**Adaptive Signal Processing**

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**Lecture - 21**
**Introduction to Linear Prediction**

So, as before we have an inner product space V over C and dimension is p less than infinity, this is giving last time just a quick recap last time what all we did?

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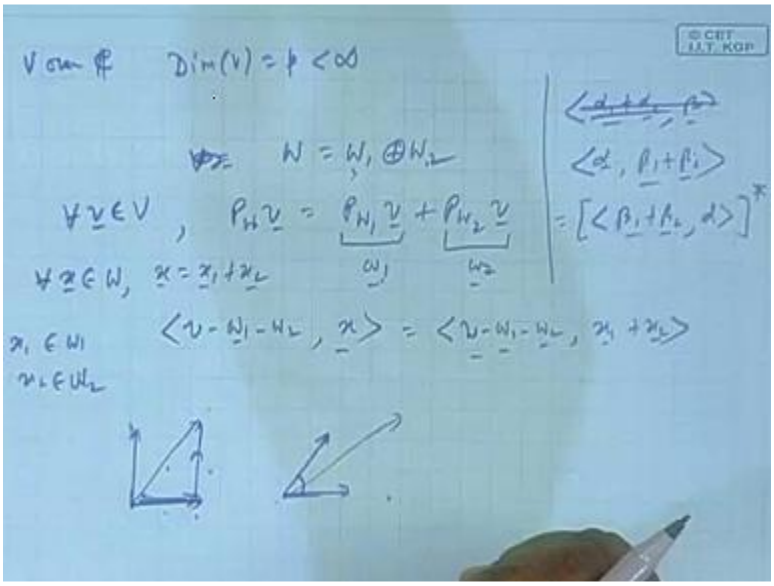

We proved we talked about orthogonal projection we said it is unique and exists, it exists orthogonal projection, it is a 1 such that the external vector minus the projection that is the error vector is orthogonal to the subspace constant. Suppose you are projecting a vector V on a subspace W, then this projection is called v cap, then V minus v cap, v cap belongs to W, V minus v cap is the error, that is orthogonal to W, and this v cap is unique it exists and unique

The existence is proved by showing that you can have a gauss by Gauss smith orthogonal process you can have any orthogonal basis or orthonormal basis for W. And using that basis we can work out a sum and expression which will give v cap in the sense that V minus v cap will be orthogonal to W that we have seen. We also showed that this orthogonal projection is such that the error vector has the minimum norm square.

If you take any other vector see V prime in W, any other vector and you take the error V minus V prime that error vector will have larger norm square or a larger norm than the 1 with respect to, when you have V minus v cap. That is why we always look for orthogonal projection because that gives the minimum norm error solutions. We also showed that, given a vector V its projection V prime is nothing but a linear operation on V, the operation that of course, it is a many to 1 operation that informally I told.

Ok after that I went into direct sum decomposition of vector spaces, where you can we considered general vector space not necessarily an inner product space, there I said that suppose there are two vector space subspaces W 1 and W 2, and they intersect only at origin at 0 vector, then and then I write W 1 direct sum W 2 it means what if you considered a basis of W 1 say alpha 1 to alpha r. If you consider a basis of W 2 say beta 1 to beta l, then you combine that two basis alpha 1 alpha r, beta 1 dot, dot beta l space span by this basis is what is indicated by that.

And they are in that space any vector is typically a combination of one unique combination because of linear independency, unique combination of one vector belonging to W 1 another vector belonging to W 2. Then we considered the particular case where these two subspaces are orthogonal, I brought an inner product space again they are orthogonal that is any vector and of course, when they are orthogonal again we proved that they intersect only at 0.

And orthogonal means, any vector in W 1 and any vector in W 2 they are mutually orthogonal, the inner product is 0. In that case again you can write W 1 direct sum W 2, I introduced the notation of perpendicular along with that direct sum just to indicate that it is actually not ordinary direct sum, it is a orthogonal direct sum. Later I will rub that perpendicular thing, we will always throughout the course we will be dealing with only orthogonal direct sum decomposition.

But anyway again W 1 direct sum W 2 in this case what again you form an orthogonal basis. Now you can talk about orthogonal basis of W 1 similarly, the orthogonal basis of W 2, I append that two, I mean take the union of that two and span of that, that is indicated by W 1 direct sum W 2. There any vector of that space is nothing but summation of, I mean one component from W 1 another component from W 2.

Then you found that direct sum any external vector is projected, then we showed that projection is nothing but summation of two individual projections. So, one on W 1 another W 2 that happens only W 1 and W 2 are orthogonal that is shown that day, and I also told you that instead of, I mean without using that gauss smith orthogonal that exact expression to prove it. That is suppose V is given to be not V, W is given to be, then show that P w V for all V element of V. So, that this is nothing but P w 1 V plus P w 2 V, you understand this notation right?

