Adaptive Signal Processing
Prof. M.Chakraborty
Department of Electronics & Electrical Communication Engineering
Indian Institute of Technology, Kharagpur

Lecture - 4
Correlation Structure

So, yesterday we are discussing the Hermitian matrices and then we discussed, what is called auto covariance matrix auto correlation matrix and all that. So, let us start from there briefly to recall, what is a Hermitian transposition? That is conjugate transposition. For say a real matrices a real vector it is simple transposition, but for complex value matrices, it is conjugate transposition. And then if a matrix a matrix you know vector is a special case of matrix is m cross n any one cross m matrix is a vector. So, I may talk in terms of matrix, but that includes vectors also.

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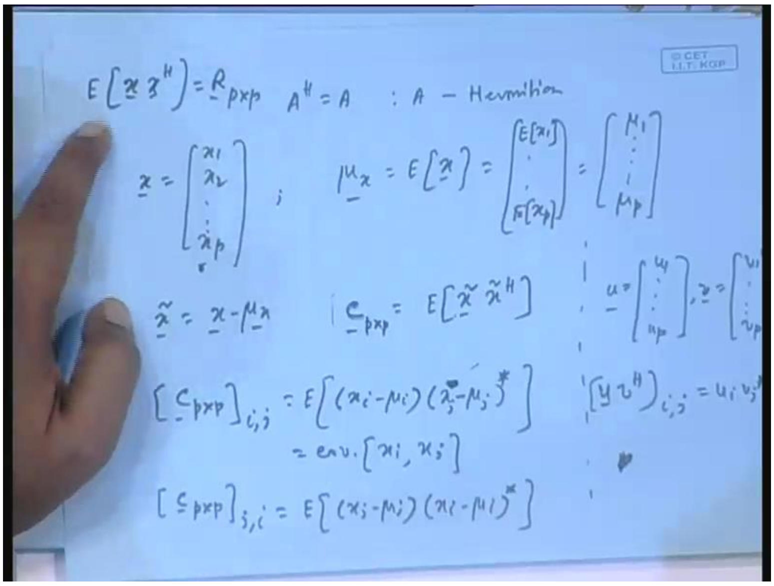

Any matrix A if A H is A then A is Hermitian A Hermitian. So, you understand for this to be true matrix has to be square A Hermitian means you are taking transpose and original matrix; if they are same means dimensionally they have to match, there will be number of row and number of column of A have to be same, which means the square matrix that goes without saying. Then, we discussed then look we said that look suppose we consider a vector x random vector means it consist of say p number of random variables; any vector or scalar under indicated by me by an underscore.

Normally, upper case letters with underscore lower case letter with underscore for vector upper case letters with underscore matrices. Sometimes, you know by mistake I may skip I might just you know you either tell me or you assume at it is as it is. Because always I may not be able to stick to this underscoring theory you know. Sometime, I may the just skip unwittingly. Then, the mean of this is there is a mean vector E of x E of x means a vector like this, where you have got E of x 1 dot dot dot. Say, E of x p we can even write it as say mu 1 yesterday I call mu x 1 yesterday I call mu x 1.

Let me, call it mu 1 because there is no other say mu p. Then I said that is you subtract if you consider, these vector say x tilde as x minus mu x vector it will give only the deviation across the mean. Then, if you take if you take this E of x tilde x tilde Hermitian I called it auto correlation auto covariance matrix; C you can even indicate c p cross to indicate the matrix. What does this matrix gives today I will not expand yesterday I expanded. Just for you consumption you remember one thing suppose I give a vector forget about all these.

Suppose, I give a vector say u as u 1 dot dot dot u p and I give another vector same length v v 1 dot dot dot v p. And if you multiply u with v Hermitian; what you get column vector u v Hermitian means it becomes row vector with conjugate. If you take this matrix and its general element i j th element what will that be. See, how the matrix is formed u one into v 1 star first element u 1into v 2 star u 1 dot dot dot v p star that is a first row; i th row means u i u i into v 1 star u i into v 2 star. So, j th element means u i into v j star u i here i th element from here j th element from here this is.

So, what is general element i j th element of course, i and j remaining within 1 to p; this is simply i th element of here j th element of a conjugate with expectation. So, that mean x what x tilde vector xi; I mean it is a component xi minus mu i. There is a i th component here of this vector and j th component of that x tilde minus mu x tilde j minus mu j star. No tilde; x j minus mu j x j minus mu j star that is coming from the tilde vector x j minus mu j star. So; obviously, this is the covariance between covariance between x i and x j. If this is zero then x i and x j are uncorrelated all those things you have seen that I am coming separately that these matrix we have seen yesterday.

Hermitian that you can see directly I told you yesterday, but I repeat again here you can see c p p i j is this. What is c? I am just trying various ways because we will be doing this kind of exercise too often. So, I am just making you familiar. What is the j i th element E of this will come first x j minus mu j and so obviously, this is conjugate of this. So, i j th element and j i th they are same, but conjugate of each other. So, it is conjugate symmetry this why you can say this Hermitian. Even, otherwise you say as I told yesterday you do not have to do all these by simply algebra if you take Hermitian of this matrix Hermitian operation you take Hermitian of this.

You can push Hermitian operation inside expectation operation because expectation means what any term is multiplied by some appropriate probability density integrated;
so, that will be replaced from i j th to j i th it will go. So, you better if move it fast and then take expectation rather than taking expectation first they move and also a conjugation, but conjugation after multiplying by the probability and integrating is same as you better first conjugate the variable and then integrate multiplied by p x because the probability is a real function. So, that way these are the mathematical things you know inside.

