Adaptive Signal Processing
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Lecture - 19
Orthogonalization and Orthogonal Projection

You continue the discussion of Orthogonal vector, so we have the vector space v over the field of original complex numbers.

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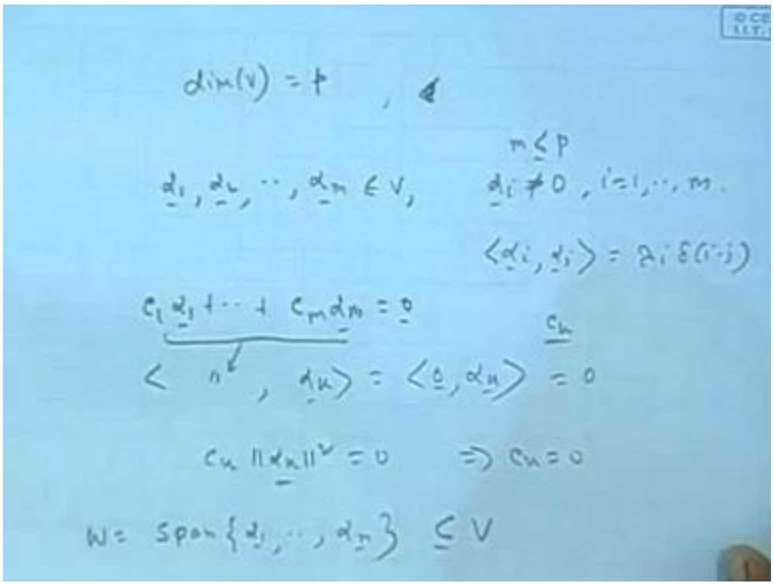

The dimension of the vector space is given to be, dimension of v is given to be say, p, there is a inner product defined on these, which takes every pair of vectors from v and use your scalar value and satisfy some constants, some that we have seen last time. In that case, suppose you have been given and we have also told that if two vectors are orthogonal, that is the inner property is 0, therefore orthogonal. And if the vector has norm equal unity and they are mutually orthogonal, then they are called orthonormal.

That is normalized, this vector is normalized, so that the norm is unity, norm means inner product square root, inner property itself positive square root of that, this thing only because of length, in the case of directional vectors. And we have also shown that given any vector, non zero vector, you can take it is norm take the square and divide the vector by the norm and it becomes unit norm. So, you can again normalize, normalization is not a problem.

Now, suppose there exist, the existence we will be seen later, but suppose there exist a set of vectors alpha 1, this is what ((Refer Time: 02:31)) we have seen this last time alpha 2 dot dot dot say alpha m, element of v and no alpha i is equal to 0, for i equal to 1 dot dot m. There is none of them is zero vector, zero vector has norm equal to 0 that we have seen, zero vector inner property itself gives you scalar 0, the normal zero vector is, you like that was one of the axioms.

The norm of a vector it is 0, if and only the vector is the zero vector, otherwise the norm is always a real and non negative number and not lesser than or equal to 0 or equal to 0, if and only, the vector is the zero vector, but I am excluding the zero vector here. So, this is giving, this is and of course I am taking, m less than equal to p; we see, that we have already seen this a reputation, but we will see why it is, saying less than equal to p. And they are mutually orthogonal, alpha i alpha j in general is what, If alpha i and alpha j i and j are different then the inner product is 0, otherwise is some constant.

Constant means alpha it itself, so it is a real non negative constant and this of course real positive constant because no alpha i is zero vector. So, if I write lambda i delta i minus j; that means when i equal to j, lambda i stands for that norm square of alpha i and lambda i not only is real, in this case strictly positive because alpha i is given to be a non zero vector, this is the case.

Then my ((Refer Time: 03:59)) was that this set is a linearly independent set, not the other way because linear it can be a linear vector space and a set of vectors linearly independent, but does not mean that they are orthogonal. If you are have the inner product defined on a vector space always, but giving that they are mutually orthogonal and nobody construes a zero vector, this forms a linearly independent set.

So, if the spanner space, this forms the basis of that space, subspace because they are linearly independent because they are doing, they will be called orthogonal basis of the span of this set. Anyway, suppose that I mean, I have to prove that they are linearly independent I form this summation zero vector, if these equation has only one solution that all coefficients are zero then they are linearly independent that was the way to prove it. If this equation has one solution possible, that is all the coefficients zero and no other solution, non zero solution for the coefficient exist then they are linearly independent.

Suppose I have to find out a particular coefficient c k, here to find out c k. what I do, I take the left hand side. Consider take as inner part of our left hand side, this left hand side with alpha k, C k with alpha k here in this summation, so I am taking inner ((Refer Time: 05:18))product with alpha k, this also 0 with alpha k .But we have already proved 0 inner product with zero vector is always scalar 0, this is scalar zero.

