Adaptive Signal Processing

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Lecture - 17
Vector Space Treatment to Random Variables

Shall go with him, there is some variance of that and all that, but that time I told you that actually this mean square estimation, is a more general mathematical problem, that of computing orthogonal projection of some element on a space span by some other elements. So, we need to deal about that, because the general adaptive filter or any mean square estimation problem can be formulated and solved using those notions know the notion of vector space.

So, I will now introduce you to this exciting topic from the linear algebra called vector space. May be today this lecture and couple of more lecture will be spent on that. Just this topic is abstract and does not require any background, but there is sometimes requires good thinking. So, I will whatever you do, today I will introduce you to this basic notions of rather axioms of vector space definitions and some properties and interesting things. This will continue till tomorrow, we will going to orthogonality orthogonal decomposition of subspaces and all that. Then finally, we will evaluate what is called orthogonal projection and all. Vector space, a vector space actually we say vector space over a field. The field is another you know something which you will not know possibly group ring and field there is various mathematical structures what is called group.

Group means a set of elements which have got only one operation is a groups only one operation and some properties. Then there is a ring, ring has got set of element with 2 operations, but one of the operation does not of the inverse. In another operation inverse exist, another operation inverse does not exist like some polynomials. Polynomials when if you add them you still get a polynomial. Set of polynomial is closed, if you add if you have one I mean.

What I say group ring field whatever it is if take any 2 element apply any of the operation on that you get another person from the same set. So, in the case of ring that 2 operations are say ring say this thing polynomial set of polynomials is a ring what are the 2 operations addition and multiplication.

See addition is simple and if you add 2 polynomial you get another polynomial. So, it belongs to the set of polynomial. Addition has inverse like negative of the polynomial all those things are there, but multiplication of 2 polynomial. That is another operation that also you get a polynomial, but that does not have inverse because inverse of a polynomial is not a polynomial one by that is not a polynomial forms that is a ring.

If you join a group it too have only one operation with inverse and all ring means it will feel more relaxed one extra operation, but does not have inverse. Then you have field 2 operations both have inverse and satisfies all the properties of commutativity and other things you know I am not getting in to that.

All real numbers integers, all real numbers, complex numbers; they all form field under the usual notion of your this thing addition and addition of decimal number multiplication of decimal real number addition of real number multiplication of real number or complex number. If you have any binary number then they form a field called modulative field. You have got dot operations and plus operation and all those

Another example of ring can you tell me ring I give you an example of polynomial another example of ring is. Integers, If you take integer you add 2 integer you will get another integer, you multiply 2 integer you get another multiple, but inverse of integer is not an integer it does not exist in the set.

Anyway these mathematical structures. Finally, generalize into another structure called vector space. Where this vector space also will be a set. It will have 2 operations defined, but this vector space itself is defined over a field. The all those we do not need for this course we will always go in for our purpose. We will be dealing with vector space defined over this field of real or complex numbers.

That will only mean whenever I bring some constant scalar. That will be either a real number or a complex number that is the only meaning, but I might write vector space defined over a field r or c r for real number c for complex numbers, in general complex. It is only within for your consumption only meaning is that all the scalars, that will be handling in this treatment. That will be either real number, if the vector space is defined over real or is complex number if it defined over c.

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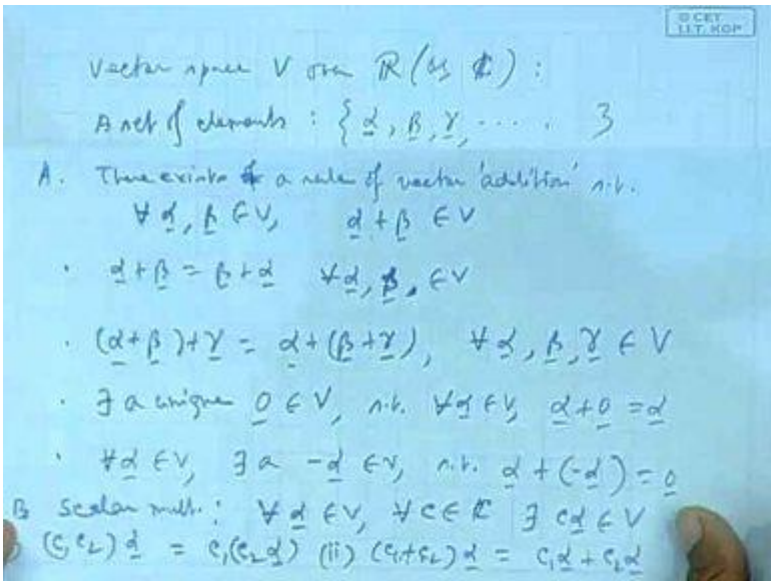

Under that vector space v over this is called R a set of real numbers or script C, this is called set of complex numbers is what. It is a set of what, elements this is what this is what the problem come. This is the set of elements means abstract elements I do not know what they are physically. So, there just and those symbols are called vectors this was nothing to do with the useful notion of the directional vector, that you know, I vector J vector dot product nothing.

It is a set of abstract elements you can thread them as symbols only nothing else. They could depend anything depending on the case to case. You can have a one kind of vector space another vector space a particular vector space and the symbols will assume some particular physical meaning and that will change from context to context. So, here a set of elements called vectors elements I am not writing everything you write.

