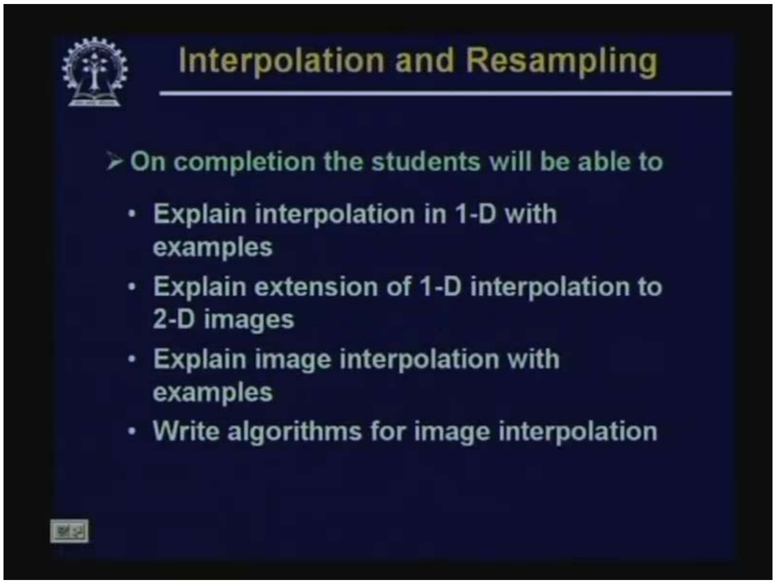

In today's lecture, we will explain the interpolation in 1 dimension with the help of a number of examples. Then we will see the extension of this 1 dimensional interpolation to 2 dimensional images. Again, we will explain the image interpolation with examples and then at the end of today's lecture, the students will be able to write algorithms for different image interpolation operation.

So, let us see that what we mean by image interpolation operation. So here, we have shown a diagram which shows the sample values of 1 dimensional signal say f (t) which is a function of t.

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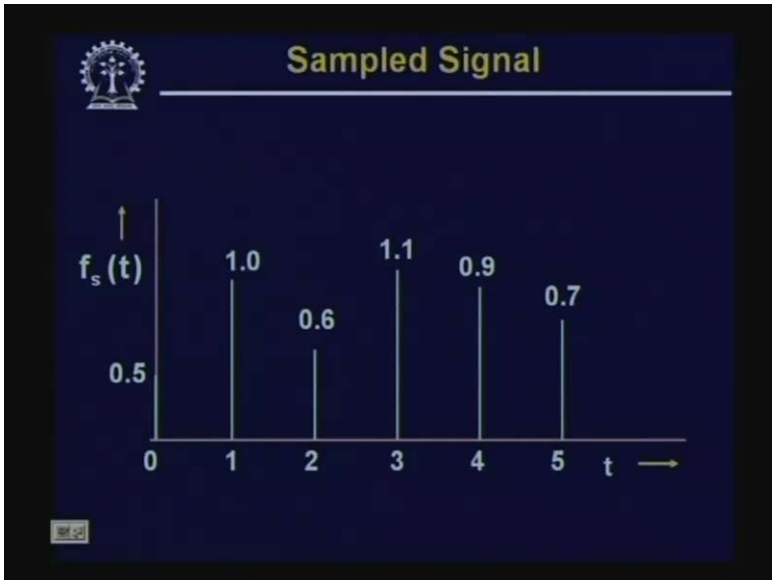

So, you find that in this particular diagram, we have a given a number of samples and here the samples are present for t equal to 0, t equal to 1, t equal to 2, t equal to 3, t equal to 4 and t equal to 5. The functions are given like time, the sample data. You find that the sample values are available only at 0, 1, 2, 3, 4 and 5. But in some applications, we may need to find out the approximate value of these functions at say t equal to 2.3 or say t equal to 3.7 and so on.

So again here, in this diagram, you find that at t equal to 2.3 is somewhere here. I do not have any information or say t equal to 3.7 somewhere here, again I do not have any information. So, the purpose of image interpolation is by making use or the signal interpolation is by using the sample values at these distinct locations. We have to reconstruct or we have to approximate the value of the function f (t) at any arbitrary point in the time axis. So, that is the basic purpose of the interpolation operation.

So, whenever you go for some interpolation, we have to make use of certain interpolation functions and these interpolations operation of the interpolation function should satisfy certain conditions.

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The conditions are; the interpolation function should have a finite region of support. That means when we do the interpolation, we should not consider the sample values from say minus infinity to plus infinity; rather if I want to approximate the function value at locations say t equal to 2.3, then the samples that should be considered are the samples which are nearer to t equal to 2.3. So, I can consider a sample at t equal to 1, I can consider the sample at t equal to 2, I can consider the sample at t equal to 3, I can consider the sample at t equal to 4 and so on.

But for approximate the functional value to approximate the functional value at t equal to 2.3, I should not consider the sample value at say t equal to 50. So, that is what is meant by finite region of support.

Then the second property which this interpolation operation must satisfy is it should be a smooth interpolation. That means by interpolation, we should not introduce any discontinuity in the signal. Then the third operation, the third condition that must be satisfied for this interpolation operation is that the interpolation must be shift invariant. That is if I shift the signal by say t equal to 5, even then the same interpolation operation, the same interpolation functions should give me the same result in the same interval. So, these are what are known by the shift invariance property of the interpolation.

And, we have seen in the last class that B Spline interpolation functions satisfy all these 3 properties which are desirable properties for interpolation. So, this B Spline functions are something like this.

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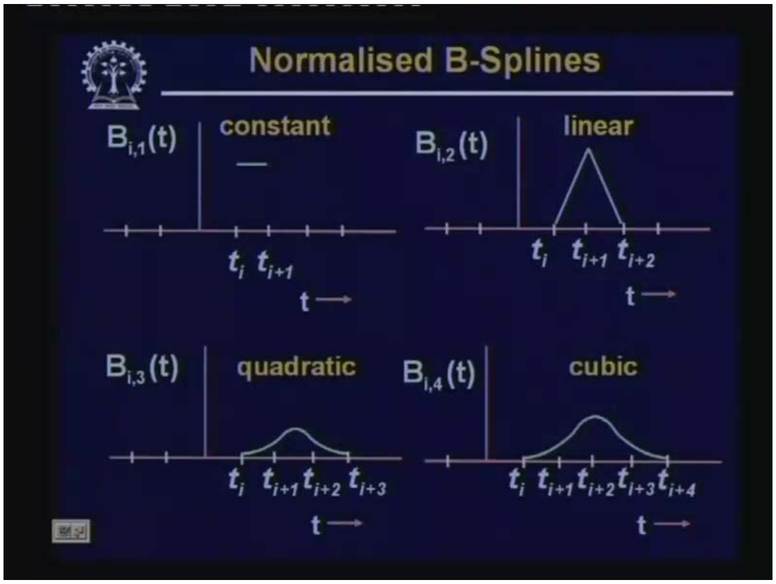

We have seen that for interpolation with the help of B Spline function, we use a B Spline function which is given by $B_{i\,k}$.

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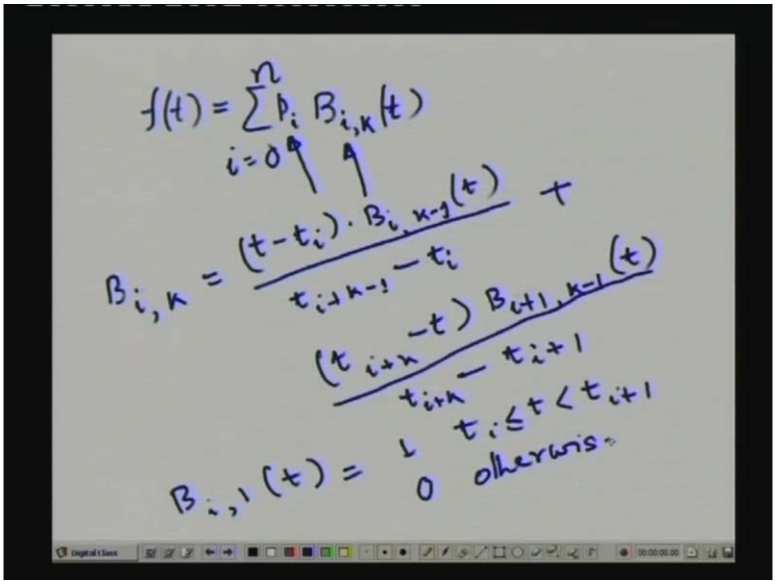

So, let me just go to what is the interpolation operation that we have to do. So for interpolation, what we use is say f (t) should be equal to some p_i into $B_{i\ k}$ (t) where i vary from 0 to say n where p_i indicates the i'th sample and $B_{i\ k}$ is the interpolation function. And, we have defined in the last class that this $B_{i\ k}$ can be defined as $B_{i\ k}$, it can be defined recursively as t minus t_i into $B_{i\ k\ \text{minus}\ 1}$ (t) upon $t_{i\ \text{plus}\ k\ \text{minus}\ 1}$ minus t_i plus $t_{i\ \text{plus}\ k}$ minus t into $B_{i\ \text{plus}\ 1,\ k\ \text{minus}\ 1}$ (t) upon $t_{i\ \text{plus}\ k}$ minus $t_{i\ \text{plus}\ 1}$ where $B_{i\ 1}$ (t) is given by it is equal to 1 for t_i less than or equal to t less than t_i plus 1 and it is equal to 0 otherwise.

