Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Digital Switching

Lecture – 30

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Okay in the previous video what we had discussed was the delta network we also had looked into the generic Banyan we also looked into how the shuffle net actually act as a delta network okay we in fact gave a prove that any kind of shuffle inter connection base network will ultimately will be a digit control routing network digit control routing networks where the networks where you actually put the output address has a tag at the input port and packet will self route through the network and ultimately reach to the output where you want it to actually go so but at this point of time they should also now try to.

Compare how the delta networks compared to a cross bar because I can make the packets which with both so let us try to see how we can actually do the performance analysis of a cross bar and delta and compare these to after this then we will move to the buffer delta network how to actually model them and how to do the performance analysis how that actually is done so this is the computational procedure which actually is used for doing the performance analysis of a buffer delta system and of course if time permit we I will do something some other things which are extra just for the sake of knowledge they will not be coming in the examination.

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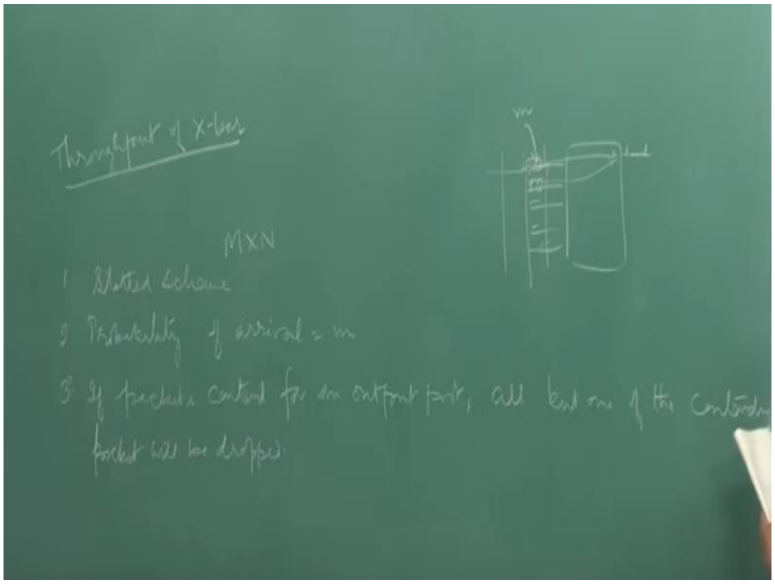

So through put of the cross bar so let us start with that so cross bar is of the size m/n that is what I am assuming it is m/n cross bar and remember it is a packet switch so what I need to find out what is going to be through put at the output so probability of arrival I am actually had of the packet at the input is a slotted system I am taking it as an m so if there is m/n cross bar so a packets are coming at the input and it is a slotted structure so in one slot packet will be coming at various input ports second slot this new setup packets will be coming in.

And the probability that a packet will be there in a slot is given by m so that is what I have written here and of course again I am assuming that there is nothing like input queuing here so if the packet more than one packet want to go to the same outgoing port I will only tell how one packet to go through the second one will actually be drop so one all but one of the contending packets will be dropped and one will be put to the outgoing port okay so that assumption so I am assuming the if the packets are contending all but one for an output port all but one of the contending packets will be dropped.

Of course in fact I should not change the number first is assumption is the time using a slotted so time slots have been define and in one slot only one packet will be coming and then to be switched out we do also do assume now.

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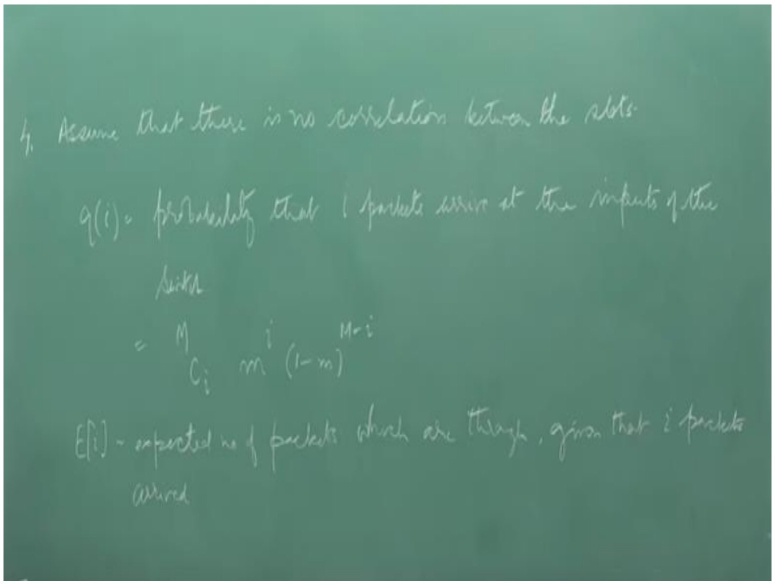