Sort of elements called vectors like these alpha and I put an underscore here. Just to indicate that this elements on the vector space or loosely they are the vectors, because after some time I will bring some scalar number some real or complex value. You will not be able to distinguish between what is which one is vector which one is which one belongs to the set and which one is the scalar. Just to distinguish I am putting underscore as a mark.

You understand. So, that you understand this a vector alpha, beta gamma dot, dot, could be infinity. So, that there are 2 operations on these. One there exist mobile should be off it just take out it. My mobile is off though, is it not? So, I should. Anyway there exist a rule of vector addition. Means this addition mind you, please note this addition is not the usual addition.

I am borrowing the word addition, it is the abstract addition symbolic addition is got nothing to with your original or ordinary arithmetic addition or algebraic addition. I am just giving a name addition to it. I do not know what it is physically what kind of operation I do not know, I do not care this also name I am giving one operation called addition. For this also I use the same symbol plus not is it not? Again it is a symbolic plus abstract plus nothing do with the real plus.

There is a rule of vector addition what does it do like ordinary addition what does it do. Take 2 number give you another number it also takes 2 vectors and give a result in the form of another vector belonging to this set. That is most important, it will not take you to some something outside the set. Take any 2 you apply the rule of addition that. So, you can form a table of addition rule. On one axis alpha beta gamma and another axis alpha beta gamma.

So, alpha with alpha where alpha with beta where alpha with gamma where. So, you can form a table that way. So, it is just mapping alpha beta this figure maps to where alpha gamma this figure maps to where. So, actually in terms of set duration this v cross v. You know v cross v, this Cartesian product map to v, v cross v map to v that way, but I am writing in pure vector addition. So, that for all alpha beta element of v alpha plus beta this is the symbolic addition. This is a vector it is also element of v number 1.

It satisfies some properties this plus is I have just said that one operation, takes 2 element and gives you another element from this set only, but it should satisfy some property what are the properties one is commutatively. That is alpha plus beta equal to beta plus alpha. I am not writing for all alpha this is true for any alpha. Any beta belonging to v I am not writing that.

Or maybe once I write for all alpha beta element of v this is true. Then associative you can understand the meaning. First you rule apply the rule of addition on alpha beta whatever you get that is the element of v with that you apply. With that you take gamma again apply the rule of addition you get something. You will get the same thing, if you first take beta gamma the result the result you compare with alpha. These are simple, but next 2 are very important.

There exist, a unique remember this linear algebra is key, in all today signal processing control and all. There is a unique 0 vector I denote by 0 0 vector this is unique only one element of v. So, that when you take any alpha beta whatever and take this fellow also and apply the rule of addition you get back that origin that guy only. So, that for all alpha element of v alpha and plus 0, that is these plus symbolic plus is alpha.

For any alpha and for alpha element of v there exist a negative. So, called negative symbolic negative abstract negative version of alpha. There exist a minus alpha, mind you this minus is minus alpha itself is the vector I am denoting there exist a vector denoted by minus alpha. This minus is just a symbol just a dash element of v. So, that when you add with this thing you get back this fellow 0.

So, this is called negative of alpha denoted by minus alpha together is the denotation minus alpha dash alpha or whatever. This four, this one respect to one operation called addition or abstract addition or vector addition or whatever, this 4 axioms. There is one more operation that is scalar multiplication, b is scalar multiplication. Scalar will come from the field either real number or complex number that is why that r or c are there you see.

Under a everything was vector belonging to this v there was no scalar here. This addition will scalar, but now vector will come a scalar will come from here. What it does it says very loosely speaking if alpha is element of v, c alpha element of v no c alpha has no meaning here because no there is these are all abstract terms. It is not the 2 into three. So, that I that is the formulate these that there is there is a rule of scalar multiplication.

Again multiplication means abstract kind of multiplication symbolic multiplication what does it do. Take up any scalar number from this field take up any vector, there is a rule by which works on them, and give you another vector from this set which I denote as c alpha. It is not is the usual sense of multiplication is the abstract multiplication. So, for all alpha element of v and for all scalar c element of I am taking complex same thing apply for real also.

There exist c alpha element of v. This is called the scalar multiple of alpha by c. Just denote it by c alpha actually some operation which takes c and alpha and give you a result. That result is denoted like c alpha. Do not be genius use the sense of multiplying alpha by c do not extend that conventional notion of multiplication all that c alpha is notation and only abstract v. Again it should satisfy some properties.

The properties are simple, one is if it is you know c if c can be broken like these C 1 C 2 alpha see it just scalar. Real or complex. So, suppose c factorize C 1 C 2. Then you will get the same thing, if you take out say C 2 first take C 2 and alpha apply the rule, you get a vector from v and on that you apply C 1. Physical what are meaning of this, alpha and C 2 you take. So, you get a resulting vector by applying the rule of scalar multiplication you denote it by C 2 alpha.

That vector and C 1 you take you get another vector again from v. This 2 are the same. Secondly C 1 plus C 2 alpha that is, if you add 2 scalar and then with that alpha you do. It will be same as if you take C 1 take C 1 alpha first C 2 alpha and add by the this addition rule. Mind you here C 1 plus this plus means, which plus this is plus for the field, because if this plus see the interesting thing.