So, you find that when we have defined $B_{i\ 1}$ (t) to be 1 within certain region and it is equal to 0 beyond that region; then using this $B_{i\ 1}$, I can estimate I can calculate the values of other $B_{i\ k}$ by using the recursive relation. And pictorially, these different values of $B_{i\ k}$ for k equal to 1, it is a constant. For $B_{i\ 2}$ that is for k equal to 2, it is a linear operation, linear function. For k equal to 3, $B_{i\ 3}$ is a quadratic function and for k equal to 4 that is $B_{i\ 4}$, it is a cubic equation.

And, we have said in the last class; so, here you find the region of support for $B_{i\ 1}$ is just 1 sample interval. For $B_{i\ 2}$, the region the region of support is just 2 sample intervals. For $B_{i\ 3}$, it is 3 samples intervals and for $B_{i\ 4}$, it is 4 sample intervals. And, we have mentioned in the last class that out of this, the quadratic one that is for the value k equal to 3, it is normally not used because this does not give a symmetric interpolation. Whereas, using the other 3 that is $B_{i\ 1}$, $B_{i\ 2}$ and $B_{i\ 4}$, we can get symmetric interpolation.

So normally, the functions, the B Spline functions which are used for interpolation purpose are the first order that is equal to 1, the second order or linear that is k equal to 2 and the cubic interpolation that is for k equal to 4.

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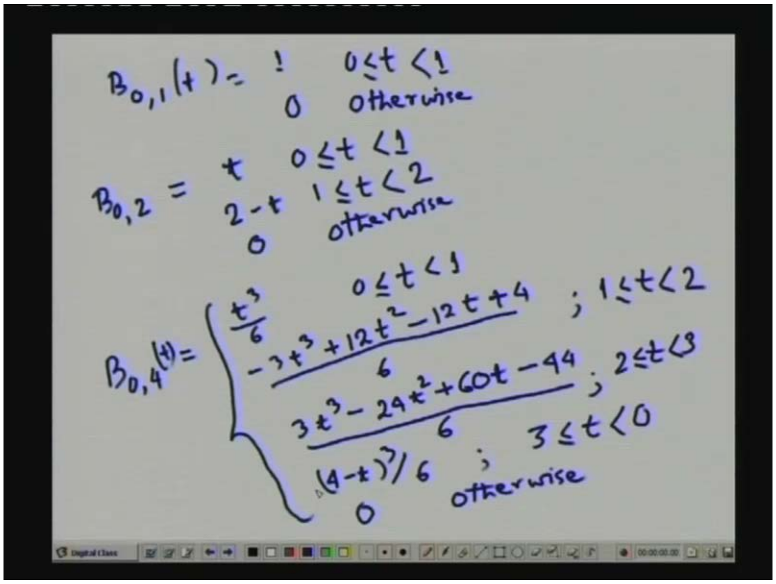

And, we have also said that these functions for k equal to 1, k equal to 2 and k equal to 4 can be approximated by $B_{0\,1}$ (t) is equal to 1 for 0 less than or equal to t less than 1 and it is equal to 0 otherwise. So, only in the range 0 to 1 excluding t equal to 1, $B_{0\,1}$ equal to 1 and beyond this range, $B_{0\,1}$ is equal to 0. Then $B_{0\,2}$ is defined like this that it is equal to t for 0 less than or equal to t less than 1 and it is equal to 2 minus t for 1 less than or equal to t less than 2 and it is equal to 0 otherwise.

So, here again you find that for the values of t between 0 and 1, $B_{0\,2}$ (t) increase linearly. For t equal to 1 to 2, the value of $B_{0\,2}$ decreases linearly and beyond 0 and 2 that is for values of t less than 0 and for values of t greater than 2, the value of $B_{0\,2}$ is equal to 0.

Similarly for the quadratic one sorry for the cubic one, $B_{0\,4}$ is defined as $B_{0\,4}$ (t) will be defined as t cube by 6 for 0 less than or equal to t less than 1. It is defined as minus 3t cube plus 12t square minus 12t plus 4 divided by 6 for 1 less than or equal to t less than 2. This is equal to 3t cube minus 24 t square plus 60 t minus 44 divided by 6 for 2 less than or equal to t less than 3. This is equal to 4 minus t whole cube divided by 6 for 3 less than or equal to t less than 4 and it is 0 otherwise.

So, these are the different orders of the B Spline functions. So, here again you find that for value equal to value of t less than 0, $B_{0\,4}$ (t) equal to 0 and similarly for value of t greater than 4, $B_{0\,4}$ (t) is also equal to 0. So, these are the B Spline functions using which the interpolation operation can be done. Now, let us see that how do we interpolate.

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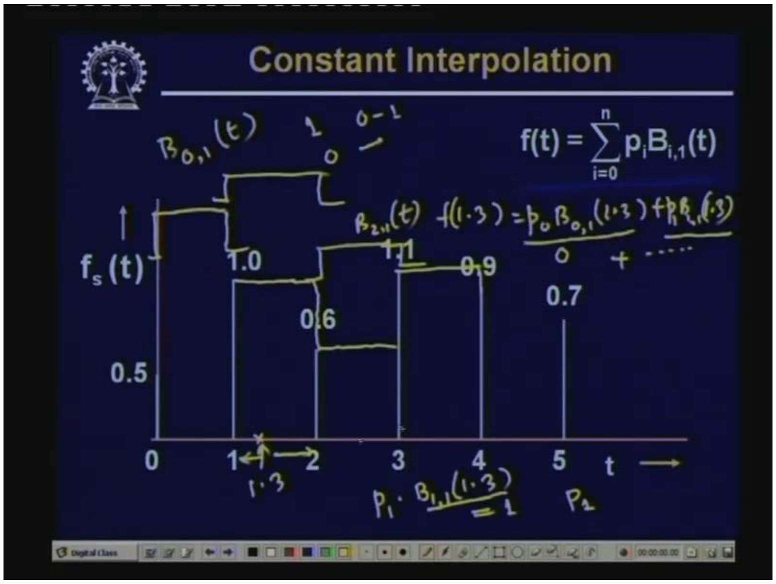

Again, I take the example of this sample data where I have a number of samples of a function t and which is represented by f_S (t) and the value of f_S (t) at present at t equal to 0, t equal to 1, t equal to 2, t equal to 3, t equal to 4 and t equal to 5. As we said that the interpolation function is given by this f (t) is equal to p_i $B_{i\,1}$ (t) if I go for constant interpolation.

Now here, $B_{i\,1}$ (t), so when I have computed, say $B_{0\,1}$ (t), we have said that these values equal to 1 for t lying between 0 and 1 and this is equal to 0 otherwise. So, if I interpolate this particular sample data say for example I want to find out what is the value of the signal at say 1.3. So, this is the point, t equal to 1.3. So, I want to find out the value of f (t) at t equal to 1.3. So, to do this, my interpolation formula says that this should be equal to f (1.3), this should be equal to p_0 into $B_{0\,1}$ (1.3) plus p_1 $B_{1\,1}$ (1.3) and so on. Now, if I plot this $B_{0\,1}$ (1.3) just super impose this on this particular sample data's sample data diagram, you find that $B_{0\,1}$ (1.3) or $B_{0\,1}$ (t), the function is equal to 1 in the range 0 to 1 excluding 1.

Similarly p_1 $B_{1\,1}$ (1.3), the value will be equal to 1 in the range 1 to 2 excluding 2 and it will be 0 beyond this. So, when I try to compute the function value at t equal to 1.3, I have to compute p_0. That is the sample value at t equal to 0 multiplied by this $B_{0\,1}$ (t). Now, because $B_{0\,1}$ (t) is equal to 0 for values of t greater than or equal to 1, so these particular term p_0 $B_{0\,1}$ (t), this term will be equal to 0.

