Adaptive Signal Processing
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Lecture - 12
Misadjustment & Excess MSE (Contd.)

As it is customary, let me quickly go through the same old drawing and a few common expressions.

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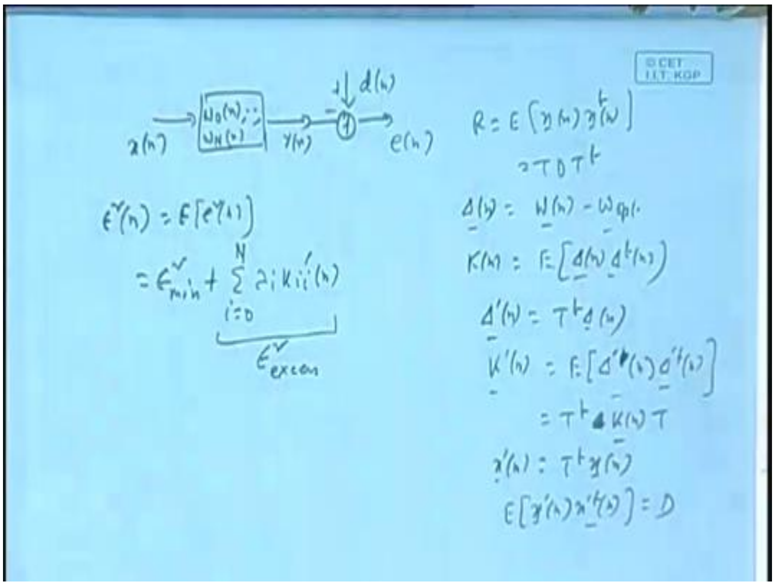

It is the filter, filter the output yn dn plus minus en, we had R as do you know what is the xn vector? I am not writing, going to all those definitions R was TDT transpose, delta N was wn minus w opt. W opt is R inverse we know. Kn was covariance matrix of this, why covariance? Because actually in the steady state delta into x, limit is 0. If the steady state whether is coherence matrix or co-variance matrix both are same, because we need 0. Then we had defined delta prime N as T transpose delta N and correlation matrix of this was k prime N.

These are ((Refer Time: 02:22)) sorry, delta prime N, delta prime cross transpose N and this is nothing but T transpose, sorry Kn T. We also defined x prime N as T transpose xn and E of X prime N X prime transpose N, that was D, D constant of the positive, real Eigen values. There we are finding out epsilon square N, which is E of e square N, this has two components. One is that epsilon square mean, which you get when you have optical weights, but since if the ((Refer Time: 03:11)) distance we did not put the actual optimum weights, but we approximated. R and P by some very you know where estimates, while estimates, what we gave, what we find is the wn dances around the optimal weight.

As a result en is not exactly, even in the season not exactly the optimal error en and therefore is variance is having an extra component, that the component was this. Therefore, we have to see is how this quantity behaves in the steady state. Lambdas are bounded you know Eigen value integrate infinite value because, matrix elements are bounded. But how this guy behaves in the steady state? How do keep this as N tends to infinity, how this quantity can be kept under check, under some bound as N tends to infinity so that, you need some parameters, we can count down the bound and keep this extra component within a limit.

This is called excess mean square error, epsilon square excess by the way, excess mean square error. This quantity we want to keep controlled, it should not grow as N tends to infinity where, you see this is, this has a recursic phenomenon, we have seen that matrix k prime N is recursively build up from its past value. So there is a recursion that you, by recursive way it can grow also which will be dangerous because, its value will then tend to, suit up to infinity. Because, we have seen that k, k prime N matrix as a recursive, we have you know its generation, we obtain the recursive relation, on k prime obtaining k prime N plus 1 from k prime N and some other values.

So their recursion has this, the possibility of what, that you know, that k prime N can go building itself path in the regionality manner, positive feedback regionality manner. So, we have to see that such thing does not really take place. We have to do things, appropriate things so that, this is kept under check, under some bound as this tends to infinity. There is a whole energies for that, we obtain that recursive equation for k prime N for your covariance I am just rewriting here again.

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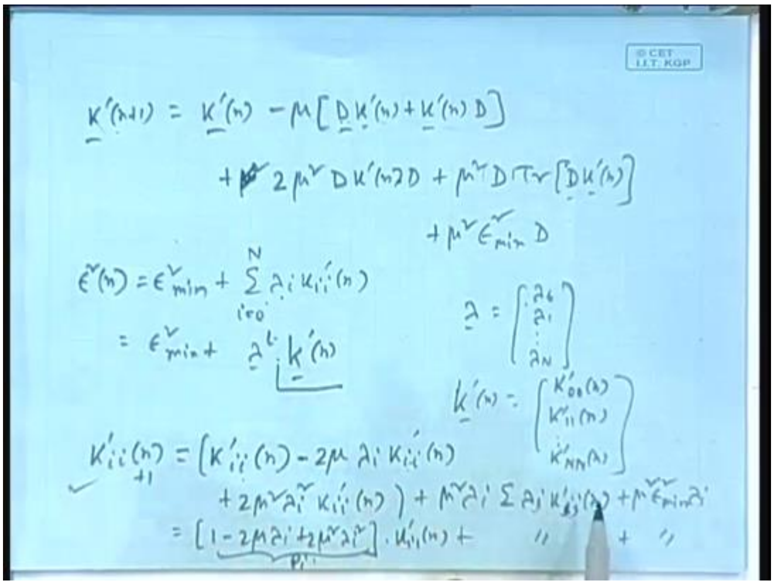

K prime N plus 1, these are recursions, what is k prime N, this is a slight for the previous D, k prime N, minus mu, D into k prime N plus k prime N D and plus 2 mu square D k prime N D, D got from the, what D came from the Eigen values of all matrix, mu square D trace D k prime N and these things. Then here come to this later then here if you take the I told you that we are not interested in the entire matrix, we are only interested in the diagonal elements. Because, this diagonal elements only matter in the excess mean square error.