The four thing we assume that there is no correlation between the slots so pure independence is the fundamental assumption which we have been making so when you are actually having a time series their multiple slots at a input when a packet arrives here and packet arrives here these two packets are independent there no way correlated it is not that have packet is arrived here there is a air chance of packet coming at this point, so there is no correlation between then it is purely independent evened so that is another assumption, so q(i) which is going to be probability that I packets arrive at the input at the inputs of the switch.

So remaining m-I inputs there is no packet arrival on those so this probability if normally will be given by mC_i m is the number of input out off this only I inputs are going to have the packets and this happens with the probability m is the arrival rate, arrival probability and with 1-m there is no arrival so this will be m-I, so that will be the probability and of course we define e(i) as the expected number of packets which are through to the outgoing port. Given that I packets arrived, so to determine this value of e(i) so we go in this manner.

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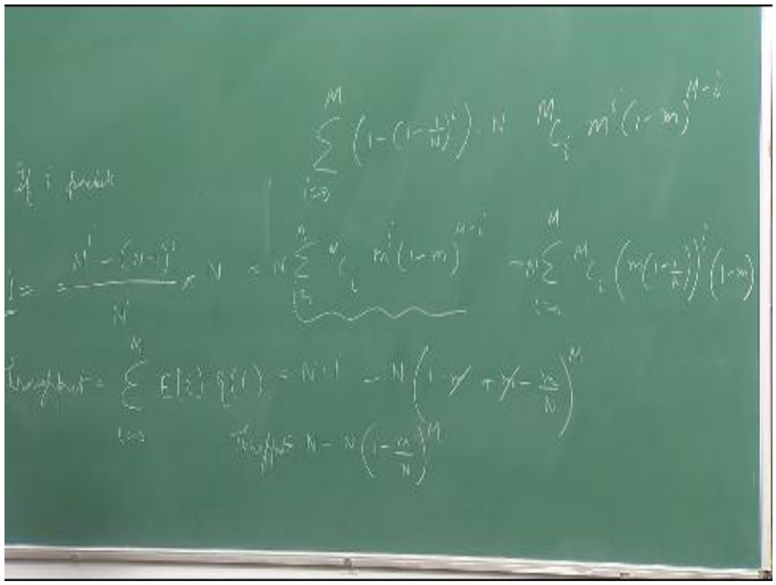

So if I packets are arriving they can be a directed to anyone of the outgoing port with equal probability, okay. So in how many ways the I packets which are coming can be mapped on to these outgoing ports so each packet can go to any one of the n ports there are n possibilities and since they are I so that the way they actually they can be mapped on to these will be nothing but n x n x n.

Because each packet can take any one of the n outputs and possible ways so these will be done I times, so which implies n^i so I packets can be mapped to all outgoing ports in n^I ways so if I look at an output and that output is not being selected in what all ways actually that can happen which actually means these I inputs are now being mapping onto these remaining n-1 outgoing ports and this will happen in how many ways.

So in n-1 ways one packet can choose these so there I packets which implies $n-1^I$ ways the mapping can happen from this I packets of these n – 1, so this implies that $n^i – (n-1)^I$ are the number of ways in which these are the number of patterns where this output is specific output will always be there a packet will atleast be mapped to this one packet atleast be mapped to this, so these are those many number of ways so total number of ways in which the mapping can happen is n^i so this should be the probability that an output will be selected, okay. This will be the probability that output will be selected and that is what is also the expected number of packets.

Which are through given that I packets are arrived so this is a probability that atleast one packet will be coming out here they total n number of ports so this must be raise power n will give me e(i) okay, so I can actually now use this to compute the throughput so throughput is summation of when I can go from 0 to m so minimum 0 packet can come in and maximum number of M which can arrive in a time slot, this expected packet which will be successfully passing through to the output and probability that you are going to have I packets arriving.

So we need to solve this so this turns out to be ∑ of, so I can actually solve this one I can take N^i out and then make a solution so this will be $(1-(1/N)^i)$.N so this N^i this once you put you put you will get 1, this can go inside so 1-1/N that is what is that total power i which will be there, right.