This plus applied for what the plus operation pretending the field of scalars like this is a concrete plus. You know this plus for decimal real number or complex number. This plus is for vector this plus defined here. You understand I am using the same notation, but the difference. So, it says C 1 plus C 2 5 plus 6 11 alpha is what 5 alpha vector addition 6 alpha. So, see this they are not the 2 side plus same plus. Symbolically they appear to be.

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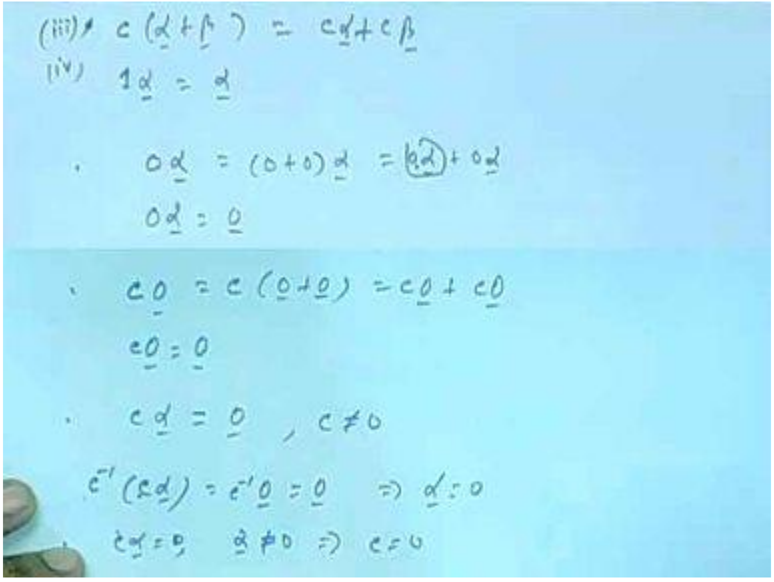

Then, this was number two, number 3 is on the other hand c the other way alpha plus beta which this is simple vector addition alpha plus beta. That is if take that, if you 2 vector alpha and beta add them by the vector addition rule. Then take multiplication of c, you would get the same thing, if you have the c alpha and c beta 2 vector and add it.

Lastly every field whether its complex number real number modulative field any every field has one you know one element called identity element. Every field of numbers if you do not want to get into field you know one number one in real number 1 2 3 that 1 or complex case also one that is the identity element. If you take that one, this rule of multiplication says that one alpha. So, b equal to alpha itself.

Rule should satisfy these then this called a vector space v over the field of either real or complex number depending on while you are taking the scalar for. While you assume the vector space for complex in general. So, complex takes care real also. Certain things we see what is scalar 0 times alpha that none of this axioms told us what is scalar 0 times alpha, but how to prove it scalar 0 is 0 plus 0 into alpha 0 plus 0. We have got these thing, C 1 plus C 2 alpha is C 1 alpha plus these axiom is given to be, that I apply here. So, 0 alpha plus 0 alpha 2 vector.

So, what is this vector. These when added to these gives you the same result. So, by definition there is a unique 0 which only it is happens. Is such a vector whatever 0 vector, such a vector which when added to somebody else gives you that somebody else only. These 2 things are same, that means, this must be the 0 vector, see how I am using those axioms. I am not assuming anything else.

So, that means by that logic this is 0 vector. So, scalar 0 times any vector times means abstract times is it not? Real multiplication anywhere, but these axioms suggest that scalar 0 and alpha, if you pick up you get 0 vector of the vector space. On the other hand, if you have any scalar times 0 vector then what you have. By the same logic 0 vector can be written as 0 vector plus 0 vector, because what is 0 vector 0, when added to anybody get you the same body.

Zero vector is what when you added to any x. So, I can write and now apply these axiom, the other one, where is the other one, this one. Again this is such something when added to x gives you the x. So, this must be 0 vector. Then suppose c alpha is given to be 0 vector and c is non zero C alpha is given to be 0 vector and c is non zero c is not the 0 of 0 scalar. Then the only solution is alpha must be 0 vector.

How do you say that, since c is non-zero since is non- zero. I can write this way c alpha this vector I can multiply by c inverse its non zeroes one by c exists here also, but just now I have proved any scalar times 0 vector is 0 vector. So, c inverse 0 vector is 0 vector and now take this axiom C one with in bracket C 2 means C 1 C 2 alpha. So, c inverse c that is one and then take this axiom.

One alpha is alpha, see mathematician find out the minimum axioms satisfies that all the other things come you are following. I can bring c inverse inside c inverse c, this is the scalar for any scalar number 1 by c into c that is 1. None of the field, but 1 alpha is alpha. So, by that logic alpha is zero. You found the other end given c non zero solution is alpha zero.

On the other hand, if it is given that is given c non zero. Suppose, c alpha 0 and given alpha is not the 0 vector then this should imply c equal to 0. That you can see easily by contradiction that suppose c is non zero. If c is non zero alpha 0 means by this theory alpha is to be 0 which contradicts. So, c has to be 0 that way or you can have a direct proof that proof we can work out.

