**Adaptive Signal Processing**
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**Lecture - 1**
**Introduction to Adaptive Filters**

So, here we begin this course and as you begin it is my duty to welcome you students to this beautiful course of adaptive signal processing.

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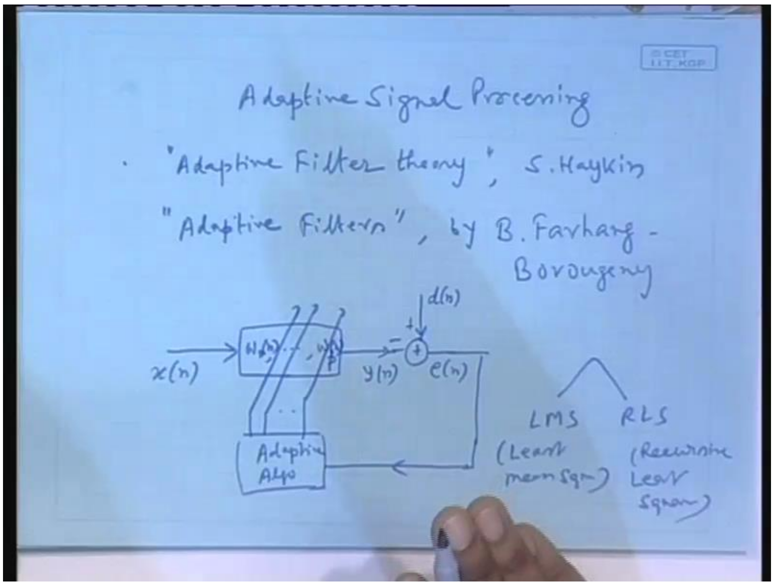


Actually, it is a course on adaptive filter. The adaptive filter algorithm and analysis - you know, this course, even though there are plenty of books on the subject, the principal book for this course is this - by Simon Haykin. Then another book or so  - it is bit handy for a particular variety of adaptive filter called LMS; for that, this book is better - adaptive filters by an ex friend of mine, by a friend of mine, with whom I worked for a while earlier. His name is, he is an Iranian; his name is Farhang Borougeny. It is a big name, we call him Farhang and both are Prentice Hall books. In fact, the first one is available as Prentice Hall of India publication.

Now, what is adaptive signal processing? I tell you that this course, which basically we will try to derive adaptive filters. This adaptive filters, I mean I would say that they are practically even more useful, because they occur more frequently than, so called you know low pass band pass filters. That is, the filters that I was telling fixed coefficient and design towards for all to have particular shape and all. Because this adaptive filters enable the system to adjust to a changing environment, changing statistical condition and all that, which normal digital filters like a band pass filter, butterworth low pass filter for a chebyshev voice of high pass filter or things like that cannot, because once I mean they are once who design the filter the coefficients are fixed once for all.

You implement the filters - its frequency characteristics will never change. So, it will remain fixed time invariant, but most of in practice will come across situations, where the signal or system environment it is a kind of fluctuating or the statistics involved of the processes of the random signals that also changes from time to time. So, in that case you cannot leave with just a filter which was design once for all. You have to make arrangement for adjusting the filter parameters to suit to the changing needs and that is what an adaptive filter does.

An adaptive filter actually, I will give some basic examples and adaptive filters what it does is this that suppose there is of input x n. What do you do? You pass it through a filter I mean for this course; we will be considering only a FIR filter not IIR filter; FIR adaptive filters only very famous; they guarantee stability and all. You pass it through as a filter w 0 dot dot dot w p so p plus 1 coefficient; so you get an output feed y n. What you do that initially there is a training period for the filter adaptive filter. During this training period what you are given is target sequences, which we call desired response, say d of n. You try to design the filter w 0 to w p so that the output y n becomes very close estimate of d n.

Now, for that what you do then this algorithm gives out at any point of time finds out the error the actual amount of error between d n and you y n. So, I want we want to minimize some measure of the error power. So, we will take it is a minimum mean square. We may be taking the error variance and try to minimize it; this error depends on filter weights because error is d n minus y n; y n depends on this choice of weights. So, this e n will be feed back into another algorithm called adaptive algo that will take this e n component.

It will adjust a filter weights that is filter weights will now become function of time w 0 n to w p n. So, at n equal to zero they have some value that n equal to one another set of value how this values change; this adaptive algorithm does this adjustment or change.

This is done during the initial training phase only when d n is given. So, you adjust the filter weights by this adaptive algorithm. So, that y n becomes a better and still better and still better estimate of d n progressively.

As you go on filtering with changing weights that is you adjust the weights in that sense. That initially sums, where initial values of weights you take you get a y n measure the error feed it to the adaptive algorithm now adjust the weights. So, that this y n is a better estimate of d n still not happy. Feed back here again feed e n back again to adaptive algorithm; further adjust the weights and so on go on this iteration till that training phase is over. So, training phase once the training phase is over and your algorithm is correct and all we can assume this filters to be reaching very close to their optimum value. So, that y n is a good estimate of d n. After this no d n will be given to you.

You have to then trust the adaptive filter you have to you have to assume that this filter is indeed a good one and you have to take y n to be a really good estimate of y d n; so that you can do the job of d n by using y n only. And then again after a while at a suppose some appropriate time if use if you find that there is a possibility of you know I mean further training the requirement of further training because may be of see may be the situation that was generating x n or d n that as changed from the external world; we again go through another training phase.