Your projection operator with respect to subspace W that is nothing but summation of that two projections, projection on this, projection on this. That was obvious when you will consider the total basis of this space that was a union of the orthogonal basis here and orthogonal basis here. So, overall projection was you can decompose into two parts. One transform to transform here another from here. So, that gives directly the two components. Even otherwise you can see just for mathematical exercise.

Suppose you do not know that form even then you can easily prove, that suppose this quantity I call small W 1 vector this quantity I call W 2 vector, then is it that this projection is indeed W 1 plus W 2, that means is it that V minus W 1 minus W 2 this error is orthogonal to W. That means this W 1 plus W 2 which belongs to W is an orthogonal projection of V on W. What I am saying is W 1 plus W 2 belongs to W. Is the error orthogonal to W? If so projection is unique, this must be the orthogonal projection of V on W. And that by definition W 1 plus W 2 there is summation of the two individual projections that is very easily seen.

Any vector, for any vector x element of W, we have got, we can write x as on 1 combination like this where, x 1, x 1 belongs to W 1, x 2 belongs to W 2 in general, is not it. So, I will consider any vector say x from W, take the inner product of this with x. Is it 0, are you following? Is it 0? if it is 0 that means, this error vector is orthogonal to any vector on W which is orthogonal to the total W right.

And that is very easily seen now V minus W 1 minus W 2, try to prove things in a more general way rather than taking that expression that is what I am saying you know. That is even if you do not know the exact expression of the projection, using the orthogonal basis and all you can still prove these things, these are better proof V minus W 1 minus W 2 and x is x 1 plus x 2.

Take x 1 inner product with x 1 this also you can write it that you can decompose it you know that we had done this axiom, alpha 1 plus alpha 2 comma beta i is alpha 1 beta plus alpha 2 beta. If it state, you have got alpha beta 1 plus beta 2, this also you can because you know this is nothing but conjugate of by definition and then apply linearity here again apply conjugate on each. So, it is linear in both.

So, here also I can, I can take the inner product of these guy with x 1, this guy with x 2. This guy with x 1 means again I break this V minus W 1 with x 1, but V minus W 1, W 1 is a projection of V on W 1 subspace. So, V minus W 1 is orthogonal to x 1 and W 2 and x 1, W 2 belongs to with this subspace, x 1 belongs to this subspace, that means they are mutually orthogonal that is 0. So, this inner product is 0 by the same logic the other 1 is 0.

Student: Sir, we do not know if w1 is orthogonal or not?

This is given and I told you that orthogonal this is given to us. This is given W 1 that then only this formula is valid, only when what is it, only when W 1 and W 2 they are mutually orthogonal then only total projection on W is the summation of individual projections on W 1 W 2 otherwise not very simple geometry. Suppose you have got two axis and there is a third vector in this world, you are projecting on this space that is same as if the projection is these that is same as this component. And this is, what is this component projection of that external vector on these physically you should be able to see, if you have got a vector like this, if you project that on this axis and this axis, you will get the same intercepts and summation of the two is this.

But, if the two vectors are like that angle is not 90 degree and we have got a third vector and the projection remains same, that is not summation of projection of the external vector on these and on these. Ordinary engineering sense tells you or can you see that, there you can see things geometrically because this is your eye, you have got your eye visually things are clear. When you see the way I have developed I have never used any geometry nothing just basic axioms that is the beauty.

So, I can extend all geometric concepts and all the results in a more general case, where there is no angle nothing. That is the beauty that they found out these basic things you know axioms that are necessary to get all those results, and which are not dependent on physical parameters like angle and distance and length and all those, this is fine. Now, whatever we do now these are very important, now suppose we did the V, I have got a set of vectors not necessarily orthogonal.

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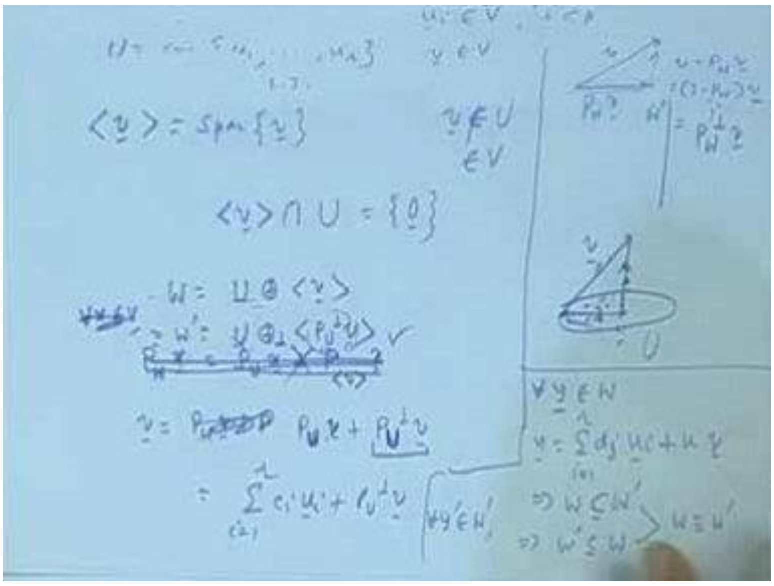

