Adaptive Signal Processing
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Lecture - 36
Givens Rotation and QR Decomposition

So, we discussed Givens rotation. Today before I jump into Givens rotation, let us consider some figures.

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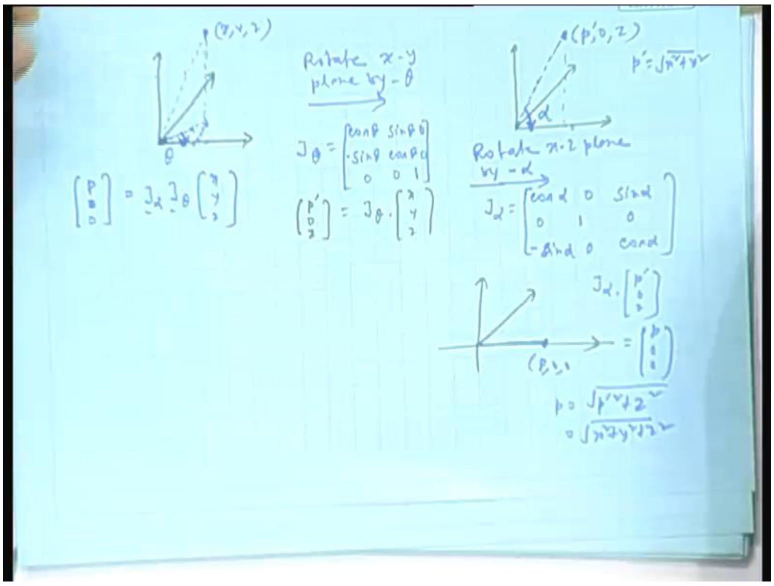

Just consider three-dimensional words to start with when I draw these lines with arrows those indicate axis. So, I have got these three orthogonal things know x, y, z and now suppose there is a vector. Any other vector I put a dot on a stop. You understand? It is not an axis. Suppose this coordinator is x, y, z. I will write in the vector form, column vector x y z, but here I am writing in a horizontal x y z. So, if you take perpendicular here this, like this, so we know this x, this y and that is z. This angle I am putting an arrow angle. This angle, suppose it is theta in the direction. So, suppose the x y plane is moved in the opposite direction by minus theta or if you find this vector is rotated by theta. This z components remains as it is. This vector is rotated and z component is dragged here.

What will be the vector? That vector, the resulting vector that is this fellow will be now translated and will be moved. It is this part will be rotated. This will come rotated here up to this. This fellow will come rotated, and this guy will be dragged on that. So, you get this vector. That vector will lie in the x z plane. Then, if I do this kind of rotation of x y plane by minus theta keeping z as it is, I will not touch z-axis and therefore, z coordinator of any vector will not get affected. I will not touch z axis. I only will keep z axis as it is. I will just rotate the x y plane. You can see that it is simple engineering thing. Roll the z axis as it is, and rotate z y plane by theta is rolled like this. For the given diagram, this is the theta and minus theta.

So, that means, this axis I mean I told you yesterday if you rotate the vector by some angle is equivalent rotating the axis by the reverse of the angle. Sometimes it is easier to understand everything if you rotate the axis, if you see by rotating axis rather by rotating of the vector. So, both are equivalent. That is why I am trying to explain by rotating the plane rather than saying rotating the vector. It actually will be rotating the vector if will the rotation operator of the vector, but what is the effect to understand that I rotate the axis. So, in this case, suppose I rotated those two axes, x axis to one side to this and therefore, y axis from here will be orthogonally pointing in this direction.

So, this comes here x axis and therefore, this fellow will lie in the x z plane. So, this fellow coordinate out of which some data will be there for x. Z will be as it is, x will be changed, this much will be x, but y fellow will be 0. Isn't it? So, I am saying rotate x y plane by minus theta. That means, I have a rotation operator now z theta. Isn't it correct? Take any vector x y z. If I want to rotate it keeping the z axis intact, I will be actually rotating the x y on this fellow by theta. By z will be so resulting. What will be resulting vector rotate these guys. So, this fellow will be pointing here on this and on that you add this vertical z component. What will become that is what I want to obtain.

For you to understand I explain it other way. Instead of rotating the vector, you rotate the plane because there it becomes easier to understand, but these were equivalent. Actually you are rotating the vector, but for you to understand easily I say that if it is rotated, so that this axis coincides with this orthogonal to that view x, sorry y. So, this will be new axis and immediately this vector will be in the x z plane, new x z plane. That is the way it is understood very easily, but actually I will be doing the opposite. Actually I will be holding axis as it is I will be rotating this vector, so that this comes in this line. Vertical component remains as it is, add that 2 and that will be resulting vector.