And this summation, it is summation, so you use one of the axioms you can still break it into a inner product between c 1 alpha 1 and alpha k ,then inner product will be c 2 alpha 2 alpha k dot, dot, dot. You understand, the first axiom was there and second axiom was there, you can separate out, it is linearly the first coordinate and again c 1 alpha 1 comma alpha k c 1 can go out by another axiom; but alpha 1 comma alpha k is 0, alpha 2 comma alpha k is 0 dot, dot, dot, only alpha k comma alpha k non zero because alpha, no alpha is a zero vector.

So, inner product with itself there is norm square is non zero positive real, so only that survives other inner products they become 0. So, what happens from the left c k norm alpha k square, equal to scalar 0 and this is positive because no alpha is 0, this has to be positive when positive only real and positive which means c k 0. So this is a proof that all the coefficients have to be 0, so it is a linearly independent set.

Now, I just taught, I just said that suppose it exist whether it exist, this kind of things exist or not we will see later little later, but suppose you take alpha 1 alpha 2 alpha m, What is the largest value of m that can be , m can be equal to utmost p. First we , if you consider w to be span of, the w is a subspace, subspace of what, all possible linear combinations of alpha 1 to alpha m, but they are mutually orthogonal and therefore, they are linearly independent.

So, that means, alpha 1 to alpha m is the basis in this case, they are linearly independent, no redundancy any vector in w is a unique linear combination of this, all that is fine. It is a basis and what kind of basis, we will give a special name orthogonal basis or even you can say orthonormal basis. If you take the norm of each, if you normalize these vector to have norm equal to one, then we call them, call it orthonormal basis of w, but w is clearly this.

Now, suppose m equal to p, suppose the m is equal to p and still if w is content in v, that means, outside w that is, if this is strictly this, so outside w inside v you can find out another vector. Suppose alpha 1 to alpha m goes up to p, firstly if m is less than p there is no problem because our theory was that, that if the dimension is p; that means, any linearly independent set can have utmost p vectors in v, it cannot be more that p, that is what I said.

Then we came to this dimension, that suppose you start with the way, you know vectors alpha 1 alpha 2, you remember, I mean you have to go at p only and then only alpha 1 to alpha p span the inner space b. Otherwise, if you stop up in between then the span will have something outside still content in v, from there you can pick up some body and applying to this alpha sequence, you can form a larger basis and so and so.

Here also suppose, you have got alpha 1 to alpha m and m is equal to p. In that case, you can easily see that w has to be equal to v, because if this w not equal to v, w is subset of w is always equal to I mean, v content in v or is equal to v, w cannot be over above v. Because each alpha is an element of v, so a linear combination of this alpha 1 to alpha v is an element of v, so w is a subspace of v. Question is this proper subspace, subspace, proper subspace or is it equal to v.

Question is if m equal to p, m cannot be greater than equal p first, greater than p because then that will highlight that thing, that vector space that has dimension p contains a set of vectors which are linearly independent, but total number larger than p, that is not possible. See the thing, we are trying to under get some inside this, we have got alpha 1 to alpha m, can m be greater than p, answer is no. Because dimensional vector space is p that means if you consequently linearly independent set maximum number that is possible is p and that will form a basis of the inter space v.

So, you cannot have more than these, m cannot be greater than p, m can be less than p no problem, I picked up one alpha 1 alpha 2 there all part of v. If m less than p is no problem, but suppose m is equal to p, if m is equal to p then my claim is that w is same as v. Suppose it is not, suppose w still a proper subspace of v then obviously, I can go outside w still depending v pick up somebody from there and if I apply that somebody, from to these, it will still be a linearly independent set but total will p plus one which is a contradiction, it is not possible.

The moment I reach p, m equal to p that means w has to be v, outside w and still inside v and there is nothing left which added to these will form a linearly independent set, an independent set. In that case, w becomes v and this alpha 1 to alpha 2 will called orthonormal basis of v itself not just of w, but v itself, but provided such a thing exist, starting here assumption like a mathematician I am speaking now.

That I said first, that suppose there exist a set of vectors like this in v, which are mutually Orthogonal may be normal, orthonormal and all of them is 0, if exist they are linearly independent. And since they are linearly independent, number m cannot be exceeding the dimension p, it is less than equal to p when m equal to p, the span is become same as v, this is pretty obvious.

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But how do you know it will exist, mathematically, how do you know it will exist, that means I have to develop a question here why we taken that, so that I can construct such things. Generally you will know it exist, that is done by what you know as Gram Smith orthogonalization, which is very common in communication particular. All of you know gram smith here, but may be you have done in the process of signals and all we have still do it in the abstract thing domain.

That will prove the existence of such kind of sets and it is not unique and you can choose, I mean you can have as many such combinations as possible, you know which makes a vector space v can have as many orthogonal basis as possible. I mean, there is nothing fixed.