I have got a set of vectors u 1, u say r where each u i belongs to V. From V, I am picking up this u vectors, u 1 to u r, r is less than p of course, suppose I am picking up such vectors and they are LI, fine? I consider the span of these, and that I call subspace U, consider another vector just one vector v element of V consider this vector v element of V. For the time being you can assume to make life simple v is not in U that is outside U, but inside v, otherwise things become difficult actually, what I am going to prove or I am going to show becomes trivial. Are you getting me? I picked up a set of linearly independent vectors u 1 to u r, r has to be less than p less than equal to p, I am taking r to be less than p.

So, there is space outside U and still inside v, from which I am picking up a v, small v then and I use now 1 notation you know, this notation without any comma that only 1 element, this is not inner product it might look same, but there is no comma it does not involve only two elements, it involves one element means, like if I say, v it is nothing but writing span of v. It is because writing span of v means, too many letters and you know it takes too much space, but this comes very easily.

So, when you have got only 1 element span of that I denote by these, but this is not inner product, inner product was a had these two symbols, but it involve two quantities and there is a comma in between that is not present here. So, whenever I do these this is span of these. Obviously if small v is outside U, small v suppose not element of U, element of

V, then these two subspaces intersect this, I mean this small v cannot be part of U, that means, these two subspaces can intersect at only at origin.

Because this subspace means any vector here is either v or some multiple of v. If they intersect at any other vector other than non zero, then that multiple of v is a linear combination of them, that means, multiple of v or v for that matter lies in u which is contradictory you understand. So, obviously, you understand or not, this is, because this span is what any vector here is a multiple of v, if these two spaces intersect at any other vector, that is some multiple of v also that is a linear combination of these, which means v is a linear combination of these, which means v belongs to U, which is not possible.

They intersect at origin, in that case, if I call W as, say first I inter ensure that they intersect only at origin, they are mutually disjoint except here. Then if I say, then inter space is called W. In that case you will know P w x for any x element of v, P w x will be, we have just said this in the beginning of the class today. P u x plus P v x projection on these projection on these, just a minute, no not this is not true, because I have not yet shown that they are mutually orthogonal, unless and until they are orthogonal I cannot write that sorry. I am going to do that I am hurry into that, but I will modify things a little.

This is suppose I am considering W like this, and I introduce another notation here, First let me explain geometrically. Suppose, there is a vector V, do not confuse with these things this is a separate story say V, this is one space span by these element or may be many elements many elements I can show like this, this space is say W, this space is W. So, projection of these will be say this amount this is P w v.

So, P w is a linear operator, given any V, P w v. For in a subspace W there is a particular projection operator P W that takes any external vector V gives you the corresponding projection. And this component is the error, there are two components projection, projection at together if you I mean you add them you get V.

So, these component is what V minus which I can write as i, i means identity operator not matrix here, there is no matrix here, it is an identity operator, identity operator also takes any vector gives you the same 1 that is called identity operator. So, I can call it i minus P w and this i minus P w, denote as P w orthogonal, because this is also a linear operator. What does this do on V, when you apply P w orthogonal V, it does not give you the projection, but gives the error.

So, I call it for orthogonal projection error operator, P w is the orthogonal projection operator, P w the orthogonal projection operator this is the projection error operator, that is if you apply that on V, you get back not these, but the error very simple. I am using the notation otherwise, every time i minus P w, I have to write, things will become very clumsy and all that, you understand. Now, come to this with this notation let us not refer to notation come to this description, here U and this span of v, they are not orthogonal agree.

Student- Sir, is it orthogonal projection error operator?

Yes. That is what I said.

So, now come to these and here I have said that U and v please understand this, this is what we are discussing now this is actually lattice filter and all that mathematically, this will come this will these things will be used again and again whatever they do now. This is given to be, but the two subspaces U and span of v, they are not orthogonal.

So, I cannot apply that theory which I was dealing here, but I want to write W. Now, as orthogonal sum of two subspaces, which was not valid here. So, that any projection on W can be writable as a projection on one subspace and projection on the other subspace that is what my game is. Now, the way to do it is this look at this V vector, if this is your U subspace U, it I am just schematically showing, it is subspace U span by all these fellows u 1 to u r, and your v fellow was here this is the origin v fellow was here.