So, this training phase is executed from time to time. So, this is that adaptive filter. In normal digital filters, you only have one path x n pass through a filter FIR or IIR and you have an output. I am not; so design the filter coefficients based on your frequency response criteria they are fixed once for all. This is not the case here you have got two processes that go on simultaneously one is called is the filtering part, which is common which gives us a convolution between x n and this weights. So, that output y is formed, but another part is the weight adjustment procedure, which takes the error e n and feedback to it and using some algorithm working on e n it adjust the weights.

There are two categories of algorithms, very famous one is called LMS least mean square and this called RLS recursive least squares. This is actually least mean square and this is recursive least squares. Again these five filters could be in a direct form or there is some other structure called lattice form and all that. In this course, we will be considering mostly LMS algorithm. LMS algorithm was derived by the farther figure of adaptive filter Stanford Professor named Widrow and it has stood the test of time. It is by for the simplest algorithm, which is very easily realizable in hardware it is very robust to input noise. It converges nicely and all that it has beautiful properties.

Recursive mean squares came up in early nineties early eighties. And it converges it gives this optimal weights in a much shorter time than LMS, but then you know its structures are not as elegant in terms of architecture as LMS is. Sometimes, they suffer from finite piece and problem that when implemented in finite registers. You know finite precision based digital hardware then the truncation random errors they go grow in this loop. Eventually they affect convergence. So, RLS also RLS is more difficult to understand, nevertheless I will cover the basic RLS, but initially I am major attention will be focused on LMS.

I will tell you beforehand that this course even though it is intensely practical because this adaptive filters are very, very useful in practice. This course will be basically focusing on that derivation of appropriate adaptive filter algorithms of one kind or another and also analyzing their performance. For that I would say that is a mathematical course. Of course the mathematics that is required will be developing in this course.

Mostly, two aspects of maths will be required one is the general idea of probabilities and statistics, where the stochastic processes whatever you required that will developed also; there will be some good deal of linear algebra and vector space theory, which again I will develop in the course. I mean, as you go along with this course so that I am not presuming any maths background as such, but then by repeat that by nature this is a mathematical course, because we will be deriving; we will be developing; we will be deriving adaptive filters algorithms one after another and also analyzing them now to give a few examples of adaptive filters.

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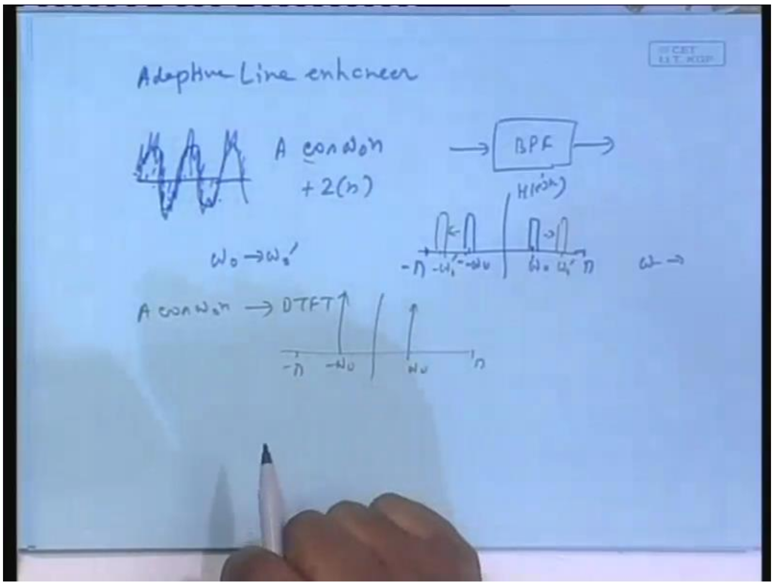


Consider first one is call adaptive line enhancer. Even you in fact, if you see these books by Hayken or Farhang and all that you will find in the chapter one plenty of beautiful examples are given and each example that could one full semester course. What we do here? Suppose, you have got a sine wave its analog, but we will discretizing it; we will be sampling it and discretizing it that way. Suppose, I have wave form is given and you take it samples. So, it becomes say some some clear cosine omega naught n; A times cosine omega naught n, but this is not a pure sinusoids. So, there is lot of noise there is some noise you know noise and the noise power could be very high.

So, the signal power may be less noise power could be very high could be high I mean it is not may not be that high. Now, suppose you want to filter out you have to extract from this thing z n is the noise and this is the signal. You want to extract A cosine omega naught n. What you will do normally is to pass it through a band pass filter. What kind of band pass filter? You have very narrow pass band centered at omega naught and minus omega naught. This is the digital filter. So, this frequency is omega, which is radian you have to plot it from minus pi to pi the filter transfer function is this; as you know its periodic any I mean any such transfer function is periodic over omega over a period of two pi.

If you pass this through this naturally the sinusoid and very little noise will be passed so far so good. But, now suppose this frequency of this sinusoid is changing from time to time or gradually. In that case, if you design the filter once for all and implement it after a while this filter will become useless; because it cannot once the frequency center frequency omega naught or carrier frequency omega naught. Signal frequency omega naught changes and goes beyond the pass band you will not never be able to recover it. There we will need an adaptive filter, which will be a band pass filter, but which will use the adaptive mechanism to continuously adjust the center frequency of the band pass filter.

See from omega naught it goes to omega naught prime; omega naught goes to omega naught prime, the filter should then move to it goes to new frequency omega naught prime. These are this way this way minus omega naught prime so on and so forth. So, it will do it on its own it is called as Q novel band pass filter. So, it will learn from the data use that adaptive filter adjustment in that loop in that loop; in that context and readjust the filter coefficients by training. So, that you get a new band pass filter, whose center frequency at omega naught prime so on and so forth.