So, it is like what will be the new component and then new component if I draw, I have not yet appoint to the vector tell me quickly why you are bothered about sign I have actually rotating the vector. I am rotating the vector. You remember what I did yesterday? I took one thing with angle alpha in that and rotated from that by angle theta. Theta started from that line. This theta only is fine. This is actually class 10 or 11 questions. You know what you are asking. I have got 1 line with that angle alpha. If I rotate it by theta, yesterday I did not do it.

So, theta starts from this line and goes like that. So, here also this vector theta starts at that. Here it so happened because of my diagram. It is in this quadrant. I have to align it to this. So, theta is pointing this way, but do not bother about the sign of the theta. I do not understand. This is a very simple question. I mean you know I am sorry that this question is. So, what did I say? This actually is wasting time. What did I do yesterday? I say that this is angle alpha and this fellow if I rotate here I start and this is how I give the angle.

Today I am saying that I am rotating in the opposite direction, but again why did you bothered about what is drawn here. Actually for this particular diagram, this quadrant, this could be this quadrant and this could be somewhere else. Isn’t it? So, it is cos theta and sign theta. So, what is reality thing? This thing I mean I thought this is very simple. These things, this rotation and all we have studied in the school actually, though I must say I have forgotten much about that. If I rotate it, so what is that after rotation what is this thing? I will be having a vector in this plane. This new coordinator will be what y will be 0, z will be as it is and what will be the x thing. That will be a length of this only. They call it p prime. What p prime is? Square root of x z square of y square. It is simple geometry.

So, that means, this vector p prime 0 z, it is obtained by multiplying by doing this. Leave it. X y z I apply on this, this z, I get this. Now, one value as be annihilated, 0 has come in. It is called annihilation. Y has been annihilated. Find the angle towards x in the x y plane. If you rotate by that the other component, you will get there is y. You are rotating in x y plane. Find the angle towards x. If you rotate by the reverse of that, this axis or the vector by that, then in that plane the other fellow y in this case that guy will get annihilated. That is the thing. That is the rule. Is that fine?

So, this is the length, this is that guy. Now, suppose this angle is alpha. Again please see the arrow whoever was asking this question please see that I am putting the arrow. So, happened in the figure I will again rotate this vector, the new vector is lying in the x z plane. I will rotate it now. So, that this comes on x, and therefore z value will be annihilated again. So, that is why I am taking the angle towards x. That angle is alpha. It is equivalent to moving the x z plane in the opposite direction by minus alpha. So, x is coincided with these. I remember y is untouched. Only x z plane is annihilated. X z plane is rotated, all right. So, that means, I will now be rotating this rotate whom x z plane by minus alpha. That is equivalent doing actually rotation of that vector by alpha in the given alpha.

So, I have another matrix here. Now, J alpha J theta is a unitary. You all agree that any rotation making vector is unitary. It is unitary 0 0 0 1 and this is the rotation part. You can easily say this is unitary J theta transpose J theta will be 1 man. Now, J alpha you now it y will be intact given any vector here it so happened this guy is lying in the x z plane. It is already annihilated, but for any other vector also I will not touch. It is y component. I am saying what I am going to do? I will just move rotate only x z plane.

So, given it any arbitrary vector, not any special vectors who it is annihilated given any arbitrary vectors also. It is y component will be untouched, only x z plane will be rotated. It will be rotated by minus alpha, right but y fellow will be intact. Then, this will be what? Cos alpha because plane is rotated by minus alpha. There vector actually will be rotated by alpha in the opposite, so cos alpha 0. Remember this is lying because y component has to be kept intact, and then this is minus sin alpha 0 cos alpha and again unitary. So, what this will result in this thing. These are things what this will result in is this vector.

What is the component? That is J alpha times your p prime 0 z, but I may take the real prime here, p prime 0, z and what will that give rise to is the new x axis, new x if you rotate it. So, this axis comes on this, and then z, then z value will be 0 where z is orthogonal, and new x value will be just the length of this. Let us call it p. So, this will be p 0 0, where p is square root p prime square plus z square which is same as the original length. So, vector you mean the vector two of its components will go by two successive unitary operations in this case rotation, where it is specific kind. Of course I have to find out theta and alpha. This is to be computed during the vector. I have to find out orientation and all these angles have been found out. That is a part of computation, mind it.

It is coming graphically to you when I have been giving the thing, but this needs to be given a vector x y z, and you have to find out those angles. That is not difficult. You know this is y, this x. So, you can find out tan inversely y by x and find out theta and all that, but in this computation, you had actually when you do adaptive filter are also that has to be done as some computation bottleneck, and there are further parcels of that adaptive filter where those competitions are also eliminated and simplified and all that. That is research. I will not go up to that extent. You know there is initial, every first categories of given rotations of r l s adaptive filter will be dealt with, but there are further research I will be telling you those competitions will be passed or simplified and all that, all right. So, obviously, you have now can write p 0 0 is J alpha J theta x y z. This is a matrix you can put away.