What is W span of that is, you take v and append v to all the elements u 1 to u r have a larger basis span of that, that is mean by this, that is the meaning of this. You are taking small v, because this is span of only 1 element v. So, you are taking that v fellow, v is the basis here for the subspace we have to give v and appending it to the basis of this I mean larger basis, earlier I had odd number now odd plus odd number take this span, that span is indicated by these.

My claim is that span will be same as these, that if I take instead of v, suppose I project it orthogonally here and take the error, then span of v appended with that basis will be same as span of this guy appended with the basis. Physically you see, why, because this guy is a linear combination is a, is an addition of these two. This guy is an addition of these 1, this guy this projection is a linear combination of u 1 to u r. So, this is nothing but a linear combination of u 1 to u r plus this fellow, that is v is P w v plus, no sorry v is. Yes P w v plus obviously.

Student- P u v.

Sorry P u, I am sorry, very sorry.

I am still having the W here, so I am carrying that W in mind projection, here it is u, P u v, P u perpendicular, v fine. Now, I want to retain this quantity P u perpendicular v as it is. And P u v is some linear combination of say some c i times u i vectors, i equal to 1 to r. That means, if you consider any vector in W, please see if you consider any vector in W, I am not writing everything manually here, I want you to hear out because you know that will be too much of writing or maybe I can write.

So, for any, for any y element of W, y is what a linear combination of y is like some coefficient set d i or say d j, u i, i equal to 1 to r, plus some constant say k times v. It is a linear combination, but that is v fellow is again can be broken like these one component here another component here, this component can be written again in terms of u i.

So, that means, for any vector y is what is a linear combination of the basis here and some multiple of this projection error, that means if I consider this. To start with you call it as W prime, then obviously this space W is content in W prime any vector in W is a linear combination of the basis of u and this fellow. And my reverse is true also, so W is content in W prime.

On the other hand, I am not writing everything, on the other hand if you take, say y prime for all y prime element of say W prime, y prime is what, linear combination of the basis here I am not writing this step then please see my logic is same, y prime is what belonging to W prime. So, it is a linear combination of the basis of U and some multiple of these, but this again is v minus projection and projection again is the linear combination of the basis of U. Essentially y prime is a linear combination of v and the basis here.

So, that gives rise to W prime content in w which means, W equivalent to W prime, are you seeing this or should, I can you see the logic? I first start with W, take any vector, there is a linear combination of the basis vectors of U and, say if I start with W prime or what else I start with W. Take any vector that is the linear combination of the basis vectors of this and some multiple of v, but v is again summation of projection and projection error.

Projection error linear as it is projection is again a linear combination of basis, essentially that element of W is nothing but a linear combination of the basis of U and these error vector, this error vector and basis of U that means, that is content in W prime, because this W is content in W prime.

I mean this time v was written as a summation and these on the other hand if I take a vector say y prime element of W prime that is a linear combination of these, but what is this element, this element now will be written as v minus, earlier v was written as plus of these 2 now this will be written as v minus the projection. But projection is again a linear combination of U, u i to u r.

So, essentially net thing becomes a linear combination of u 1 to u r and this error fellow, sorry u 1 to u r and this v because v minus projection, I am replacing the projection error by v minus the projection essentially is a linear combination of the basis of u 1 to, I mean U that is u 1 to u r and v. That means, that vector also belongs to W, that means W prime content in W, in 1 case replace v as a summation of the two in another case replace this as v minus this.

But, this is how the mathematical proof works. So, this content in this and this content in this, that means, they are same. But, advantage is in this case the two subspaces are mutually orthogonal. So, If you want to take any external fellow say x and project it on W or W prime is same, I have to project it on W, if I use this form advantage is that projection will be summation of projection on these and projection on this.

Projection on single component will be what I know the general expression, when you have got so many, I mean I mean many orthogonal basis vectors what is the projection, I know with each direction you take the inner product divided by the Norm Square of that vector times the vector, I have got only 1 element. So, projection of any x with this will be what, x inner product with P u perpendicular V divided by the norm square of P u perpendicular V into P u perpendicular v.

Please understand things might start becoming complicated now. Any external fellow x if needs to be projected on this guy, is the only 1 fellow in a basis. So, what will that be x there will be a constant, what is the constant inner product between x and this guy divided by the norm square of this guy that is a constant into this vector itself. And if U has an orthogonal basis now you can assume that U has an orthogonal these are orthogonal.

So, projection on U can be easily evaluated using that expression. Say suppose you have already calculated the projection on U, and you ask this question I want to have an extra element v. So, that my subspace is bigger than U. What is the new orthogonal projection, suppose I post this question that is lattice filter actually. What are updating. This is you know I mean suppose you will amount to say some p th order filter or r th order filter I will bring in another element and you find new estimate, estimate is orthogonal projection, what is that?