So, you took the highest diagonal entry, ith element of this is ith element here minus mu times. They are two matrices diagonal times k prime N k prime k prime tends say diagonal, we saw the, that time that diagonal ith diagonal entry in either case will be same. Lambda I times k prime i i n so twice that, minus twice mu lambda k i prime m mu plus 2 mu square and this. Again k prime N D lambda 0 times first column, lambda 1 time second column dot, dot, dot and then lambda 0 times first row, lambda 1 times second row dot, dot, dot. So, ith row ith column that time either lambda I square only. So, lambda I square mu square mean, 2 mu square lambda i square same K i i prime N and then here, next 1 mu square these are scalar quantity, trace.

So, ith diagonal entry of this diagonal element is D, which is lambda I times this trace, and trace is what? This is the diagonal matrix multiplying these and then taking the trace, diagonal matrix means first row times lambda 0, second row times lambda 1 dot, dot,

dot. So that means, lambda j times k prime j j, do not use i previous or the other I you do not in to use the same index i because, i was also figuring out i is constant. It is better to use j or r or k or a or anything like that, that is the small mistake which you know skip by attention.

So, this thing and plus this and so this quantity k i i prime N, if you take common you get 1 minus 2 mu lambda I, plus 2 mu square lambda i square this quantity, I am calling rho I, this quantity I am calling rho i. So, rho i times k i i prime N plus this, plus this from we built up or I will do now. These diagonal entries I will put them in a vector form, that is what I was doing towards the end, I just hurried. So, I thought that I would again start from there because, that time I was really hurry because they give me a signal to wind up.

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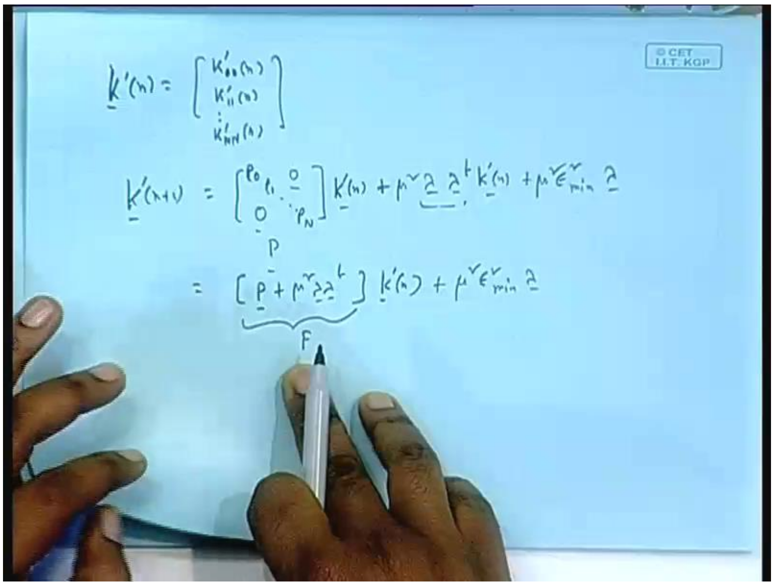

So if I put them in a vector form, if I put them in a vector form what I get? That means I am defining this vector, this is a lowercase k, k prime N it will consist of capital k that matrix 0 0 with element, first diagonal element dot, dot, dot nth diagonal element. I am putting that just in a vector form, here I had a recursive because in terms of ith element and i can be 0, i can be 1, i can be 2 dot, dot, dot putting in the vector form. In that case, if I call it all the vectors form all the elements here form the, I mean denote the vector here. Here also rho i times K i, i prime N.

So, what happens K prime N plus 1 are you following me? This is K i, i can be 0 1 2 I am putting them in a vector form, that is equal to what, rho 0. Another vector rho 0 times k prime 0 0 n, rho 1 times k prime 1 1 N dot, dot, dot. That you can write as a diagonal matrix, we can call this matrix P 0 0 times the same vector, is it right? Rho 0 times, k prime 0 0 N, rho 1 times k prime 1 1 N like that. So, if you write like that k rho 0 times k prime 0 0 N, rho 1 times k prime 1 1 N like that, plus look at this quantity mu square lambda i. So for i equal to 0, this quantity scalar quantity trace, constant.