This called line enhancer line because you know whenever you have got a frequency like this; whenever you have got sequence the sinusoidal signal if you take its Fourier discrete time Fourier transform you will get lines. A cosine omega naught n; if you if you take its DTFT discrete time Fourier transform; this way you will got the A one impulse at omega naught another at minus omega naught. This follows from basic DSP. So, these are lines, but these lines were buried in noise that time. So, this adaptive filter what it does it enhances the lines that is remove the noise an adaptively. This one good example of adaptive filter another example is echo canceller for telephone modem and data modem.

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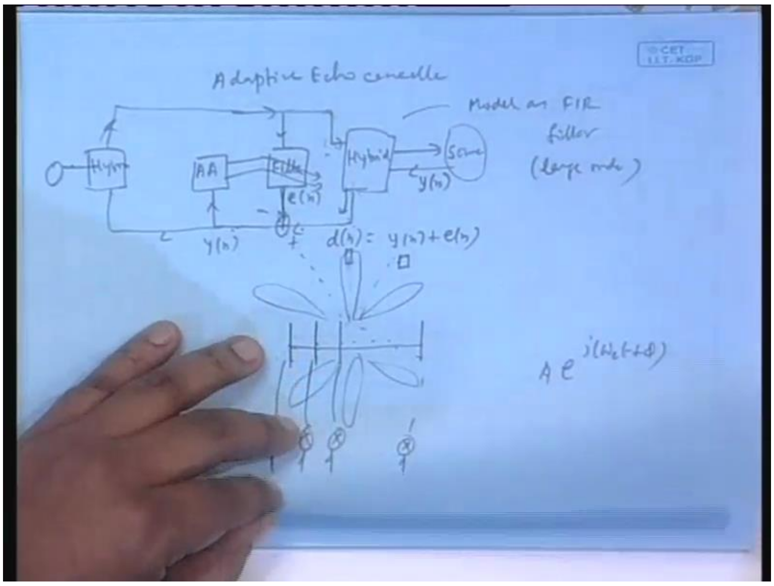


In these modems, what happens through one wire a signal comes and there is a hybrid. The hybrid what it does, whatever signal is comes here comes as an incoming signal it diverse the signal to this side to the speaker; it is a speaker or source on this side. So, there is a speaker which listening to the signal from this side. Hybrid will transfer the signal to this side and if the speaker say something it will divert it to this side. It will not allow this a signal spoken by the speaker to come back by this line or vice versa. The signal which is incoming to the hybrid that should entirely be divert it to the source to the speakers, so that hears we can hear it and nothing is diverted back down the process.

This is not only useful in your telephone system, but most importantly it is this kind of modems occur in data modem, where incoming data is to be sent to this side and again outgoing data from here is to be diverted to this path. Now, ideally there is no problem, but you know most of the hybrids they from time to time have some kind of leakage; so that part of this incoming signal whether it is speech or data instead of going fully to the source part of it comes back to this side.

So, what basically comes out here is you can say d of n, which is this fellow signal y n plus a part of this incoming signal, which you call echo; because that will go back to the speaker on this side, who spoke those because this is coming from hybrid here also and here is another source. So, this echo part of the leakage that the leakage to the hybrid that is; what that is say part of the signal coming from speaker at this end; that will go back to that speaker only after a delay. So, that will be an echo along with that echo the actual signal y n transmitted by this source generated by this source that also will go.

So, what will go is d n equal to y n plus e n? Now, if it is purely telephonic conversation the e n of course, is a disturbance is an irritant; it will create some problem, but still you know human beings are intelligent using their brain and all they can find they can somehow ignore that and listen to only y n. But it becomes a more acute problem; if you are dealing with data modem where there is no human being and computers are processing, receiving, transmitting. That time the data that is transmitted through this line is not only is never the one that should have been transmitted; this extra component which occurs noise or disturbance which as interference that comes up; as a result we need to remove it.

So, what we do there you module this hybrid as an FIR filter. Model as FIR filter a very large order large order and suppose you know the filter, you then connect that filter here. Filter, by this FIR filter what you are modeling actually not the hybrid, but the leakage generation echo generation mechanism. That mechanism you are modeling by an FIR filter. In fact, if that mechanism is a linear system; you can always approximate to a by a very high order FIR filter. Any linear size linear time invariance system can be approximate it.

Why a by a very large order FIR filter? Because even if it is IIR if it is a stable system it will die out after a time, so naturally if the order is large you will take almost all the samples of the impulse. Once you if you put at large order FIR filter that will approximate or that it will model the echo generation process. Suppose, I construct that here and then that will generate the same echo e n and I simply subtract it. So, what will go here is again y n? Echo is gone. Now, how do you do that? You do it adaptively it is because you the hybrid characteristics may change from to I mean that echo generation process can differ. Can change from time to time and you change the hybrid again it changes.

So, you cannot design the filter once for all. So, you use an adaptive mechanism this is your desired response. You try to match that desired response you know then it can be shown the best you can go is when this is equal to e n; this is that is theory. So, this is that error and that error the filter output and its desired response or target sequence that the difference is the error that error will be put in that adaptive algorithm; AA adaptive filter algorithm. This will go here and this will adjust the weights. This is called adaptive echo canceller.

On this again plenty of work has taken place and this is only a very basic concept basic scheme. But actually there are many you know detail issues involved in echo cancellation; you can read books on echo cancellation. They are coming courses purely on echo cancellation; I am just giving a basic theory to you to generate motivation for this course. Then we take another example, suppose we have got some antenna dipoles and then arrays like you know your TV antennas; we have seen them now that here you the antenna some dipoles array of dipoles.

