

**Indian Institute of Technology Kanpur**

**National Programme on Technology Enhanced Learning (NPTEL)**

**Course Title  
Digital Switching**

**Lecture – 29**

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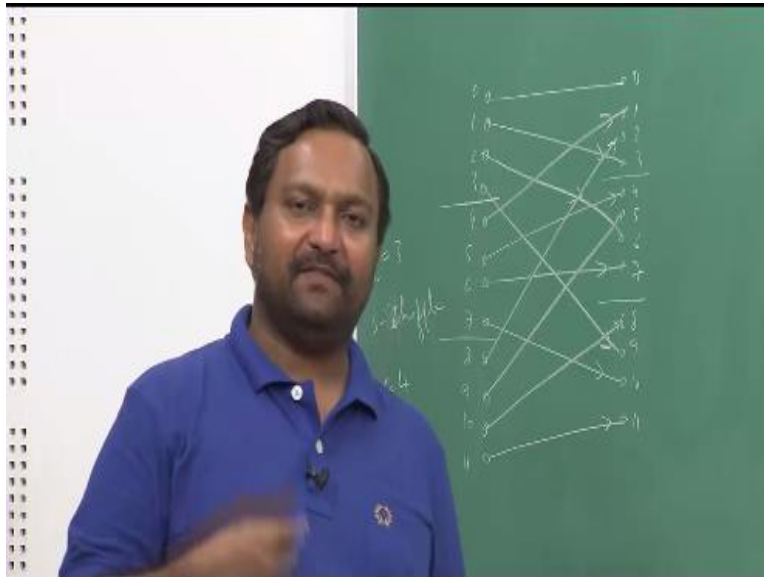
So in this video we are going to continue from where we from left in the earlier one so in the last video what we had done was we are looked into the Banyan networks they are essentially basic definition and we had also looked into how you can actually create a  $\Delta$  network so  $\Delta$  network is a special sub class of a Banyan network configuration so Banyan network are the networks we are there is exactly one path from any input to output so all input output pairs will have a different paths.

And they are bound to be blocking networks by design  $\Delta$  networks has only one specific property that at any stage in any switch all the inputs which are coming to that switch will be coming from the same output levels so it may from the previous stage all output 0 is of every switch or some of the switches it will be coming to the same input so or it maybe 1 or two so if this property just being taken care of in the structure we will actually have digit control routing networks are you we also call them  $\Delta$  networks.

Okay we had also looked into how switching elements will be required in a  $\Delta$  network which is a symmetric one say  $n/n$  or  $r$  raise power  $k/b^k$   $k$  kind of thing switches so where number of inputs and outputs are not same we have also looked for that scenario then I had asked a question that in most of this  $\Delta$  networks is it possible that can I actually have same inter connection pattern being repeated between two consecutive stages so among all the stages it is done is it possible through build up a digit controlled doubted networks and in so we can actually do it using a shuffle inter connection so let us start with the what is shuffle inter connection and then get into the proof of

that how shuffle inter connect network actually is a digit control routine network so a shuffle network a shuffle is a pattern.

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Is basically there is an object correction of objects I can shuffle them so basically when I shuffle I will rearrange so they will just get mapped in some fashion so I have to know define this mapping actually but before I go to the mapping let me show you how actually the shuffling happens so if there are shuffle actually word has come from the card cards the game of cards so where I you normally take two stakes out of it then you actually take one card from here one card from here.

One after another and all 52 cards are being put back again so that is what is called shuffling so basically what you are doing is for example if these are these many objects are there so there 1 2 3 4 5 6 7 8 9 10 11 so let me make it 12 so I will actually put 2 batches of six objects each and then I will take one object from here and then one object from here one from here and that is what the shuffle inter connection I have actually created two shuffle here I have created two objects one six objects here.

I shuffle them together so it is a binary shuffle which has been created for 12 objects so in general I can actually define a Q shuffle for  $Q \times R$  objects so here  $Q$  is actually  $=2$  and  $R = 6$   $Q \times R$  is the total 12 objects which are there okay so we will define this q shuffle s shuffle QR and the entries which I will be making for this QR objects they go from 012 and so on till last one will be  $QR - 1$  that is how we will be indexing the entries and I can define this shuffled pattern basically for any given input identity I should be able to which will be I will be my input I will be able to get SQ a star R which is the index number on this side, so we can define this things is, SQR as function of input.