So, these are extra result that come from these mind because if I after equation I fed 0 alpha equal to 0. I have already proved. Do not extend you usual notion of multiplication. So, and 0 into is 0, because is it not? Usual multiplication usual addition. So, you really need this results which looks similar to our conventional business.

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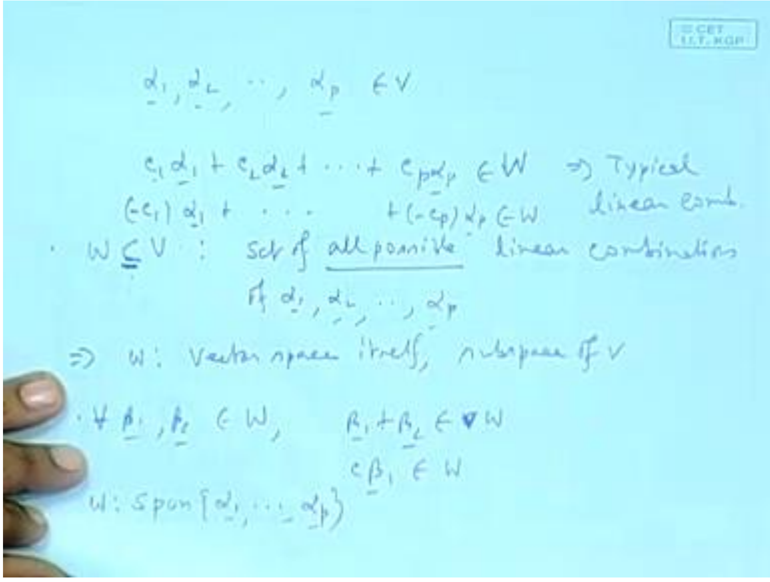

Next, given a set of vectors alpha 1 alpha 2 dot, dot say finite number of vector alpha p element of v. A typical this is kind of combinations are called a typical linear combination. This is element of v you agree or not because each term scalar multiplication is an element of v after addition also remain same v. So, it is an element of v. This is called typical linear combination.

Now, suppose let w be a subset of v. Obviously, it will be subset of v which is set of all possible. Please understand these all possible linear combinations of alpha 1 alpha 2 dot alpha p. That means w typically has elements of this kind of form, w is a set and it set consist of what and you are given this fixed vectors. Just p number of vector you are forming all kinds of linear combinations possible collectively, each is a member of v also.

That means this set w is a subset of v about that about there is no problem, w is a subset of v because a v element is a linear combination which is an element of v. So, w is a subset of v a set of what any element is of this kind of form. It is a set of possible such linear combinations not just few then my claim is w also is a vector space by its own, v is a space within that w is a subset which is also a vector space is called as subspace of v.

Then w we will see these is called subspace of v why. Firstly, take any 2 element suppose beta 1 beta 2 for all beta 1 beta 2 element of w. Beta 1 plus beta 2, what will be beta 1 plus beta 2. That also will be a linear combination that also will be a linear combination. Obviously we have C 1 alpha 1 plus these and say beta 1 and d 1 alpha 1 d 2 alpha 2 and you add that 2.

Then using those axioms you can make cone alpha 1 and d 1 alpha 1 means within bracket C 1 plus d 1 into alpha 1 using those axioms. So, as another linear combination. So, that means, for any beta 1 beta 2, they are beta 1 plus beta 2 element of, sorry element of w. Any c beta 1 say element of w any scalar c beta these take these multiply by a c.

Use those axioms C into 1 vector 1, vector means C 1 these C 1 these C 1 these like that. You said again another linear combination C C 1 alpha 1 C C 2 alpha 2. Using those axioms you can make within bracket C C 1 within bracket C C 2. So, another linear combination that is part of w very simple not only that commutativity is very simple for addition.

Take any 2 beta 1 beta 2 beta 1 plus beta 2 beta 2 plus beta 1, you get the same thing associativity also very obvious 0 vector. If I take all the coefficients to be 0 I have already proved 0 times any vector is 0 vector. I did not prove it just for fun. These are sequence I am going into scalar 0 times. Linear combinations also include just those that particular case where all the coefficients are 0 that also a linear combination, but I have already proved 0 times a vector is 0 vector. So, all are 0 vector at 0 plus 0 plus 0 plus all other be 0 that is the property of 0.

So, 0 exist and then if you give me these I take another linear combination where instead of C 1 the scalar is minus C 1 instead of C 2 the scalar is minus C 2 dot dot dot. That is also linear combination part of w, but when you add that 2 you get what. Suppose, instead of these I take these, that also is part of w. If this is part of w this also part of w is a linear combination.

If you add that 2 what you get you get 0, using those axioms you can take alpha 1 out and C 1 and minus C 1 within bracket is 0, 0 times a vector is 0. So, everywhere 0. So, that means, for any element of w is negative exists. Other properties of the scalar multiplication follow easily. Other property scalar multiplication, what are the property you take any element multiply by a constant C resulting thing will be c 1 alpha 1 C, C 2 alpha 2 like that another linear combination. So, may part of w.