So, you can push Hermitian operation instead of v operation and if you do that if you take this polar x Hermitian x x tilde x tilde Hermitian Hermitian of that; first second term will come first Hermitian Hermitian will cancels you get back x tilde. First term, will go second with Hermitian x tilde Hermitian, which get back the same thing and that prove that it is Hermitian. If mu x if you take only this vector this much only x no subtraction of mu no subtraction then it is called correlation matrix. R matrix p cross p correlation matrix this two become same this also Hermitian you can easily see by the structures here.

Expectation of something into something transfer Hermitian and this becomes same as this when your mu x is zero for zero mean process is zero mean random variables correlation covariant are same. As a special case, now consider this that supposes there is a discrete time random process x n.

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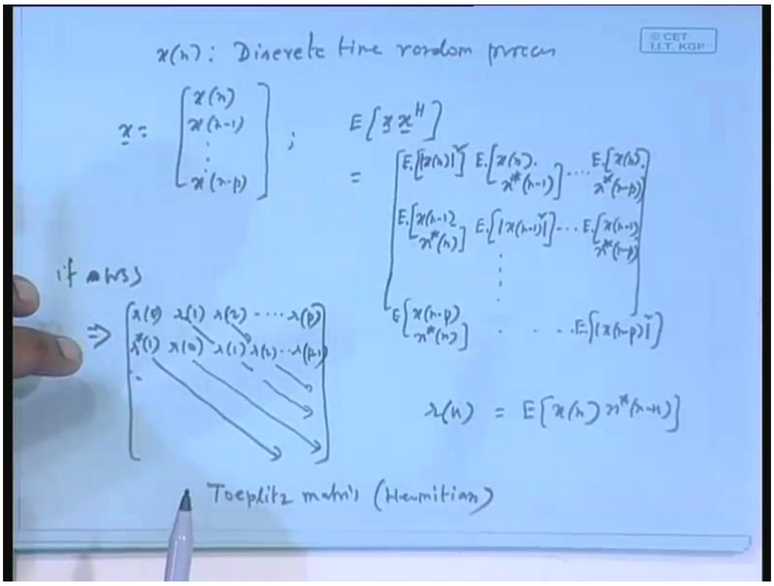

So, x n x n minus 1 x n minus 2 that is now this is given. Stationary or not I am not speaking so far. I am just taking some samples. Please note all these things carefully because later in the analysis of adaptive filter; I will make use of all these results frequently. So, I will not repeat rewrite this. So, actually p plus 1 element have taken in n minus 0 n minus 1 up to n minus p. So, it is not p p plus 1. Suppose, I form a vector like this I call it x. I can if I take this E x E x yes E x and x Hermitian; this will be a correlation matrix and if mean of this samples is zero, then there will be covariance matrix also.

What it will look like now I write it in expanded form first term will be mod x l square x n into x n then will be these are very important matrix. So, that is why I am doing this way x n into x star n minus one dot dot dot E x n x star n minus p. Next, row will be x what you cannot see this is a conjugate of this, you can otherwise you can take because of the Hermitian property. This conjugate will come here n dot dot dot. So, for there is something special this just a correlation matrix you only in this earlier it was giving general variables x 1 x 2; now the specific terms x n x n minus one x n minus two so and so forth.

So, in terms of those or other in terms of the correspond correlation this matrix elements will be obtained. That is all that is only, but now suppose and this is always Hermitian.

Now, suppose I say this x n also is wide sense stationary; that means, correlation is dependent only on the gap. So, then it takes a very very interesting structure. Firstly, you see even if not stationary this matrix if I know one half diagonal and upper half lower half can constructed because on the Hermitian property. Conjugate transpose, one half is enough for me upper triangular part with the diagonal or trigonal for the diagonal. Now, you see if there is stationary if stationary if WSS is this leads to what and I write separately here I define E of x n x star n minus k correlation it is equal to r; I should write r x x, but for the time being I am dropping x x because I am handling only one random process; so no point in writing too much.

So, you can easily understand what I am referring to so this is r k this is the definition with this definition in mind. What is in matrix? First term will be r 0, which is expected instantaneous power r zero, then will be please see this what is this r 1 then r 2 dot dot dot r p and obviously, this will be r star 1 Hermitian property. Then, again r zero then again I am not shown it here what will be the x 1 r 1 then r 2 dot dot dot r p minus 1. You will see along this diagonal r 0 will propagate because of stationary T, where there is E of mod x square E mod x n minus 1 square or E of mod r 0.

Similarly, along this sub diagonal; r 1 will propagate along this sub diagonal; r 2 will propagate so on and so forth. On this side, whatever be the element that also you propagate, this kind of matrices are called toeplitz matrix; this a special case you have a semi this Hermitian toeplitz. In the case of toeplitz this may not be r star 1 it could be anybody. So, a a a this next one could be b b b b that is the general toeplitz matrix, but this is totally toeplitz. This is why another structure the Hermitian that this element and this element are conjugate of each other. This guy and this guy are conjugate of each other there is an extra thing, which is a Hermitian property.