That is say I am trying just throwing hints here. So, you understand that I am not just showing maths for maths. So, new projection will be what if I already know say advantage is I already know the projection here on this, I do not have to recalculate it. So, previous estimate I have to just add an increment that is the extra quantity coming from these guy, that will be the new estimate. So, I can go on having more and more and more elements V, V prime V double prime like that, and I have to just I do not have to recalculate projection on this subspaces, are you following? Some tricks will be.

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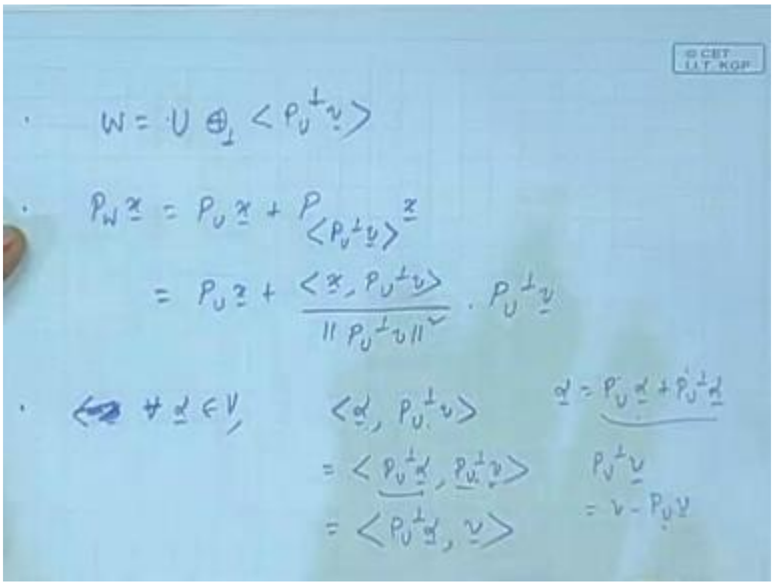


So, this is very important then that W can now be written as, never forget this W can be because and do not ask me question on this later on the two things I shown now.

W can then be written as U this actually this orthogonal, but I am dropping the notation I will drop the notation later, because every time I do not want to write this orthogonal, because in my life here every direct sum decomposition will be orthogonal from now onwards. Then span of how to remember these thing take this orthogonal of with what take the projection on this P U, P U put an orthogonal sign.

So, that will be pointing to perpendicular direction of the U of that vector v. Earlier you had U, you wanted external vector v to be appended to U, and take the span of that, and that you call W. W can be decomposed like that. Take U go in that direction perpendicular to U that is P u perpendicular v, then v if you project in that direction, there can be many perpendicular directions that suppose this x is perpendicular to this way this way this way, but you have got only one view, only on v.

So, take projection on that and taking that direction then that will contain then that and this will contain v also. Are you following? A simple geometry, but only thing is without taking any course to any angle, any length or anything simply, algebra and few axioms, that is why all geometry concepts can be applied to functions, matrices, all vector spaces. You can define pseudo angle between functions. Here I will define there is no physical angle that angle can be obstructed because this is very general.

This is one thing you know this can be, that means, P w x will be what? P u x plus P of x which means P u x plus, this is called ordered update relation, you had odd number of terms, I added 1 more element. So, ordered if the number of term is order, order was r in new, odd number of dimension was r, I added one more, not odd number of terms, but our basis had odd number of terms basis of U the dimension was r.

I added one more external element. So, now the basis has I mean how many components r plus 1 so number of element that is dimension if you instead of dimension you call it ordered set, using filtering your DSP language order was r, order became r plus 1 old projection, new projection, new projection in terms of old projection plus an extra term, this is called ordered updating the previous projection, this was previous projection, new projection is what your ordered updating, updating this update term adding an update term to the previous projection you will get the new projection.

They are forget this I will take the use this many things. Now, I show some tricks on this inner product, any vector not just like here any vector, for any vector and that is why I should write may be I suggest some vector say for any x y suggest something u v u, u and u r, I have already used u, v, w, I have used, for any vector could be what say for any alpha, whenever you are not happy with you know English characters ((Refer Time: 32:12)) for any alpha, element of V. Suppose you are carrying out an inner product like this, alpha with some P u perpendicular v, then my first claim is this P u perpendicular you can repeat on this, this is same as if you repeat the P u perpendicular on this, and is next claim is you can drop this from here.

Firstly what does it mean, alpha was just a free fellow. Nothing do with u and all, but you are taking the inner product with alpha and the projection error, v was another fellow projected on some u to the error between them you are taking the inner product, no relation nothing. I am saying that will be same as inner product between this error and also if you now take alpha and project on the same space u and take the error between that 2. And then I am saying this will be same as keep this error as it is, instead of these just v.