And this the N, so this total is E[i] and then of course I put in the value of q_i which is M_{Ci} m^i 1- so this is what is going to be the throughput, so this now can be written as with this one, so N will be this N I can take out M_{Ci} -, so I can explain this, so this value is nothing but $m+(1-m)^m$ which will be 1 so I can multiplied by 1-, so again this will be this plus this entity raise power m, so this m cancels so you will have N-N, so that is what will be your throughput, for m/N cross bar.

So fractional throughput if you want to estimate you can actually get it so fractional throughput will be.

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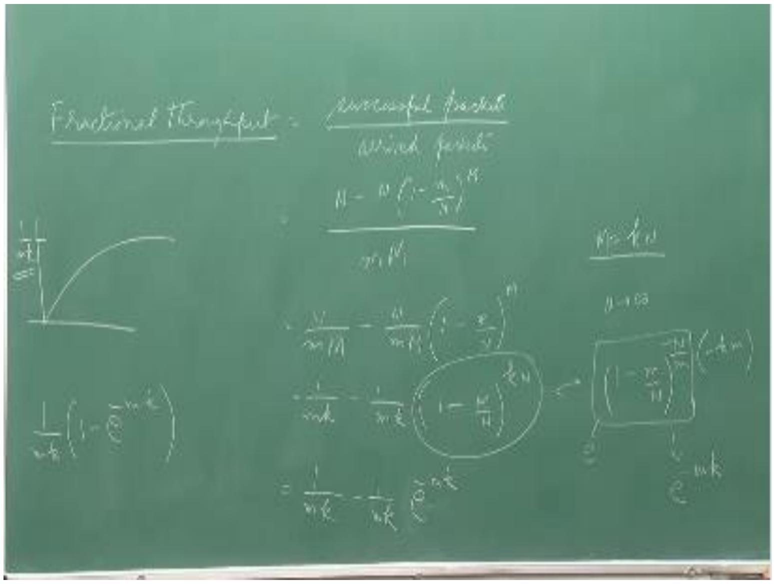

So fractional of whatever is coming at the input how much of that is going out that is the fractional throughput, this now can be estimated as successful packets divided by total arrived packets of the input remember the packets which are not in the condition which are not successful they are being dropped out okay, so that is why a fraction of the whatever the incoming number of packets only it is going out that is the fractional thing.

So this will turn out to be whatever is the through put which we have that is the total number of packets which are going out will be $n\text{-}n1/m^{m}$ / total number of packet switches are coming m x m okay so this value will be n/m_n $–n/m_n$ sorry n_m so when m_n is a basically symmetric square cross bar then n will be equal to m and then correspondingly things will change there so this will be 1- so you can I can always simplify this thing to figure out the fractional through put has for a very large values.

So I can write m = k(n) so this ratio is fixed and then I can take n goes to infinity And then actually solve for this expression so I will get this thing is $1/m_k$ $-1/n_k$ so I can define this m/n so this particular part is going to change when I am going to take n goes to infinities so I can write this thing is $1\text{-}m/n\ ^{n/m}\text{-}k_m$ so in fact we can see this particular fact will be nothing but 0 when n goes to infinity this will get converted to e.

So this whole thing will now be e^{-nk} so when n goes to infinity the, what you will get this m_k - 1(mk) this will be e^{-nk} that is what will be your expression for the fractional through put okay so that will be the fractional through put so in fact we can write it has 1/mk1-e^{-nk} so m is a probability which goes from 0 to 1 and we can see has ma actually grows when this is 0 this value will be 0 when m actually grows this will actually take a maximum.

Then again it will drop it will infects, when this is infinite, this will go to zero, one over a case, so this technically goes from, it will saturate and this value will be one over mk on saturation. So that will be maximum thru, put which you can have and k will be 1, when m is equal to n ,so one over m will be maximum throughput which is possible.

So now let's look at delta network, so this value when m is growing can take maximum value of 1, when m will be equal to 1, when k=1, you will get maximum value as, one over 1-one over e, which can get as you're throughput as you can get your m, for a very large size switch and this will be for k=1, when k is not equal to 1 the value will change correspondingly. Now look at the delta network, how you will find out, the similar kind of throughput performance for a delta network.

So once you have this, you can actually compute, for delta network you have to do a computation, to find out the ultimately the throughput. So let's see how it can be done and then you can do a recursive computation and find out the throughput, and once you plot they will compare with each other, so let the delta network will be of size, a^n * b^n, this is basically going to build using a/b switches, multiple of them, so there will be total n stages.

Each one will consist of a/b switches, so we can take the cross bar result for a/b, so a/b is a cross bar, so the output, arrival rate at the input is m of a/b, so m is the arrival rate here. What is the outgoing rate that we have to compute, so this will turn out to be nothing but, we can actually use, same, similar computation which we did, this will be b- by this, so what is it mean is, when a packet is going to come, it comes with n and this is going to be directed to one of the b's.