The input index this will give me  $QI + IR|QR|$  so let us see what is going to happen if I am looking into this case, okay. So here actually it is a  $Q = 2$  in this case that total 12 objects, okay 1 to so I have to count from 0 to 11 that is why my index center should be there so QR is 12 in this case so R is 6, QR is 12 so if I is 0 so  $0 \times 2$  gives me 0 and this is a floor function so unless i value actually crosses 6 I do not bother actually.

So when it crosses 6 I smaller than R this value will be 0 when  $I > R$  but less than  $2R$  it will be equal to 1 it is a floor function this technically means if you put any real number take a floor thing this actually means the largest integer more than or equal to R where R is the real number, okay. That is the floor function so that is what we have been using here so if I do this  $0 \times 2$  will give me 0.

So I am actually connecting to 0,  $0/R$ , R is 6 here the floor function is also 0 and  $|QR|$  okay. So if I take  $I = 1$  so  $1 \times 2$  will give me  $2/6$  floor function give me 0 so if 1 has to be connected to 2 so I have connected to 2 I can take it for value 2 to bigger connects to 4, 3 will connect to 6, 4 will connect to 8, 5 will connect to 10 and when I go to 6, 6 will connect to  $6 \times 2 = 12$  and  $6/6$  so now the floor function will give me 1.

So  $12+1$ , 13 QR is 12 and is a  $|QR|$  operations  $13|12|$  will give me 1, so the 6<sup>th</sup> one gets connected to 1, okay. 7<sup>th</sup> one will get connected to 3 and so on and the 11<sup>th</sup> one will get connected to 11<sup>th</sup>. So this how the shuffle is actually is been represented mathematically,  $Q = 2$  is

a simple one this we have come and use this kind of thing in the when we play the game of cards but game of playing cards actually.

We can actually try even for 3 shuffle or 4 shuffle or 5 shuffle kind of things also can be done, so idea is basically you will take multiple objects, okay and then of course for example I can take 12 of them in this example same the way it is actually  $4+4$  yeah is a total 12 objects I can create 3 groups is a 3 shuffle actually now I am talking about. So R is now in this case will be equal to 4,  $4 \times 3$  QR objects will be 12.

And I will be able to map it to again 12 objects and they are numbered from again 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 and 11 so I can use the same formula only think now q will be equal to 3, r will be equal to 4 so once I do this 0 gets mapped up to 0, 1 will get mapped to  $1 \times 3$  actually so I am now skipping 2 and then connecting it to the third one that is what we are doing, second one will get connected to, so we have to skip 2 and then connect to the last one sixth so  $2 \times 3$  will give me 6 and of course  $2/4$  the floor function gives me 0, so  $3 \times 3$  so again I have to skip 2 and 3 will give me here 7,8,9, 10 and 11.

When I come to 4 so  $4/4$  will the ceiling function will give me 1,  $4 \times 3$  is 12,  $12+1$  is 13 modulo of 12 qr objects will give me 1 so 4 gets connected to 1, so I give basically you start connect with 0 you skip 2 connect with third, you skip 2 connect with 6, skip 2 connect with 9, skip 2 next is already connected so connect with 1 that is what it means, now you skip 2 this is free so I connect fifth to this one, you skip 2 this get connected to this one, so you can actually even count from there that  $4 \times 3$  is 12 so it is goes to 1,  $5 \times 3$  is 15 and  $5/4$  floor function gives me 1,  $15+1$  is 16 modulo qr, qr is 12 so it becomes 4 so 5 is connected to 4.

Similarly  $6 \times 3$  are  $18+1$  19 so it trans out to be 7, 7 will get connected to 1,2 and so 7 will get into 10 8 will get connected to we count from 11 and then of course I will skip one more this 1 I will not count this 2 so this will be connected to 2 it will get connected to,  $9^{\text{th}}$  will get connected to  $5^{\text{th}}$ ,  $10^{\text{th}}$  will get connected to here,  $11^{\text{th}}$  will get connected to here and that is the three shuffle, so similarly I can define four shuffle, five shuffle and so on. There is an alternative expression also which is feasible.