So, this constant mu square so we have a very simple thing, lambda 0 for i equal to 0 lambda 1. So, it will be a lambda vector only, i define a vector lambda vector like this lambda put Eigen values in a vector form lambda 0, lambda 1 dot, dot, dot lambda N. So, I have got lambda 0, lambda 1, lambda 2, lambda N, one vector will come up, isn't it? Multiplied by a scalar mu square and the, another scalar, at this scalar mu square lambda vector from here and in this scalar element, this scalar element can I write like this. Lambda transpose, after look at this lambda 0 times 0^{th}, lambda 1 times first diagonal entry, lambda 2 times second diagonal element, isn't it.

So, diagonal entries in a vector form and then lambda 0, lambda 1, lambda 2 rho vector, row into column then you will get this and once you have that, you get clubbed down these two together. And last one is again very simple, last one is mu square epsilon square mean which is scalar, lambda 0 for i equal to 0, lambda 1 for like dot, dot, dot lambda vector. So k prime N, I can take P plus k prime N, k prime N you take common one diagonal matrix and lambda column vector times, row vectors as a matrix actually it is a reinforced matrix, times the scalar mu square.

Together it is a matrix, that times k prime N plus now those who are from electrical background, I do not know how much controlled theory they have studied, but they are they should be pretty familiar with this discrete time takes place equations. But students who are coming from our department electronics are all they may not be. So, for general interest I will give some general, produce some general results now, which may be known to that segment of this audience, which comes from electrical background. Because, they will be extensively this system theory or I hope so I do not know what exactly they do though.

These are discrete, you see this matrix is known, P is known two us, P came from this and rho came from what? Rho it depends on mu and lambdas, all mu's, lambda's are suppose known to us. So P is known to us, mu is known to us, lambda vector known to us, P F is a known matrix. So, it is nothing but this equal to F into k prime N plus this, can you please see that F is also a real symmetric matrix, Hermitian matrix here. F consist of what, diagonal matrix plus lambda, lambda transpose. I told you any time you have this kind of form lambda and lambda transpose, there is a symmetric matrix.

ijth element is given by lambda i lambda j, jith element lambda j, lambda i, but their products is same. In any case if you take this matrix, take this transpose you will get it back. If you take this if this and take the transpose, transpose means transpose of P because, diagonal matrix get it back. A transpose of this part, lambda and lambda transpose, you will get back lambda, lambda transpose. So F is a symmetric matrix, real symmetric matrix, I call it real Hermitian matrix.

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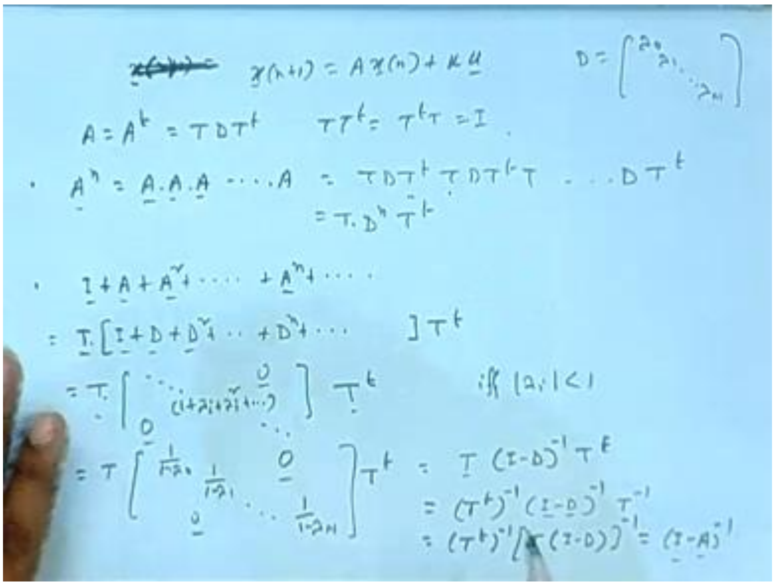

Now suppose, I give some other results now. Suppose we have so this is the departure I am trying to get into something else. So, do not get confused to the rotation, this x here is not the x we are dealing with earlier the generally. Suppose, you have got an equation like this or say xn or maybe okay, x starts with xn plus 1, plus some scalar k times some vector u, u is a constant. A is giving to be, is a real matrix real so it is a Hermitian matrix. We have all real valued quantities, A is the symmetric matrix means A is a real

Hermitian matrix. So, it means A can be written as some T D T transpose where T T transpose, this T and previous T are different.

These are separate things I am doing all together to some results, do not thing this T has got anything to be the previous T, which came from diagnolizing the auto, this correlation matrix or these are different result, nothing to do it or previous derivation. This T transpose T is I, that is matrix to send the result because, this is A is a Hermitian. So, it can always be read it like we have seen earlier. Now, two things you see what is A to the power N, that is A, A, A dot, dot, dot N times and there is a meaning of A to the power n. How do it will that be, TDT transpose, TDT transpose, TDT transpose like that, always these two will cancel, you will cancel like that.

Finally, D and T transpose will come, can you see? So, it will be T D to the power N T transpose and what is D to the power N, these are diagonal matrix, these are diagonal matrix. Suppose D is lambda 0, again this lambda is nothing to have, is not the same lambda which we are dealing with earlier. We say ((Refer time: 16:00)) contacts here lambda 0, lambda 1 say lambda N. So, D to the power N means all Eigen D into D means what, lambda 0 square, lambda 1 square dot, dot, dot lambda N square. So, D to the N power means just square of the just raised, which Eigen to the N nth power.