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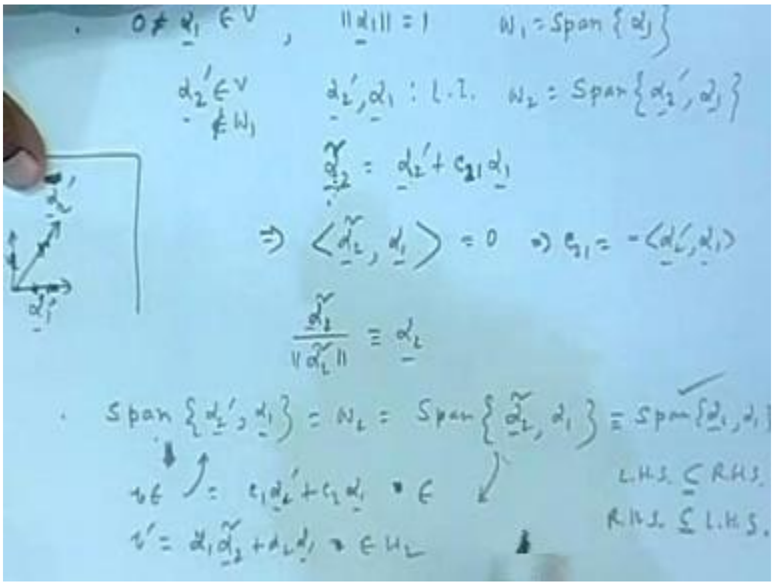

Then, suppose I take a vector alpha 1 and alpha 1 non 0, obviously none of them will be 0 ,element of v, alpha 1 I have normalized so that, this not necessary for this proof, but it makes life little simple. This is equal to 1,even if it does not having unique norm I can always divide by the norm and make it unique norm and call it alpha 1, that much is fine and w 1 is the span of alpha 1, then take some alpha 2 prime, element of v not element of w 1.

Obviously you have seen earlier here alpha 2 prime and alpha 1 L I, you have seen earlier while proving dimension and all that you have seen earlier and you consider the span. Now, consider develop a vector say alpha 2 tilde, this is tilde as some linear combination of this two, but in the linear combination may be, I choose things like this. If it is c 1 alpha 2 prime plus c 2 alpha 2 prime, what am I doing, I am trying to economize here, I can take c 1 common, so it is c 1 times within bracket alpha 2 prime plus some constant times, alpha 1.

So, I, for that timing forget about c 1,because ultimately this will have, c 1 will only changes the norm of that and ultimately I will make it unique did not dividing the, dividing it by it is norm, so why you have to bothered about c 1.You understood my explaining, you can delete two coefficients, but I will for that time being, even if you write c 1 alpha 2 prime plus c 2 alpha 2 alpha 1, I will take c 1 common and alpha 2 tilde by c 1, let that be new alpha 2 tilde. If I find out alpha 2 tilde and finally I will make it unique norm.

So, once you make it unique norm, whether you put the c 1 there or not, does not matter, is it not. So, I will for the time being, keep the coefficient here one and say you r coefficient c 1 Student conversation ((Refer Time: 14:56)) c 1 1 I am putting, c 1 1 because or see just a minute, c 2 1, because this first index will show the iteration number, iteration one was here w 1 you consider, this is iteration 2, w 2 that time you have got a coefficient. So, this will indicate these 2, c 2 and 1 means I have got only one term alpha 1.

Suppose I construct a term like this, you understand alpha 2 tilde, alpha 2 tilde is content in w 2 anyway .Firstly, how to choose the coefficients, this is content in w do you agree, I choose the coefficient, so that the inner product between alpha 2 tilde and alpha 1, alpha 1 has only with here. Just like this you know, I mean draw a figure, suppose this is your unique norm alpha 1, this is your alpha 2 tilde, I am linearly combining them to get a vector in this direction.

This is not alpha 2 tilde, so this is alpha 2 prime, this is alpha 2 prime, this is given, these two are not orthogonal, this was spanning a space along this line, I went outside brought this fellow, these two are linearly independent together the spanner space what, this plane; that is w 2, within w 2 I am finding a alpha 2 tilde vector, so that, that is orthogonal to these. So, that will be a linear combination of two. So, I choose the coefficient with this to be one and some coefficient here.

So, I will still doing this direction provided, I chose the coefficient correctly and then I will normalize it. So, this with inner product with itself would be 0, so this I want to be 0, that means, if you replace this by here alpha 2 prime and this thing c 2 1 alpha 1, what will you get alpha 1 with itself is 1 because normalized, so this will give rise to some coefficients c 2 1 as ((Refer Time: 17:13)).

Student: ((Refer Time: 17:13))

Assume that the alpha 2 prime is here, it was it, I mean by chance you could have taken alpha prime itself in this direction, that is alpha 2 prime could have been taken by you, as orthogonal to alpha 1. In that case, obviously, this is zero and this is some constant times alpha 1 and this constant times alpha 2 prime and this is alpha 2 prime only, Is it not and alpha 2 prime you seen that normal direction orthogonal, that is the special case mind you.

When you do not have that, it will, you will have nonzero value, when you do not have that you have the nonzero value otherwise, zero value, it cannot have zero value. I am just giving you more implication, if you have zero value here, zero value means this and this are same means because you is you, you took these two be orthogonal to this, this also will be same as this orthogonal to this and then I will normalize this, normalization is no problem. So, I will get, then alpha 2 tilde by its norm, I will be calling the giving the name alpha 2, unique norm.