J alpha J theta, this product matrix is also unitary. Obviously, you take the transpose of that and p multiply will be unitary. Isn't that a unitary operator? So, that means, the norm square of this and norm square of this are same. You can easily see norm square of this is p squared. This is p squared. Length has not changed rotation after all. Those are fine. Now, instead of 1 such vector x y z, I will play a game. I will have the two vectors. Now, two vectors I will give you and I will do rotation jointly on these. It is little complicated than this. Then, we can easily consider that case of mm numbers of vectors, not just one or two m numbers of vectors. So, now let us consider that case.

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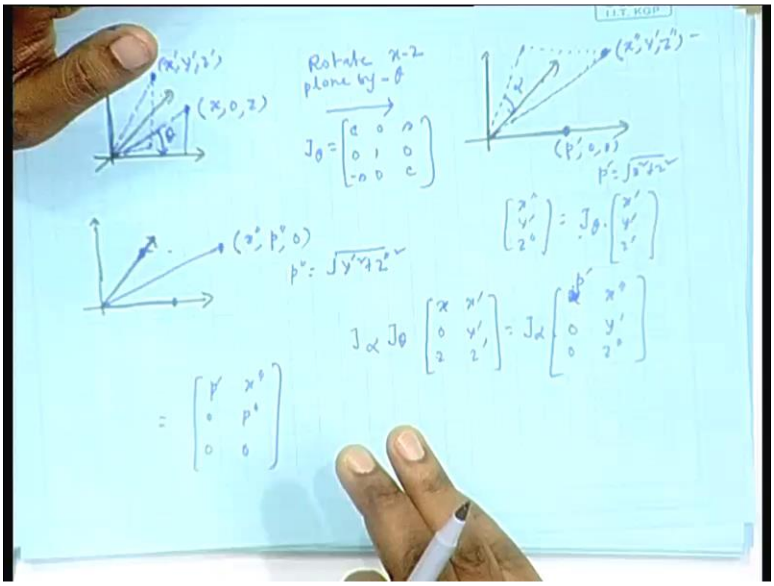

Suppose I have to use these colors. I wish I had some more color pencils. This is given now two vectors. Out of two, the vectors I am given one I am making life little simplified. One vector is given already in x z plane. No need to rotate it. I mean one vector is already given in x z plane like this. This is one vector. X z plane, its coordinates are say x 0 z. This is already given. Already 1 y component is annihilated here. It is given to you. X z plane is nothing in y. No, nothing in that distance, just lying in x z plane, but the other vector that will be given is naught. So, simple it is a general vector.

It is like these you know x prime, y prime, z prime. These are situations. What I will be doing? I will be first considering on these guys. I will consider this angle theta. I will be rotating the x z plane without bothering about the other vectors. Whatever happens to it let it happen. I only rotate this x z plane by minus theta, so that even this z is annihilated. Obviously that will change the coordinates of the other vector also. Nothing will be annihilated in general, but coordinate will be, of course but y prime will remain same. I am only rotating x z here, y prime will remain same.

I am saying rotate. That means, say again I write the matrix J theta. We wrote it whenever cos theta comes you know let write c and whatever sin theta comes every time cos and sin, but angles may differ. Sometimes J theta, sometimes alpha you substitute that. You have it in the mind that c does not mind same cos everywhere. So, just to you know I mean to make the notation spot, I am writing the book also cs minus s minus s and c, but here it will be just with the angle theta c means cos theta. If afterwards the angle is alpha, again I write c. Do not think it is the same c because the concept that is important here, not the exact values. That is why I will just shorten the notations.

So, it will be c 0 s c means cos theta in this case. I hope you understand this. This theta from subscript of the resolution J theta, so borrow the theta, apply that inside c means cos theta s sign theta sign theta cos theta. Theta comes from here. So, obviously, what you get is? So, you get that vector y prime. That remains same. Somewhere my figure may not be very good. I mean do not try to find out the accuracy of my drawing. See the concept because if I have to make an accurate drawing, I need to. So, I am just saying that this vector gets back to another vector. Just another vector, just z double prime y prime z double prime, but I might draw it in a wrong place. That will not change anything. Do not find fault there.

So, it will be in general somewhere, say here I am not sure whether it will be here or in some other place length. Also by mistake I am making much longer than this. Do not get into those things. This is that vector. What is important is what I write here, x double prime y prime. Y prime is important. I maintain the same y prime z double prime, and this guy will be here somewhere here. Actually I am rotating this fellow by applying a rotation operator on this guy and therefore, on this guy also, but effect will be easily understood if you view as though the plane is rotated. These vectors are held in constant in the position. That is an easier way to understand. That is why I am always going by that. Root plane is rotated by minus theta. Then, we easily understand the orientation and all that, but actually x is not changing. You are applying rotation operator in this vector.