So, this is I am writing Hermitian; this matrix place a very key role in all signal processing application statistical signal processing adaptive signal processing; linear prediction that is filters, because we often handle stationary processing, where model the model the process to be stationary. Generally, we proceed further immediately the correlation matrix we get that is a toeplitz matrix. This matrix has beautiful property that in the sense and if you know one row or one column entire matrix is known you can construct; so that means, this matrix has lot of redundancy, which means if you are solving an equation a x equal to v and a is matrix of this kind.

Very first algorithms can be developed, because there is too many too much of redundancy in a in the matrix elements. Those algorithms exist very powerful algorithms, which come in various context of signal processing involving stationary processes. This is a Hermitian toeplitz matrix, this special case I have discussed. Again, I come back to general Hermitian matrices; general, Hermitian matrices of which correlation of covariance matrix are some examples.

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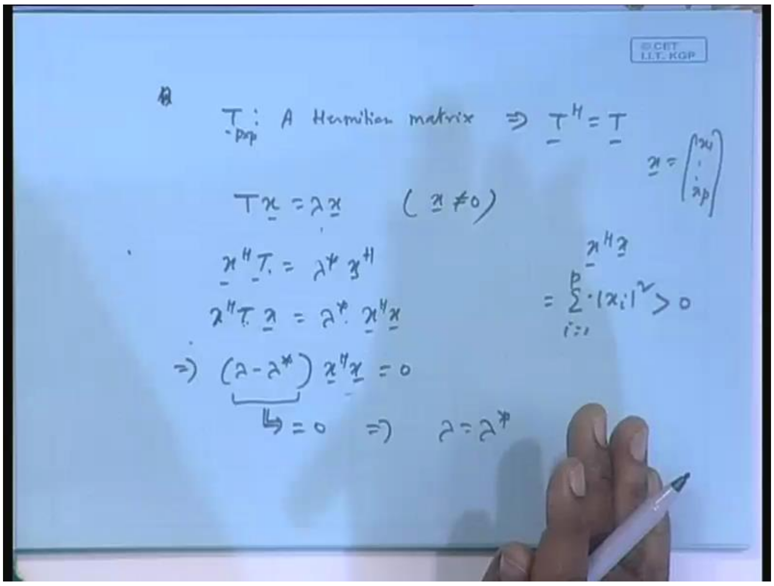

Suppose, not H suppose T Hermitian matrix T you call p cross p. I am writing p cross p, but when I go to the adaptive filter part; it will become p plus 1 into p plus 1 you know like it become last time. Simply, because last time if I have taken if I had taken form x n to x n minus p if I wanted to make it length p and not p plus 1; I would have to put n minus p plus 1. So, again everywhere you know n minus p plus 1 writing too much. So, I better make it p plus 1 and go up to n minus p just for my writing convenience writing nothing else.

Suppose, it is the Hermitian matrix in generally p cross p Hermitian matrix complex value T in general; Hermitian means it implies T Hermitian is T. I am skipping this underscore often please excuse me. Then first consider is Eigen value Eigen vector problem, suppose there is you are solving this equation. You know Eigen vector Eigen vector product that firstly, I this vector x must be non-zero. This is this equation is valid for any zero vector. So, you cannot find out a lambda. Eigen vectors are by need definition non- zero vectors and you are equation like this. What is the meaning of Eigen vector? It is such a vector that when you multiplied by T.

You do not get any arbitrary vector; you can make the same guy. Just scaled up a down by of element called lambda, there is in a geometrical sense it was pointing in rejection after you operate it. There are many operation of vector in a rotation and translation, but in such an operation that when you operate on that particular vector not for any vector on that particular vector. That is given an operation it has got such specific vectors called Eigen vectors. We are taking one Eigen vector, if you operate that Eigen vector within that operator. It will point it will dissolve in a vector pointing in the same direction its magnitude will change.

It will change by a vector lambda could be complex in generally, this called Eigen vector, Eigen value problem and x cannot be zero. Suppose, T is Hermitian now let us derive some interesting property and very simple properties very very simply derived. Suppose, I take Hermitian of both side; I can do that. So, if I take Hermitian of left hand side what will I get; x Hermitian T Hermitian and T Hermitian is T because T is the Hermitian matrix. So that means, x Hermitian T and Hermitian of right hand side means lambda star. You will take conjugate every element is multiplied by lambda you will take conjugate and transpose lambda will be replaced by lambda star.

There again pull out lambda lambda star. So, it will become lambda star x H. So, x Hermitian is a row vector multiplied by a matrix. So, real thing will be a row vector. The row vector if I now multiply here, this is a row vector; if I now bring back x here then row into column it will be a scalar. And lambda star, but T x is lambda x; see this matrix multiplication is commutative or distribute associative it is. So, you are multiplying this with this and then result with this you can also do this with this followed by this with this. But T x is lambda x. So, bring that lambda x here lambda is a scalar take it out.

So, you get back lambda x Hermitian x, on this side lambda star x Hermitian x. So, now, this result in this thing lambda minus lambda stars equal to zero. I am not showing see they are simple very silly very simple steps. Now, x is a non-zero vector what is x Hermitian x like you know I mean if x is of this kind; x 1 dot dot dot x p. What is x? Hermitian x summation mod x i square i from 1 to p; this is always real non- negative, if it is zero every term has to be zero because every term is mod square term. If the entire summation has to be zero every term has to be zero. Every term has to be zero mean each x i is zero, which means x is a zero vector, but that is ruled out because x is an Eigen vector x cannot be zero vector.