Now I am finding this coefficient, now one thing you see, span of I make that length span of this thing alpha 2 prime, alpha 1 which I call w 2, that is same as span of alpha 2 tilde alpha 1, that is span of these two lines is same as span of these two lines. You understood what I am try to say is, span of these two is same as span of these two, but these two are orthogonal, so that means for I will prove it, so that means for w 2, we started with a non orthogonal basis and got an orthogonal basis.

And this is very easy to show, consider any vector of this, consider a vector; any vector v element of these, that will be a linear combination of these, is it not. That will be a linear combination say c 1 alpha 2 prime plus c 2 alpha 1 in general, but alpha 2 prime, you have already find found out alpha 2 prime is this minus this, alpha 2 prime is alpha 2 tilde minus. So, basically you replace alpha 2 prime by that I, I am not showing some extras ((refer time: 20:16)) you see, alpha 2 prime if I replace alpha 2 prime by alpha 2 tilde minus c 2 1 alpha 1.

So, this again becomes a linear combination of alpha 2 tilde and alpha 1 only, so this content in this, of the element of this is writable like this and alpha 2 prime. You write as alpha 2 tilde minus c 2 1 alpha 1 as you get a linear combination of alpha 2 tilde and alpha 1 only finally, which is then content in this, that means left hand side, that means LHS content in RHS another end take any value from here, alpha it will be a linear combination of the two.

So, if you take the prime as some d 1 times alpha 2 tilde plus d 2 times alpha 1 and alpha 2 tilde you replace by this, so of this is content in this, content in w 2, is it clear or not that means RHS also content linearly with this ,so that means RHS and LHS are same, Is it not. Any question on this, any see alpha 2 prime is linearly related to alpha 2 tilde and alpha 1and vice versa, alpha 2 prime is linearly related to alpha 2 tilde and alpha 1 and therefore, where ever you get alpha 2 prime you replace it by a linear expression.

So, that linear combination becomes a linear combination of alpha 2 tilde and alpha 1 which is part of this and vice versa. Just like this, these two span of this two, at there you replace these as a linear combination of the two, so that means, if you take any linear combination of this fellow and this fellow that effectively becomes a linear combination of this fellow and this fellow. So, span of these two is content in the span of these two and vice versa.

So, I have w 2 and w 2 span by equivalent, I can also write in the normalized form w 2 is now span by alpha 2 alpha 1, instead of alpha tilde I just normalize it so alpha 2 alpha 1. So, this is important for me, so I had one subspace w 1, I got a vector of unique norm which is span in that. Then, I have w 2 which is span by an orthonormal basis out of which one is alpha 1 another is alpha 2, they are mutually orthogonal mutually unique norm and w 2,w 2 contains w 1 of course. Now, I will go outside w 2, I will do that procedure, I have said w 2 pick up another fellow call it alpha 3 prime. From that I develop alpha 3 tilde and then tilde I will finally normalize, I will call it alpha 3. This is another step after that, it will be very clear.

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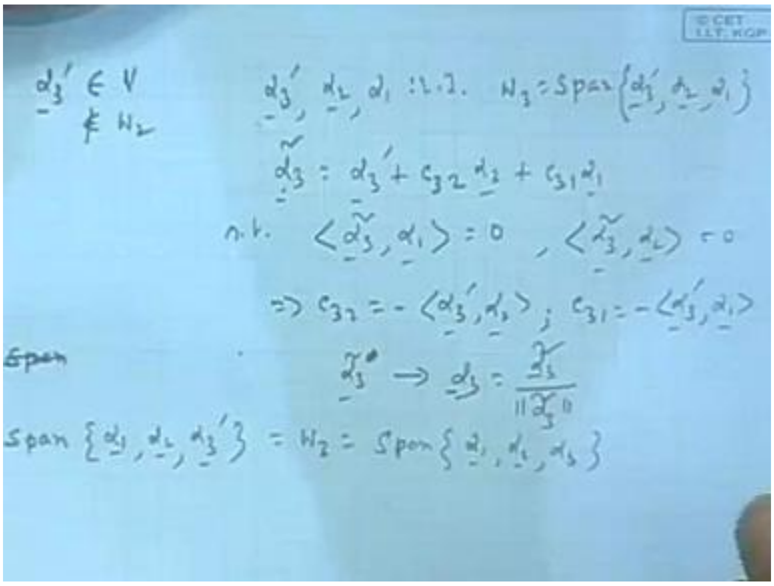

So, now I take alpha 3 prime outside, there is content in v not content in w 2, you understand I am saying, this is the same thing like I took alpha 2 prime, now I am taking alpha 3 prime, content in v not content in w 1 earlier, now w 2. So, if I append that, if I append this will be, that this will linearly independent set alpha 3 prime, alpha 2 alpha 1, these one will be orthogonal to this, just like it after all having done these, I am now going into the third dimension and not necessary I have, I am not getting a vector directly perpendicular to this, I am getting a vector at an angle, calling it alpha 3 prime.