Normally, if you give some signals to them this array and there is some phase difference. You adjust between the signal given here and there distance is I think lambda by two lambda is lower length or lambda by four; some specified distance and phase difference given to the signal here and here. I mean phase difference between the signal same signal is given to all the dipoles, but with a phase difference; then what you can happens is you can have some kind of beam forming. That is most of the energy if it is transmitted antenna it might go in one direction; in few directions; or in this directions nothing.

So, those in these are these directions I have null and in this directions we have got beam. If it is a receiver antenna, then again if you then you will not be giving signal rather you will be receiving signals at this dipoles. You have if you multiply those signals if you adjust the phases of the signals and then add them. Again by that process the reverse will take place. You will be receiving signal mostly from this side or this side or this side nothing from this side there is a reverse. That is again beam forming.

Now, since signals are complex signals in general the signals are complex valued you know. You know the phase the phased things know it is called j omega c t plus phi; these are typical wire with some amplitude A. It is a typical complex valued signal. So, if i have to adjust the phase of the signal I will multiply by a complex number. So, from phi 1 to say from phi 2 i want to go phi prime. So, i have to adjust it and multiplied by e to the power j within bracket phi minus phi prime minus phi.

So, suppose you adjust these signals by multiplying by some weights; this weights can be complex number though; you can view as the as though they are like filter weights. Incoming signal is coming you are multiplying by this some filter weights depending on the coefficients exactly the phase difference that you are giving will be determined. And depending on that only the orientation angle of the beams; whether beam will be that when beams will be like this or it will somewhat tilted that will be determined.

Now, suppose this is very useful this beam forming in mobile communication; suppose in the same sale, two users are given the same frequency; then how to separate them, because you are using the same band. So, in that case suppose one user is a here another here. One person is here another person is here. I will adjust the weights such that a beam is formed in this direction null in this direction; so that even though both are using same frequency band; I only listen to this guy and not to this guy. So, it is called special filtering separation those signals are a coming in the same frequency band; I am not using that those kind of filters you know low pass band pass, but it is a special filter.

In the space in one direction signal are allowed another direction signals are filter out. Now, if both are in motion it; this fellow is starts moving I should be able to lock on to it so that means, I should adjust this filter coefficient adaptively. So, that my beam latches on this guy and null latches on to this guy. This is the basic philosophy of smart antenna, which gives rise to this special filtering. So, again you need to adaptively adjust the filter coefficients.

There are plenty of such examples you know, but since my purpose of the course is not to a take of examples; it is just to give some motivation; plenty of examples you can take a have a take a look at the book by Hakin by Farang and then by the book by Widrow, which is been much more rich in terms of examples. Widrow’s book is the first book on adaptive filter there are plenty of copies available here. So, you can have been look and you know it discusses many examples; another thing, which is particularly valid for communication systems. Let us consider and that is what I considered somewhat in detail.

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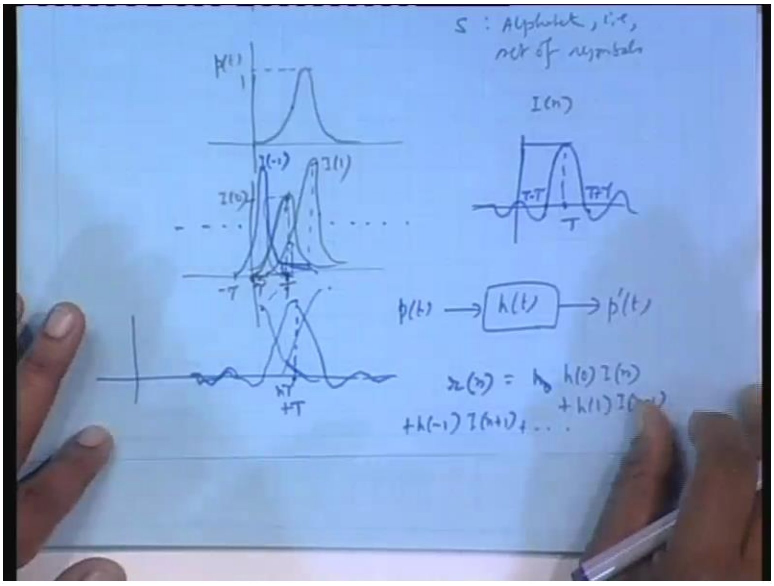


I will be considering, it is called adaptive channel equalization. You know we all talk of digital communication. What in what you do in digital communication is this? The binary bits are coming 0 0 1 1 0 0 1 like that may be you can see that look I will handle four bits at a time, so four another four another four. So, four means how many combinations two to the power sixteen. So, you can determine sixteen discrete levels; each level is called a symbol. So, there will be sixteen different real valued voltage levels or maybe you can make complex valued.

So, real part taking eight and imaginary part taking eight or no four and four; four into four. Four into four four and four they can within complex number; in our course I will assume to be real. So, it says a sixteen different voltage levels fine. But if you want to transmit it to the other side it is not that I say there looks level A and the other side receives it. Because who will take it down the channel; channel is a physical media, which could be a free space.

When you use antennae to antennae communication or which could be a wire telephone wire copper wire, which could be optical fiber which could be say fluid like water so that for submarine to submarine; you know under water communication takes place this kind of things. But all physical media, which constitutes communication channels they are analog systems. They are not discrete they are all analog system. So, you can give analog wave from cosine omega t it will go, but if you say one or two or four nothing will go they are only numbers. So, you need to have an analog waveform so for transmitting those symbols the symbols are fixed say sixteen levels.