Replace into the simple geometry, those who you cannot see, for them I am showing what is this? alpha can be written as alpha can be written as you see very nice P u alpha and P u perpendicular alpha, any alpha at any subspace u, you can show that alpha is nothing but, a projection and the error, whatever be the subspace I can write. So, replace that alpha by these here, P u alpha and P u perpendicular P will cancel, because one is perpendicular one is another is u. So, that you are you will get these. You will get these, you replace alpha by these here and P u perpendicular v, P u perpendicular v and this fellow this is P u this is P u in perpendicular immediately they will cancel.

One is in U another is pointing perpendicular I mean perpendicular to U. So, they will cancel. So, left with only this fellow and this fellow that is what it is, and then again here P u perpendicular v on the other hand you can write as v minus P u v, v minus P u v. Actually these two is from I mean from this only you can say that exchanging the result interchanging the relation between U and alpha that is not symmetry, you can say like here from alpha, I put P u perpendicular alpha with same logic I take out. But, any way if you cannot see that you know P u perpendicular v is again original v minus the projection. So, replace these by these, replace P u perpendicular v by these and what happens immediately one is P u another is P perpendicular they cancel So, you are left with P u perpendicular alpha with v.

This is not only proof of projection operators you know in general any hermesian, if it is a hermesian operator any of the hermesian matrix and all this will be true. There is some operator called hermesian operator, we will touch upon it later. Projection operator is actually a hermesian operator, if you see in matrix form the matrix is hermesian matrix has to be and this is, no later not now, I do not want to take to the specific form of that matrix and all this is a very general vector space all x y j and alpha beta are abstract symbols, no any no relation with physical entity.

So, I will make this kind of manipulations time and again that, you have got this I will shift from this to here or I will make repetition this kind of things in a specific context of vector space this is a very general case. Now, after this I come to our real thing, will be dealing with in this course vector space of 0 mean random variables. That space if you take up any two random variables say x and y. Random variable means there is a probability density and all background, because when that whenever you bring in that is name random variable I know all the stop that goes with the definition of random variable.

But, is the vector space in the sense that what is the meaning of then x plus y, what is the meaning of addition, rule of addition? It means x plus y by these I will assign it to another variable, whose value will be what every time you are conducting experiment, we are finding some value of x some value of y. So, just add the value assign to a variable, that variable will be called Z so obviously, Z will be I will denote that by Z.

So, Z will be denoted as x plus y because every time you conduct experiment, you add the find values of x y add them you get a value of Z that is what is meant by Z equal to x plus y. But, then Z is also random. So, Z is also belonging to the same set which I considered the set of all possible random variables. So, it is closed under addition for any x and y belonging to that Z, x plus y belongs to that Z, same for c into x, c into x you understand, c is a constant every time you measure x find the value multiplied by scalar c as I need to say some Z. So, Z also is a random variable.

So, it belongs to the same set. What is a 0 random variable, there has to be a 0. So, that what is a 0 random variable, it is such that when you add two another random variable every time it is ((Refer Time: 37:41)) to be only the value of x that means, 0 random variable is 1 variable which always takes 0 value in all the experiments. Then you can then you can ask the question as to what is it that I told what is it how is it that there is a random variable which always takes only one value then what is random about it.

To answer that for this say mathematicians or math stats fellows, they said it is a random variable which takes probably this value 0 with probability one, impressive probability, integrate is one probably it could be a delta function, that kind of thing that is all mathematical things that they said. So, that you know things are correct and one can proceed. Then what is the negative of a random variable obviously, you take any value of x whatever is coming negative that value give it to another variable, that variable will be the negative of x. So, obviously when you add that two you get 0.

So, you get the 0 random variable or 0 vector. And see this also a random variable negative of that it is also belongs to original set similarly, for the scalar multiplication those rules are obvious, scalar one times any random variable will give you the same random variable. So, one alpha is alpha, then you remember c 1 plus c 2 into alpha that is trivially obvious c 1 alpha plus c 2 alpha or c into alpha plus beta c alpha plus. All those things are predictable in a vector space.

So, that you can take any two, any few number of vectors odd number of vectors, I can define the linear independence dependence all in the same manner, but those additions are mind you additions means, always means like there is an experimental experiment going on, there I am getting the numerical values those values are added, every time at the result is formed and that result is meant by the linear combination, that is the physical meaning of the addition there. Then not only vector space I will introduce inner product there, and this will be called as Hilbert's space, Hilbert's space of random variables.