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$$S_{qr}(i) = \begin{cases} qi \bmod (qr-1) & 0 \leq i < qr-1 \\ i+qr-1 & i=qr-1 \end{cases}$$

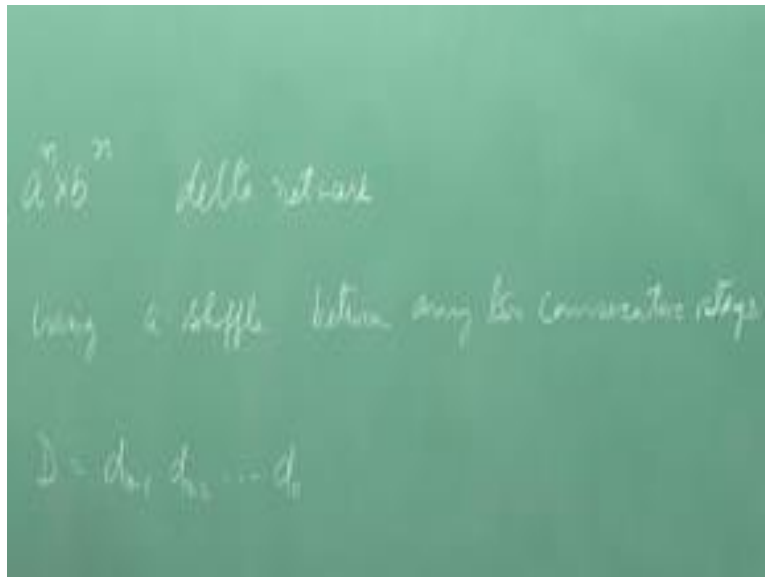
$$S_{qr}(S_{qr}(i)) = i$$

inverse

So I can write  $qr$  objects is a  $q$  shuffle can also be written as  $qi \bmod (qr-1)$   $m=i$  when this for 0 less than and when  $i=qr-1$  this another way of expressing the same shuffle thing. So I am not actually adding this floor function here, so it is a modulo  $qr-1$  sorry this has to modulo  $qr-1$ , so we can actually even tested from here so look at the three shuffle, the first object  $1 \times 3$ , 0 goes to 0 1 goes to 3, 2 goes to 6, 3 goes to 9, the fourth  $4 \times 3$  is 12 modulo 11 which becomes 1 so actually fourth gets connected to 1 and that is what is happening in this case from here to here, okay.

So this is an alternative form of putting it, and of course there is one property which we can actually prove that  $S_{qxr}$  I do it  $S_{rxq}$  this should be equal to I, so remember this is  $r$  shuffle of same  $qr$  objects and this is  $q$  shuffle, you end up and getting back to the same thing. So we can actually now use this formulation I will typically use this formula to actually prove that shuffle net or shuffle inter connection actually leads to self routing so it is a kind of  $\delta$  network so we will be able to prove that having similar inter connection between consecutive stages lead to a  $\delta$  network if given a certain number of stages. So we have to show that  $a^n/b^n$ .

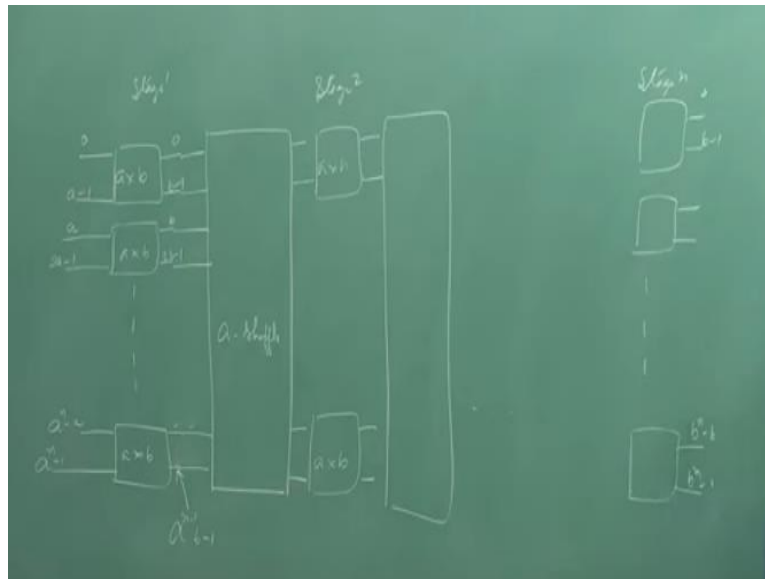
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Delta network can be cosseted using a shuffle between any two consecutive stages so the network will be of  $n$  stages and between any two consecutive stages I am going to use a shuffle and what I will get a delta network we will be able to constructed destination address  $d$  will be now representing by a number this number will be consisting of because in every enthuse stage I will be using one of these destination digit.

So it has to be base  $b$  digit so I will be using numbers which will be  $d_{n-1}$  in the last stage  $d_{n-2}$  do this will be in the first stage this digit will be used and so on. So that will be the destination, so let us see what will we will be doing and normally the way we will be representing this switches will be like this.