Those set of symbols is called alphabet S; S is the set of symbols. That is set of symbols and you are transmitting every time one particular level from here. Some level it can be this it will be that like that, but how to transmit it; because just transmitting the number as no meaning you have to have an analog waveform. So, what they do they choose a basic pulse say p of t you can mean take the height to be 1. What you do? You trigger the pulse; switch on here at this point switch on and you multiply the pulse by the particular symbol you want to transmit now.

Suppose, you are transmitting a symbol your symbols sequence I denote by I n that is I 0 is the symbol transmitted at zero th clock cycle or zero th symbol cycle symbol period. So, you pick up some guy from this set transmit it. So, that time you multiply this p t by that so that means, what you have is something like this height goes up to I 0; you are multiplying the pulse height was one now height was goes up to I 0; you switched on here. When you receive it you measure the pulse at this point when it is highest.

You receive fine I get I zero is the height; that means, what you transmitted is I naught. So, I found out the symbol. Ideally, this should be the case then after tau second tau is my symbol period after tau second you switch on the next pulse. So, it goes on like this and you multiplied by the next symbol. Next symbol is I 1 because during the zero th symbol period; you transmit it I 0 multiplied by that pulse; after tau gap of tau second you take the next guy next symbol. Again, somebody from this set is I 1 some level one of those sixteen levels multiply this by the pulse by this.

So, you switch on the pulse again here multiply it; tau seconds earlier you switched on the previous pulse; something like this I minus 1 dot dot dot dot. So, what goes on the goes through the system is super imposition of these pulses. Now, the problems is if at the receiver I want to find out I naught; you told me to measure the sample at the center point capital T, but what I find already a future pulse already had start it before T this point is arrived that is carrying the information of I 1.

Already, some contribution from previous pulse is coming which carries intermediate function of I minus 1; they all have some contribution. This this pulse has this much of contribution here. You see at T point this much and this pulse has this much; so I will get contribution from all of them which means I will not only get I naught I will get other components, which are mixed up thing. So, I am lost I cannot find out because there is interference from future and past symbol are coming because of the overlapping on the pulses.

You can say that suppose and not only that I want to transmit with faster and faster. So, this tau will be made narrower and narrower smaller; the moment switch on immediately after that switch on another one immediately after that switch on another one. So, there will be overlap of many pulses; pulses are very close to each other now not very far away. Because you are not waiting you are transmitting again transmitting again; transmitting again transmitting. So, it is very very closely space, which means there will be too much of interference from future symbols and past symbols.

Once again, you can say that look as you are decreasing tau transporting fast you also compress the pulse. So, that they do not come close to each other, but I cannot compress the pulse the moment you compress the pulse. If you shrink in time domain it will require more bandwidth, but you have got a band limit. You cannot occupy consume more band more bandwidth then is given. So, that i cannot task this even I have to bear with I have to bear this; I have to bear with I have to bear this intern signal interference so do something to avoid it.

So, what they do is this they design the pulse so that when you have sampling this that time this fellow should go through zero. But this fellow also should go through zero. Other fellows also to the right and to the left should have should go through zero; so they do not interfere then. If that; that means, after all this pulse also this one this also this one this also this one same pulse only. That means, this pulse should be such that at T it should have value one, then from T if you go tau seconds to the left like this point from the center point; how much you go the left tau.

So, if you go tau to the left T minus tau it should have a zero; if you tau to the right T plus tau it should have be zero like this some like this; T plus tau T plus two tau T minus tau T minus two tau like that. So, all this fellows will have zero crossing there. This kind of pulses, which satisfy this zero crossing is called Nyquist pulses. This properties actually the zero crossing property, we enforce this zero forcing condition on the pulse by this design. This should be fine if we design the pulses are transmit like this there is no problem.

I sample at T I get back this i sample that T plus tau I get back this so on and so forth. Problem is the channel is not and ideal channel. Channel has got its own distorting influence; channel is band limited first has a telephone wire has very bad frequency responses is not flat, band limited also very bad frequency response.

So, naturally the pulse will get distorted, pulse p t will pass through the channel; channel has an impulse response h t and the convolution between the two will give you a new pulse p prime t. So, this pulse p t will get distorted p t had the zero forcing satisfied, but after distortion zero forcing will be gone. So, I will back to this again and therefore, there will be interference. What kind of interference that if you are in the if you are somewhere here; you are taking up the n th symbol n th. So, the sampling point will be n tau plus T n was zero for the zero th.

So, it was only T then T plus tau T plus two tau T plus n tau for the n th symbol. So, I should have I of n only, but what is happening instead of having this I of n the pulse p prime T no longer p t. So, its height itself is not one height itself is distorted you can distort version of this. That get got multiplied by I n. So, you will get something here of that distorted pulse may be something like this. You know something like, but distorted.

Then, again you will have a component here you will have a component from this guy they will not have zero crossing and this point because p prime t it is not p t. So, you will have some contribution from the next symbol from the previous symbol next to next symbol from the previous to previous symbol and all that. When you measure net height what you will get that is if you measure the net height and you call it r n ideally you want r n I n, but you will not only get you will get I n times that new height which is not to one. So, some constant times I n that constant you can call the h 0 say h 0 some constant into I n and then some constant times next sample some constant times the previous sample. So, h 1 times previous sample h minus 1 times next sample,, so I both sides. So, this give rise to a discrete time model and this is a non-causal model because contribution is coming future and past. Physical channel is causal, but the model is this is the only a model equivalent discrete time model came out of this analyses that is non-causal, because contribution from future and past both are coming.