So, that means, this guy will be say p prime 0 0. P prime is simple square root of just length of this and your J theta time is what we had. Now, please see what I do is very interesting. I have annihilated. You already gave me one value annihilated. I annihilated the other value also without caring for this guy, so that it become something 0 0 and is data. Now, what will I do is without disturbing this, I will annihilate z double prime here. How I will now say that I do not want to touch this guy. So, keep x axis as it is. Do not touch it. Just go rotate y z axis. How you take the projection of this? This much is x double prime. You agree vertical. I have drawn. Is it not? This much is your x double prime. Let it remain intact and find out this angle.

This angle; let it be alpha. See if I rotate the plane, y z plane by minus alpha, then this axis will coincide with this. Z will be orthogonal to that. So, that means, in a new coordinate system, this length only will be the y value. Z will be 0; x will be as it is. Agreed? This simply means operation. This is rotation, nothing else. Can you see this, what I am doing? I am now only moving the y z prime. Now, x z and yz z is common. z is my target. We will see z is the last guy is my target always. If I had x y something and z, then x z y z that something z is the last guy. Keep that in mind. Z is common. Z is always to be annihilated. That is why x z I did it was already annihilated here and then here in this vector I do not disturb x double prime, I do not disturb these x values.

So, only y z plane is rotated. How I find out what is the alpha? So, minus alpha direction, so plane is rotated and equivalent the vector is rotated by alpha in the given alpha direction. So, obviously that will give rise to this figure. This is the thing. This vector will be somewhere here. This fellow, I mean do not please see the size and all you know. I mean I made so big here unfortunately. This will come on sorry. It will be the x y plane. Sorry, this will be somewhere here. This will be the x y plane. I agree this projection, actually this I should have say this projection comes here. It is just a slip of tongue. This projection I mean instead of pointing at this point there and this projection will be on this because projection was in the y z plane. I rotated y z, so that this y x comes along with that projection thing.

So, this is the projection thing. X value remains intact. So, it will be somewhere here and original guy is here. So, this will be what? Its x value remains as it is. X double prime y prime will be what? Now, y will be what? That is a p double prime and 0. What is p double prime? It is length of this projection norm square of this projection because once y x comes over this, this much becomes the y value, new y value. This is the length. The length will be what? Euclidian norm that square root of that apply and all things, all right. So, this is the situation. So, let us sum up them. So, what all we did is suppose we have got two vectors put side by side. One was the given x 0 z, another is x prime y prime z prime, something like this given to you. I apply plus J theta on this and this is a J alpha. What do I get by applying J theta on this? This 0 gets annihilated and immediately I get x double prime. This is not x. This becomes p prime. No, this is p prime norm of this, length of this.

See I apply this z square. This becomes some general value x double prime. Y prime remain as it is and this z double prime and on that. So, mind you this data 2 zeros, but data and then this fellow will be intact. This fellow will be intact here. What will come? P prime 0 0 x double prime, here p double prime will come here. P double prime is square root of this square and this square and this guy will become 0. So, what is the shape upper triangular. Isn't it a chunk of zeros? So, given this kind of thing, I will first rotate, so that this is annihilated. Let the effect of rotation be something on this. I do not care and then on this. I will not touch the first guy. I will take these two again rotate.

So, first component will remain as it is, but rotation of these two means 0 will come, and this will be annihilated, fine. This guy that is when you rotate this vector by alpha, this is already 0. That will not give rise to anything and this coordinate is untouched. Its effect will be on here, but these are norm 0. Out of these, this will be annihilated. Then, this will be replaced by the length of this part. So, I can now generalize this. Now, you have understood this. So, I will write the very general case now.

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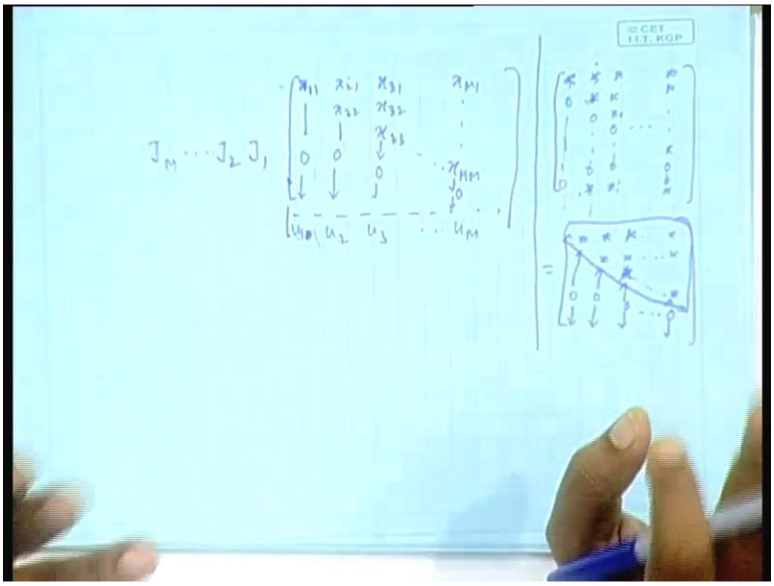

Suppose we are giving a situation like this. I have general matrix giving like this that a vector say x 1 1. This is x 2 1, x 2 2, x 3 1, x 3 2, x 3 3 0 0 dot dot dot say x m 1 dot dot dot x m m. This is given to you zeros and then zeros. Then, what is happening I am suppose appending one row here, one extra row say u 1 1 say u 1 naught 1 u 2 u 3 dot dot dot. Suppose this is the matrix. This is coming. How it comes that we see in the direct to filter context. We consider a special case of this, where we had only two columns and immediately below this, we had this u 1 u 2 u 1 u 2 x 0 y x prime y prime z prime. That is the decision we took there. Isn't it and more general?