Now, when I compute this, p_1 into $B_{1\,1}$ (1.3), you will find that this $B_{1\,1}$ (1.3) is equal to 1 in the range 0 to 1 to 2 excluding 2 and beyond 2, the value of $B_{1\,1}$ (t) is equal to 0. Similarly, for values of t less than or less than 1, the value of $B_{1\,1}$ (t) is also equal to 0. Similarly p_2 $B_{2\,1}$ that value is 1 within the range 2 to 3. So within this range, $B_{2\,1}$ (t) is equal to 1 and beyond this, B to 1 equal to 0.

So, when I try to compute the value at the point 1.3, you find that this will be nothing but p_1 into $B_{1\,1}$ (1.3) and this $B_{1\,1}$ (1.3) is equal to 1. So, the value at this point will be simply equal to t 1;

so in this case, it is f_S (1). And, you find that for any other values of t within the range 1 to 2, the value of f (t) will be same as f_1 or p_1. So, I can approximate this or I can do this interpolation like this. That is between 1 and 2, all the values the function values for all values of t between 1 and 2 is equal to 1. Following similar argument, you will find that between 2 and 3 the function value will be equal to f_2, between 3 and 4 the function value will be equal to f_3, between 4 and 5 the function value will be equal to f_4 and it will continue like this.

So, this is what I get if I use this simple interpolation formula. That is f (t) equal to p_1 p_i $B_{i\,1}$. Similar is also the situation if I go for linear interpolation. So, what I get in case of linear interpolation?

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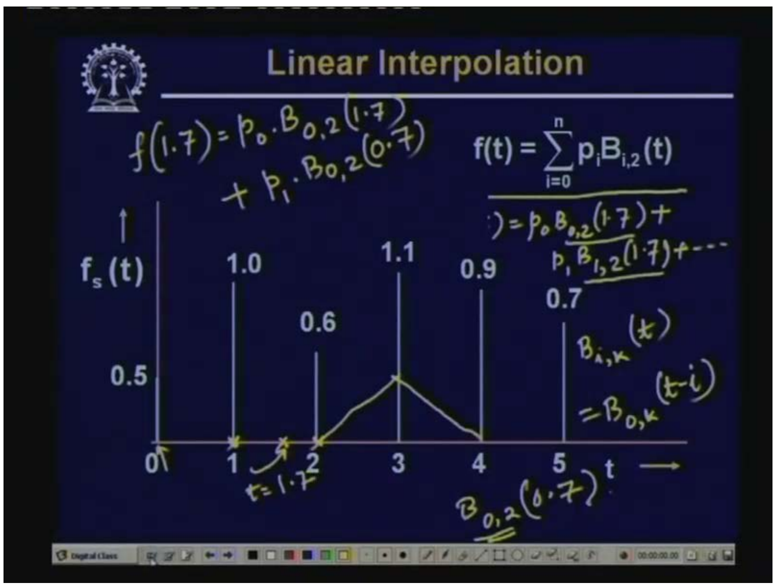

In case of linear interpolation, f (t) is given by $B_{i\,2}$ p_i into $B_{i\,2}$ (t) where we have to take the summation of all these terms for values of i from i equal to 0 to n. So, what we get in this case? You find that we have said that $B_{i\,0}$ (t) is nothing but a linearly increasing and decreasing function. So, if I plot p_i $B_{0\,2}$ (t), $B_{0\,2}$ (t) is a function like this which increases linearly between 0 and 1. At 1 this reaches the value 1 and then again from t equal to 1 and t equal to 2, the value of $B_{0\,2}$ (t) decreases linearly and it becomes 0 at t equal to 2.

Similarly, $B_{1\,2}$ (t) will have a function value something like this. It will increase linearly from 1 to 2, it will reach a value of 1 at t equal to 2 and again from t equal to 2 to t equal to 3, it decrease linearly. Then at t equal to 3, the value of B_1 to t becomes equal to 0. So, here again if I want to find out say for example, the value of function at say t equal to 1.7. So, I have the functional values at t equal to 1, I have the value of the function at t equal to 2. Now, at t equal to 1.7, I have to approximate the value of this function using its neighboring pixels.

Now, if I try to approximate this, you find that using this particular interpolation formula; here again, 1.7 is to be computed as p_0 $B_{0\,2}$ (1.7) plus p_1 $B_{1\,2}$ (1.7). So, it will continue like this. Now, here we find that the contribution to this point by this sample p_0 by this sample value p_0 is given by this interpolation function $B_{0\,2}$ (t) and by this, the contribution to this point t equal to 1.7 by this sample f_1 or p_1 is given by $B_{1\,2}$ (1.7), contribution to this point by the sample value f (2) is given by $B_{2\,2}$ (1.7). But $B_{2\,2}$ (1.7) is equal to 0 at t equal to 1.7 because $B_{2\,2}$ (1.7) is something like this.

So, value of this function is 0 at t equal to 1.7. So, only contribution that we get at t equal to 1.7 is from the sample f (0) and from the sample f (1). So, using this, I can estimate what will be the value of $B_{0\,2}$ at 1.7, I can also at estimate what will be the value of $B_{1\,2}$ 1.7. And in this particular case, we have seen a property of this B Spline function that is we have said earlier that $B_{i\,k}$ (t) is nothing but $B_{0\,k}$ into t minus i. So, that is a property of this B Spline functions.

So, when I do this, you find that this $B_{1\,2}$ (1.7) is nothing but $B_{0\,2}$ because this is t minus i and value of i is equal to 1. So, this will be $B_{0\,2}$ (0.7). So, if I simply calculate the value of $B_{0\,2}$ for different values of t, I can estimate that what will be $B_{0\,2}$ (1.7). And, in that case, the value at this location that is f (1.7) will now be given by; if I simply calculate this, so in this case f of 1.7 will be given by p_0 into $B_{0\,2}$ (1.7) plus p_1 into $B_{0\,2}$ (0.7) because this is same as $B_{1\,2}$ (1.7) where value of p_0 is equal to 0.5 which is the sample value at location t equal to 0 and value of p_1 is equal to 1 that is the sample value at location t equal to 1.

Now, here we have find that there is a problem that if when I am trying to compute the value at t equal to 1.7, the contribution only comes from the sample values at t equal to 0 and t equal to 1. But this interpolation or this approximate value does not have any contribution from t equal to 2 or t equal to 3.

So, your interpolation or approximation that you are doing is very much burst because it is only considering the sample values to the left of this particular point, we are not considering the sample values to the right of this particular point t equal to 1.7. So, that is the problem with this basic interpolation formula.

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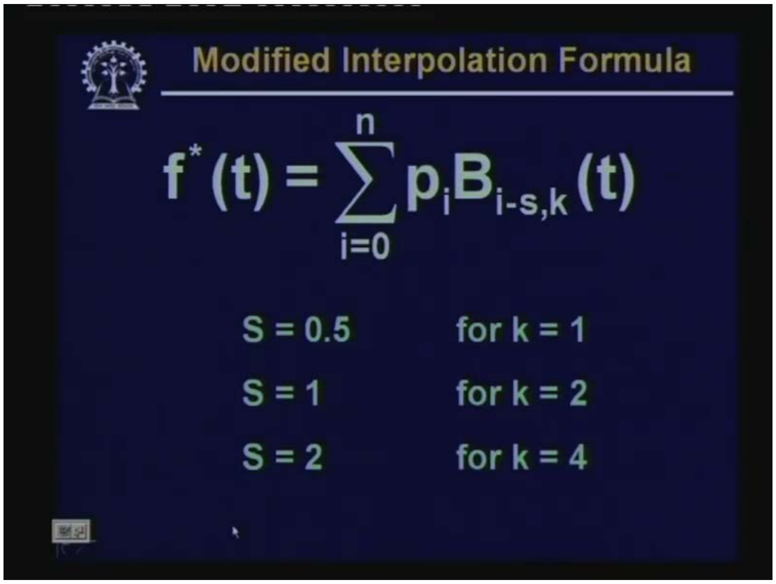

So, to solve this problem, what we do is instead of using the simple formula that is f (t) equal to summation p_i $B_{i\ k}$ (t) where i varies from 0 to n, we slightly modify this interpolation formula. We make say, f star t is equal to p_i $B_{i \text{ minus S}, k}$ (t) where again i varies from 0 to n. And the value of S, we decide that for k equal to 1 that is when you go for constant interpolation, we assume value of S to be 0.5. For k equal to 2 that is for linear interpolation, we assume value of S to be 1 and for k equal to 4 that is for cubic interpolation, we assume value of S to be equal to 2.