So that means, this is always greater than 0. So, in this equation then this is equal to zero this implied, because this cannot be zero. This implies lambda equal to lambda star, which means lambda is a real value. So, even if matrices Hermitian and complex valued Hermitian matrix Eigen vectors are in general complex values. Eigen values are all real of a Hermitian matrix. So, for any correlation matrix if you find out Eigen value there were real.

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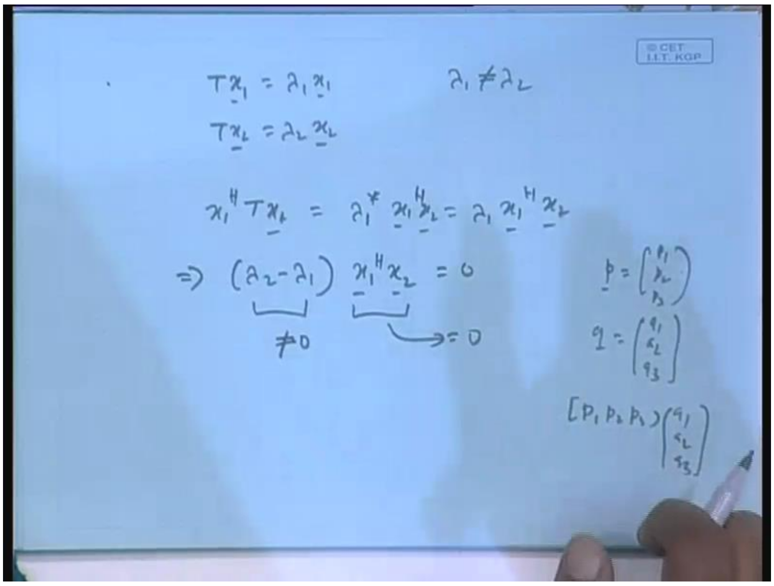

Another property, suppose I have got two distinct Eigen values T x 1 as say lambda 1 x 1 T x 2 lambda 2 x 2 and lambda 1 and lambda 2 they are not same two distinct Eigen values. Then, I can do one thing on this matrix just a minute; you take the Hermitian of both side of the first equation. So, you get x 1 Hermitian T Hermitian, which is T and on this side lambda 1 star lambda 1 star x x 1 H, but I have proved lambda is real. So that means, this is equal to simply lambda 1 this is this is reason, we have proven that. This now on this side I multiplied by x 2 so result are multiplied by x 2 here also.

Now, T x 2 is from the second equation lambda 2 x 2, so take out take that out. So, this will lead to this lambda 2 minus lambda 1, but I told you I am taking distinct Eigen values lambda 1 and lambda 2 are not same. So, this cannot be zero; that means, this is equal to zero. So, this is zero; that means, we say there orthogonal; two vectors in our definition if they satisfy this kind of theory you know to vectors x and y. If you do the x Hermitian y or y Hermitian x x Hermitian y then is zero that will mean y Hermitian x also zero; it will be the Hermitian of both side there, then x and y are called orthogonal.

Actually, you have done dot product of vectors is a three dimensional wall. So, p vector and q vector p dot q p suppose was p 1 p 2 p 3 q say q 1 q 2 q 3, what is p dot q p 1 plus q 1 plus q 2 p 2 q 2 plus; that is p transpose this p 1 p 2 p 3 transpose of this into q 1 q 2 q 3. That was the dot product, but if they are complex value you can generalize. These are generalized definition of a dot product, where there are not only transpose there conjugate transpose. Then, take these that is the dot product between two vector p and q; in a generally algebraic sense not in the three dimensional wall because there is nothing complex there.

You can generalize from three dimensions to any dimension. So, there is a given two vectors x and y x Hermitian y is a dot product and dot product, there is called inner product. This is more general term than dot product inner product; I will discuss all this later at length, when we come to vector space theory is dot product and dot product zero means orthogonal. Like p and q, if there are dot product means that is orthogonal, but there is a geometric sense that directly you can see visible geometrics, which I would say.

Geometry is there everywhere, whether is a visual geometric as you can see the angle is ninety degree and all those things. So, x 1 Hermitian x 2 0; now supposed to start with T is such a matrix p cross q, I have matrix how many Eigen values it can have at the most p Eigen value; p cross p matrix you solve the characteristic u s and you can have at the most p roots and for the time being assume that all the p roots are distinct. When not distinct what appears I will just tell few lines I will let prove, but that is heart of the linear algebra thing you could just take down my I mean take down what I say, but just to start with assume all the p roots are distinct.

So, I have got lambda 1 up to lambda p all real and I have got these equations T e k as lambda k e k; lambda k is the k th Eigen value given, lambda one not equal to lambda two distinct not equal to dot dot dot not equal to lambda p. These are case I am handling first so in that case can I write here directly.