Suppose here I consider these two fellows. This is in one plane for this axis, and this axis other values are 0. There I rotate it, so that this is annihilated, so that some data comes. Data is what square root of this square of this and square of this sum, and this guy gets 0. The effecter rotation on this fellow I do not care. Let it be anything. Those rotations however see some rotating consists of this plane and this plane. That will not affect these values here. This is extra thing that is coming. This will not affect these values. I am saying I do not care for the same, but not that zeros will become norm 0 because I am not rotating this plane.

So, this coordinate will change this coordinate will change, but these values will not change. The zero structure will not change after doing that. So, everywhere only first and last, first and last, first and last guy will change. Intermediate guys will not change. So, these zeros, zeros, zeros, they will remain as it is, but after doing the rotation it will be data and 0 here on that. Suppose I do this J. Some J 1, J 1 means you understand how to construct J 1. Some c 0 0 0 s 1 1 1 diagonal entry again minus is 0 0 0 c. You understand? I hope I do not have to write it. You understand? What will be the structure? I mean cos theta, then all zeros from this sin theta. Then, I will lower identity matrix and then again minus sin theta 0 0 0 cos theta that I find out that matrix I am calling J 1.

J 1 will work on this guy. It will give you a data and 0 here. Data will be square root of this, but I am not bothered so much about what data comes, you know what it is. That is all, and first and last guys of other vectors will change. Determined things will not change. That means, in short I will have a structure after this. This is my, I will have a structure like this. I will not write the exact value. I will have a star. Star means norm 0. Please see star means norm 0 in my notation and then zeros, zeros, and here will be star, star 0 0 0 star again star star star 0 0 star dot dot dot star star dot dot star 0 and star, right. This is my structure. Exact values I do not care.

Now, target this second guy. Second guy I will be considering only these two and I will rotate, so that this guy is annihilated and this is kept fixed. I will only rotate this plane. The plane containing this plane, the corresponding will be this plane. If I rotate this and keep other plane, other direction intact, those coordinates will not change. This value will not change, this value will not change, and other values will not change in all the rows. Only the second guy and last guy will change. Second guy is already 0 here. That will not get altered. Last guy is the second and last I will be rotating. So, I will annihilate this and this will become square root of this and square of this.

So, that will be J 2. I am not writing J 2. Do you understand? You can put cos theta sign theta in appropriate places. What will be 1 0 0 0 0? Then, 0 cos theta 0 0 0 0 0 0 1 identity matrix change 0 0 0 minus sign something second last, yes second and last second and last. See you can prove that cn and s appropriate, so J 2 J 1. So, that means, after that this will be as it is then this 0 0 0 0 0 totally mean these all zeros, this, this, all zeros and then I will target this guy, this guy, these fellows. This axis I consider the corresponding time will be rotated, so that this is annihilated. Other values will be a. So, this is dot dot dot J m. If I do you understand this, then that will give rise to a structure like this upper triangular structure, then zeros, zeros. This is the upper triangular structure; this is the upper triangular part.

So, just by m successive Givens rotations and given this kind of matrix, I can give this kind of matrix. Only these are the given structure. How it comes, we will see, but given this kind of structure I can make it just r and this chunk of zeros. So, keeping this is mind, now let us come back to that adaptive filtering. There will be zeros because I need not tell you how many rows are here, how many zeros. Those rows plus one extra row, those will be that will be given to you, but here this was not upper triangular, upper triangular plus 1 extra row, and some rows of zeros. This will be contra into just upper triangular matrix and rows of only zeros.

This is not a square matrix. So, I need not give you a square matrix into I will not be able to give you square matrix. This will come from data. Tell me. Here also it is very simple. When I take this two, I will find out as though this is x axis, this is y axis, I will find out z inverse y by x cos theta sign theta. Why that will be a unitary matrix? All those properties will follow in here also.