From that, I will try to get a vector perpendicular to this plane and then normalize it and again I will show, that if you take this vector which is angular, I mean which is at some angle not ninety degree with this plane, that and this two, just span the same space as a space span by a this, these and the one perpendicular to these. What I did in two dimension, I am doing three and then it will be clear, so these three form a linear, linearly independent set because after all alpha 3 prime was taken out from outside w 2.

So, if you append that to the basis of w 2, they forms a linearly independent set, say L I. Let w 3 be span of, just span and alpha 3 prime, from these I want to develop a vector alpha 3 tilde, as a linear combination, so that one of these three, so that, that is content in w 3. What kind of linear combination, In general, it could be some c 1 times this plus c 2 this plus c 3 times these, c 1 I will take common, so that this leading the I or if you call it c 3, c 3 times alpha 3 prime plus c 2 times alpha 2 plus c 1 times alpha 1, c 3 I will take common.

So, that this guy has coefficient one, others has some coefficient and c 3 will not matter because finally, I will be normalizing this vector, so that this has unique norm. You understand and so only similar step, I am doing up to this level, so that I mean, you understand how recursively this will go on. So, alpha 3 prime will be of this kind of form, alpha 3 prime plus some coefficient now, c 3 1, c 3 2 iteration number 3, c 3 2, so that , so that this is orthogonal to both, mind you.

Earlier it was alpha 2 tilde was orthogonal only to alpha 1, now this must be orthogonal to both these, so that alpha 3 tilde with alpha 1 0 and alpha 3 tilde with alpha 2 0 both. So now you can generalize if I have more w 4 w 5, this leading guy should be orthogonal to the rest, which are forming a mutually orthonormal set already, because of previous stage of construction alpha 1 and alpha 2, they form a mutual, I mean orthogonal set already we have orthonormal set we have already seen, alpha 3 prime which you got, it is not orthogonal to them.

So, I am forming a linear combination which is orthogonal to them and then after that I will normalize it. So, that it has unique norm, essential thing is, this still linear in w3, so from this equation you can easily see if you take, you replace this here, with alpha 1 if you take the inner product alpha 1 and alpha 2 are orthogonal that term will disappear, that is the important thing, so that means c 3 2 will be what, minus alpha 3 prime with alpha 2, very easy to write down the expressions, c 3 1 will be minus ((Refer Time: 27:53)).

If it so happen, you took this vector alpha 3 prime which is orthogonal to at least one of the two, then the corresponding coefficient will be 0, it is fine. If it is already orthogonal to both, both the coefficients are 0 and alpha 3 this, this fellow which I want to be orthogonal to both alpha 1 and alpha 2, will be same as the one you have picked up. So, you got this coefficient and now I will be again make the statement, that span of and from alpha 3 prime you normalize it, alpha 3 tilde.

From this you go to alpha 3 equal to by its norm, so unique norm, symbolically it might look very complex, but it is nothing I got this vector taking norm and divide. So, that it becomes unique norm so so for we can have alpha 4 alpha 5 and I make this statement span of alpha 1 alpha 2 alpha 3 prime, which is your w 3 ,which is same as span of alpha 1 alpha 2 alpha 3.

Obviously, any vector here is a linear combination of the three, but alpha 3 prime can be retained as a linear combination of alpha 3 tilde and therefore, these alpha 3 also, because alpha 3 tilde is nothing but, this norm times these. So, any alpha 3 prime in a linear combination of these three can be replaced by a linear combination of alpha 3 alpha 1 alpha 2, is it not. So, that means, there will be a linear combination of this three only, so this entire set is content in these and vice versa.

Here, if you take the span, if you take any linear combination of these three, alpha 3 is this by these and this is equal to this linear combination, so essentially it will be a linear combination of these three elements. So, that means, that any vector here is content in the span here, so this span is content in the span here, so a content in b and b content in a means a and b are same, so both are same. So, I start with w 1 picked up somebody from outside got w 2, then picked up somebody from outside got w 3, but each time I am constructing an orthonormal basis for the respective subspaces from w 1 to w 2 w 3.

But as you know, this process will finally terminate at n equal to p, p has a state because dimension of the vector is p, after that w p will be same as v, I will not have any vector which lies outside w p and still inside v, for that, by that time by this process, I will be constructing an orthogonal basis for w p is alpha 1 alpha 2 alpha 3 dot, dot, up to alpha p, they are orthonormal.

In fact, which will be the span of, which will then be the basis for v also, because w p and v are same. So that means, we can always construct such orthonormal basis or orthogonal basis or any vector space v and that choice is not unique, I took any alpha 1 for that I consider w and then I went outside w 1 to any ((Refer Time: 31:13)) then you remember the what, any w 2, any alpha 2 prime and then constructed w 2. And then went outside and took any alpha 3 prime, remember the word any, as a result there is no unique choice.

