Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Digital Switching

Lecture – 08

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Okay, so in the previous lecture what I had discussed was M/N composites switch and then of course, call congestion and time congestion for that M/N composite switch okay. So how do essentially make understand the blocking probability in that kind of thing. But remember previous so that I was talking about a strictly non blocking property of the crossbar. And there was an issue which I actually question which I raised that can actually, because I cannot unendingly keep on increasing my cross bar size, because it becomes use then it is going to be very difficult to manage through a control system which snaps the various cross points.

So can we take very small ones and then create use multiple of them and create a bigger switch. Then we tried creating using a twister interconnection okay. But then we figured out that if I want a strictly non blocking property I will end up in the same cross point complexity ON^2 again. So number of cross points will grow as the N grows the growth rate will be N^2 so if you have 10,000 you require 10,000 square number of cross points if you have 1 lakh input, output ports you have 1 lakh input a square number of cross points will minute it okay.

And of course, can I still improve the situation, so possibly this was a question which was there for a long time.

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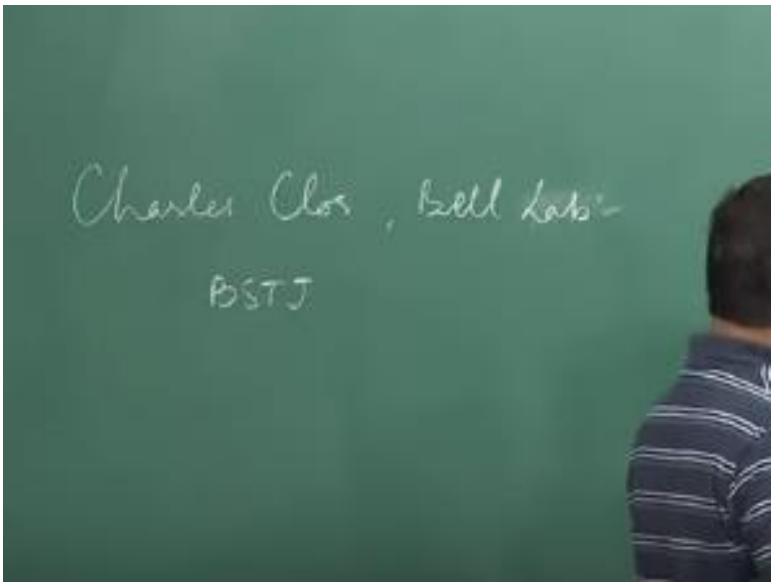

Then of course, one gentlemen Charles Clos actually came up with the first configuration where it was possibly was actually with Bell labs and his paper which was there in BSTJ, Bell system technical journal, this was published in 1953 and the paper was originally submitted in 1952. It is also one of the actually there are many publications just come out in BSTJ which were remarkable which actually had an impact on the technology this is actually one of them.

There are many others like cloudy sentence, information theory paper, Bard John Bardins paper on trimester and so on, there are many work which actually come out in that. So this is one of those seminal papers which actually came in that. So it is Bell labs contribution. So now it is of course, known as Nokia Bell labs, because Nokia is acquired a cat allusion which was actually owning this lab earlier.

So he gave the configuration and this is a pretty simple one, so let me explain the logic, how I am going to create the rest number of cross points in the cross point complexity will reduce, it will no more view and square. And I can actually still build a strictly non blocking switch. So let us see how it happens.

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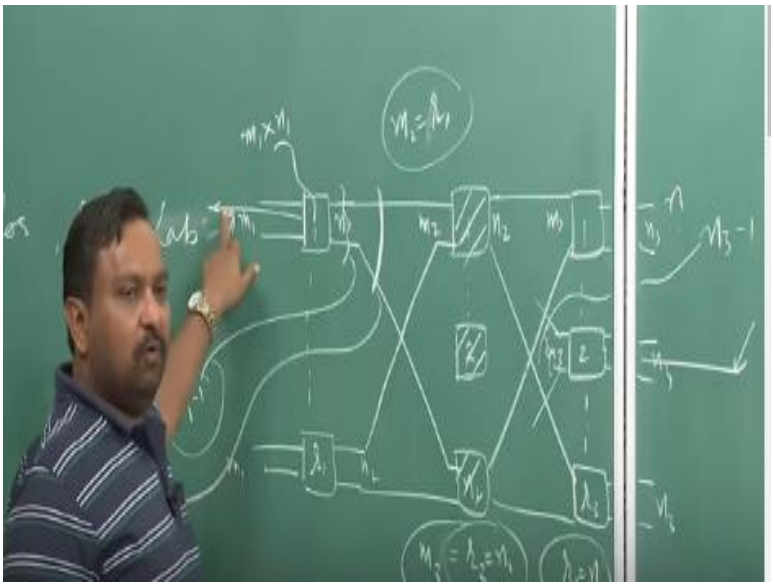

So he actually explained it, the clue actually comes from two stage configuration. So remember let us see where the problem happen in two stage configuration. So the problem happened because when I have been all these n guys were trying to communicate to this n output ports, so initially I thought if I have for example, four switches in each one of these two stages, I will take one fourth here, I will take one fourth here, I will take one fourth here and I will take one fourth here and so on.