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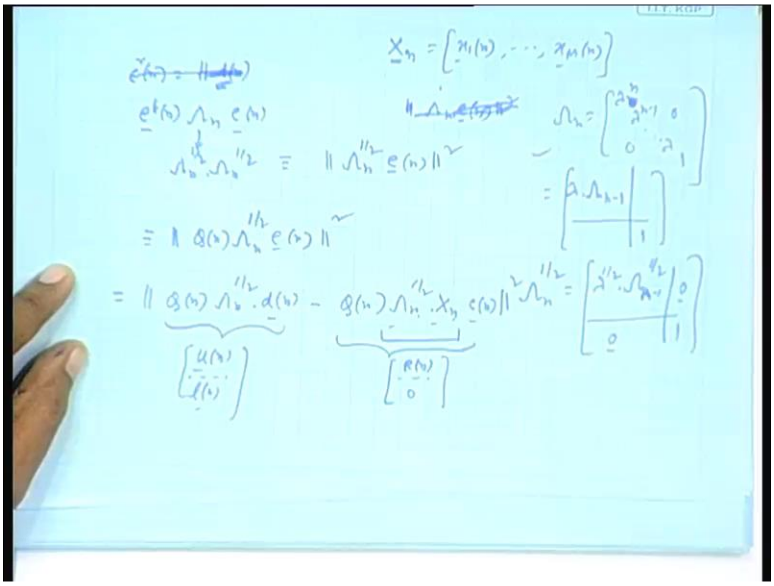

Now, come to that optimal filtering case. What I said yesterday that I had very quickly let me write down x, I gave x n. Now, what is it? Bracket n or subscript n data matrix x n was subscript n. This was the data vector. I told you in the filtering case, these are not I mean different data vectors x n vector z inverse x n the z inverse 2 x n like that. Remember? Is it not? but here I am still trying more general as though n different columns are given, so that becomes a special case of this. No problem. X n and then I wanted this epsilon square n was d n desired response vector. This norm I conversing in the conventional norm by writing out that delta part explicitly.

Actually you take d n minus x n minus x n into c n that error vector takes the norm square. In that norm square, your lambdas would come. You remember? Norm square of any vector in my definition will be what that vector pre-multiplied by your lambda diagonal matrix 1 lambda square dot dot dot dot lambda to the power n. I give it a name capital lambda subscript n. That times the given vector. So, the given vector is c n lambda n. That is very quickly this norm square means when I use this norm square, this is the usual norm square. That is just square of the terms, this term and n and this lambda n was this matrix. Sorry, lambda to the power n, isn't it? Remember one thing. You see one fact, I can write this matrix like this. I will be using this factor, this kind of you know I mean division of the matrix. So, I will be time updating from n minus 1 to n. That is my goal.

It was not lambda n. Actually it was, sorry it was lambda into the power half this part is correct, let me correct this part. What we do there actually e transpose n. This is our actual norm square. Isn't it? First term here, they multiply by lambda to the power n and likewise. Then, I say I factorize it as lambda n to the power half into lambda n to the power half by simply taking a matrix, and this lambda replaced by positive square root of lambda. If you multiply by that with itself the same way with itself, you will get this and these are all the symmetric matrices become diagonal matrices. This is the entire thing turned out to be equal to this lambda n to the power half en norm squared.

So, here lambda n I wrote like this very quickly. Then, lambda n to the power half you can easily see can be written as lambda to the power half into, instead of lambda; it becomes lambda to the power half, right. This quantity we want to say we know that this norm square is a usual norm square. So, that norm square will remain intact even if you pre multiply it by unitary matrix. So, this will be same as some Q n times. The same thing Q n has to be, any Q n has to be any unitary matrix, but we will choose it appropriately. You break it up. Q n into this d n minus Q n c subscript n or capital N. What did I choose? C parenthesis n or parenthesis n. These are norm square. This is what we minimize to get I mean optimal combiner coefficients. Is it not?

Then, I said that you should choose Q n very quickly. What do you say? You should choose Q n, so that this matrix is converted into an upper triangular form by Q r at least

Q r factor exists. We know from that orthogation point of view, I gave you that existence actually that you can really have those orthogonal vectors and you get a Q r form. I mean this is a square. So, this will give rise to, this entire thing will give rise to what one matrix like this is yesterday's, but I need to recall that to apply today's theory R n and the zeroes, R n was that upper triangular part. Isn't it? This times c n and Q n times, this vector I decomposed into two parts, upper part and lower part.

Upper part which means this m cross m, so upper m m into one part, I call it small un upper. Lower part was l n and this entire norm square. What is the minimum? When it is minimum, then only I will get the optimum. The entire norm square will be what lower part square, lower part norm square this thing. I would have written like this un dot dot dot l n. So, what is the total norm square? That is l n minus 0 l n and un minus R n c n. That is the upper part and norm square of this lower part l n square which is independent of c n plus upper part. So, that means, you choose c n square upper part norm square as minimum un minus R n c n. So, minimum possible is only when 0 norm square cannot be negative.