We can always construct your own kind of basis, orthogonal basis ((Refer Time: 31:34)) So that means, such an orthogonal basis exist there, basis is not unique, but orthogonal basis exist for a product basis, at least for finite dimension basis where dimension is p and p is a finite number, P is a finite number. Now, why you had so bothered about orthogonal basis or orthonormal basis. Suppose you have a vector, you have a space v. You have, suppose a space v, vector space v and you are finding out a real giving an orthonormal basis for v.

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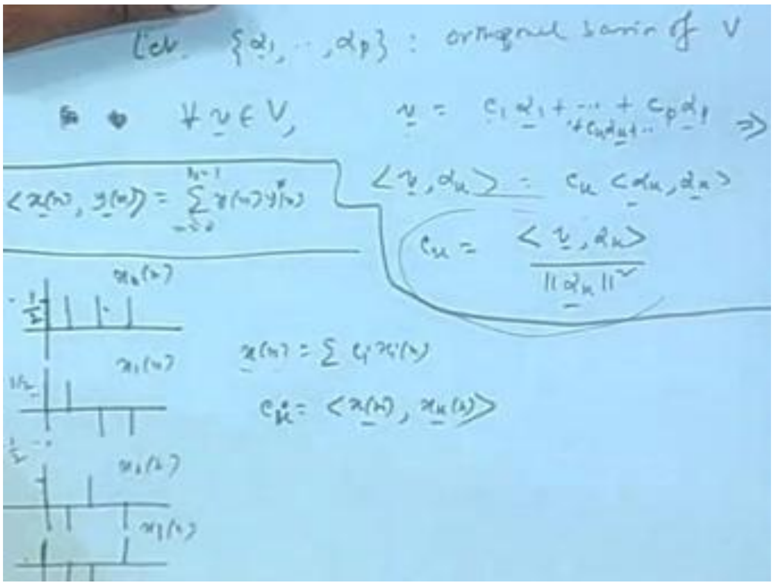

That is let, orthonormal or orthogonal basis of v, that means for any v or all v vector, element of v will be linear combination of these terms, is it. For any vector belonging to this vector space v, vector space v, it will be a linear combination which is always happens in the case of basis, see this after the basis, it is not only orthonormal basis ((Refer Time: 33:16)) basis. So, any vector belonging to capital V will be a linear combination.

In the case of basis, not the thing the orthogonal basis, just when we are doing this simple vector space and no inner product defined, V was given, but how to get this coefficients from v, we did not study because that was difficult, but when inner product is defined and based on that inner product you got an orthogonal basis and using this orthogonal basis if you linearly combined the elements and especially any vector v, then I tell you each coefficients can be obtained very easily.

Because, suppose you have to find out a particular coefficient c k that is, there is a term here dot, dot, dot plus c k alpha k plus dot, dot, dot, you have to find out what is c k, immediately I will do these inner product of these with alpha k, the corresponding basis vector. Here also, you understand here all other terms will go only this ck alpha k comma alpha k will survive, c k alpha k comma alpha k.

If they are orthonormal, this would have been one normal within, I mean, norm inner product with this. In general case, where they not orthonormal ck remember is this, this relation you have studied in DSP without understanding, this is called inverse transform relation and this relation is called transform relation like inverse d f t, d f t inverse d c t, d c t ,all class form.

Suppose, if you generalize from here to continuous time or function and all that, this will give us to inverse transform, where a given function will be expressed as a linear combination of some basis things. And you combine a coefficient will be obtained from the given one and the basis particular basis content by a formula. I use some example then you will understand.

Consider, though it is not part of this course otherwise it will become bit dry, so you understand the utility of the orthogonal transform, I can easily directly get this coefficient very easily. Now, I consider DSP, I consider sequences, so if to start with real value sequences, of length say four, length four. In general, for real value sequence of length capital N, dimension of the space is n, you have seen, real value sequence is real or complex does not matter of length capital N means dimension n.

And they are inner product between two sequences if I did the, denote the vectors as this manner, in this manner x n is a sequence y n is a sequence. I say, I define a inner product, you look term wise multiplication, sample wise multiplication, zero is a sample, zero is a sample after conjugation at. In general, for the complex case real means no conjugation, when the real conjugation does not matter, but otherwise this.

You can easily see the, this will be satisfying all the four axioms of inner product. Firstly, if it is instead of x n, if we make it c 1 x 1 plus say x 1 plus x 2 ,two sequence, here you put within bracket x 1 plus x 2, you can separate out one summation with x 1 another summation with x 2 place. So, that means, inner product is separable one will be inner product with x 1 and y 1 another will be x 2 with y 2, Is it not.

Like this you can see the linearity here, if instead of x it is x 1 plus x 2 , you put that here, you can separate out that summation into 2, so you get two inner products one between x 1 and y another between x 2 and y. So, number one satisfied, number two is what, instead of x if it is c times x where c should be, could be taken out. If you put c x n here c can be taken out in the summation c times the inner product, then another is, if you inter change that two, real thing inner product will be the conjugate of the earlier one.