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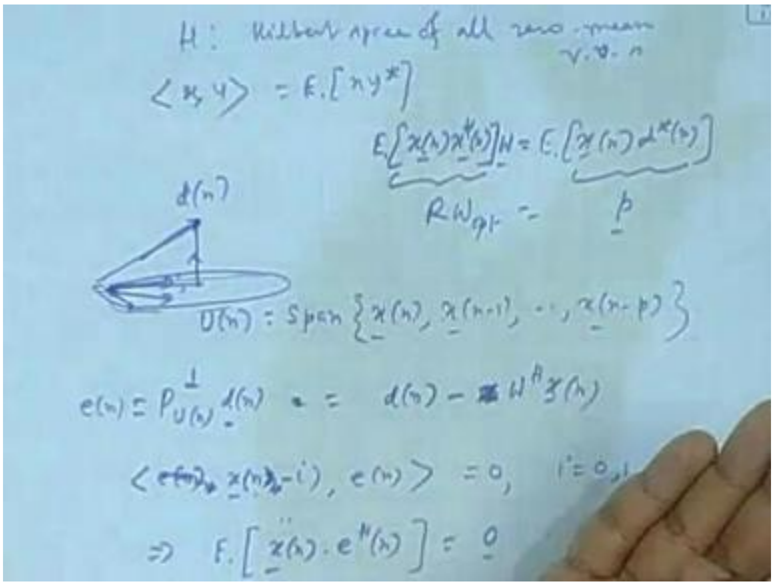

Inner product I will define x and y as and there in general complex value the correlation between them and correlation covariance are the same because they are 0 mean. You can easily see, it satisfies all the four axioms of inner product. Firstly, instead of x it is x 1 plus x 2, you can separate it out linear in first coordinate, if x is c times x, c can come out because after all this is nothing but multiplying by joint density and integrating. So, c can come out of the integral.

Then x y and y x they are conjugate of each other you can see. How? y x means E of y x star, is that conjugate of E x y star, little extra maths is required. What is E x y star? See you have to be like think like a mathematical logic should be correct, E of x y star means you are multiplying these by a joint density, and E y x star means again multiplying by same joint density, but joint density is a real function that is why it will happen.

So, please see once one level below and see the logic, you are multiplying by the joint density and integrating you get these, y x means E y x star say again y into x star you are multiplying with joint density and integrating. See joint density is real this integral will be what just conjugate of the previous one, if joint density were a complex function then this would not be conjugate, because P remains joint density remains as itself it does not get conjugated in this process.

Ok not understanding. I am multiplying this by joint probability density integrating, next time I will take y into x star multiplied by the same joint probability density and integrate these two are conjugate of each other, because joint density real function, so whenever you conjugate the first integral joint density can remain as it is. Do not jump at whenever you have to see them mathematically, because joint density is a real function even when the variables are complex probability is real. And last one is last one is very important, inner product with itself that gives you U of mod x square which is real nonnegative which is called the variance.

I also indicate by norm square real and nonnegative and equal to 0 means, only when expected value of mod x square is 0, that is possible only when x takes always 0 value, then only expected value is 0, because mod x square expected value 0 means you cannot ask that x sometimes take positive or sometimes negative or it is 0 whether I am mod x or mod x square rather, if that is 0 then x has to always take 0 value only there is no escape.

So, x is the 0 random variable, ok with respect to this then I can have orthogonal sets and all that so that means, two random variables are orthogonal means they are uncorrelated because correlation is 0. So, previously I used to call them uncorrelated random variable now, they will be called orthogonal random variables. But, provided 0 mean because actually uncorrelated means E of x y is E x into E y or E of x y star is E x into E y star, that is uncorrelated the covariance is 0, there I am saying 0 mean that is why E of x y star is 0 orthogonal means it is 0.

So, that two things coincide when there are 0 mean and I am always dealing with 0 mean cases. So, uncorrelated random variables and orthogonal variables in our case are same. Where is the constant term? Constant term the meaning is, what is the meaning of constant term? I am dealing with 0 mean cases, you cannot have constant term, constant, but I am dealing with 0 random variable. So, constant case does not appear.

Realize the answer, constant term does not appear, constant you give me that I am not taking. I am doing the 0 mean case because variance means you have to take difference from the mean, not just square of the square constant value and you get some value not that. I am taking 0 mean cases, that is then only I am calling it variance and all that, suppose they are not 0 mean U of mod x square is never the variance you understand that is the answer, when there is 0 mean then only I give you I go forward 1 step forward call it variance giving the name variance, but if I, Orthogonal means this inner product 0. Or provided this is equal to E of x into E y star, but if they are 0 mean they are both the things are same.

Now, consider that optimal filtering problem which we dealt with. No they will have hold, whether there are irrespective of 0 mean or not, that axioms will always work, but I am, I want orthogonality and uncorrelated to have the physical meaning actually orthogonal means just orthogonal, but if it 0 mean they also be uncorrelated, then there is a physical meaning that comes out, the random variables which is no correlation among them and all that. That is why I am, I would have that physical thing you know, otherwise and are all is purely mathematics orthogonal means correlation 0, but if you make it 0 means it is also uncorrelated.