So, here again we find that I have not considered k equal to 3 because as we said that k equal to 3 gives you the quadratic interpolation and in case of quadratic interpolation, the interpolation is not symmetric. So, what we effectively do by changing $B_{i\ k}$ (t) to $B_{i \text{ minus S}, k}$ (t) is that the interpolation function or the B Spline interpolation function, we give this is a shift by S in the left direction while we consider the contribution of the sample p_i to the point t the for interpolation purpose.

Now, let us see what we get after doing this. So, as we said that with k equal to 1, I take the value of S is equal to 0.5. So, in the formula that p_i $B_{i\ k}$ (t) when I consider the contributions of sample p0; in the earlier formulation we had to use the B Spline function B_0 k (t). Now, if k equal to 1, this is the constant interpolation. So, I have to consider $B_{0\ 1}$ (t). Using this modified formulation, when I consider the contribution point p_0, I do not consider the B Spline function to be $B_{0\ 1}$ (t) but rather I consider the B Spline function to be B minus 0.51 (t).

Similarly, for the linear operation when I take the contribution of point p_0 to any obituary point along with B p_0, I had to consider as per the initial formulation $B_{0\ 2}$ (t). Now, using this modified formulation, I will use minus B $_{\text{minus } 1\ 2}$ (t). Similarly for the cubic interpolation, again with p_0, I will consider the B Spline to be B minus 4t instead of $B_{0\ 4}$ (t).

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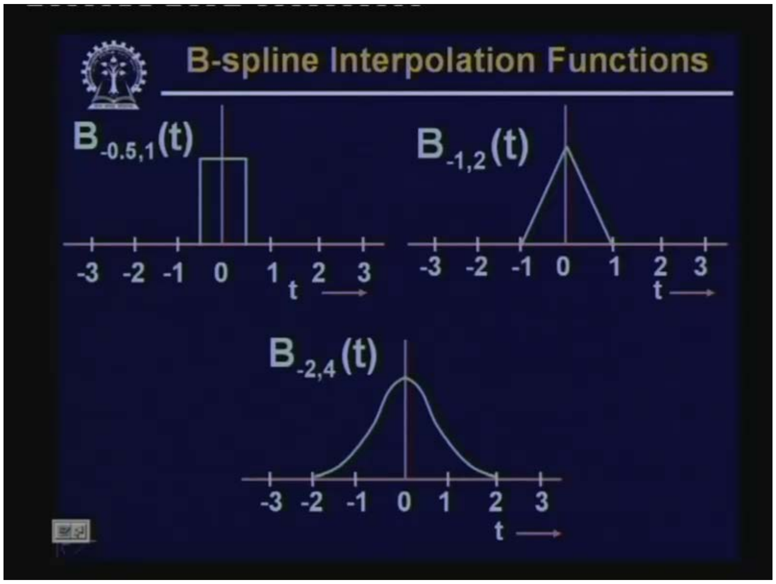

So, here we find that in this particular diagram using this formulation, effectively what we are doing is we are shifting the B Spline functions by the value of S in the left word direction. So, $B_{0\,1}$ (t), in the earlier case we had $B_{0\,1}$ (t) to be 1 between 0 and 1. So, $B_{0\,1}$ (t) is something like this.

Now, along with p_0, I do not consider $B_{0\,1}$ (t) but I will consider $B_{\text{minus } 0.5,\,1}$ (t) and minus 0.5, 1 (t) is equal to 1 for values of t between minus 0.5 and plus 0.5. And, value of $B_{\text{minus } 0.5,1}$ (t) will be equal to 0 beyond this range. Similar is also the case for the linear interpolation, the linear B Spline and it is also similar for the cubic B spline that is $B_{i\,4}$ (t) and in this case it will be i minus 2, 4 (t). So using this, let us see that how it helps us in the interpolation operation.

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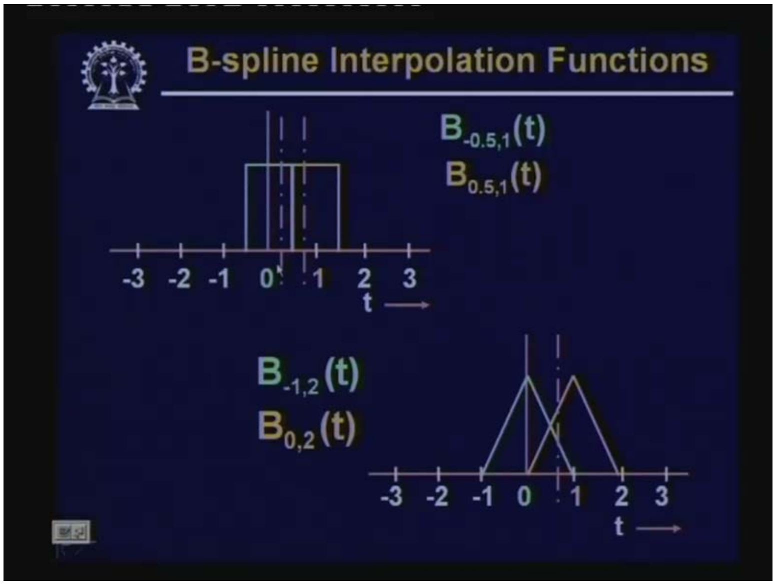

So, as I said that for interpolation, when I consider the contribution of p_0 to any particular point; along with p_0, I will consider the B Spline function to be $B_{\text{minus } 0.5,\,1}$ (t) for constant interpolation. So, it appears like this that for contribution of p_0, I consider $B_{\text{minus } 0.5,\,1}$ (t). To find out the contribution of p_1, I consider the B Spline function to be $B_{0.5,\,1}$ (t). Similarly, in case of a linear interpolation, to find out the contribution of point p_0, I consider the B Spline function to be $B_{\text{minus } 1,\,2}$ (t) and to find out the contribution of p_1, I consider the B Spline function to be $B_{0\,2}$ (t) and so on.

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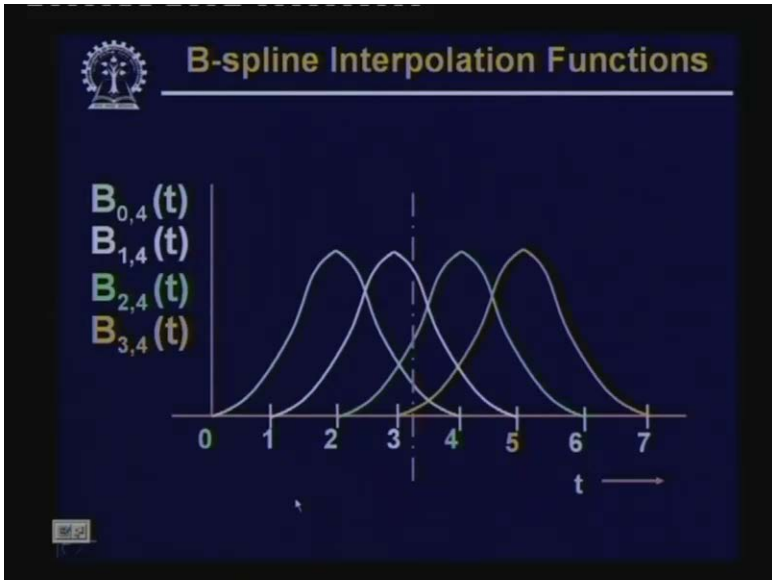

Similar is also case for the cubic interpolation. Here again, to find out the contribution of say p_0, I have to consider the B Spline function of B $_{\text{minus } 2,\, 4}$ (t). To find out the contribution of p_1, I have to consider the B Spline function of B $_{\text{minus } 1,\, 4}$ (t). To find out the contribution of p_2, I have to consider the B Spline function of $B_{0\,4}$ (t) and so on.

Now, let us see that using this kind of using this modified formulation; whether our interpolation is going to improve or not. So, let us take this particular case. Again, we go for constant interpolation.