Then, if is it not? That thing you know C 1 C 2, if you multiply the entire thing by a product say k 1 k 2. You will get the same thing as k 1 separate and then k 2 times these and then multiply by k 1 that was very pretty clear. Those axioms follow easily I would say. You know there is one axiom C 1 plus C 2 times, alpha is C 1 alpha C 2 alpha. So, if you have k 1 plus k 2 times these you can separate out or a k times one such combination.

Another such combination, you can separate out k times these and k time other combinations. The other one is one times these must be these, obviously, one into C 1 1 1 into C 2 1 C 2 by inspection only it is very clear. So, w is a subspace. Then w is called subspace spanned by we write this way span of alpha 1 to alpha p. That means, the subspace span by these means set of all possible linear combinations of alpha 1 to alpha p. I will give some examples then you will see, let we go through this little bit more. All real life beautiful examples we will follow.

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No. There occur so many other elements in v, which are not covered. I will give an example why I just picked up alpha 1 to alpha p taking their linear combinations. So, this logic only says that each combination is part of v. Therefore, w also is a part of v. Whether w excess v or not in some case that it could I some case that I am not coming to be yet. To tell you before w is a finite dimensional space dimension I am not coming to.

Suppose, dimension you know dimension w is a finite dimensional space dimension equal to p. They are so called linearly independent that that not proved all that. They form a basis in that case, but when they say all that. I picked up arbitrary p element I mean do not what is called dimension p elements I am considering their span. Logic says w must be you can say these way at the most.

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Like that, I put these way this better, it is called span. Next thing is this alpha 1 to alpha p is that there is redundancy in them. This can one be written as a linear combination of the rest or it everybody can be independent there that is the thing.

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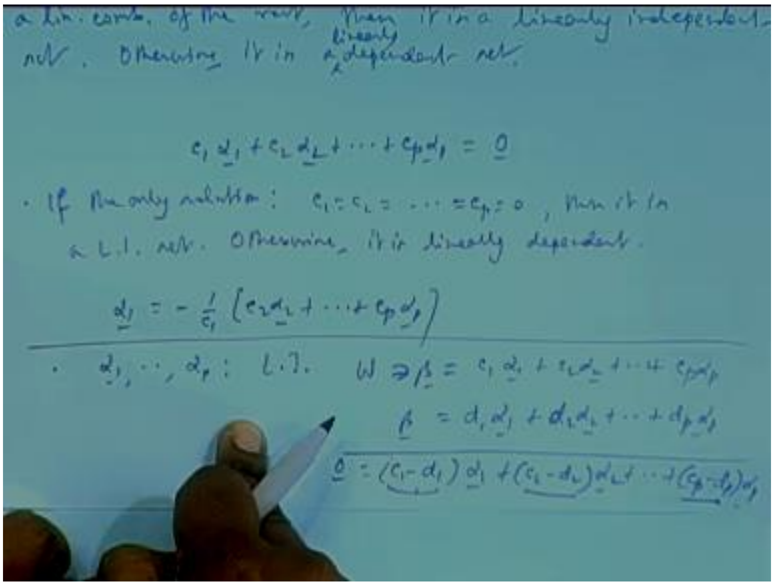

If given, if no alpha I can be written as a lin comb. I am writing lin comb means linear combination of the rest, then it is a linearly independent set independent. Otherwise that is, if that is at least one of them can be written as a linear combination can be written as a linear combination of the rest. Then it is linearly dependent.

If not dependent, then independent or if not independent then dependent. It is a dependent set linearly dependent. There is so many types of dependence you know statistical dependent and also physical; these are physical thing none of the alpha I should be writable, say alpha 1 you should not be able to write as something times alpha 2 plus something times alpha 3 plus dot something times alpha p.

If it not I mean alpha 1 I was trying to say that you can write it like you know something times alpha 2 and something times alpha 3 up to something times alpha p. Even you do not have to take up to alpha 1 to alpha p if it less than that. So, alpha 2 and alpha 3, even that will not work. If you have that alpha 1 is a linear combination of alpha 2 and alpha 3, even that will not work, then it is dependent or alpha 1 is just a multiple of alpha 2. Even then it will be dependent, linear combination of the rest means you do not have to always take the rest elements.

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That is yeah that is. So, always assume as 0. How to test, given this set this is a pure English statement that if, but how to really test it. This is an English statement that if none of them is expressive will as the linear combination of the rest. This is the statement in English, but how mathematically, I mean verify it. What is the mathematical condition for this or that. The mathematical condition is that suppose I form a linear equation. Take a linear combination and equate this to 0 vector and want to find out the solution for C 1 C 2 C p for which this is satisfied. Now, one solution all of you know this is what all 0 solution.

Obviously, if C 1 is 0 C 2 is 0 c p is 0, you already have seen 0 alpha I is 0, alpha 2 is 0, alpha is 0 and 0 plus and 0 plus. So, all 0 solution is a trivial solution always exists. Now, if the only solution is C 1 equal to C 2 equal to dot, dot equal to C p equal to 0 scalar 0 then it is a l I set otherwise it is linearly dependent.

We can see this way suppose. Firstly, we have this equal, this satisfied. You find some non-zero solutions for which also this is true. Suppose, this equation is true for some nonzero values of C 1 to C p at least some of them are non-zero or will not have to be non-zero some of. Suppose to start with all are non-zero.