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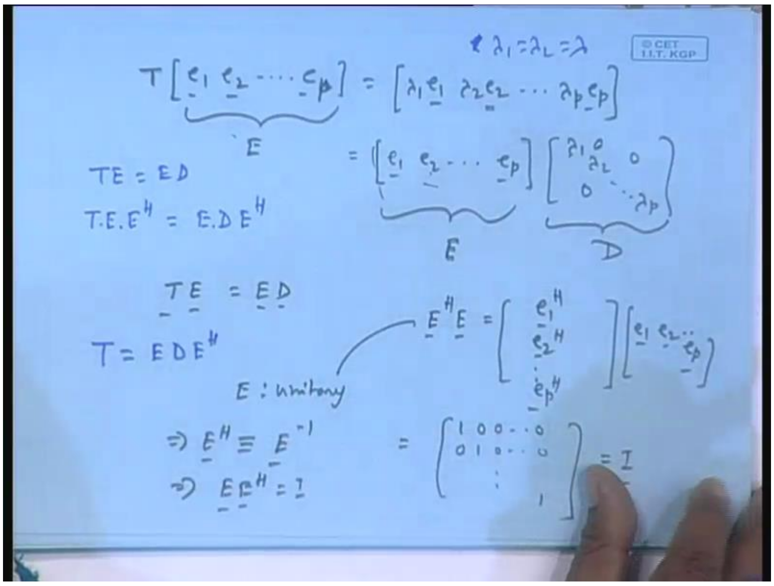

Suppose, I make a I just put the Eigen vector side by side e p, what will the this be T into e one will be the first column; T into e two will be the second column so on and so forth. And T e 1 is lambda 1 e 1 lambda 2 e 2 these are all column vectors. These are all column vectors now my claim is I can write it like this. The same vector if I bring in e 1 e 2 dot dot dot e p into lambda 1 lambda 2 dot dot dot lambda p a diagonal matrix. These will give this. No, can you see that?

How do you do when you do matrix into matrix multiplication how do you do? Suppose, A and B are two matrices I come back to some rough work done for you benefit; I am try to give you some extra things regarding you know basic matrix via operations.

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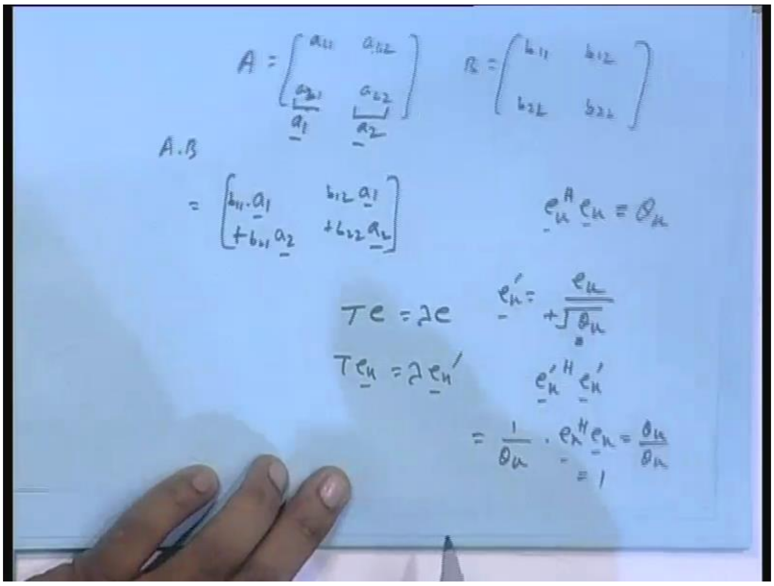

Suppose, you are given one matrix A, say element like a 1 1 a 1 2 a 2 1 a 2 2; another B I am taking two by two from that you will see what I trying to mean B 1 1 b 1 2 b 2 1 b 2 1 b 2 2 and you have to do A into B. How will we do normally what you do? Tell me. You do like this a 1 1 into b 1 plus a 1 into b 2 1 term a 1 1 into like that. So, from to today class you should never do that, which is not the way we should do that is done in the school level and all that level not any one now. My claim is if you call this column is a column vector a 1 first column a 2. Then, this will be a matrix again two column; first column will be this column vector times b 1 2 scalar plus second column times b 2 1.

First series, if you take this column b 1 1 times this b 1 1 into this b 1 1 into this and take this column b 2 1 into the this b 2 1 into this add do you get back the same quantity. Can you going to see that, b 1 1 times this column vector. So, first element a 1 1 b 1 1 stored, what is this a 2 1 b 1 1 stored and then this guy into second column, b 2 1 a 1 2 stored on top; b 2 1 a 2 2 stored and I you are adding. So, you get back the this. That thing a 1 1 into b 1 1 plus c 1 into b 2 2 b 2 1 are you convinced or not. Similarly, here you call now combined with this b 1 2 into again a 1 plus b 2 2 a 2.

So, this you may forget about matrix if it is a vector only for any column vector matrix terms column vector. What you are doing? You are taking the first element multiplying first column with that. Second element multiplying the second column with that likes that and then adding; that will be the net thing that is this matrix into that vector. Again these matrixes into this vector do accordingly; so that means, you linearly combining this column vectors by the elements of this column here. Linear combinations of this columns by the elements here that will give you here take you here. Again take that next fellow, linearly combine the columns b 1 2 times this column b this time this column if there are more columns this into third into third fourth into fourth.

Then, add that will be the corresponding resulting column vector. Are you convenience? Now, come to this. I had please have a look, I had lambda one scalar times e 1 lambda 2 scalar time e 2 dot dot dot and say either can write like this. Very simply first this by this is a matrix first column second column p th column this is a matrix. That times say first column here that will gives rise to the first column of the resulting product matrix that should turn out to be this, but that is obvious lambda 1 time e 1 other fellows are 0. So, they are left out lambda one into e 1 comes. See, how easily it comes if you know that then here you have zeros zero into e 1 forget it.