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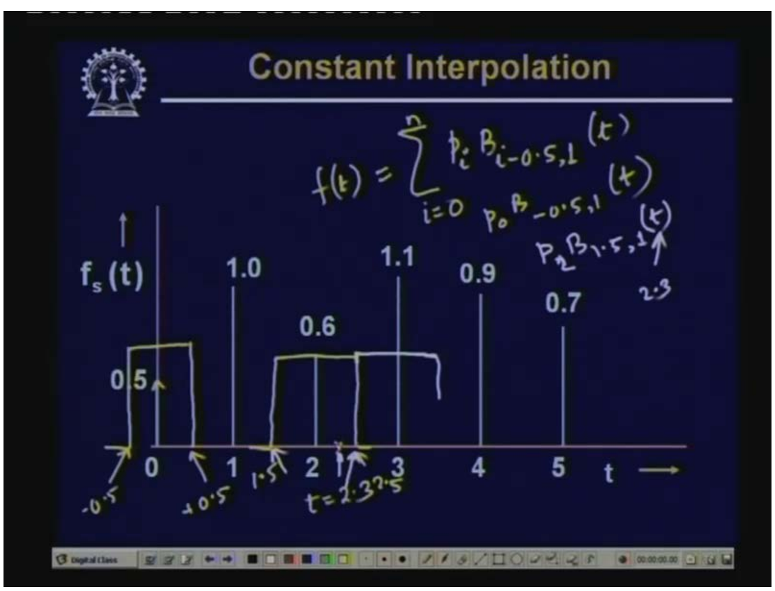

Here again, I have shown the same set of samples and now suppose I want to find out what will be value of the function at say t equal to 2.3 and for considering this, I will consider the equation to be p_i into $B_{i \text{ minus } 0.5, 1}$ for constant interpolation value of k is equal to 1(t) and I will take the sum for i equal to 0 to n. So, these will give me the approximate or interpolated value at t.

So, again you find that coming to this diagram, as we have said that when I consider $B_{i \text{ minus } 0.5}$ (t) in that case, $B_{\text{minus } 0.5,1}$ (t) is equal to 1 between the range minus 0.5 to plus 0.5 and beyond this range, $B_{\text{minus } 0.5}$ (t) will be equal to 0.

So, when I compute p_0 $B_{\text{minus } 0.5, 1}$ (t) for computation of this particular component along with this sample p_0 which is equal to 0.5, I have to consider the Bezier function. The Bezier sorry B Spline interpolation functions as this which is equal to 1 from minus 0.5 to plus 0.5. So, here we are going to find that because this $B_{\text{minus } 0.5, 1}$ (t) is equal to 0, beyond t equal to 1.5, so this p_0 does not have any contribution to point t equal to 2.3because at this point the contribution of this or this particular product p_0 B minus $_{0.5,\ 1}$ (t) will be equal to 0.

Similarly, if I consider the effect of point p_1 to t equal to 2.3, the effect of point p_1 will also be equal to 0 because the product p_1 B minus 1 p_1 $B_{0.5, 1}$ (t) is equal to 0 at t equal to 2.3. But if I consider the effect of p_2 which is equal to 0.6 here; you find that for this, the B Spline interpolation function has a range something like this, the region of support.

So, this is equal to 1 for t equal to 1.5 to t equal to 2.5 and it is equal to 0 beyond this junction. To find out the contribution of p_3 which is equal to 1 point 1.1, here again you can see that to find out the contribution of this particular point, the corresponding B spline function that is $B_{2.5}$ (t) is equal to 1 in the range 2.5 to 3.5 and it is equal to 0 outside.

So, even p_3 does not have any contribution to this particular point t equal to 2.3. So, at t equal to 2.3 if I expand this, I will have a single term which is equal to p_1 into B sorry p_2 into $B_{1.5,\ 1}$ (t) so where t is equal to 2.3 and the same will be applicable for any value of t in the range t equal to 1.5 to t equal to 2.5.

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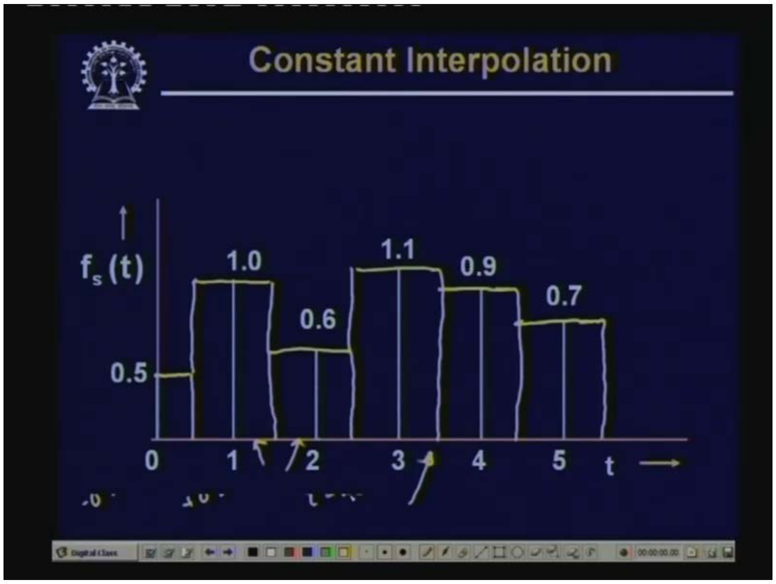

So, I can say that using this formulation, what I am getting is I am getting the interpolation something like this. After interpolation, using this constant interpolation function after our modified formulation, the value of the interpolated function will be say from t equal to 0 to t equal to 1.5, the f (t) value of f (t) will be equal to f (0). Between 1.5 to between 0.5 to 1.5, value of f (t) will be equal to f (1).

From point 1.5 to 2.5, the value of f(t) will be equal to f (2), from 2.5 to 3.5, the value of f (t) will be equal to f (3), from 3.5 to 4.5, the value of f (t) will be equal to f (4) and from 4.5 to 5.5, the value of f (t) equal to f (5) and this appears to be a more reasonable approximation because what we are doing is whenever we are trying to interpolate at a particular value of t, what we are doing is we are trying to find out what is the nearest sample to that particular location of that particular value of t and whatever is the value of the nearest sample, we are simply copying that value to this desired value of t.

So here, for any point within this range that is for t equal to 1 to t equal to 1.5, the nearest sample is f (1). For any sample from 1.5 to 2, the nearest sample is p_2 or f (2). So, this f (2) is copied to this particular location t where t is from 1.5 to 2. So, this appears to be a more logical interpolation than the original formulation of interpolation. Similar is also case for linear interpolation.

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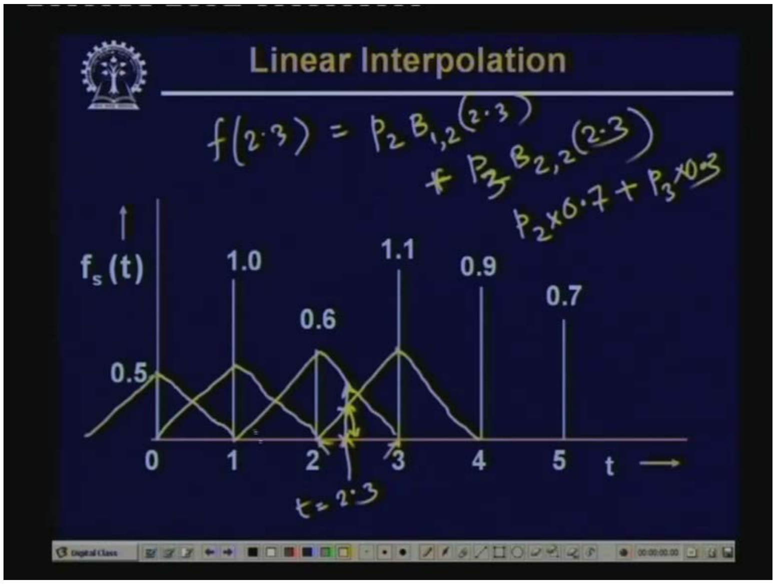

In case of linear interpolation, when I consider the value of p_0, what I do is I consider B $_{\text{minus } 1,\ 2}$ (t) and B $_{\text{minus } 1,\ 2}$ (t) is something like this where from minus 1 to 0, this increases linearly, attains a value of 1 at t equal to 0. Similarly, if when I consider the contributions of p_1, the corresponding Bezier interpolation function that I have to consider is B_0 to t which is something like this.

So now, if I want to find out what is the value at the same point say 2.3, you find that to find out the value at point 2.3, the contribution of p_1 will be equal to 0 because the value of B_0 to t is equal to 0 beyond point t equal to t at t equal to 2. And by this, you will find that the only contribution that you can get to this point t equal to 2.3 is from the point p_2 and from the point p_3.
And, in this particular case, I will have f (2.3) which will be nothing but p_2, then B because it is I have to make it i minus, 2 in i minus 1 in this particular case. So, this will be equal to $B_{1,\ 2}$ (2.3) plus p_2 into B sorry p_3 into $B_{2,\ 2}$ (2.3) just this.

So, the contribution of this point to this point t equal to 2.3 will be given by this value and the contribution of point p_3 will be given by this value and here you find that because the function increases linearly from 0 to 1 between t equal to 2 and t equal to 3 and it decreases linearly from 3 to 0 when t varies from 3 to 4; so here, you will find that the value that we will get will be nothing but p_2 into the value of this function in this particular case will be equal to 0.7 plus it will be p_3 into 0.3.