Now, if you interchange this two, it becomes y n into x star n summation that is conjugate of this one and last one is inner product with itself that is real non negative number and equal to zero, only if the vector is zero. Now, if your inner product with itself it becomes mod x n square sum, mod x n square is always positive, sum; the sum will be zero means each mod x n is zero, that is each sample is a zero; that means, this sequence is all zero sequence. So, it is a zero vector for the sequence space.

If it is not, that summation will be non zero and positive ((Real Time: 37:38)) real positive, this will real always it will be positive. So, that means, ((Refer Time: 37:43)) this actually means, the inner product I can find out some sequences which are orthogonal that is, whose this, I mean you compute this kind of inner product between those sequences and they are orthogonal then they find, I can construct orthogonal basis of signal spaces and depend on the basis only you have DCT, DFT and all transforms.

I can construct, I can take up an exercise here. Suppose, I am taking length four, suppose I taking, you can play around with this you know, suppose I take length four sequences, that is this capital N is four. So, only four points, 0, 1, 2, 3. Suppose, I take one basis as these, this is the length four sequence; one basis not normalized ((Refer Time: 38:27)) not normalized because if you do the norm of it what you get, if it is 1 what you get, 1 into 1, 1 into 1, 1 into 1 is your 4, is it not.

So, norm square is 4, norm is 2. So, if you divide by 2, 1 by 2, 1 by 2, 1 by 1 by 2, there you see, one inner product with itself will get 1 by 4, 1 by 4, 1 by 4 at it becomes 1. If you want I can put this way 1 by 2 everybody, then take another sequence length 4, which is the orthogonal to these and unique norm by yourself doing it, many choice, are they orthogonal, this two otherwise sample wise multiplication and add this into this, this into this, whatever you get, you get the negative of that here add zero, is it not?

This two are orthogonal, but total how many are possible, how many should be possible, four because dimension is 4 .Theoretically, I can take the span of these go outside get the, give the name of alpha 3 prime to that vector, from that I construct alpha 3 finally which is orthogonal to the both, but find that way, by the intuition only you find out now, that was theoretically by intuition. Now, let we find out them, that only tells you that you can take, go up to 4.

So, this is not enough, but how to get 3 and 4, you use up your intelligence, suppose I take, is this orthogonal to these two both, orthogonal; height is of course half. So, that when you multiply the sample by itself, you get 1 by 4 always and some it becomes 1, so this, this is orthogonal to these three and now find out another one which is orthogonal to three both and all the three and therefore, I get a complete set because more than 4 is not enough, it is not possible and dimension is 4.

That means, this will be an orthogonal basis of the sequence space for sequence of length four, is it not and what will be that, again there are many choices you take this two way. Let, these be these and these be these, thus take this and you have see, see whether this is orthogonal to these three, sample wise multiply and add, plus plus plus minus minus minus like that, are you following, these are called walls fundament((Refer Time: 41:13)) basis.

So, any length for sequence I took just 4 here, it could be generalize to n where n is a power of 2.((Refer Time: 41:22)) you know, these kind of basis decomposition is fine, if the length is power of 2 that way either 4 or 8 or 16, not for 17 or not 19 , you will get that. This does not matter but it has a length, any length for sequences can be a linear combination of this four, that we will call the inverse transform relation.

If you call them as 0 n x 1 n x 2 n x 3 n, then any x n will be a linear combination of c i x i n, this is the inverse transform relation, inverse part of transform or inverse square, where given vector is represented as a linear combination of some orthogonal basis vectors, how to find out c i, particular c k or c i whatever you use the formula and they are now orthonormalized, is it not. They are normalized already so this denominator is 1, you should be have this numerator this is the inner product between x n and a particular one , x k n. So, whether if it is x k in these put that back here, you get the corresponding coefficient. If this is these, put the sequence here and then carry out the inner product in this manner, sample wise multiplication, addition you will get c 2, like that this will be called transform coefficients.

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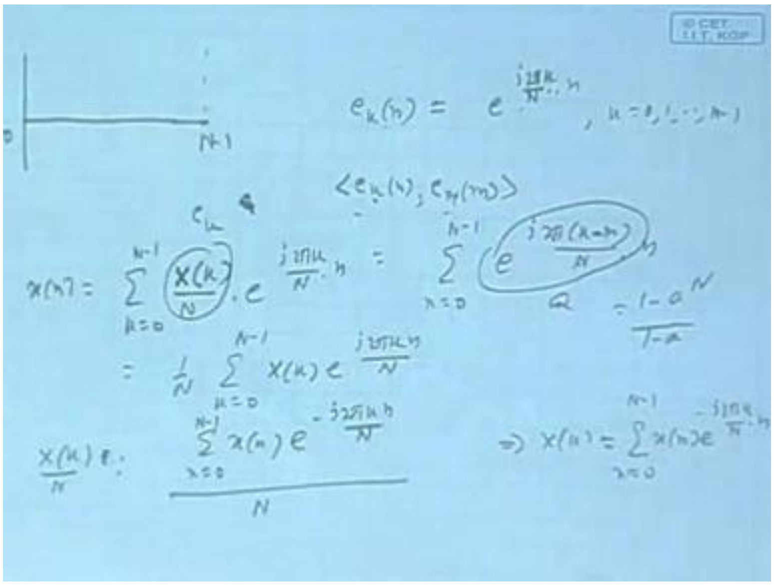