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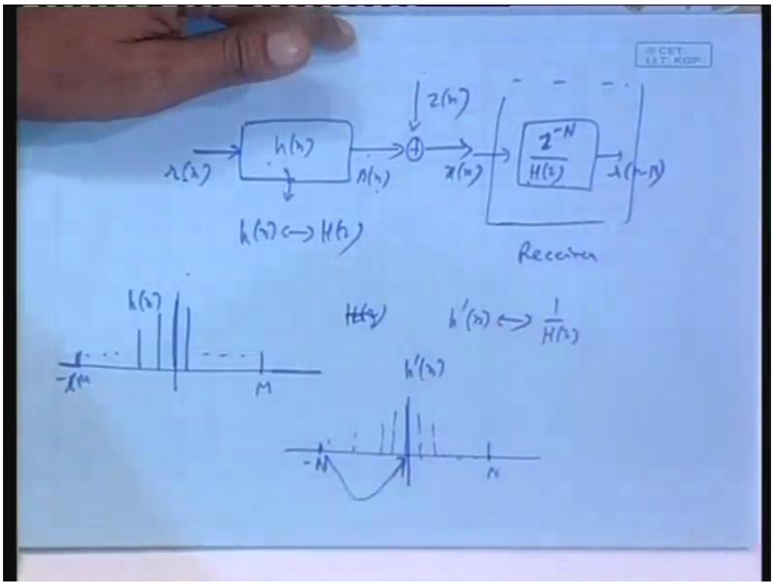

So, then you generate this is my channel model h 0 h 1 h 2 is minus 1 n minus like that you transmitted r n these what you are receiving. Also along with this the channel is noisy, so noise signal also gets added, but you what I do normally we combine that net effect of the noise and show it as a separate noise component being added here. Noise is there all along the line, but we take out the n net effect of noise as a single source of noise and what about here comes get this to that we add the noise this is a model.

So, there is suppose noise is z n; this is what you receive. This is the signal part the noise part and this is what you receive may be this is x n purposes, how to get back r n from x n for the time being assume no z n for the time being. Then suppose h n is h n has got z transform H z. Then, if I passed this on the receiver side this is the receiver side if I pass it through a system one by H z r n through h n give you S n; that is in terms of z transform capital R z into capital H z that give you S z. So, S z into one by Z will give you r n back R z back. If there is no noise, otherwise they will contribution of noise. So, this is the basic equalizer you have to construct this.

Now, there is one they are certain details about this let me just quickly tell you h n what kind what is h n like. How did h n come up, but again you have a look at this figure; h 0 came from this central value of the pulse; h 1 came from this this pulse, which was passing through which was having this value. So, it came from this value of the p prime t is suppose, p prime t is this something like this. The central value it is no longer one whatever value that much was h 0. Then, if you go tau to the left whatever the value of the pulse that was h 1 h minus 1, because that got multiplied by the symbol.

Whatever the value of this side that was h 1 that got multiplied by because you see from the from behind this side is coming in this side, from the right side this side coming in. So, this much height was here and that was my h minus 1 because, this pulse corresponds to this symbols somewhere like they are like that. So, what are these h 0 h 1 h 2 there are just values of the pulse at this point this point maybe here like that. So you understand that they will progressively die out. Obviously, pulses which are situated very far away they cannot overlap only nearby ones. So, as you increase go on taking h of n for higher and higher n final it will die out.

So, h n will be a dying out sequence. Typically, it could be like this; maybe you take it up some value say minus m and plus m and then zeros. So, if you take this h n. So, this is typical h n. So, h n is FIR h is FIR only thing is its non-causal; it does not matter. If it is they FIR you will have H z it is z transform. Z transform we will have only zeros because if I have filter only it has zeros no poles; so that means, one by H z has the problem it has only poles and there comes a problem of causality stability poles must lie within unit circle.

Then only it is causal and stable, but I have no guarantee because they are coming from the channel I am not continue the channel. So, some times in the most general case, some poles could be inside unit circles some poles could be outside unit circles, which means system is stable, but its non-causal this system is stable, but the non-causal. So, 1 by H z this system if you if its impulse response is h prime n. This will be stable where non-causal so that means, h prime n could be something like this; it will go in either direction because non causal. But, it is stable for any stable system you know impulse response; if you take the mod value and go on adding it has to be finite that is a condition. So, then this also should die down if it does not died down, if it remain steady. It will never we sum of all for a stable system impulse response should be absolutely sum of all that is if you take the absolute value mod value and go on adding final it will be finite. That can happen only if this dies out; if it remains constant you go on adding it goes on increasing. So, if it dies out say something like this up to here say minus capital N then on the side up to N. This is what I have to design here, but problem is this is non-causal I cannot implement it non causal system means contribution for future comes to genuine present, which cannot be done it, becomes causal if the origin were here. So, what I do I take the same data, but apply a shift on it to the right by how many points capital N points. So, that this guy goes here then it becomes causal and shifting to the right by N means if z domain I multiply by this. So, this filtered I construct what is the problem; what is output then output will be not.

What are you got when it was 1 by H z that time strictly r n, but you are delaying means r n also what you get that also will get delayed. Suppose, you have got a transfer function Hz, your input is x n output is y n. You say instead of H z I call it H z into z to the power minus two. Then, what is output? Earlier output was y z was Hz into x z that was y z. Now, it is H z into z to the power minus two into x z. So, z to the power of minus two x z means delayed input and therefore, output also is a delayed by the same amount.