So, if I simply replace the value of p_2 which is equal to 0.6 and p_3 which is equal to 1.1, I can find out what is the value of f (t) at t equal to 2.3. So, similar is also the case for cubic interpolation.

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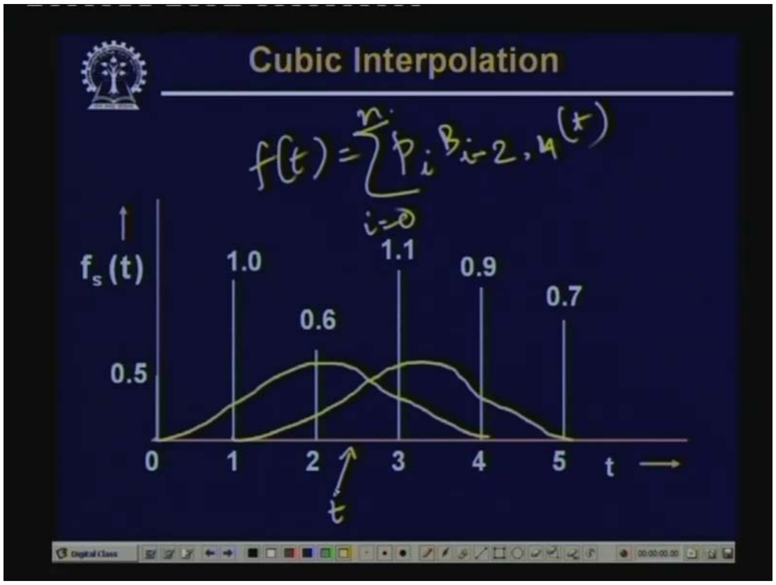

And in case of cubic interpolation, you find that your region of supports will be something like this. Sorry so the region the nature of the region of support will be something like this and again by using the same formulation that is f (t) is equal to p_i $B_{i \text{ minus } 2}$ in this particular case into k value of k is equal 4t, I can find out, I am taking the summation for i equal to 0 to n; I can find out what will be the value of f (t) for any particular time instant t or any arbitrary time instant t.

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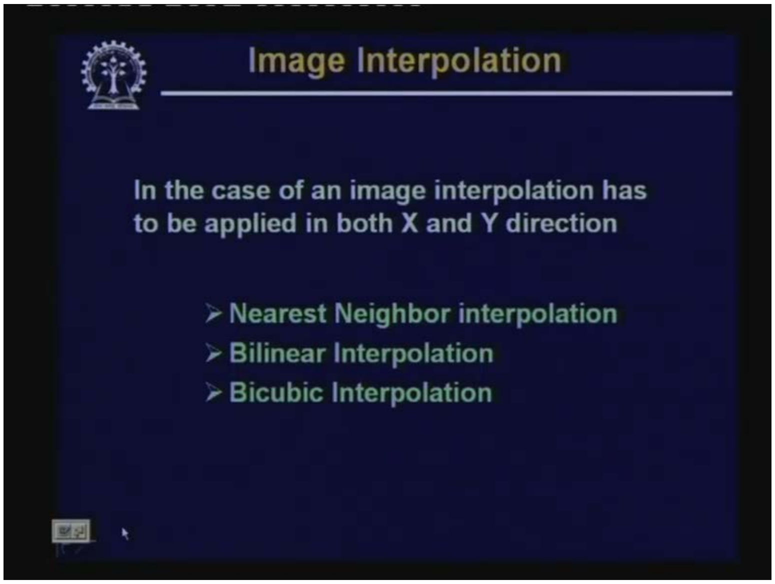

So, by modifying this interpolation operation, we can go for all these different types of interpolation. Now, to explain this, let us take a particular example. So, I take an example like this.
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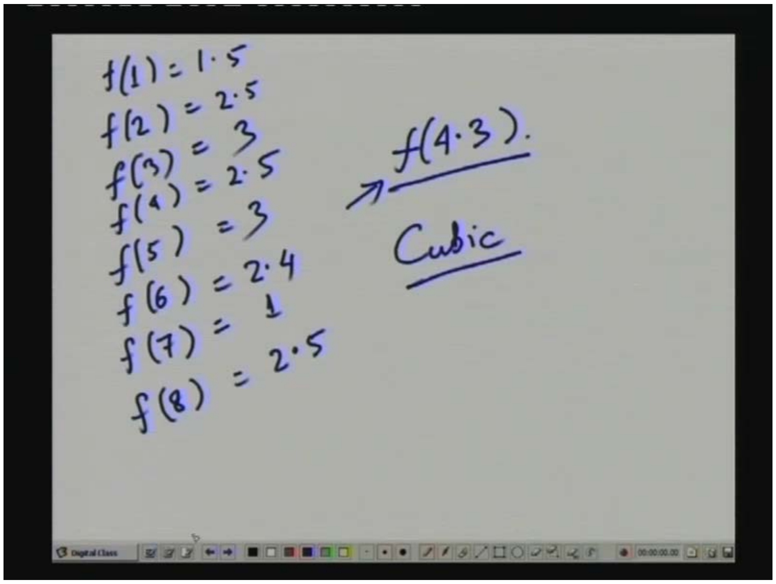

I take a function say f, the function values are like this that f of 0, so I take f of 1 is equal to say 1.5, I take f of 2 is equal to 2.5, I take say f of 3 is equal to say 3, I take f of 4 is equal to something like say 2.5, I take f of 5 is equal to again 3, f of 6 will be something like 2.4, I can take f of 7 to be something likes say 1, I can take f of 8 to be something like 2.5 and I wants to find out the approximate value of this function at say t equal to 4.3.

So, given this sample values, I want to find out what is the value of this function at t equal to 4.3. So, I want to find out f (4.3) given this sample values. Now, suppose the kind of interpolation that I will use is a cubic interpolation, so I use cubic interpolation and using these samples, I want to find out the value of f (4.3). So, let us see that how we can do it.

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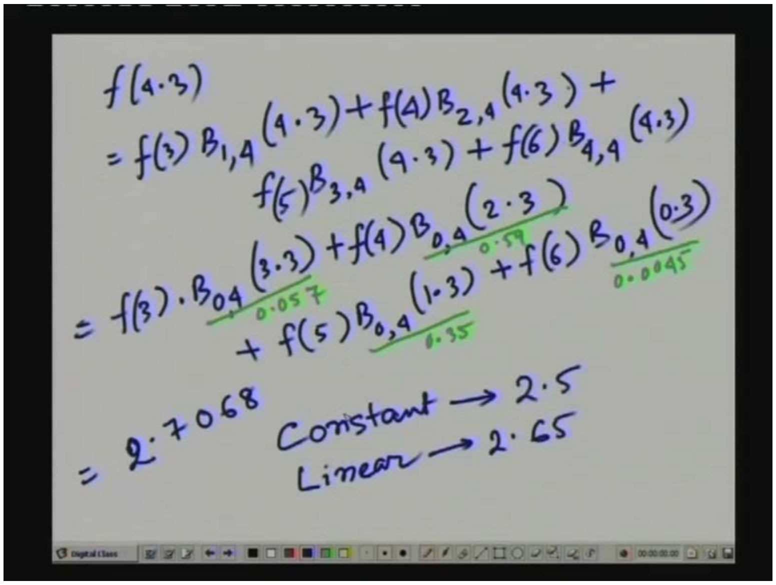

Here, you find that f (4.3) can be written as using considering the region of support, this can be written as f (3) into $B_{1\,4}$ (4.3). So, you find that since this f (3) is nothing but p_3 in our case, so this B_i becomes $B_{i \text{ minus } 2}$. So, 3 minus 2 is equal to 1, so I am considering $B_{1\,4}$ and at location t equal to 4.3. So, this will be plus f (4) which is nothing but p_4 into $B_{2\,4}$ (4.3) plus it will be f (5) into $B_{3\,4}$ at location t equal to 4.3 plus it will be f (6) into $B_{4\,4}$ at location t equal to 4.3.

Now, as we said that we have told that $B_{i,\ k}$ (t) is nothing but $B_{0\,k}$ (t) minus i. So, just by using this particular property of the B Spline functions, I can now rewrite this equation in the form that this will be equal to f (3) or p_3 into $B_{0\,4}$ and this will be now t minus i, i is equal to 1, so this will be 3.3 plus f (4) into $B_{0\,4}$ (2.3) plus f (5) into $B_{0\,4}$ (1.3) plus f (6) into $B_{0\,4}$ and this will be equal to 0.3.