And I will do the same thing with all these other switches also, but the problem happened is when all these n wanted to make a connection here, so the input is free, the output is free only ¼ of these number of ports could be connected remaining they could not that is let to blocking you can actually take a clos from here probably if I actually instead of using this configuration I actually use a configuration of this kind and allow ¼ here ¼ coming from this side ¼ from this side ¼ from this side maybe I can root it through multiple middle state switches.

So if actually configure create three stages it will be possible to create a strictly on blocking configuration yes that is true it is possible to create a strictly non blocking configuration if I cannot, I can actually keep on increasing more number of switches in fact in two stage I end it up

creating a more number of outgoing ports then the input ports when I was trying to create a strictly on blocking configuration we did that calculation.

Okay so we in fact had if there four stage a 4 switches here if this is a m I need 4n here for a strictly non blocking configuration in a two state system in three stage I can keep on increasing my number of switches and that is where the clos actually came off with the idea that it came up with the condition when this will be a strictly non blocking and then of course a generalized it for a various different configurations various different numbers.

For example here it can be n_1 here it can be n_3 anything in these cases when this particular kind of three stage configuration will be a strictly non blocking he came up with that idea so let us see how it comes before we actually if we do the cross point complicity analysis and then of course we save the switches not going to go of that kind how to estimate a blocking probability we will come to that okay so the cross configuration technique means there will be m_1 inputs and n_1 outputs.

I am taking a general configuration so this 1 is m_1/n_1 cross bar first stage will consists of r number of switches r_1 actually 1 r_1 so I am making very generic definition we will come up with the conditions when it is going to be a clos, so clos network is a very, very special clos where by in fact here you see there more than 1 connections which are connecting a switch from stage 1 to stage 2 and stage 2 to satge3.

Here what we do is actually that is generalization but we can actually prove it using that is going to be exactly one connection from a switch in the previous stage to the switches in the next stage exactly one connection between any pair of switches which I take which actually means this number should be equal to n_1 so r_2 the switches as to be equal to n_1 okay r_2 will be the number of switches same is 2 here so number of ports which will be there in the switch, there should be exactly equal to r_1.

So remember these are the defining conditions for a clos network if I create a third stage so my outgoing ports are n_3 so how many outgoing ports has to be here because each one of them is

going to be connected in this section so you will be requiring her n_2 and n_2 should be equal to r_3 m1 should be equal to r_1 m_2 should be equal to be r_1 and n_2 should be equal to r_3 and this is $_{n3}$ so m3 should be equal to r_2

So this is the condition 1 condition these three conditions should be satisfied and this kind of switch configuration is what is known as Clos configuration, now the question arises, when it is going to be strict in one blocking? So we will actually formally prove it using a Clos theorem later on, once I described you a method known as pause matrix method, for analyzing this kind of switches.

A three stage interconnection networks, in fact important thing it is in odd number, now each one of them is a strictly if this can be a strict non blocking switch I can put this as a strictly non blocking switch I can put this as a strictly non blocking switch again by using three stage, so I will have I can actually keep on I can have large number of stages but there all generalization of a three stage Clos network, okay.

Multiple stages are feasible in fact we will be proving that a four S stage system in fact a five S stage system will give me something which is known as white sense non blocking switch, okay. So will come across as we go along these things, so now if the question is when the syllabus strictly non blocking? So we have to take some port which is free, I have to take some port which is free and I want to connect these two ports without disturbing any existing connection.

So in worst case this m3 can be greater than n3 but then it means only utmost n3 connections can be made but since one of the outgoing port is free only n3 – 1 connection can actually be emanating from here, so what is emanating from here is, m3-1 connections. From here the connections which will be emanating is m1-1 connections in worst case, okay. So that is the maximum which is feasible.

Now how this connections are been made? For example if I want to connect this, this has to connect to any one of this lines come to the middle stage switch it has to connect to an appropriate line and go to the outgoing thing, so it is possible that it will occupy out of these

lines m1-1 lines will be occupied, remaining will be free, okay. Out of these remaining will be free.

So you will have m1-1 lines which will be free, okay. So technically if m1-1 lines are occupied these switches m1-1 switches are being used to route the calls already set up calls, the one I am interested in setting up the one which is free here, same is true here m3-1 middle stage switches are occupied, if I keep on increasing my switches it is possible that it is like connecting the middle stage once.