Therefore, if it is that each Eigen value of this matrix A suppose by some ((Refer time: 16:30)), I make sure that its value magnitude Eigen values are real here, isn’t it. A may not be positive derivative, I am not assuming its positive derivative, I am not assuming too much, this Hermitian is fine. Whereas, because of this Hermitian Eigen values are also real and therefore, their magnitude suppose lies within 1, that is that value lies between 1 two minus 1. Less than 1, greater than minus 1 if that be they are going to raise them to power N, its value becomes still less and as N tends to infinity the value the approaches the 0.

So, this matrix will approaches 0 matrix as N tends to infinity because, D to the power N all the N it will become a all 0 matrix. This is very common in control theory, this is very I mean this bread and butter they are actually, there these things in system theory and all, you understand this, this is one result. And another result is suppose I have got this quantity, identity matrix plus A plus, A square dot, dot, dot, dot that is A, A square means A into A another matrix, A into A into another matrix and you are adding. There is no guarantee that it will, it will be converging to a matrix of finite elements.

That question is open, I am still writing it dot, dot, dot it may diverge it may converge I do not know. But this I can always write as A is T D T transpose, A square is T D square T transpose, this is T D to the power of N T transpose, I is T into T transpose. So I can write like this, T into I into T transpose, I can write this I this T transpose we take on this side D, I am forgetting the underscores they are there. Now I the, these are all diagonal matrices so that means, you have a got a thing like this T and a diagonal matrix where, ith element will be what? ith element will be this is 0 here, 0 here, 1 plus lambda I.

If lambda I less than 1, if and only if then only this will be finite, 1 by 1 minus lambda I, isn't it. Then only this matrix will be, this summation will be convergent, then convergent to where, if this is so this is convergent these are necessary and sufficient condition. If and only if each Eigen value has magnitude less than 1, then this will happen because, you see all diagonal address will be 1 by 1 minus lambda or something. Lambdas are finite and therefore, you get finite numbers and so this convergent only this happens, fine. In that case what is this matrix, this matrix is 1 by 1 minus lambda 0, 1 by 1 minus lambda 1 dot, dot, dot 1 by 1 minus lambda N, right into T transpose 0 0, isn't it.

So, I can always write as T I minus D inverse T transpose, can I write, what is I minus D? Diagonal matrix 1 minus lambda 0, 1 minus lambda 1, 1 minus lambda 2, dot, dot, dot, those are the diagonal. Now you take inverse so they will become 1 by 1 by o1ne by and these again you can write T transpose is same as T inverse. So you can as when take this to be inverse, you can do this way T transpose, this T you can write like T transpose inverse because, T transpose is T inverse. This inverse this T transpose I can always think of T inverse. You take these two first A inverse B inverse means, B A inverse isn't. So, T will come first actually if you have story I can write this, straight away like the last expression, these are the silly things actually.

Student: ((Refer time: 21:55))

This term taking.

Student: ((Refer time: 22:02))

That is what, but I am going step by step. I do not know how prepare people, I am going step by step. This inverse, this inverse T into I minus D whole inverse or I get these two, B inverse A inverse means A B inverse. So, T transpose will go here and then what happens, T within bracket I minus D T transpose T into T transpose is I and T D T transpose is A. So, this becomes I minus A inverse is very much like what you have in normal scalar thing, 1 by say 1 minus k inverse, 1 by 1 minus k is 1 plus k plus k square plus dot, dot, dot it mod k is less than 1.

Here instead of A, A cannot be less than 1 is a matrix, you see that highest Eigen value and I am assuming some structure here, I am assuming A to be symmetric with Hermitian all those things. As there I am saying all the Eigen values less than 1 in magnitude this will happen. This region does not required by the way that it should be for our cases fine, but this region does not require that it should be Hermitian because, this is I use the fact this Hermitian. And therefore, I could write A in this kind of form and do all these things you know to my convenient, but if you know A is not Hermitian this question of TDT transpose and all these does not arise.

You can then this will work out I minus A inverse will be I plus A plus A square provided, the norm you have to define something called norm of a matrix, norm of the matrix is less than 1. One norm of the matrix is called as spectral radius, spectral radius is the Eigen value of the highest magnitude. Matrix A may not be diagonal element, may not be of this form, but it has some Eigen values. The Eigen value which has the highest magnitude is called spectra radius of the matrix, that works as a norm of the matrix. Norm and all these will done little later in this course earlier from me.

Student: ((Refer time: 24:16))

The spectral radius, it satisfies the norm properties. So, that is working here actually because the Eigen, all the Eigen values are less than 1 means, what spectral radius are less than 1 and this is what it is. These two I will use here now, I come back to our business.