Now, we can compute the values of this $B_{0\,4}$ (3.3), you can compute the values of $B_{0\,4}$ (2.3), you can compute the value of $B_{0\,5}$ (1.3) and you can also compute the value of $B_{0\,4}$ (0.3) using the approximate analytical formula of $B_{0\,4}$ (t) that you have given you have seen that this is nothing but a cubic formula of variable t. So, if I do this, you will find that and using the sample values, this $B_{0\,4}$ (3.3), this gets a value of 0.057. $B_{2\,4}$ $B_{0\,4}$ (2.3), this gets the value of 0.59, $B_{0\,4}$ (1.3), this gets the value of 0.35 and this $B_{0\,4}$ (0.3) gets a value of 0.0045 and you can verify this by using the computation the analytical formula that we are going to learn.

And by using the values of f (3) f (4) f (5) and f (6), if I compute this equation; then I get the final interpolated value to be 0.7068 sorry 2.7068. Now, if I do the same computation using constant interpolation and as I said that the constant interpolation is nothing but the nearest interpolation; so, when I try to find out the value at f (4.3) the point t equal to 4.3 is nearest to point t equal to 4 at which I have a sample value.

So, using nearest neighbor or constant interpolation, f (4.3) will be simply equal to f (4) and which in our case is equal to 2.5. Whereas, if I go for linear interpolation, again you can compute this using the linear equations that we have said that using linear interpolation, the value of f (4.3) will be equal to 2.65.

So, you find that there is slight difference of the interpolated value whenever we go for constant interpolation or we go for linear interpolation or we go for cubic interpolation. So, using this type of formulation, we can go for interpolation of the 1 dimensional sample functions. Now, when you go for the interpolation of image functions, you find that images consist of a number of rows and a number of columns. Now, the interpolation that we have discussed so far, these interpolations are mainly valid for 1 dimensional.

Now, the question is how do we extend this 1 dimensional interpolation operation into 2 dimensions so that it can be applied for interpolation images as well?

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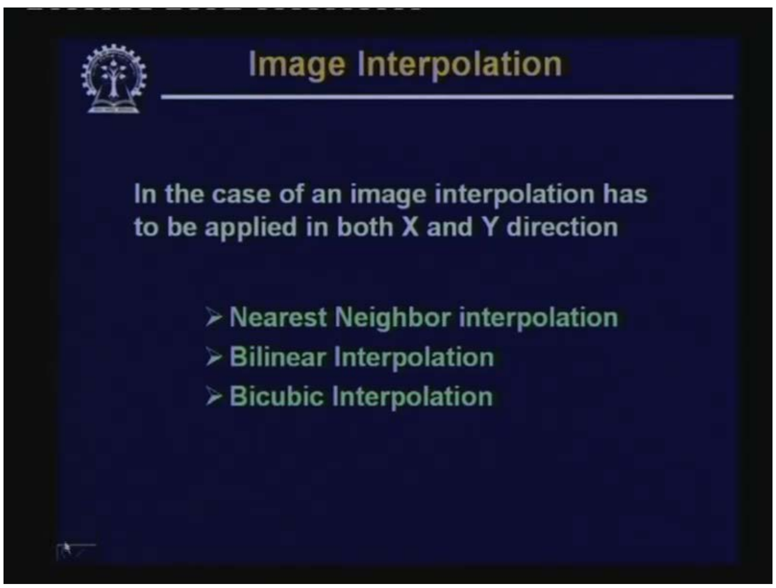

So, in case of image what we will do is as the image is nothing but a set of pixels arranged in a set of rows and in a set of columns.

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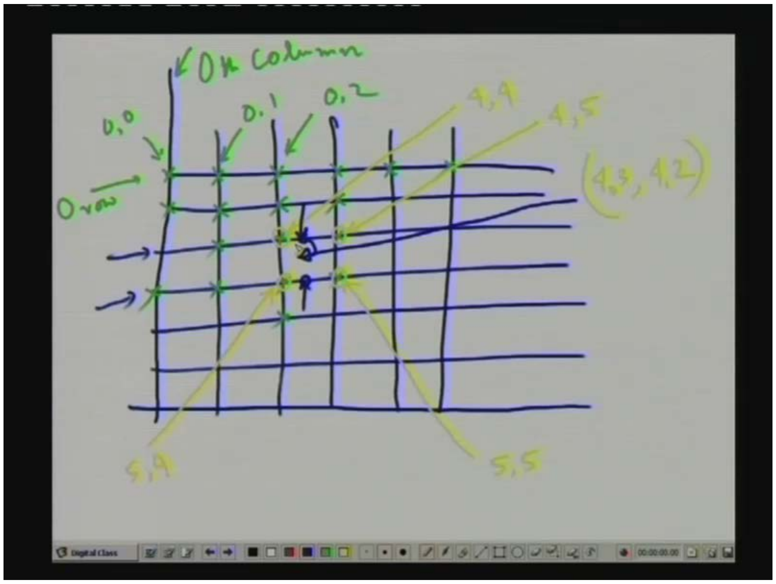

So, let us consider a 2 dimensional grid to consider the image pixel. So, I have the grid points like this. So in case of an image, I have the image points or image pixels located at this location, located at this location. So, these are the grid points where I have the image pixels. So, it is something like this.

So, I said this is location (0, 0), this is location say (0, 1), this is location (0, 2) and so on so that I have these as the 0'th row, these as the 0'th column and similarly I have row number 1, row number 2, row number 3, row number 4 and column number 1, column number 2 and column number 3, column number 4 and so on.

Now given this particular situation; so, I have the sample values present at all these grid points. Now, given this particular pixel array which is again in the form of a 2 dimensional function matrix, I have to find out what will be the pixel value at this particular location. Suppose, this is say location (4, 4) let us assume, this is at pixel location at (4, 5) - 4th row, fifth column, this may be a pixel locations say (5, 4) that is fifth row 4th column and this may be the pixel location say (5, 5) that is fifth row and fifth column.

So, at these different pixel locations, I have the intensity values or pixel values and using this I want to interpolate what will be the pixel value at locations say 4.3 and say 4.2. So, I want to compute what will be the pixel value at this particular location 4.3 and 4.2. So, now you find that the earlier discussion what we had for interpolation incase of 1 dimension, now that has to be extended for 2 dimension to get the image interpolation. Now, the job is not very complicated, it is again a very simple job.

What we have to do is I have to take the rows one after another. I also have to take the columns one of after another. So, first what you do is you go for interpolation along the rows and then you try for interpolation along the column and for this interpolation, again I can go for either constant interpolation or I can go for linear interpolation or I can go for cubic interpolation. But now because our interpolation will be in 2 dimensions; so the kind of interpolation, if it is linear interpolation, it will be called a bilinear interpolation, for cubic interpolation, it will be called a bicubic interpolation.

So, let us see that what will be the nature of the interpolation if I go for a constant interpolation. So effectively, what we will do is so in this particular case, we will interpolate along the row 4, we will also interpolate row 5. So, we will try to find out what is the value, pixel value at location (4, 4.2). We will also try to find out what is the pixel location at (5, 4.2). So, once I have the pixel values interpolated values at this location and this location that is (4, 4.2) and (5, 4.2), using these 2 sample values, I will try to interpolate the value at those this location (4.3, 4.2). So, simply extending the concept of 1D interpolation to 2D interpolations.

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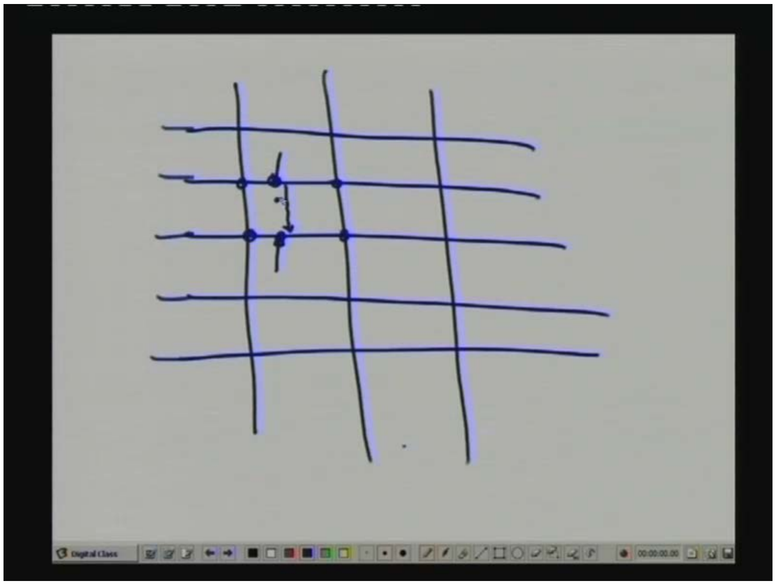

Now, what will be the nature of this interpolation if I go for constant interpolation? So, as we said the constant interpolation simply takes the nearest neighbor and copies to the particular, the arbitrary location which does not occur on the regular grid.