Other zeros forget only e two picked up by lambda two that comes here so on and so forth. All the Eigen values are real you give a name to this matrix D, diagonal matrix D. You give a name to this matrix say E and this matrix E. So that means, I can write T E is E D. What are these columns that the Eigen vectors corresponding to distinct Eigen values. Distinct Eigen values this is for lambda 1; this is for lambda 2, that is how these vectors are obtained and you do T into this you get this. So, there are mutually orthogonal e 1 Hermitian e 2 0 e 1 Hermitian e 3 0 only Hermitian with itself will not be zero. Because the vector is not 0, I told you.

So, Hermitian with itself will be that summation of mod square terms, which cannot be zero because an every term has to be zero, which is a contradiction. So, all cross terms will be zero. So, if I say if you take E matrix if you take E Hermitian into E if you taking Hermitian, what will happen you have to take conjugate transpose of this matrix? There first column will become first row transposition and then conjugate of that. So, here you will have e 1 Hermitian this guy becoming row if you take this first column that will become row and every element conjugate it.

So, what it is that actually that row is nothing, but original e 1 the Hermitian of that; then e 2 Hermitian of that dot dot dot e p Hermitian of that that is for E H. E is e 1e 2 as before e p nephew to the product; row vector column vector row vector column vector. So, what do you see here? It will be diagonal matrix; e 1 Hermitian with e 1 only non-zero e 1 Hermitian with e 2 2 e p 0. Similarly, e 2 Hermitian with e 1 0 with e 2 non-zero, but others zero; so it would be a diagonal matrix. Here, I forgot to do one thing. So, I come back when I was solving this equations you know T e equal to lambda e this kind of equation.

You got lambda 1 upon lambda p and e 1 to e p; whenever you get a vector say e k e k Hermitian with e k; it is dot product with itself. It is called actually now a term norm square in our vector algebraic term, it is a square of the length of that vector a norm square. This always positive we have shown because there is none of them is a zero vector. So, if you call this quantity as norm square is the scalar terms you give it a name say theta theta k; then instead of e k if I take e k by square root positive square root of theta k. If I take a vector like, this also is an Eigen vector with Eigen value lambda T of this. There is a dividing I mean multiplying both side by one by square root theta k.

Here also and that you can pull inside below e here also below e. So, you can this you can call a new vector say ek prime I am saying T e k prime also lambda e k prime first thing. You must know, if any ax equal to lambda x x is an Eigen vector then two x three x four all are Eigen vectors for the same Eigen value scalar multiples. So, this, but what is the advantage of this e k prime is this what is norm square of e k prime? e k prime with these are scalar and what kind of scalar positive real. Do you cannot conjugate it? e k of Hermitian means when you take the Hermitian of it.

This will become row vector with conjugate. So, this will get Hermitian transposed, but this only remains as it is, because this real fellow. So, that will become an from this e k prime from e k prime also another square root theta k will come. So, it will become theta k square root will go and then e k Hermitian e k and e k Hermitian e k itself is theta k; so theta k by theta k which is equal to 1. So, this call normalization you can always find out the norm square of that take this positive square root of that divide the vector by that. That vector also is an Eigen vector, but its norm square is unity.

I will forget to mention that when I am dealing with this e 1 to e p. I normalized each to have norm equal to 1. So, e 1 Hermitian e 1 is 1 e 2 Hermitian e 2 is 1 so and so forth.

So, in that case this matrix becomes simply identity matrix e 1 Hermitian e 1 all are zero; e 2 Hermitian e 2 second diagonal entry that is 1 others 0. So, it becomes 1 zero zero dot dot zero zero one zero zero dot dot zero. Any matrix and that has to be square matrix because otherwise this product cannot take place. Any matrix may be may not be Hermitian because e is a is not a Hermitian matrix.

Here, the E there is not a Hermitian matrix any matrix says E; if it is satisfied property that E Hermitian E is identity, then this matrix is called unitary matrix. And E Hermitian E is identity; that means, E Hermitian is nothing, but equivalent to E inverse; if I need a square root two square matrix is A and B and A B equal to y you know B is A inverse and A is B inverse that we know. So, that means E Hermitian to inverse such a matrix you really do not have to do a enough computation that in inverting matrix it is a big problem. If a matrix is given to be unitary you just take a Hermitian transposition immediately get its inverse.

Therefore, this also implies that instead of E H E if I make it E E H that also is identity, where E H is inverse, so that means, T E is E D and E is unitary T is unitary. Here, I am writing the separate part from this equation, what I found TE is equal to ED and E is unitary D is a diagonal matrix of real diagonal values. So, if I now take this E square matrix T square E square and bring another matrix here E H here also E D E H, but E E H cancels I have told you unitary as well EH is the universe of p. So that means, T is this is the main result T can be written in this kind of form E D E H, where E consist of the Eigen vectors normalize to unity.

D consist of the corresponding Eigen values and this is the Hermitian transposition of that E consider Eigen vectors and E is a unitary matrix here. E is a unitary matrix. Now, you can ask me that I assumed this thing that lambda 1 to lambda p are distinct and therefore, e 1 e 2 e p; they are mutually orthogonal, but if lambda 1 to lambda p are not distinct. Suppose, lambda 1 and lambda 2 are same. So, e 1 e 2, where from you get and are they orthogonal because the correspondence same Eigen value lambda 1 equal to lambda 2.