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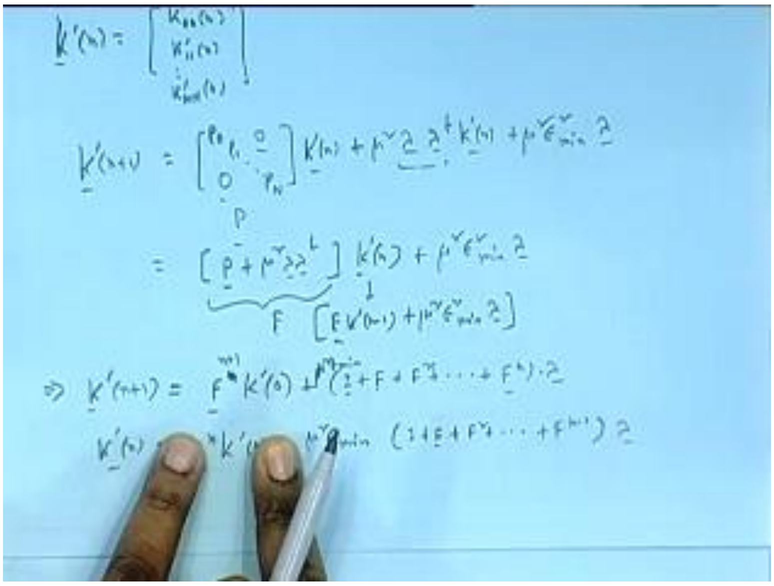

What was our equation, this is our equation k prime N plus 1 is F into k prime N plus this. Now, k prime N if you see you may get replaced by the corresponding recursion, what is the corresponding recursion? This will be F into k prime N minus 1 plus mu square epsilon square min lambda and go on doing it, you go on doing it again replace this. So you can see what will, what will obtained is, what will happen? F into k prime N then F square k prime N minus 1 and F into this term, this mu square epsilon square min lambda that is common that is there, F times that is there, this time F square of that will come up.

This will give rise to finally, if you go to N equal to 0 go back to see from N to N minus to N minus 2, that is how going back. Backward recursion and finally, if I stop at N equal to 0, you will see you will have these. Take a minute of yours and see that you will obtain these, it will be a to the power N or N plus 1, N plus 1 and this will be I that mu square epsilon min that will be there already here, mu square epsilon min and lambda. Just look at the expressions, F into k prime N, k prime N you are writing as F into k prime N minus 1, that you are further writing as F into k prime N minus 2 so and so.

So, finally k prime 0 into F, into F, into F, into F, into F and what is the other term? This is one term free of F, then one term F into this F into this then another term F into whatever comes out of this. Then again another F is there so F square that is why I F F square, it will be like this. Instead of k prime N plus 1, that we come back to the convection, k prime N with this F to the power N k prime 0 instead of N plus 1, I am now taking N. F is please see this, I told you F is a real symmetric matrix here. So, all my previous results can work out, if the Eigen values of F after all the F comes from P and lambda, lambda the Eigen values of all matrix, together is a new matrix that has a new set of Eigen values lambda prime.

Lambda 0 prime, lambda 1 prime dot, dot, dot if those Eigen values if you adjust, so that those Eigen values are having magnitude less than 1. Then obviously, F to the power N as N tends to infinity becomes 0. So, this part time dependent I as, I am bothered about only the time dependent part I should not go. So, I can make it die down to 0, if I make sure that the Eigen values of F are in general the norm of F is less than 1, but in this case I talked directly in terms of the Eigen values. Where, it is symmetric, real symmetric matrix, real Hermitian matrix.

If the Eigen values of F that is lambda 0 prime, lambda 1 prime, lambda 2 prime dot, dot, dot all of them are kept within 1 in magnitude. Then, as N tends to infinity F to the power N will tend to 0, a 0 matrix that you have seen in the previous just a little while back, which will make sure that this quantity will not grow. K prime N as N tends to infinity converge on something constant, these on the other hand, if the Eigen values of F are having magnitude less than 1, this will converge to what we have seen.

Student: ((Refer time: 29:06))

What did you see here? I minus A inverse when all the Eigen values are less than 1 in magnitude this happens. So, this will converge to I minus F inverse. So, this quantity will be the balance amount, I cannot through it away. I can make it less by taking mu to less. In fact it is not so easy because, F itself depends on mu, if there is mu square here, there is a lambda this is mu square here, itself it is not consist of mu square, it is not if you see mu square outside, mu square inside also. But somehow I will, I will find out the expand this expression and see it is dependent on mu and all that.

So, by choosing mu appropriately we can still keep it check, but it is a constant quantity we have leave with it, that is all. This one you have to leave with, by at least it will not be too instability, it will not lead to a situation where that error variance each element of the, this vector is starting at infinity. There is a stability issue so in a variance is remaining bounded, that will happen only in the Eigen values of this matrix F are kept within check, within their magnitude lies within 1 to minus 1, magnitude lies within 0 to 1.

Now, for that I am not showing here that definitely if you, then what you have to do you? You have to take this matrix, study its property this is a diagonal term what is your rows, lambda into lambda transpose, mu square, have to see this matrix study its properties, evaluate the Eigen values, it is a bit clumsy things and all that. And then impose the condition each Eigen value magnitude less than 1 from there you can generate a condition on mu, that what should be mu. Finally everything will be well known in terms of mu, but it will be, it just here me out we can show that if you take that previous bound, 2 by lambda max or to be on the safe side 2 by trace are that is a stronger bound this is enough.