So, at this location I have a pixel value, at this location I have pixel value, at this location I have a pixel value, at this location I have a pixel value. Now, if I want to find out what will be the value the pixel at this particular location, what I do is I simply try to find out which is the nearest neighbor of this particular point and here, you find that the nearest neighbor of this particular point is this point.

So, all the points which are nearest to this point within this square, all the pixel values will get the value of this particular pixel. So, it will be something like this. Similarly all the pixels, all the points lying within this region will get the pixel values of this particular region. Similar is the case here and similar is the case here. When you go for bilinear interpolation, when I try to interpolate the pixel at any particular location like this, so again here, incase of bilinear interpolation if I want to find out the pixel value at any particular, any arbitrary grid locations something like this, so somewhere here, what I have to do is; I have to consider this pixel and this pixel, do the bilinear interpolation to find out the pixel at this location. I consider this pixel and this pixel do bilinear interpolation to find out the pixel to value at this location. Then using this and this, doing bilinear interpolation along the column; I can find out what is the pixel value at this particular image and the same concept can also be extended for bicubic interpolation.

So by this, we explain how to do interpoltion, either constant interpoltion which we have said is also the nearest inerpolation, we can go for linear interpolation. In case of image, it is bilinear interpolation or the cubic interpoltion. Incase of image, it is bicubic interpolation where you have to do interpoltion both along the rows and after the rows, you can do it along the columns. It can also be reversed. First, you can do the interpolation along column, then using the interpolated value along 2 or more columns, I can findout the interpolated value on any row location which does not fall on any regualar grid point.

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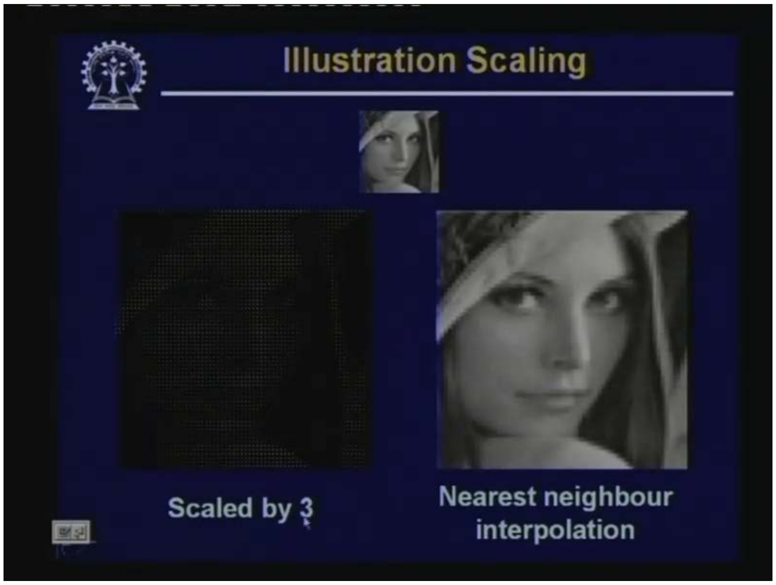

So, now let us seen that whatever results, these results we had already shown in the last class. So, find that the first 1 is interpolated using nearest neighbour interpolation and as we have explained that because the value of the nearest pixels is copied to all the arbitrary location; so, this is likely to give blocky … defect and in this nearest interpolted image, you also find that those blocking defects are quite prominent. We have also seen the output with other interpolation operations.

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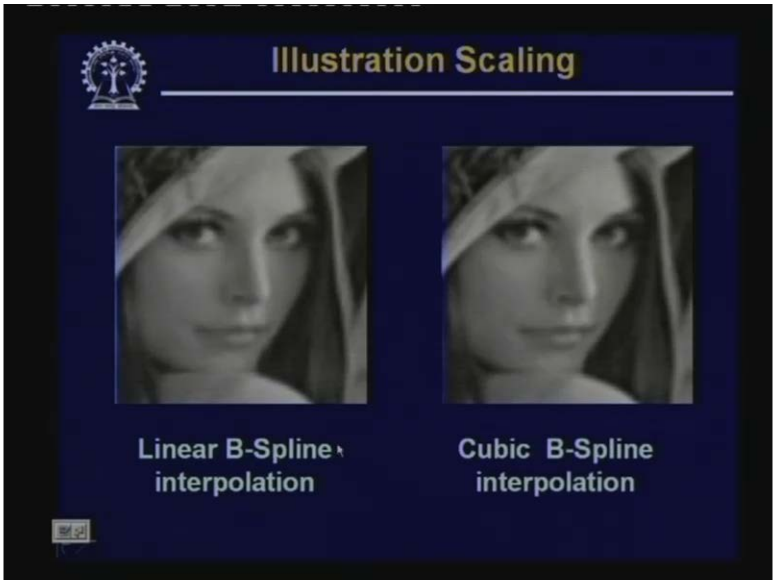

We have shown the output with Linear B Spline interpolation and also shown with cubic B Spline interpolation.

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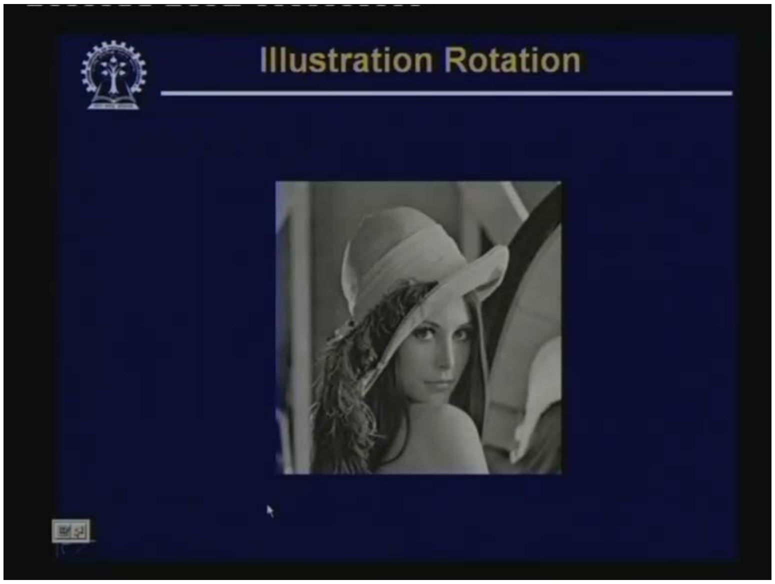

So, this is case with rotation.

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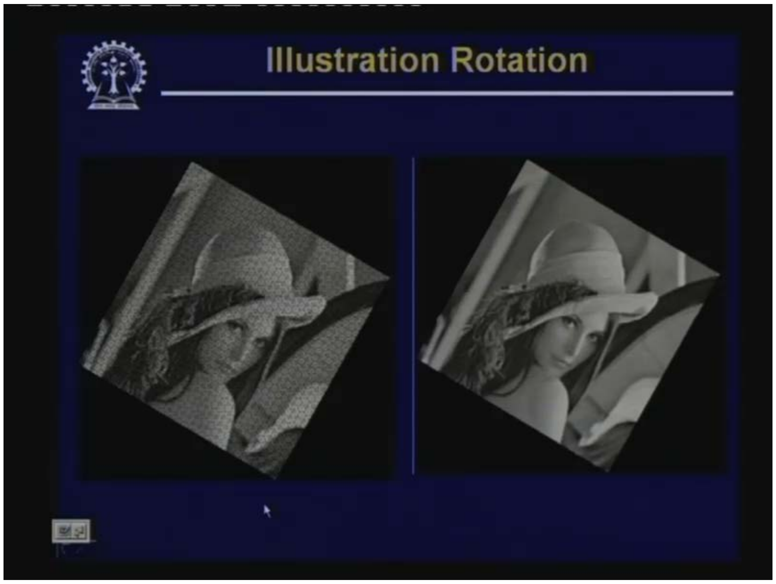

Again when you rotate, if you do not interpolate, you get a number of patches black patches as shown in the left image; if you go for interpolation, all those black patches will be removed and you get a continuous image as it is shown in the right image.

Now, this interpolation operation is useful not only for this translation or rotation kind of operations, you find that in many other applications, for example in case of satellite imaginary. When the image of the earth surface is taken with the help of a satellite; now because of earth rotation, the image which is obtained from the satellite, the pixels always does not follow regular.

So in such cases, what we have to go for is to rectify the distortion or to correct the distortion which appears in the satellites images and this distortion is mainly due to the rotation of the earth surface. So, for correction of those distortions, the similar the type of interpolation is also used.

Thank you.