Do you mean if you can find out by solving equation R n c n equal to un, then that is the optimal one, isn't it. So, that is how you find out I said that is what I said yesterday, and this square of norm square of l n will give you the corresponding error with those projection error. That is where I stopped yesterday. Now, suppose by hook or crook you know appropriate Q n at nth index n minus 1th index. So, that is n minus 1 cross n minus 1 matrix, but that you know. What was that matrix that took delta n minus 1 to the power half x n minus 1 that matrix, it converted into this kind of form upper triangular R n minus 1 and 0.

This is always m cross m because number of columns in this is not changing. Number of rows may change, but number of column is m and upper part is m cross m and that is not changing. Number of coefficients as same you have to solve this part. So, this size of the equation does not change. This length of the zeros change I do not care. Suppose by hook or crook I know that Q n minus 1. If I know that then what I do? I form first one matrix.

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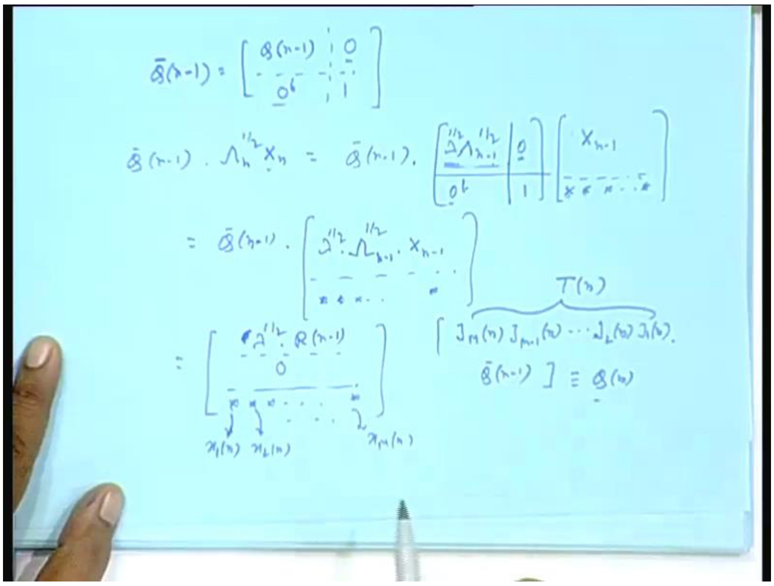

Q 0 0 actually it should be 0 transpose. This is a row vector and this is a 0 column vector, and you call it Q bar. You can put it in n minus or n. So, it is Q bar n minus 1. If I have taken this first, if I take that matrix and apply that on this part instead of Q n, I have to come at the appropriate Q n. I am coming step by step. First take that matrix, apply on this guy. What is the result? Let us find out. Now, that is Q n minus 1 into delta, this is lambda n to the power half x n. What is the appropriate Q n is our goal. What is appropriate Q n given Q n minus 1, what should be that Q n, but I am just doing some experiment. Let us think. Suppose I construct a matrix like this. See it is effect. What will I get? Look at this structure Q bar n minus 1 into lambda. This we factorize. We have already seen, and this Q bar n minus 1 and your x n matrix is what x n minus 1, and some data nth index value of x 1 n nth index value of x 2 n. This I am writing separately no problem. This x n, what is the x n matrix? After all upper part I write x n minus 1 data collected up to n minus 1th index and then this is x 1 n x 2 n dot dot dot. Instead of writing all that, I am putting star.

Now, consider this. This you should be able to understand. This part time x n minus 1 and 0 will multiply this that will be the upper half, and then zeroes will multiply this one. That is how this product is formed. Forget about the lower part. First consider the upper part. This term times this and 0 into this, this will not give rise to anything. So, this into this, there will be upper part and that will be again multiplied by this. 0 will multiply the lower half. So, can you see this or we have to work out the steps. We have to work out these steps or for some of them, I know you are very fast, but for others I am not. The silence does not much you know. So, this will be lambda to the power half, sorry this is lambda. You correct me.

As a matter of fact, I am teaching this first time. You know I do since this I never taught this in the first and this star, star, star right. Isn't it? Now, you understood? Now, you substitute this. Those zeroes will multiply the last column. You forget it Q n minus 1 times this. What will that give rise to? What is Q n minus 1 which worked on this guy lambda to the power half? You take out, it is a constant. So, Q n minus 1 worked on this, but what is Q n minus 1? It is 1 which was correct and which worked on this matrix. Make it upper triangular 1 and some zeros. Upper triangular part was m cross m in size. That is never changed. So, that will give rise to what lambda to the power half R n minus 1 and then there will be zeroes, but zeroes total number of zeros 1 less and then thse stars, this row times, this columns. If you have these stars will come up those last data, all right.