This is sufficient to guarantee that Eigen values of this matrix will even bound it, within 0 to 1 in magnitude. I am not showing that if time permits maybe some other day I will come back, I will do analysis myself. The books just mention this, I need do analysis myself in terms of spectral radius actually it will come, but our previous bound for convergence that is good enough for this to happen. If you analyze these and all that you can show, it can be shown that if you take lambda mu within this range or rather mu greater than 0 less than 2 by tracer, that is a very stronger bound 2 by tracer is are less than 2 by lambda max.

So, you have proved this will this Eigen values here will be bounded within 0 to 1 in magnitude and therefore, this time dependent term will disappear and therefore, will get these. So, that this condition is sufficient for us you can leave happily with this condition. Question is then let us study this excess error this, this quantity. Why this quantity we remember, what was the excess min square error, this much. Epsilon square N plus epsilon square min plus this and this you can write as lambda transpose, k prime N k prime N. Therefore, I am interested at here in epsilon square N as N tends to infinity, that time this quantity does not grow, but attain some particular finite value which is this this side.

So, I have to see this quantity as N becomes infinity that will be the excess min square error, the minimum error, but this is the excess quantity, excess min square error in the steady state. So, it tell me N equal to infinity. So, after that study lambda transpose k prime infinity.

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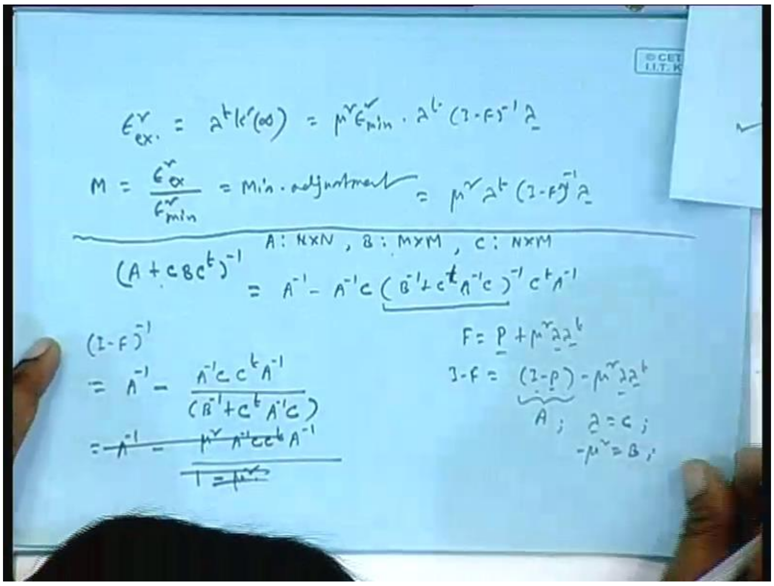

You see this, what is epsilon square excess, ex for excess that was from this lambda transpose k prime N. This is excess as infinity, epsilon square excess if you want I can put with in bracket infinity, but I can drop that I can just put there in the definition that, epsilon square excess is steady state excess min square error. That is this quantity as N tends to infinity, what is this quantity lambda transpose k prime infinity and with this quantity with I minus F inverse, dot, dot, dot infinity means I it becomes I minus F inverse, right. Lambda transpose so that means, mu square this lambda transpose here and I minus F inverse, this I lambda interesting, matrix column vector column vector row into column scalar, this quantity.

Remember, F itself has mu square inside. Often, we are interested in a quantity which is a ratio of this excess min square here divided by, this is called m is adjustment this is just a definition. So, you see this epsilon min, this will cancel, this is equal to mu square lambda transpose. Now, how to analyze this quantity, how to analyze this we will again you know invoke something which you can verify, but we will not prove anything, that is there some it is called matrix inversion lemma. A something very tool which is very commonly used in so many papers I find. So, many bounds in common cases a signal process control also.

It says let we write from the book that suppose, you have got some matrices all square matrix A. Suppose you have got a square matrix A, A say is N into N this is there in all books actually, is called matrix inverse lemma. There is a matrix B, which is say M cross M and there is a matrix C, which is N cross M I suppose, yes, N cross M. Suppose three matrixes are given, which will come across a term like this A plus C B C transpose inverse. This kind of expression comes very often C into B into C transpose, C is N cross M, N cross M, N cross M, N cross M. So, N cross M, M cross N inverses N cross M.

There, this will be equal to some big expression it will involve many inverses and I am assuming the inverses exist. Under that assumption only the formula is valid, isn't it. So, it will be equal to A inverse, this sometimes difficult to name, but some book actually. I suggest you can verify if you multiply this, by this quantity A plus C BC transpose if you multiply this and simplify you have lot of terms and some calculations, finally, you will get identity matrix. This is how to, but how this was derived I do not know, that may be some something actually, I do not know really.

Student: ((Refer time: 37:48))

C trans, C B C transpose.

Student: ((Refer time: 37:52))

C transpose A inverse C, yes this is transpose. If you take this quantity, multiply either here pre or post and do all the term by term multiplication and you can simplify, I am sure we will finally, get I matrix. That means, these will be this into this will be I means there is a inverse of this. This is very well known matrix inverse lemma. Now, we have got this quantity I minus F inverse, our problem is I minus F inverse. Now what is F, by your definition of F the, what did you do this F thing, F is this. F is this, for your benefit I am writing, what is F, F consist of that matrix diagonal matrix, P consist of the row values rho 0, rho 1 plus mu squared lambda, lambda transpose.