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I will have  $a/b$  I will count from 0 to  $a-1$  these are the first switch the next one will be starting with  $a$  this will go to  $2a - 1$  again  $a/b$  switch the last one will consist of  $a^{n-a} a^{n-1}$  this will also by  $a/b$  the switches inputs will be numbered in this passions from top to bottom I will also will numbering these outputs correspondingly in the same passion there will be number from 0 to  $b-1$  this will be  $b_2 2b-1$  and last one will be  $b^{n-b} b^{n-1}$  and then I will be now connecting a shuffle inter connection here.

So these will be number from 0 to  $b^{n-1}$  I will be doing a shuffling here and I will get the out puts okay and these again will be connected to  $a/b$  and so on okay and then we will have again a shuffle inter connection network and so on till we go to the these the first stage one stage 2 ultimately there will be the stage  $n$  and they will be going from okay sorry this will not be  $bn-1$  it will be different number this one will be that is what 3will be the number okay sorry this will not this here you will actually have 0 to  $b - 1$   $b^{n-b} b^{n-1}$  and each one of these will now we represented by a  $n$  digit number each digit will be a base  $b$  digit which will be used as an declinational address so now the thermo actually which go's I like this  $a$  is power  $n$  and  $b$  is power  $n$  delta network which uses a sufferable as inter stage like Patten between any costume stage's can

connect any input to any output even by this by switching the input to output and output port  $d$  – I at each stage I so where the labels are been assume away are

Be showed going from top to bottom okay in the first stage this particular digit in second stage I will be using this last stage this particular digit for deciding which output the packets should be routed and once this is done I will able to create a safer output.

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$$\text{Source } S = (s_{n-1}, s_{n-2}, \dots, s_0)$$

↑  
base a

$$D = (d_{n-1}, d_{n-2}, \dots, d_0)$$

↓  
base b

So let us start with the proof so source has now will be return as  $s_{n-1}$  as  $s_{n-2}$  so on remember each one of this is base  $a$  that a digit in that system base digit there will be in such because I am actually using each one of them  $E$  power  $n$  representation so there is  $0$  to  $a-1$  total  $n$  digit so a is power  $n$  inputs can be uniquely undefined by this particular digit similar I actually representing destination  $d$  as which I have return there  $d_{n-1}$   $d_{n-2}$   $d_0$  this will be based  $b$  system okay.

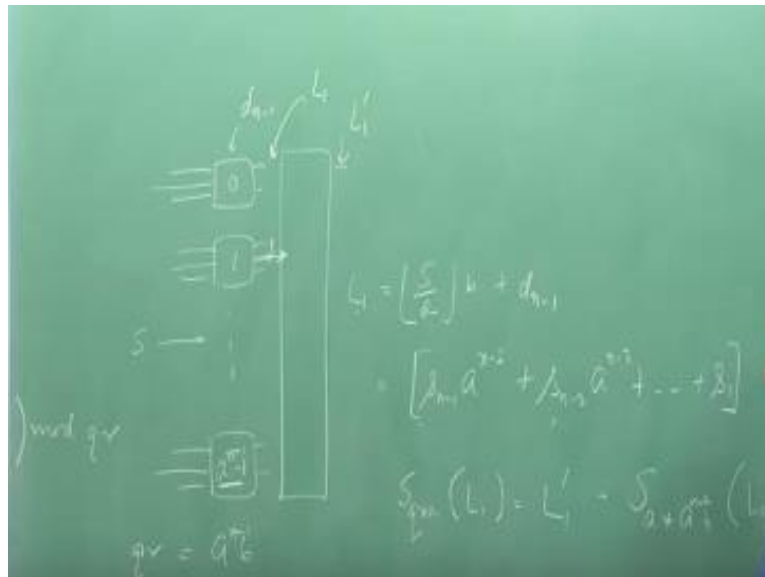


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$$S = \underline{s_{n-1}} a^{n-1} + \underline{s_{n-2}} a^{n-2} + \dots + \underline{s_1} a + \underline{s_0}$$
$$0 \leq s_i \leq a-1$$
$$D = d_{n-1} b^{n-1} + d_{n-2} b^{n-2} + \dots + d_1 b + d_0$$

So in fact the source  $s$  now can be returned as a raised power  $s_{n-1} a^{n-1} + s_{n-2} a^{n-2} + \dots + s_1 a + s_0$  so if I count it in decimal number this  $s$  the count from the top to bottom so I can represent this base  $a$  digit number to decimal number using this expansion formal which is a stranded form so this what represents the input in this case similarly the  $d$  the digit which is there so I can decimal this will be now  $d_{n-1} b^{n-1} + d_{n-2} b^{n-2}$  and so on till  $d_1 b + d_0$  okay so in this case  $0 \leq d_i \leq b-1$  will now change from  $0$  to  $b-1$  in the first stage.