So, that means what you get here is not r n, but r n minus capital N in the absence of noise, which is okay for me because this delay I do not care as long as correctness of data is present. As long as I get the same data, but only after a deal of n n clock cycle I do not care. Problem here is so far so good, this is called a linear equalizer. Only thing is to know capital H z I must know what are small h n I must know what is capital H z? Therefore, I can take the reciprocal, but who will give me h n. Perform h n came up it came up from not p t, which was known to me, but p prime t, which basically is a combined effect of the channel and the pulse there is convolution between p t and h t.

So, I if I know the channel it is if I do not know the channel; I do want to p prime t and more over channel may be changing it is characteristics h t may be something now something else after a while. So, p prime t there is no guarantee of you knows remaining fixed of fixed shape. So, since I do not know p prime T i do not want know those values h 0 h 1 of all that, so that means, I have to every time after some you know every half an hour or twenty minute or fifteenth minute I have to train. So, that I learn from the incoming data about this channel and adjust accordingly.

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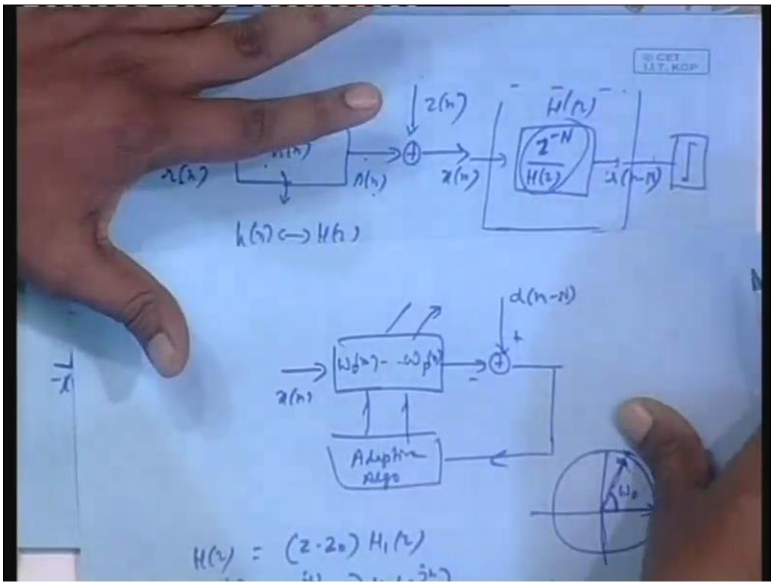


So, what I do in equalizer? There is you know may be say after every half an hour there is a period of training handshaking period, that time transmitter transmits r n which is a known sequence. Universally fixed known standard sequence called pilot sequence, which the transmit receiver also knows. So, receiver knows what is transmitted. So, receiver only finds out some x n and it gives it to a filter adaptive filter may be w zero to say w p. This output it want to be what was transmitted that delayed by n transmitted sequence suppose d of n because that is known.

So, it tries to it knows what is d n, because what is transmitter transmitted that is inversely known that sequence a pilot sequence call it d of n these are response d of n. It just delays it by this amount capital N and finds out the error use it in an adaptive algorithm. These are adaptive filter philosophy and go on adjusting it that means this output it is trying to bring as close as possible to this. So, that this filters finally, becomes really something like this. So, that this is indeed N cycle delayed version of what was transmitted. till that happens it goes on training it and if you give me sufficiently long training period; it will be indeed be the best one very close to this and this output will be indeed be very closed to this.

That training is over relay on this for a while that may be after sometime whether needed or needed go through the training again like that, this is called adaptive equalizer. This adaptive equalizer and now point is if you take the effect of noise, then what happens?

This noise now is present here z n goes through this you can call this entire thing as H prime z z is it goes through this. So, in the output there will be a component now suppose there is no adaptation is done. I have got a trained once good once I am living with them, but that will also possess the noise. So, in the output there will be noise component.

But that is because you know ultimately I will pass it through a hard decision device. Because after all you transmitted sequence of samples, but samples you are not taking any arbitrary value one of those sixty levels pre design fixed. So, you have a hard decision device look at the value here and find out which level it is closest to assign that here. That to also some effect noise will go; suppose, there are two levels and this kind of gap.

Suppose, there is level like one volt two volt three volt you received you would have got two here, but because of knowledge noise it becomes two point two, but so what even from two point two you can say that its very close to two. So, I equate it to two, which means the effect of that point two whole noise effects will disappear. So, noise will be largely filtered out by hard decision device. Now, this kind of equalizer had one problem sometimes, you know sometimes this channel H z shows the presence of zeros close to unit circle. H z for this channel suppose it has got a zero say z minus z dot into say remaining part H 1 z this z naught could be very close to unit circle suppose here the unit circle. So, what is H e to the power j omega? If you take the Fourier e to the power j omega minus z naught into H 1 e to the power j; omega will go from say zero and take a round go up to two pi periodic over two pi when omega takes this much.

So, there two lines are aligned; two phasers are aligned and that time the difference between the two will this much. Initially, when omega was zero this was one phaser; this was one phaser. So, difference was this much which as a good magnitude, but as the omega goes on increasing increasing increasing like that when you reaching here.

The difference becomes minimum and it is really very small then this part has very small magnitude. So, if you takes its mod and plot it that at omega equal to if you call it omega naught; there will be very low magnitude their will dip. There will be spectral dips or spectral nulls something like this at omega naught like that dot dot dot a pi minus pi this is for H e to the power of j omega. Now, this system the equalizer is reciprocal of that so;

that means, at omega naught and minus omega naught the equalizer will a picks and this noise, which does not go through h n it only goes through this.