Then, at least between these two I cannot have this kind of thing you k now that cross terms canceling on inner product with itself remaining one. So, diagonal matrix is formed that kind of that property would not hold good. They are you have to just here me out that in such cases for Hermitian matrix T. The other way out mother matrix is Hermitian matrix and finding an Eigen value vectors and Eigen values of Hermitian matrix only. So, for Hermitian matrix if suppose an Eigen values repeated; suppose e 1 suppose lambda 1 equal to lambda 2 equal to say some lambda. Then, given this lambda you can find out two Eigen vectors if it is repeated twice, you can find out two Eigen vectors, which are mutually orthogonal.

That you put here, they will be orthogonal with the rest, because there Eigen values at distinct from this lambda. They are mutually orthogonal to this; I would able to prove now with that will required lot of things whether linearly lot of things. So, remember given at Hermitian matrix T, whose examples could be correlation or covariant matrix not necessary stationary; if that is of stationary and extra structure comes of top linear.
I am not taking any advantage of top of these property. I am only seeking Hermitian matrix, I even took a general Hermitian matrix.

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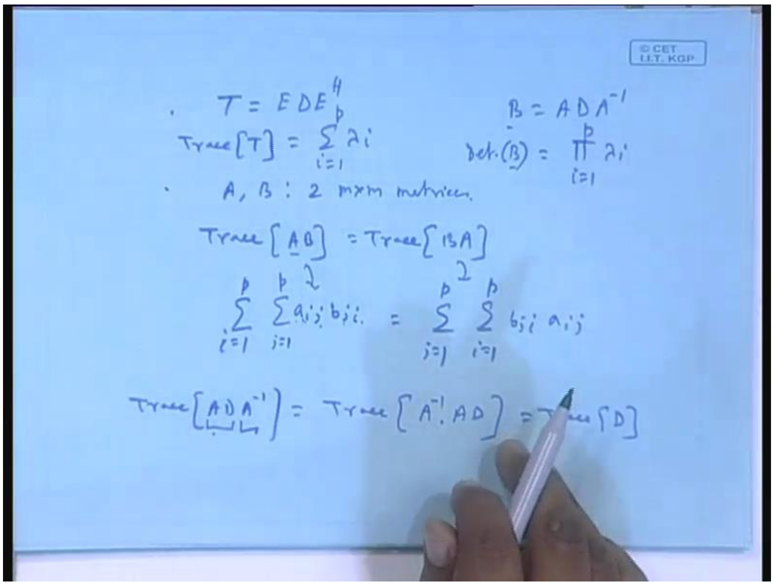

That can be written as; this is what we have proved. This kind of transpose, you know transformation E D in fact, E inverse in general here; because it is T Hermitian E here Hermitian here are the Hermitian E is a unitary matrix. So, E Hermitian is a universe of p all those things nice properties are there. But in general, if you have got structure like this that you take one matrix say A; then some diagonal matrix D I am talking of a very general case I am give an extra input to you. A some matrix not unitary nothing just A into a diagonal matrix D into A inverse A is invertible that is only thing I am assuming A into D into A inverse.

You call the product as B, then B and A they are solve they are similarly related. I mean this is called similarity transformation on B. In such cases that is suppose B is in general A D A inverse, whose special case is this, because A here is E; A inverse; obviously, E H; D is the diagonal matrix of Eigen and T is its stands for B. In this case if you take determinant of this matrix B what you get you know determinant of a product is able to determinant of the individual matrices. So, determinant of A into determinant of D into determinant if A inverse.

Now, determinant A inverse that is one by determinant of A do you know all these. So, determinant of A and one by determinant of A they will cancel. So, determinant of B will be simply product I mean this determinant of D and what is determinant of D is diagonal? So, simply product of the diagonal elements in this case Eigen values. So, determinant of a Hermitian matrix is that is a the product of the Eigen values. See, if all the Eigen values are non-zero; then determinant is non-zero it is invertible because determinant is non-zero. On the other hand if any Eigen value happens to be zero. Then that matrix B is not invertible. So, this is a product of this is a product sign i equal to one two p.

Please understand these are the core of statistical signal processing we need this often. Every now and then I want you to be absolutely conversant with this properties. So, there is no scope of re thinking how this how that all that. We find this extremely useful in communication controls single processing this three subjects, lambda i in this case lambda is real and all that so we get a real product pi. So, it will just product of the Eigen values that is one thing. Another thing, do you know this that suppose A and B are two square matrices two say m cross m matrices.

You know the trace of a square matrix, what is it mean trace of a square matrix is summation of other diagonal elements. Then do you know this trace of A B and trace of B A they are same. You may not know. So, we just it is not difficult. What is trace of AB if I the find out the diagonal entries? What are the diagonal entries? A B is a product what is the i th diagonal entry i th diagonal entry is A i i th row of this guy. So, a i j and b j i j as to be move from 1 to p; there is the i th A B product I am talking of the AB product. I further the product matrix what is the I j th entry.

So that means, i th row of A j th column of B i th row of A means a i 1 a i 2 a i 3 dot dot and they are multiplied by j th column. No, i th diagonal i comma i comma I may wrong statement. I am bothered about the diagonal element only. So, the product matrix not any arbitrary I j i comma i th element, i comma i th element so that means, I start with i th row and i th column i th row of A; i th column of B, so i comma 1 1 comma i i comma 2 2 comma i. So, that will be the i comma i th element i th diagonal entry and now I have to sum all the diagonal entries; so then final summation over 1 to p.