Then, what is I minus F? It will be I minus P one diagonal matrix minus mu square lambda, lambda transpose, no nothing conceptual. This is diagonal matrix, what are the diagonal entries here 1 minus lambda 0 1 minus lambda 1 1 minus of the dot, dot, dot minus another square matrix. This two whether is I minus F, I can call this to be A this A, I can take lambda to be my C N cross 1, N cross 1 there is a lambda transpose become

C transpose. Then what is B, B will be 1 cross 1, B you take to be minus mu square, minus mu square is B.

I can do that, are you following me? I minus this diagonal matrix very easy to handle so I am keeping that aside. I am giving A the role of A to that, inverting that is no problem when A inverse comes so many, so many times. Rest, lambda if I call C, N cross M means lambda is a scalar column vector that means, M is 1, B will be 1 cross 1 this is a scalar. So, minus mu square these together is my scalar B and C transpose, lambda transpose, if you do that put back in the formula, then what you get? You get I minus F inverse is equal to A inverse the, with these definition of A, A inverse where, A is this minus I am following this, for this result.

Minus A inverse before that you look at this quantity here, C transpose A inverse C. C is a column vector, A inverse is a squared matrix. So, matrix into column vector column vector row into column scalar and scalar. So, this is a scalar inverse these 1 by that, isn't it. So, it can be taken out of this entire product this be just, but our product matrix comes A transpose C into C transpose A inverse A inverse C C transpose A inverse, whatever comes that by the scalar number. That is all the elements of the polar matrix to divided by the scalar that means and that is, this quantity I have just put it back here. B inverse, B inverse means.

Student: ((Refer time: 42:38))

B is a scalar. So, B inverse is a scalar and these are scalar, matrix, isn't it. C transpose A inverse C, what is C, what is A inverse, A inverse consist of what? A consist of diagonal matrix, A is the diagonal matrix so diagonal elements 1 minus row 0 1 minus row 2 dot, dot, dot. So, inverse that 1 by that and then so what you get do actually you replace B inverse by let we proceed this way, B inverse by 1 by minus mu square. Just a minute, this is a bit clumsy, so let we work out. So, 1 by mu square so mu square will go up also minus 1 by mu square, so mu square will go up also. This, take this conclusion, I am doing next step over here, so some signs and all I might just, some positive sign could be negative, but negative sign is positive, just correct if I make any mistake.

Mu square into this quantity, C transpose A inverse C, C transpose A inverse C, C transpose A inverse C will be what? C transpose means lambda transpose, lambda transpose diagonal matrix lambda that means, there will be a summation isn't it. There will be summation or just a minute, just give me a minute. [FL] I have just trying to take one thing here.

Student: Sir, totally lambda 0 square by 1 minus.

No, before I do at all, all those suppose I carry out this part A inverse into B inverse, I have just trying to see something here. A inverse into B inverse plus A inverse into this quantity minus this quantity, a normal algebraic addition. My I am just trying to see that something will cancel here, there is a possibility.

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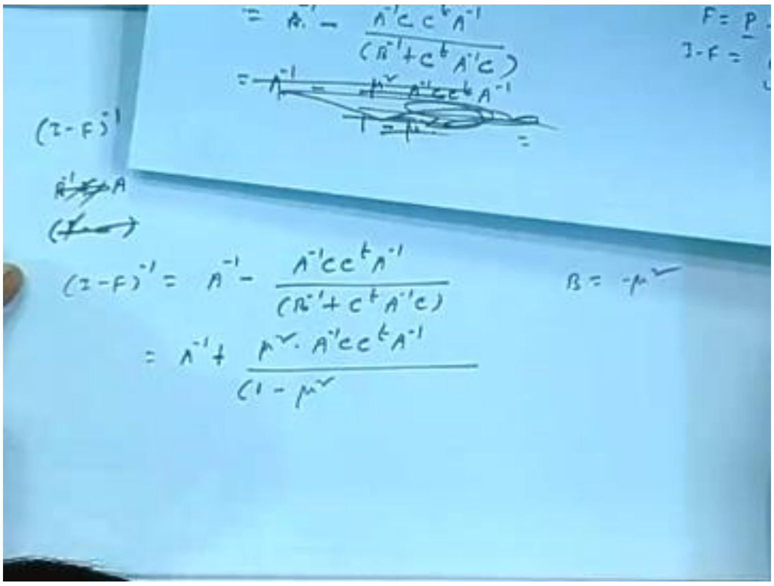

This, what I am say is I minus F inverse A inverse into B inverse plus A inverse C transpose A inverse C minus A inverse C, C transpose A inverse. These two quantities should be same according to me, A inverse C is a row vector, is a column vector and this is a row vector. This is a matrix because, this is a scalar this there is a mistake somewhere.

Student: ((Refer time: 47:45))

No, no, no this is general term A inverse C, C transpose A inverse. So, just a minute this turning out to be just A, A inverse C transpose A inverse C that somewhat scalar here isn't it. But it is not scalar because, this is A inverse B inverse, a matrix this quantity. Yeah what you were saying?

Student: ((Refer time: 48:16))

I am taking a most general lemma ABC.