So, this will be amplified and there will be huge noise; here this called noise enhancement effect. This enhance noised will be eliminated for those kind of situation this occur frequently in mobile communication; the spectral null phenomenon for that this equalization is not so good. Then there is something called decision feedback equalizer.

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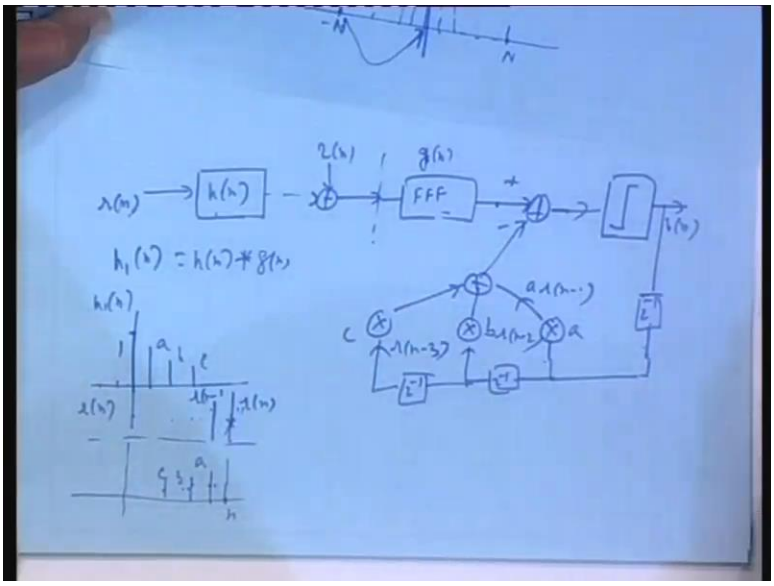

I just briefly tell you, what is done their very briefly you design suppose this is your channel h n. This is a receiver side for that time being I mean I am adding the noise, but for the timing forget the noise. These are receiver side you design one FIR filter say g n it is called FFF feed forward filter. So, that when you cascade the two resulting things becomes causal h n was non-causal in general letting the contribution of a future and a past, but when you design the two net effect net cascade of the two becomes causal.

That is if h 1 n is a convolution between the two there is equivalent system forget the noise part. The equivalent system is convolution between the two it should be like this. Say, some value that may be say a may be b may be c may be there after the zero. After all both are FIR, so convolved will be FIR. So, I take only three say you can also adjust the gain here. So, that this this leading coefficient is one if it not one suppose it is ten you can divide this every everything by ten. So, this becomes one this is normalized to what that a b c.

So, this is the net thing what you do is; suppose I am going in the backward way this is your is where your output decision will come up output decision that what was transmitted. Suppose now this I called v n if you pass it through a delay and suppose you will assume that so far at least for last three cycles correct decision were made by hook or crook or somebody that is v n minus 1 v n minus 2 v n minus 3 are correct. What i will do then if i take this v n minus 1 then multiply it by a then pass it through a delay? So, it is v minus 2 multiplied by the same b and again passes it through a delay multiplied by c and adds them.

What will happen? h 1 n is a net thing that was convolved with this side r n that was convolved with r n. In fact, that, so you r n convolved with r n. So, h 1 n convolved with r n what is how you know how to do the convolution you have to reverse it and then shift. So, if you receive it a goes here b goes here c goes here and then go on shifting this way or that way depending on the n at which you want to carry out the convolution.

Now, suppose you want carry out the convolution at a n th n th point. So, you have to take one here then a b c and this you go on with this sequence r n; r n will be r n here r n minus one here dot dot dot. You multiply top to bottom sample wise at add that is convolution; if you do that then you get what is obtained here because this two together was h 1 n r n input. So, what you get here is what you obtain by convolution. What is that r n times one this is one r n times 1 plus a times r n minus 1 b times r n minus 2 c times r n minus 3.

What you get here a times v n minus 1, but I am assuming v n minus 1 is same as r n minus 1 correct decision. So, what you get here is a times r n minus one itself. What you get here is b times r n minus 2 itself correct decision; so v n minus 2 one delay one delay. So, if v n minus 2 I am saying v n minus 2 and r n minus are same correct decision was taken. And then here c times this is your r n minus 3, because it is v n minus 3, but I am saying that also was correct decision. Suppose, it is so this three means this part this part this part a into r n minus 1 b into r n minus 2 c into r n minus c that part is coming out here.

If i take this and subtract what i am left out is this component r n into 1. So, that goes here and r n is what I want and that goes to this and device that will give me r n only. So, for n th cycle also correct decision is made and therefore, in future cycle are like that. And this time the noise does not go through a filter which is inverse of the channel. So, the pick phenomenon does not occur here. So, the noise enhancement phenomenon does not occur. This called decision feedback equalizer; again here if you know h n than only you can first construct g n.

So, that it is causal and once you know this over all h 1 n then only you know a b c you can construct this. So, you to construct these two filter; you need channel knowledge, but I told you channel knowledge may not always be available. So, here also you have an adaptive mechanism, s, that this filter coefficient actually obtained from the incoming data by an adaptive algorithm. That is called adaptive Bayesian equalizer.

I think that is enough for today. I suggest that you go through this these examples and also take the book by Hackine and also Farang take for that matter many of the examples like say adaptive linear predictions, adaptive interference cancellation. There are plenty of examples and you know very nicely thing as we do sometime let in this course you can run some basic program and implement this filter MATLAB based very simple and you will how beautifully they work. So, that is all for today. So, I would suggest that we begin from this for the next class.

Thank you. Thank you very much.