Suppose, in that case I just keep alpha 1 on the left hand side cone is non-zero. See you can say division by C 1 is possible then the rest C 2 alpha 2 plus C p alpha p. So, that means, this is a linear combination of the rest. So, that is dependent when all the terms are 0 all the terms are non-zero.

I am saying that suppose a solution exist for which is it not? That I mean all the terms are 0 it is a non-zero solution can be any kind. I am starting with the case where every coefficient is non zero then you can write like these now suppose every coefficient is not non-zero some are non-zero. So, this is 0 others are non-zero then only this term will go I can still write alpha 1 like that or only suppose only these 2 C 1 and C 2 non-zero others are 0 c 3 0 c 4 0 C p 0.

Even then if C 1 and C 2 are non-zero alpha 1, can be written in terms of alpha 2 still dependent. Suppose, not even 2 only one, see C 1 is non-zero others are 0. In that case 0 alpha 1 is C 1 alpha 1 0 and C 1 non zero. That means, I have prove already alpha 1 must be 0. If alpha 1 is 0 vector in this set still it is a linearly dependent set why 0 vector can always be written as a linear combination of the rest.

What kind of linear combination 0 times alpha 2 plus 0 times alpha 3 plus dot plus 0 times alpha p. Addition of a 0 vector in a set makes the set dependent because the 0 fellow can be written in terms of the rest as a linear combination with 0 coefficients. So, what does it show that the moment we have any non-zero solution. Either for all with all non-zero or part of them, non zero or at least when one of the non-zero coming up.

This set becomes dependent. Only when you have none of only when this is this is there then it is been dependent. Why because if with these is it not? Independent that is it is still dependent. That means, I can find out it is a contradiction actually I can find out some non-zero solution for these to express one element in terms of the rest. So, that means, a non-zero solution of this will appear are you following me see the proof of this is these I first started with non-zero possible solutions.

I showed that any non-zero solution, if it exists where all the terms are non-zero or part of the terms are part of the coefficients are non-zero. Two coefficients only non-zero or if it one coefficient non-zero. It is a dependent set what happens when this is the only solution then is it independent. Answer is suppose in that case that you are saying this is the only solution and still is it not? Independent you are saying. That means, it independent.

Either independent or dependent there is nothing third part there is no third solution. Suppose it is dependent, that means, any of the vector say alpha 1 or could be any body start with. For example, say alpha 1 should be expressible as a linear combination of some of the rest if not all of the rest. So, that means I can put that terms together and make it equal to 0.

Suppose, alpha 1 is 2 alpha 1 plus 3 alpha 2 alpha 2 plus 3 alpha 3 and other terms are not present. I can write as alpha 1 minus 2 alpha 2 minus 3 alpha 3 equal to 0. So, that means, one here minus 2 here minus 3 here equal to 0, but you are saying these are the only solution. So, that is a contradiction. Again this is the only solution and still it is dependent then they are contradiction come because I form an equation here with some terms non-zero, because you are saying that dependent out of that dependency equation will come up.

So, one is writable in terms of the rest using that like that the equation will come up in non-zero terms non-zero coefficients. That means that will contradict in this. So, this is the test for linear independence. Now, coming to these. If you are considering the span of alpha 1 to alpha p. Additionally, it is given alpha 1 to alpha p are linearly independent that is there is no redundancy in them, everybody is important.

None can be written as a linear combination of the others that is everybody is important mind you. That is why they are linearly independent. Then this set is called the basis of the space w. As such they were spanning w any element of w was writable as a linear combination of the rest, because that is how w was formed, w consisted of all possible linear combination of these. So, any element w is a linear combination of these, but on term of that, if they are linearly independent then this will be called the basis of w.

If they are linearly independent then the why we are bothered about linear independence. I have in pure English told you that there is no redundancy. All that everybody is important, but mathematically you know that, suppose you forming a linear combination. Suppose, alpha 1 to alpha p given to be l I.

You are forming a linear combination say some beta element of I am writing this way beta element of w I am writing these way beta element of w. Some kind of linear combination C p alpha p because any element of w is a linear combination of those told me that they are linearly independent Then my claim is these coefficients by which you are combining alpha 1 to alpha p to get beta these coefficients are unique.

Please understand these if alpha 1 to alpha p are linearly independent. Then these coefficients by which you are combining them to get a given beta of w this coefficients are unique that is. If I want to have something like these, this also true. That is you find another set of coefficients d 1 d 2 up to d p. So, that you linearly combine you still get the same fellow beta.

Then my claim is this is not possible C 1 must be equal to d 1 C 2 must be equal to d 2 c p must be equal to d p because this is linearly independent. How do I say that very simple you subtract let beta from beta you get 0. Here you subtract using all those axioms I can write like that C 1. What you have told me this, is a linearly independent set. So, if any this is a linear combination of alpha 1 to alpha p. This is one coefficient, this is one coefficient, this is one coefficient. Please lift your head and see this.

This is a linear combination of alpha 1 to alpha p left hand side has become equal to 0 you told me alpha 1 to alpha p linearly independent. This is a linear combination of alpha 1 to alpha p that is equated to 0. So, only solution for them is this is 0, this is 0, this is 9. That means, C 1 equal to d 1 C 2 equal to d 2 c p equal to d p.