So that the probability that the packet is arriving here for this outgoing port is m/b, so the probability that the packet is not going to arrive at this port for this one is one over 1- m/b, so that's the probability, no packets will come here, this does not happen in any one of the incoming port, that probability will be a, so and then of course 1- of this means, there is going to be at least one packet coming for this outgoing port, so that will be the throughput.

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There is a probability that the packet will be available here, so the overall throughput will be if I consider all these b ports, I should simply multiply by b, so that will give me the total throughput at this outgoing port, and that's what this throughput is all about. So the probability that the probability will be here, so I call it, if this is the j switch, this one particular switch so if I call it m in m out will be given by this expression.

So I can write down m of out is 1 – m in by b is power a ok so ones I have this wt I can do is I can do a recursive calculation so we will actually do thermos here which will justify this arrangement this basically comes from there so if I know the input I call I m 0 I can find out what is m 1 ok so remember that each one of the input packets are each one of the inputs are coming which are coming independently so these packets since there is uniform distribution uniform probity packet is going out anywhere so I can find m 1 this m 1 should be same for all out going ports after the first stage all packets since the input here in next stage going to be connecting having to path only from a certain thing certain inputs and this size of the inputs are going to be same so the probity of the arrival to be same here so this premise can hold recursively and I can find out

m 0 to m 1 m 1 to m 2 ultimately I can find out what is going to be m of n – 1 okay there are total m of n sorry m of n this n stage this is first second in the n stage once I no m n that's is probity that a packet will be available here an since total number of outgoing port will be b refer power n so through put should be power n b is power n m n * n m that's the total through put and off course the relative through put now be if I know the input and total number of incoming

Port is e power n and we had a packets coming m 0 okay are we call it m actually here o this will be the so this will be the relative through port so this is the basically number of fraction of incoming packets which has been pushed out as an function of m 0 so but this as to be done computer okay so now we can compare relative through put so if you compute and plot so I can just plot for you these are relative input for cross bar as well as delta so how they compute so

So computationally there graph will look something like this o it will start from 1.0 this is proximity is there where you start which 1 and 2 and 4 and 16 64 256 1024 409 this will be roughly saturated is about .5 6 7 8 9 roughly it will start stat tarring here this will be the cross bar Okay we can also built delta 2 ,delta 4 basically depends on weather i am going to build by 2/2 elements or I am going to build by 4/4 ,so if you take delta 2 this will be something like this ,and if it is delta 4 it will be something like this ,so cross bar is always give you the best performance. But of course you need a more central CPU which is going to do the header processing that need to have higher capacity to process the more number of headers.

You actually do a distributed computation here you don't required a centralized CPU to do all the computation ,you can just put that tax at the input shelf routing is going to happen you can build larger switches ,so that independence is at the expense of your relative throughput dies away but you get more independence and more resolution system .

Even if few switches some CPU fails still part of the switch will keep on functioning, okay, so that's and of course the cross bar is fully strictly non blocking network but this one are the blocking networks, okay, so infect if you want to estimate the permutations, between the crossbar and the delta i think i can do that, so if you have a cross bar of n/n, so how many possible connections pattern which can exists.

So one input can get connect to n of them, the second one connect to remaining n of them and n-1 so it will be n (n-1) (n-2)...1 so this will be n factorial. So n factorial ways the basically connections factorials are being supported by this ,while if you look at the delta network delta 2 for example and if it is ,$N=2^n$ with that capability ,this actually means 2/2 you are going to have ,n stages ,okay, and each one ,each switch will have will you have n/2 switches .

Total number of switches is will be actually $n/2*\log_2 n$ and each switch can actually have two states either cross or bar so total number of possibilities will be $2^{n/2}*\log_2 n$, now this number, these are the total number of permutations or combinations for any banyan networks, is not only delta any banyan network this are the possible permutations input to output maps which can be handled.

This number is far as smaller then this number ,this number is much larger actually okay ,so n factorial is much larger then this and because of which the blocking will happen and you will have a poorer performance ,okay, because we remember we assume the containing packets will be dropped ,as I go from delta 2 to delta 4 delta 8 ,this will become as good as a cross bar ,okay, so cross bar actually said this so it should keep on increasing your delta side the output "b" number delta "b" goes from 2,4,8,16 ,,it will tend to become equal cross bar .So that's how the cross bar will deserves delta net work performance actually can be compared.

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