This you get on the left hand side, but I told you there are finite sum. So, you can interchange the any DSP also you know the next step is there is not DSP, but often double summation means you interchange the if you interchanged the two, the result comes immediately b j i I write like this a i j. So, j a th row of B and j th column of A they are multiplied and summed over all j. So, this is what does it imply here? In general, A D A in I again I am try to give in more general case, A D A inverse instead of here instead of here whatever is true for this that is true for this do you agree is it. So, I am that is why taking of this. No, for us this is important not that.

So, A into D is a square matrix. So, you break it like this trace of this into this is simply trace of A inverse into A D. Trace of this part into this part is same as trace of these goes first and this go second, but you see A inverse and A will cancel. So, simply trace of D this is result you do not forget, I make use of this result many times. So, Hermitian matrix its trace will be same simply summation of the Eigen values. So that means, trace of T is summation of lambda i’s. Please remember this, I will deal with trace of correlation matrix auto correlation matrix, so that time I simply summed Eigen values do not ask a question then get conversant with all this. Now, I come to a special kind of Hermitian matrices that is called positive definite matrix. Have you heard of this terms positive definite matrices? May be in way I mean some communication course or some course positive definite it may.

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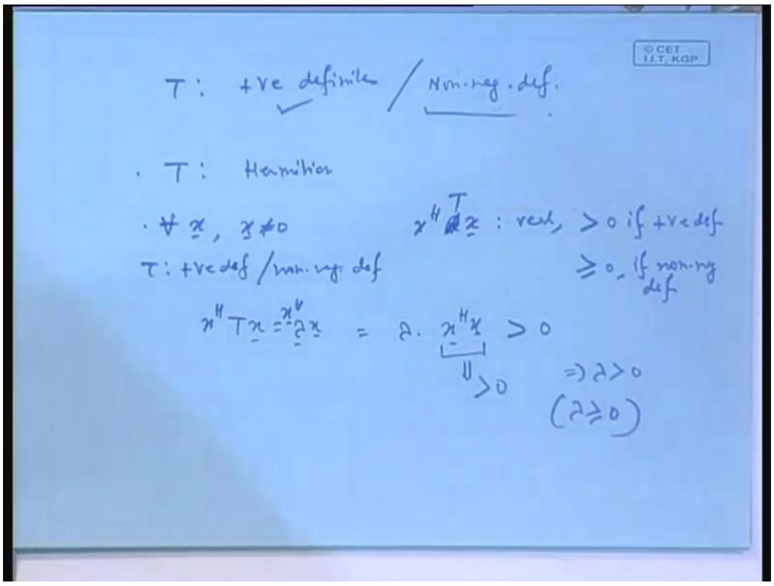

So, T positive it can be positive definite or sometimes positive semi definite, positive semi definite is also called non-negative definite, positive definite or case is non-negative definite. Non negative definite, which is also called positive semi definite, there are various ways people write you know. So, this case this second half this is more general I mean, I am considering on this and whatever I say I will just modify a little, so that will apply here. In either case T first be a Hermitian if not Hermitian then we cannot do anything; if not Hermitian then we cannot be anything. So, T first be Hermitian T must be Hermitian. Secondly so Hermitian we have already studied. So, we have there has to be something more.

So, what is that for you know for this sign for all? All x where x is a non-zero vectors this is important. If you take this product x Hermitian R x; R x is some vector y x Hermitian that thing this must be real and greater than zero, if positive definite and greater than equal to zero if non-negative definite, positive definite or non-negative definite also called positive semi definite. If this extra property holds, then it is called our correlation matrices or covariance matrices they are not only Hermitian they are positive definite; always non-negative definite, but we model them as we can also take them as positive definite unit under some assumption, which is in general valid.

This is T, this is T this not R this T, because I started with T; I always have in mind correlation matrix are some pre are, x transposed T x. In such case before I wind up within one or two minute just one property you see. T is Hermitian right T is Hermitian now again come to that Eigen value problem. Suppose, x is an Eigen value Eigen vector of Eigen value lambda for T; lambda is real we have seen and x is a particular Eigen vector non-zero Eigen vector, where x cannot be zero. X is an Eigen vector x cannot be zero say x is a nonzero vector. So, I told for all x where x is non-zero.

So, now I pick up x itself multiply here, then I can I have to multiply this also by x H, but x is a non-zero vector. So, then this must be real which is of course, the case because this becomes lambda into x Hermitian x and x Hermitian x is always real. But, if it is positive definite suppose T is given to be positive definite or non-negative definite since this Hermitian by its definition I took T x equal to lambda x lambda real, then I multiplied by x H here also x H x is a non-zero vector, which is again vector on the right side I get lambda into x Hermitian x, but by since it is also positive definite.

This entire thing since x is non-zero vector this product must be real really it is coming in any case, because this is real Eigen value is real, but this must also be greater than zero or greater than equal to zero. Now, out of this this is strictly greater than zero. This is norm square strictly greater than zero; that means, lambda the simplest lambda greater than zero or lambda greater than equal to zero. So, if it is positive definite Eigen values are not only real they are all positive. If it is positive semi definite or non-negative definite Eigen values are not only real they are non-negative, but non-negative means zero can included, which means matrix T cannot be may not be invertible, because that determinant, which is a product of the Eigen value.

Some Eigen value can still be zero. So, that is why we will often make as an assumption by which T will be your this thing positive definite. So, I wind up today. So, this week we are not meeting anymore because we have taken three classes. So, next time we meet that is on Monday three thirty.

Thank you very much.