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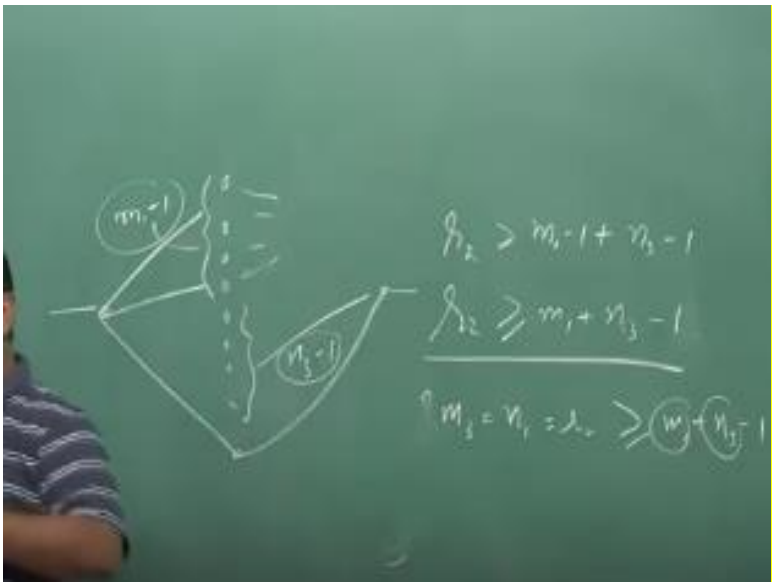

So this is actually setting up the calls where this m1-1 this guys actually using this n3-1, m and if my number of middle stage switches has one more than that so this free and this free thing can be used these free input and output ports can be route can it be actually a call can be set a bit in them using this free middle stage switches, is a worst case occupied switches. So I am not worried about how the others output side is been connected.

So they can be so I need not even bother about that, so, so far my number of middle stage switches are more than this plus this I will be able to always set up the connection, so switch is

going to be a strict in one blocking, so when this is going to happen? When R2 will be greater than m1-1 + n3 -1 if I put equal to this will become, become equal to 1, okay. So first this condition is satisfied.

You will be having a switch which is strictly a non blocking, okay. Which is a switch which will be strictly non- blocking okay which is a switch which will be stricken on blocking so in fact you can see this and once it is you call R_2 and $M_3 = R_2$ both the n and m_3 are greater than M_1 and n_3 okay so what I am saying is since R_2 is greater than these $m_3 = n_1 = r_2$ are all of them are greater than $r = + n_3 – 1$ and they must individually greater than these actually and that is what was the clause condition this was the basic idea on which the things actually your based we will formally prove it later on so now let me formally figure out how many cross points will required in such a scenario.

So let us compute in fact this is for a uni-casting connection uni-casting actually means one input and one output we are only setting of the connection between that if from one input I am going to setup connections to multiple of the outputs it is possible that I take a switch I put one output the same output is available one this input is connected to all these that is a multi casting so I am not looking into multi casting scenario so in that case this result does not hold okay this problem was not actually take.

Tackled by clause but later on by other mathematicians and we will I will give you a bound on this that what is the bound will be actually derived for multi casting case also but let us do it first fall only for a uni-casting scenario so for a uni-casting a strictly known blocking switch what will be my cross point complexity that is a question.

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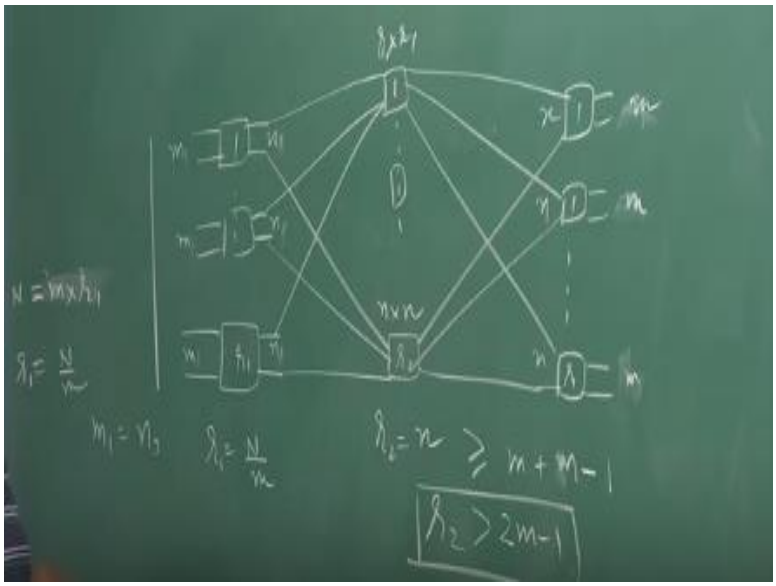

So you will have a switch consisting of m_1/N_1 in fact let us take a symmetric switch that is a still much more logically in general I have just change these numbers so symmetric actually means the number of inputs and outputs if you look from this side or look from that side this was just look same which actually implies that my environment should be equal to n_3 okay was that condition is there I can in fact replace these by m/n and these switches will be R_1/R_1 because this will also turn out to be R_1 this will become m.