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In the first stage I am actually using  $d$  of  $n - 1$  for making the decision so if my source is somewhere here somewhere in between I call it as which is represented by this number so this number will be now connected to which one of these switches I need to find out after switching what will be the output number what is the number here what is a wire number from top to bottom when I am counting to which this packets will be routed okay so this will be now connected a switch which will be  $s/a$  the lower floor function so if  $s$  as a value from  $0$  to  $a - 1$  it is connected to switch  $0$ .

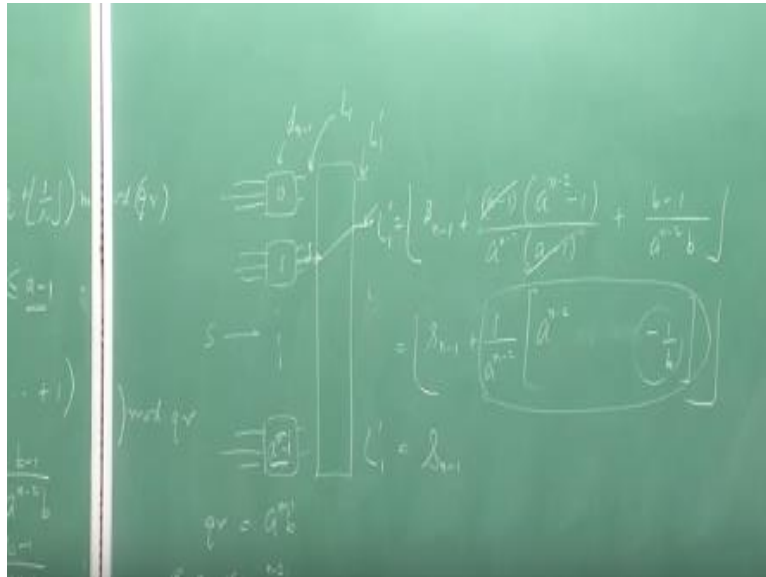
So I am counting from  $0$  to  $a - 1$  switch numbers and this will be  $a^{n - 1}$  switches okay so when I divided by  $a$  I will get the switch number so if  $s$  is say  $2$  as less than  $a - 1$  it is connected to switch  $0$  so this will give me the switch number multiplied by  $b$  so if I am actually here so  $b$  this  $b$  count will be there and remaining will be this number so which particular outgoing port will be decided by  $d_{n - 1}$  so the line to which I am connect I call it  $L_1$  so  $L_1$  will not be presented by  $s/a$  lower floor function into  $b + d_{n - 1}$  okay so  $d_{n - 1}$  because that is a digit being used for routing in the first stage.

So I can now expand it I can put this  $s$  from here and divided by  $a$  and get the floor function so I will get  $s n - 1 a^{n-2}$  I have divided by  $a$  remember so this is  $n-1$  will become  $n-2$  this become  $n-1$  this will go away and  $s0/a$  will be left in the end so I will also have  $s0/a$  and I have to take the floor function of this now this  $s0$ ,  $s0$  can take a value only from  $0$  to  $a-1$  look here and I am dividing by  $a$  so this value certainly lower than one it is a fraction rest everything will be integer complete, complete integer so when I take the floor function.

This will be going away so the movement I take floor function so this will no more be there this will not be there and a state so this whole thing multiplied by  $b + dn - 1$  that will be the number to which the input will be routed after the first stage okay now this is going to pass through a shuffle inter connection here so now I have to look into the what is the line number so I define this thing by as  $L1$  prime after shuffle inter connection so I can actually take this  $L1$  and  $SQR$  so total number of objects I need to know.

Based on that I will have  $R$  and I have to do a shuffle so total number of objects which are present here total switches are these so I have  $a^n$  number of switches into  $b$  that is a total number of ports which are there or lines which are being counted so  $QR =$  this and now  $Q = a$  so which implies  $r$  is  $a^{n-1} b$  okay so with these two parameters I can now take  $L1$   $n$  will give me  $L1$  prime okay so in fact now I can write  $s \times a$  since it is a shuffle  $r$  now will be given by  $a^{n-1} b$  sorry this has to be  $a - n - 1$  so this has to be  $a - 1b$  so this will in turn will be  $n - 2br$  so this has to be,  $n - 2$  okay into the  $L1$  so we need to solve for this.