Student: ((Refer time: 48:21))

But it is if you assume specific form for CBC, CB if you assume some very specific from like you know if you take something to be identity and all that, this is I am trying to give you the most general result. This is a most general matrix inverse or lemma.

Student: Sir, basically we have used that lemma so many times, but.

What is the form you are familiar with?

Student: So, we are B inverse is equal to A inverse plus C transpose B inverse C you can write.

B inverse what?

Student: B inverse is equal to A transpose A inverse B inverse equal to A inverse plus C transpose B inverse.

No how, how what is that the…

Student: No if you can write this, that

No, no like here I have I minus see, you cannot just add some result you have to concentrate, we have to restrict those, these thing, I minus P diagonal matrix minus this term. This is A this is B there you see same, do not come with just some generally value which you want to study somewhere here and there. This A, this B I am trying to make use of the fact that the A is diagonal. So, A inverse can be easily computed that is my goal from the beginning. I am just making some, just writing some, making some silly mistake is because A inverse B inverse is fine, there is a matrix. So, A inverse C transpose A inverse C that also should be a matrix, is it not?

Anyway if it does not cancel I cannot write, I will just proceed the way I was doing, replace B by minus mu square. I just thought that you know some clever cancelation will takes place. So if it does not, no harm.

Student: Sir, A inverse into C transpose we can multiply and take it as column.

Which one, Here?

Student: A inverse and C transpose we cannot club?

A inverse and C transpose you cannot club, yeah.

Student: Because C transpose is a column. ((Refer time: 50:42))

Yeah, this is the problem. Anyway, anyway I, I, I just thought of some clever cancelation just my, just my you know I am kind of anticipation out, I thought something might cancel. Is something cancelling?

Student: No I have not computed, but it is coming as summation lambda A square by 1 minus rho I.

No it's not, it's much more complicated than that.

Student: No, no that C transpose A inverse C coming as that.

That is how we asked, you transpose the you see whether that is how we asked, that 1 by 1 minus rho, that's that one by 1 minus rho that term will be we know lambda A square will be on the top and summation of that, that is too obvious that is okay. I want to no, I thought that something might cancel and A inverse B inverse by you know. Anyway, let me directly put because I know the result, let me again write A inverse minus this is the result we had, right. This is the matrix C, C transpose matrix, matrix, matrix okay B was minus mu square.

So, if you put that back here you get it becomes plus or minus, 1 minus isn't it, into that summation thing. Is A or is a symmetric matrix, isn't it, A is diagonal, but A is a symmetric matrix also so inverse of A also is symmetric, right. I am still, excuse me I am still not able to give up the temptation here, I mean whether we can clear around with this.

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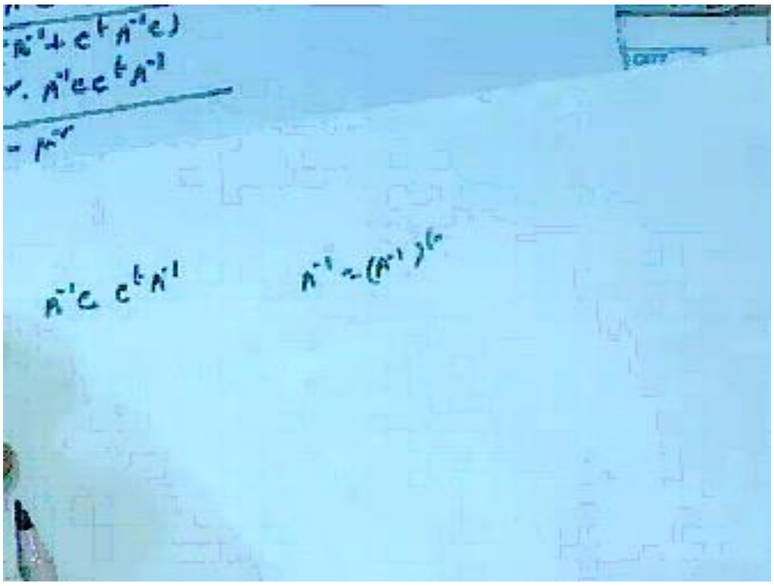

A inverse C, C transpose A inverse because I can there very little there for the final result and A inverse is same as A inverse transpose. So, this is matrix into I do not know some cancelation has to take place on top. What I suggest is this, let we do from here in the next class, that will be better, but I will stick to this form. This I can, I do not know how you are suggesting to simplify it. This is the most widely used lemma.

Student: This is the most widely used lemma [FL]

And stick to this, we have this this form A minus mu square this part.

Student: Sir, A B C dual matrix.

Ha dual matrix

Student: [FL]

No, no that form will come by simplifying this, the, the form, that will come that is written in the book, I am not writing it. That, that, that.

Student: How that bond is coming.

That, you do not that is not a big proof you take this multiply, this you will get identity.

Student: No, no, no sir that is the inverse proof, ((Refer time: 54:48))

Anyway, are you all start with this, are you start this is by start because, if you leave take that special case you know, what this diagonal and the scalar and all that, it is better you know this proof, know this inverse lemma. Just a little bit is left agree that. So, I mean getting this matrix and all that and finally, I depend on mu. Just two more lines are require actually, but I have to go through a derivation, So, I will do that in the next class, alright then…