Because of symmetric this will be n and these are the R2 switches so R2 should be equal to n okay and as per the condition which I have write this should be equal to m+ m -1it should be equal to 2m-1, that is a clos condition and each one of them is a cross bar let me see how many cross point will be require by us. So let us see so the total switch size here is now m x r1 okay so in fact I should call it there arer1 switches the total size is n because I am going to essentially define my cross point complex in terms of n.

So you will have now n is the output so m x r1 is equal to n so I can very well write r1 has n/m okay so that is one thing which we get so this r1 is n/m. okay so same is true here so how many cross point will be required in this switch so I have to find out the number of cross points so the

cross points which will be required remember this is 2m-1 okay. So m x 2m-1 switches that is the switch dimension how many such switches n/m switches will be required now come to the middle stage what is the size of this switch n/n this is the n/n switch okay and n is defined us 2m-1.

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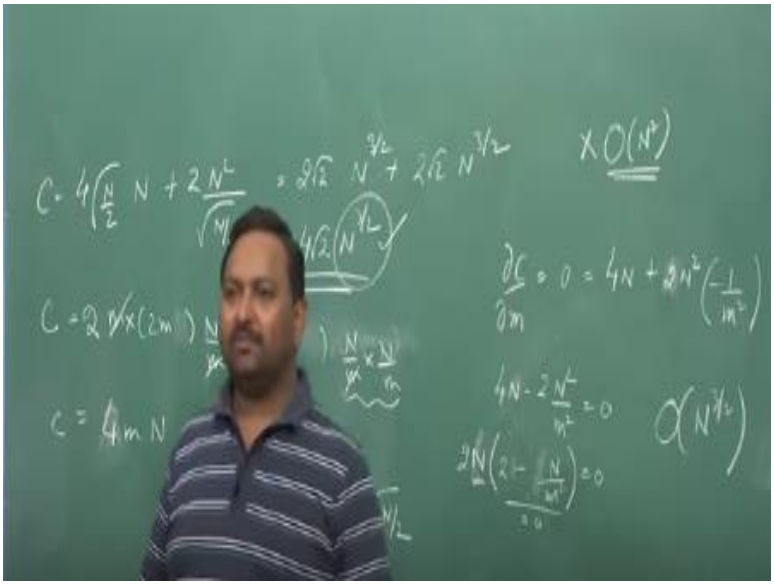

So this will be (2m-1) (2m-1) in fact I will I can make my life simpler if I and how many such switches will be there? Should be equal to 2m-1 switches okay. Sorry number of switches is 2m-1 the switch dimensions here has to be govern by this number which is n/m so it is to be n/m/n/m.

That is the number of cross point in a single cross bar in the middle stage the traits stage is exactly same as what was your first stage so I can actually just double it up so that is a total number of cross point which will be there in fact it does not matter if I actually I add one more switch so this one can be done away way. Okay I can approximate this is a bound which I am estimating so I will be doing away with this particular thing.

So what is my outcome? Now 2 n cancel with this is become 4nm number of cross points plus this same cancels with this, $2n^2/m$ that's the total number of cross point which should be required but we don't know we only know that n is fix but what's the optimal value of M we can take a derivative of this and estimate ,so which will turn out to be and make it equal to 0 for optimum so it is for n +this value this will turn out to be so either n should be 0 that's one possibility or this should be 0 if this term does not give you the solution I am not looking for that .

So n is some finite values so this is the only thing which can become equal to 0 which will give me $\sqrt{n}/2^2$ so m should be equal to $\sqrt{n}/2^2$ if I put that value what's a cross point the number of cross points which will required by me, so let me put 4 so this turns how to be ,now the important thing is this result so number of cross points are not growing as 1^2 ,the number of cross points are growing with $o^{3/2}$.

So using a three stage multi stage interconnection network using a cross bar ,smaller cross bars on a specific configuration it is possible to reduce my cross point complexity and of course is modularly you can build up a larger size which using a smaller once of course you pay for the complexity here the connection set up ill go to them is required in this network it is always possible to set up a connection without disturbing the existing ones that's why restrictive in non blocking that is proven by this result okay.

But formal proof will come across this particular thing so one thing which we conclusive here is that i can reduce my cross point complexity so from n^2 I get become and $s^{1.5}$ okay so that's an improvement by given by the Clos.

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