So, uncorrelated has the solid physical meaning, that two variables are uncorrelated. They are varying purely randomly without any relation between them. I want to make use of that. So, that you get a physical meaning of say wide noise, other things you know with process. Ok now, come back to this optimal filtering problem now. Forget about all our derivation of linear filter and all that. If you have class you can go suppose, you have got one random variable and this space i will denote by H, Hilbert’s space of all 0 mean random variables. This is random variables.

Suppose you have got a vector d n and you have got a space, span by some elements. Dot, dot, dot, this say u n is a span of, I am introducing the notion n here, because everything time is coming, u n is a span of some dotted sequence x n. Each is a random variable, forget about n. For each n this is a random variable means one vector, this is one vector, this is one vector and assume there is no linear relation between these random samples, but is they are linearly independent.

When you get purely random better there is no linear relation between them. So, they are linearly independent. So, there is they form a basis of U n. And that U n, is this, and I want to estimate D n, as a linear combination of these fellows, but estimation should be best in the sense that the variance of the error should be minimum. Error power, but variance of the error means norm square of the error. Norm square of the error minimum means this linear combination has to be the orthogonal projection of D n on U n.

So, essentially I have to find out orthogonal projection of D n on U n. So, to find out this guy, I am now putting it D n here, to indicate their vectors also, but actually they are scalar, but in a vector space treatment they are to be treated as vectors they are abstract vectors. This I have to find out, that will be what, that will be such now, if they are orthogonal suppose, they are orthogonal, then that projection is finding is very easy as I dealt with case

Take everybody anyone inner product between that two. In fact, inner product of D n itself, I gave you that expression of orthogonal projection D n itself with any of them divide by the Norm Square of that times these, and same for all. If they are orthogonal, but they are not orthogonal. Lattice filter orthogonalize a set, in a Gram Smith manner, that we will see lattice filter will generate, orthogonal sets out orthogonal sequence out of these that is an uncorrelated sequence.

We can also say it generates quite sequence, it can use you can take any correlated sequence which of I mean generate a wide sequence out of it uncorrelated, uncorrelated means wide. All those things I am just dropping hints. This have to find out, but when they are not orthogonal then I do I cannot use that formula, then how to use? I only know this should be such that the error is orthogonal to the entire U n. What is the error? Error is this.

So, that is D n minus, some components I write like that, linear combiner coefficients instead of writing W 0 x n W 1 x n minus 1, I start with W star. Well that, does not matter, it is a constant only instead of W my notation is W star, you remember the way I did complex elements algorithm and all. So, that same thing W star W 0 star W 1 star dot.

So, this W H x n vector. I do not have to give you new definition of x n, we have all done this again and again, in our elements theorem the same thing. This is your error vector. This is the error, and the error must be orthogonal to the entire space. If that is satisfied then this will indeed be corresponding to orthogonal projection. Error orthogonal to the entire space means error orthogonal to individual basis components then ((Refer Time: 50:26)). That means, if I take e n, not e n just a minute. If I take say x n, no x n minus any element, x n minus i with e n that should be 0 for i equal to 0 1 dot P.

So, I put the inner product in a vector form, inner that is, inner product of e n with, like I mean if I take what is the meaning of this expected value. Expected value of, these into e star n 0, for i equal to 0 for i equal to. So, I can always this means E of e star n, I can also write as e h n 0 vector.

Now, it is statistics, no longer vector space. I am putting this all the inner product, if the inner product in the correlation form. Instead of taking individual element, I am putting a vector here, what is that vector? First element is x of n e of x of n e star n, the next element here is f x n minus 1 is e star n like that, all the inner products, is it difficult to see? I am not showing those intermediate steps this will give rise to what, u of x n e star n 0 e of x n minus 1 e star n 0 e of x n minus 2 e star n.

So, I am just writing that the entire thing in the vector form. Now, it is no longer vector space, it is simply correlation at e h n is what, you will take e h n. So, h on these, immediately you get D h n or D star n and x h w, put that here and what do you get E of… That is equal to 0 and then now we have to simplify 0 is there on the right hand side.

E of x n x n into e h n e n is this h of this you replace here h of this. So, D h means D star, these are scalar. Minus the other term, other term h of these means x h w x h w. I have x already. Take that to right hand side, or left hand side whatever, that is x n and W can go outside, because it is not random. This is interpolation matrix R, this is that vector p, and R W what? This is what is it, it is we are that time you are minimizing variance explicitly and all that, but that will give you nothing but orthogonal projection, projection is unique.

So, you get back to original filter, but that time within the vector space you have to do all that stuff, but anyway that time you will learn this matrix differentiation all that. So, I stop here today from here I will go to the linear prediction thing. Please refresh your mind with today's lectures. In fact, today and last lectures you know, because those are the lattice relation, linear prediction lattice relations.

Thank you very much.