So, what kind of form upper triangular lambda to the power half you forget or you are observing this. So, what kind of form upper triangular followed by a chunk of zeroes, and this is your x latest data x 1 n, this is your x 2 n dot dot dot, this is your x m n. See if I construct this matrix, apply on this, I get this. Now, I apply Givens notation, I take first row, annihilate this guy. Sorry first column and annihilate this guy, one rotation and then take the other row, next row. Second row means, it is a next column, second column, second row fellow. This and last fellow I consider that and annihilate the last guy so on and so forth. So, I apply the sequence of rotations, Givens rotation n number of rotations I get back a perfectly upper triangular matrix and then followed by chunk of zeroes. So, that means, if I now take this and have j m, now I put n by the way because all dependent on data up to index n j m n j 2 n j 1 n into this Q bar n minus 1.

This part let us call it T n, T n into Q. This is my Q n. So, coming to this equation, this is your equation. I know what is Q n. Now, this is equal to T n. This is product of those rotations which is unitary part T n is unitary, and just remember one thing. I forgot to inform this matrix is also unitary. Just a 0 and 0 and 1 does not change anything. This still remains unitary. I observed it in an identity kind of matrix. Isn't it? So, this product is unitary after its rotation only.

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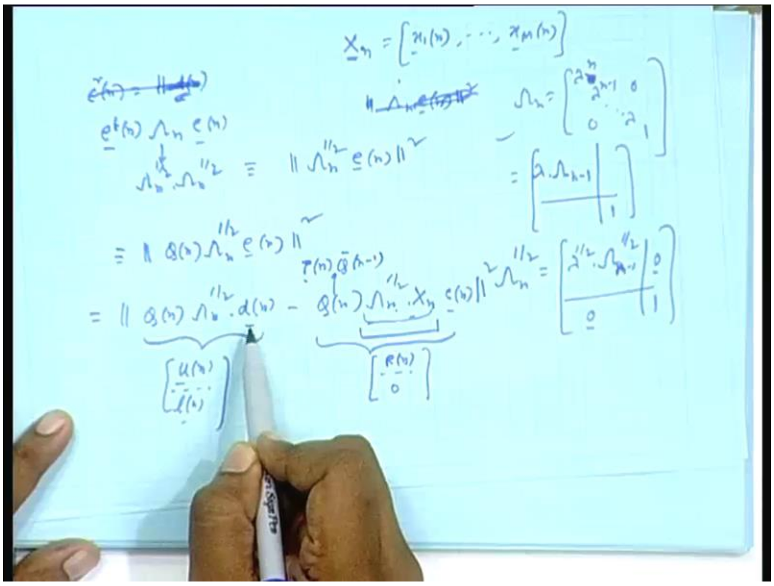

So, this is T n into your Q bar n minus 1. You know the structure of Q bar n minus 1 Q n minus 1 0 0 1. So, that means, the same applies to this Q n also. Isn't it? This lambda n to the power half d n is this again by the same logic.

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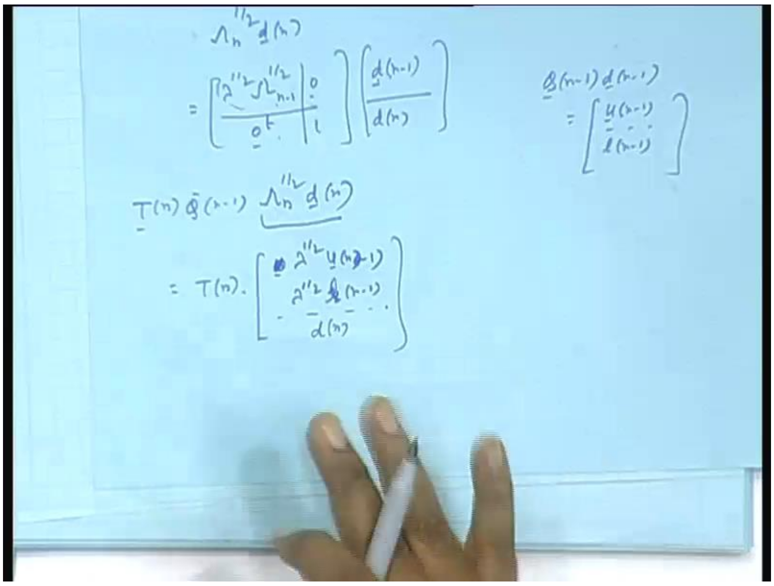

We can write the same absolute same logic you know. We can write as lambda to the power half and then on this if you apply this T n like take just 1 or 2 minutes on this, then what you get? This part is this. This into this 0 into this does not matter and then 0 into this and 1 into this. So, this real thing is vector where this and this comes on top. That will be multiplied by first Q n minus 1. That component you already obtained earlier, that is if you know Q n minus 1 d n minus 1, you can call it what the upper half lower half. Isn’t it?

So, that means, this will be what T n lambda factor will come up. That lambda to the power half, sorry un minus 1. This is l, the last guy is d n. This will be the thing. From here we will develop the adaptive algorithm in the next class.

Thank you very much.

Please come prepared with this. I do not want to you know time is running short. 1 or 2 lectures of course we will be finished. Please come prepared on this.