That is the advantage of linearly independent. So, if alpha 1 and alpha p has spanning w. They are linearly independent then I can come to this bold conclusion that any element of w can be written as a unique linear combination of alpha 1 to alpha p. You can have another set of linear combiner coefficients as still giving you the same element no not possible. Every element of w has a unique representation unique set of coefficients that can be used to combine alpha 1 to alpha p to get with that element. Conversely if they are dependent then there is nothing unique.

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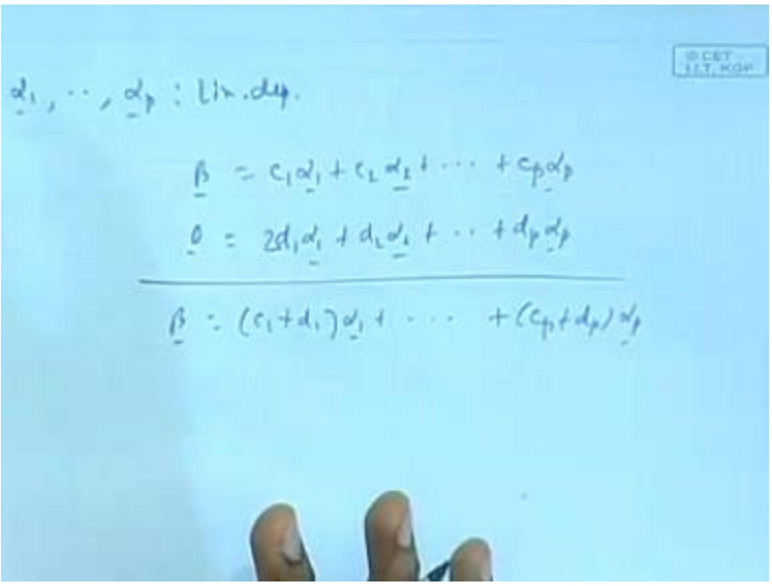

Alpha 1 to this also you can see alpha p they are dependent linearly dependent. Suppose, you are having beta as some comp C 1 alpha 1, but you are saying that dependent. That means, if I form a combination like d 1 alpha 1 plus d 2 alpha 2 plus dot plus d p alpha p equate to 0. You are telling me that all the solutions, I mean there exist non-zero solutions for d 1 to d p or it is part of them, because they are dependent.

That was the definition, if you equate it to 0 you may get non-zero solution for all the coefficients or part of the coefficients, but for which also this true. So, suppose I choose both combiner coefficients and now I add this still remain beta, but this will become C 1 plus d 1 alpha 1 dot, dot c p plus d p alpha p. So, I get beta into another set of coefficients.

At least one of them is non-zero. So, one coefficient is new and if it is true instead of these I can take it 2 d 1 2 d 2, 2 d p still remains 0 vector and add again. So, you get C 1 plus 2 d 1 c p plus that way there is infinite possibility. So, coefficients are not any. There you make if they are linearly independent that is on linear independence. See if alpha 1 to alpha p are linearly independent.

A space w then they will called a basis for w they are linearly independence. In that case any element of w is a unique linear combination of alpha 1 to alpha p. Now, we will show that number p for w that is a magic number you can choose other basis also I will show, but always you should have the same number p and that is called the dimension of w.

Alpha 1 to alpha p, how many p, that p is some kind of a magic number it does not depend on the choice of this basis alpha 1 to alpha p. From w I can pick up other fellows say gamma one gamma 2 dot which also will be linearly independent which is also span w I will also show how, but it will. So, happen total number will remain same that will never cross p and that p is called dimension. That way dimension is independent of choice of basis.

Basis can be many way, I will show you how to construct basis there infinite way of constructing basis, but number of basis element is fixed here, mind you we are dealing with finite dimensional vector spaces. In real life many vector spaces are infinite dimension, specially all the analog signals. That time you remember Fourier series and all you have got summation going up to infinity. You might not understand what I am saying now, but infinity is because we are in infinite dimension of space there, but in all these matrix vectors every things, where these are finite dimensional. We will see choose of examples I am just giving you brief hint.

DSP is that way easier because everything turns out to be most of things turns out to be infinite dimension. All orthogonal transform basically they are you know, you define some new or new kind of basis. You get one transform, you say another kind of basis by which you linearly combine and get another sequence get a sequence. You have one kind of transform by you that time that is I will then explain. I will give some examples and all tell you that is how basis transform come actually. After all these I will work out some examples for you. Otherwise it will still leave any abstracts, but before that I have to still go some other for.