Consider now, in a general case since I have done this topic in a different way on that time I promised, so that I come to this and finish it properly. Suppose, I consider the both general case of 0 to n minus 1 length n, sequences which are in general real complex value could be real valued also. I define a set of sequences e k n as this, take a basic thing e to the power j 2 pi by N, 2 pi by n is a digital frequency k times that, if you call j 2 pi by n as the j 2 pi by n as the fundamental frequency k times that and n is the called time index, you call it as time index, this is the frequency.

Then firstly, they are, I will show that and how many are there, j equal to 0 1 dot dot dot to n minus 1, then we will see that any two sequence is a orthogonal, any two given sequence is a orthogonal easily, you say inner product between e k n e n n, you will call them vector. What will that be, second term has to be conjugated is it not and then sum, Summed over this index n that was the repetition of the inner product, sum over the index n, time index n and first one will be as it is and second one will be conjugated of that, so e to the power j 2 pi k minus m by N into n .

See this thing, where did I do all that ((Refer Time: 45:16)) here, sample wise multiplication after conjugation of the second guy, is it not, after conjugation of the second guy .So, sample wise multiplication you just conjugate these, first one is not conjugate so 2 pi k by n, this k that remains k when it is m, instead of k you have m, e m n so it becomes m that is conjugated so minus sign and you add them, you know this is zero, I told you know this ((Refer Time: 45:43)) equation is .

If you consider this factor and I call it a, it is a small n it is a ((Refer Time: 45:47)). So, its summation is 1 minus a to the power n 1 by a, that is, if you raise a to the power n, n and n will cancel, e to the power j 2 pi into an integer which is 1, 1 minus 1 0. Otherwise, also it means what, you have taking a complex sinusoid and you are summing, you are summing the samples, what is the critical number of periods, so that becomes 0. Is this orthogonal, not normalized, if you take the norm of this, what will be the norm by the way, what is the norm inner product with itself will be equal to what, n.

Because these with its conjugate and summation over small n for n equal to 0 to n minus one, conjugate and this will cancel each other, you get 1 and 1 1 1and n times n and it is not orthogonal, I agree. But at least any x n can be written as a linear combination of this kind of vectors, instead of linear combination coefficients, instead of denoting the combiner coefficients c 1 c 2 c 3, I denote them by see k is the index here, I denote them by capital X, otherwise I denoting by c 1.

So, this 1 was coming in the subscript, instead of subscript I shall put it here, within bracket and instead of c, I put capital X, no problem. The coefficient and also I put a capital N here, this is by combiner coefficient into corresponding term, this N can go out, this is my combiner coefficient, this vector times and coefficient. If the entire thing is actually c k, c k times these, I am denoting like this and this is your inverse d f t relation. Then, how do you, how do get the x k, you use that formula. Inner product of the given vector that is x n with these divide by the norm, norm square, is it not?

So, inner product of x n, you remember, inner, so inner product of x n, x n times alpha k, I am interested in finding out x k, the corresponding basis sequence is these, so inner product of x n with that, so it will be coming with conjugate summation over n and this will give me x k by n by the way, this is the coefficient x k by n, x k by n will be what, inner product between x n and the k n basis divide by the norm square of the k n basis, what is norm square, norm is n , the norm square is N.

What is x k, you get your d f t, always remember transforms like these. I will not continuing any more now, from this orthogonal , these thing we will go for orthogonal projection, that is we have shown orthogonality as shown as utility that any combiner coefficient can be obtained easily efficiently by some formula, orthogonal basis exist we have seen, Gram smith orthogonalization by next time covered in detail. Examples of orthogonal basis we have seen in from DSP all these transforms.

Now, what we have to do is estimation, we are much interested of estimation we will, as I told you in the case of optimal filtering that linear mean square estimation is basically an orthogonal projection computation. So, now we will be going to orthogonal projection, that is we will show that in a vector space v there, if you take a subspace w and your outside w, you take a vector v with in capital V, but outside w you take vector as small v, your for small v within w, there is a unique vector which I call it is projection or shadow.

So, that the difference between small v and that shadow that is orthogonal to the entire subspace w perpendicular and that has the minimum norm, there is a difference between the original one and the shadow, if the shadow is taken to be like that. So, the difference is orthogonal to the entire space, then that difference will have minimum norm square, that is the advantage, that the difference will have minimum strength.

And that is why you consider the shadow orthogonal projection, you take these unique and using… And then we go for projection decomposition orthogonal projection based decomposition of subspaces that from that we will much to again our optimal filtering and all using this ((Refer Time: 50:34)). So, that is all.

Thank you very much, I need to have your roll card quickly.