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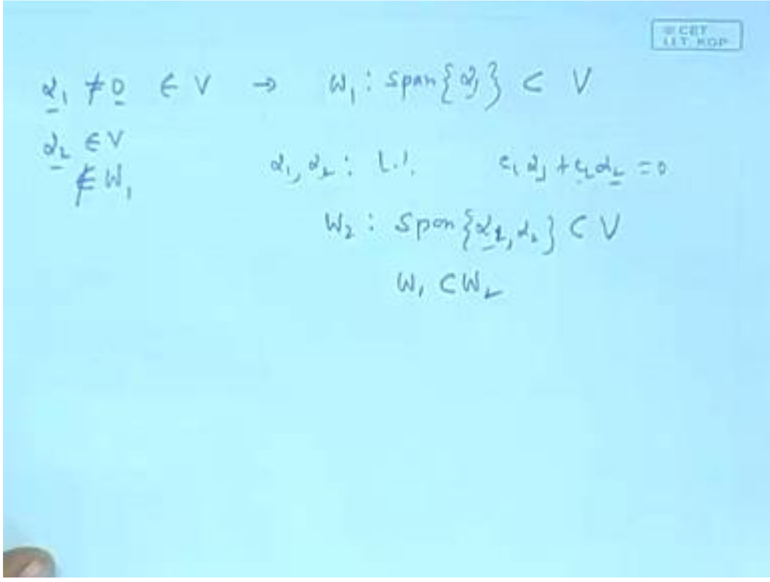

Suppose, I am in v I take one alpha 1 and mind you I coming to this dimensional proof purely a shortcut way without diluting anything mathematically. I am not diluting any no mathematician have challenge making found some weak foundation here and there, but this constructed by me for my students. So, I can quickly come at dimension without so many theorem of problem here this and that. So, you just take notes here.

Suppose, I take an alpha 1 not equal to 0, 0 vector I will because 0 means dependence. Anywhere you add 0 it becomes a dependent set and non-zero alpha 1 I take out of v, no problem. Then I consider w 1 is what span of just this fellow only. Span of one element means I told you span of alpha 1 to alpha p means set of all possible linear combinations. Here we have got one element. I cannot do anything with these 5 minute.

If there is only one element there, that means, typically any element typically what kind of linear combination just constant time alpha 1 C 1 alpha 1 or c alpha 1. So, just multiples of alpha 1. Set of that is the vector space by the same thing, w was a subspace that time for all possible linear combination. So, this is special case.

You take any 2 multiples add them you get another multiple of alpha 1 that way you know. So, this is subspace of v fine. See this subspace this subspace has 0 vector because 0 times alpha that also is part of it this. Any space, all multiples of alpha j alpha 1. So, 0 times alpha 1 is also part of it which is a 0 vector any vector space or subspace has to be 0 vector and 0 vector is unique for v is 0 common, v or it is subspace w. They are at the same 0, they intersect only at that 0 they intersect at 0 mind you.

If it v has to different subspaces w 1 and w 2 mutually disjoint. They have one thing common 0 is unique is subspace has to have 0 vector, where vector space has to have a 0 vector where 0 is unique for the other space v. That means, they intersect at 1. So, w 1.

Then suppose I take a vector alpha 2 which is element of v and not element of w 1, that means, I go outside w 1. I take look at w 1, I take anybody outside w 1, means not present in w 1, but present in v then my claim is alpha 1 alpha 2 l. I linearly independent very easily you can see alpha 1 and alpha 2, they are linearly independent why C 1 alpha 1 plus C 2 alpha 2.

If you get equate it to 0, is it the only solution that cone and C 2 0 where non zero solutions exists firstly. First suppose non zero solutions exist for C 1 C 2 both so that means, you can write alpha 1 as some constant times alpha 2 or alpha 2 as some constant times alpha 1, but alpha 2 lying outside w 1. So, alpha 2 cannot be part of w 1. So, alpha 1 cannot be a multiple of alpha 1 because then that would have been part of w 1.

Alpha 2 is not contain it. So, that is not possible. It is like this you know on this is one axis span by this one axis is span by one element. So, every element can be vector is just constant multiple time that and I take another fellow this way, outside these in the bigger space in the plane lying, but outside. So, this can be never be a multiple of this, they are pointing at a like that I am doing.

So, alpha 2 both cannot be 0 both cannot be non-zero because then alpha 1 is just minus some constant times alpha 2 or alpha 2 minus constant times alpha 1. Means alpha 2 is part of w 1 which is a contradiction not possible. Then is it possible to have C 2 0 C 1 non zero C 1 non zero, C 2 0 means some non-zero into alpha 1 equal to 0, but that is not possible because alpha 1 is non zero vector.

Some non-zero into alpha 1 0 that is not possible, because alpha 1 is non zero that would mean alpha 1 0 is it possible to have 0 here and non-zero here, non zero into alpha 2 equal to 0, that means, alpha 2 0, but alpha 2 cannot be 0 because 0 vector is such already part of w 1. So, alpha 2 is the outside w 1. So, alpha 2 cannot be 0. So, only possibility is C 1 and C 2 are 0, that means, they are l i.

So, I consider now bigger space. I will just 2 3 minutes I will stop, alpha 1 alpha 2, this is v this still be a linear combination of them. Linear combination of this is part of v, this part of v this part of v, but you remember you understand w 1, content in w 2. I think we stop here you just go through these. Refresh your mind and then I meet on Thursday.

We continue from here. We will arrive at dimension give some example, then go for inner products dot products, dot product more general notion of dot product over linear product. With that we will define orthogonality orthogonal projection all I will need more lectures probably see more or 2 and half more lectures over all these. Should I will do everything in the context of random variables, vector space can be of any type that will be basically in the case of random variables, 0 mean random variables.

Then bye, thank you.