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So this will be so  $L_1$  prime will be  $L_1 a$  I am going to using the shuffle formula which was as into QR I will use this so it will become  $L_1 a$  is a A shuffle and  $i$  is  $L_1 + L_1/a - 2b \bmod$  total number of lines which is  $a^{n-1} b$  so this is what will be  $l_1$  so the number of line so as a shuffling this will be now routed to somewhere here so I am trying to find out that line number when counted from top, so let us solve one by one so let me take this floor function first, what is the value of this, okay. So  $\lfloor \lambda_{n-1} / a^{n-2} \rfloor$  so what is this, so this remains floor function  $l_1$  is now  $(s(n-1) a^{n-2}) b + dn - 1$ , okay. This I have got from there.

I have to divide this whole thing by  $a^{n-2} b$  and take the floor function so when I do this I still have to keep the floor function first, so if I am going to look into this particular thing B will cancel I will divide by  $a^{n-2}$  in fact  $s(n-2) a^{n-3}$  okay plus if I look here  $a^{n-2}$  is here  $n-2$  is here I can very well now cancel this term so this will go out and I can only use I can convert into this thing, now interestingly what is the maximum value of this  $s_{n-2}$ ,  $s_1$ ?

If I look at the definition as I can go from 0 to  $a-1$  so this is a maximum value which is feasible so let us put the maximum values here, so I am let us looked this one is complete so this will be actually certainly will be there in the floor function I have to only check for these noe, so if I put

$a-1$  here,  $a-1$  here for all the terms let us put the maximum value of  $dn-1$  which will be  $B-1$  here, okay and then let us compute the whole thing.

So on computation so for this part we will have  $a-1$  which will be coming  $s$  is out plus  $a^{n-3} + a^{n-4} + 1/a^{n-2} + b-1$  which is a largest value here, okay. So now for this series what is the value we have to figure out, okay? So this series consists of how many term there it is  $n-2$  terms actually  $n-3$   $a^0$  so this is nothing but a geometric progression and I can replace these thing by so series will be first term into this is the multiplication factor for every term.

And total number of terms is  $(n-2) - 1$  divide by whatever is the factor  $-1$ , okay. So that is what you will get plus  $B-1 a^{n-2b}$  so I can now take this okay I can write it on the top may be here, so this value will turn out to be so this cancels with this one so I can put this whole expression here in this and I can write this  $l1'$  as floor function  $s(n-1)$  so this one actually cancels out you will have this.

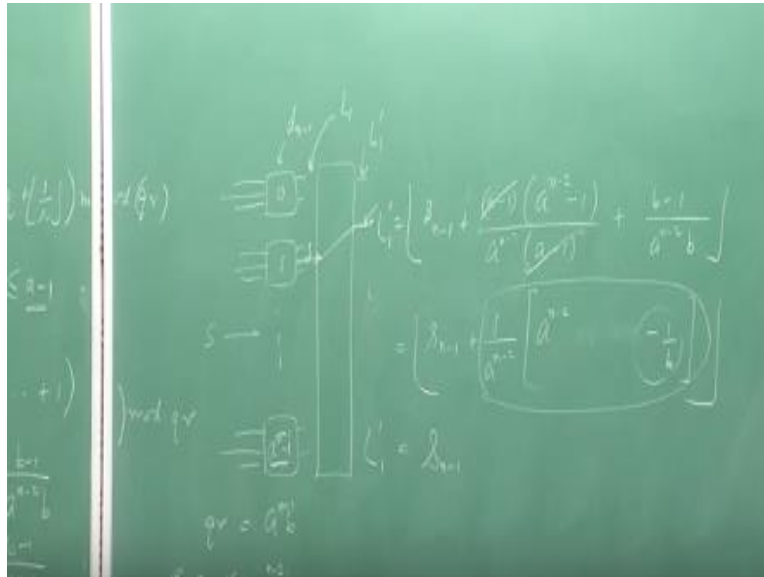
And this certainly if you compute if you look into this aspect the numerator is smaller by this much amount and denominator is  $a^{n-2}$  so this has to be a fraction so when I am going to take a floor function I should result into nothing but  $sn-1$ , okay. So that is what will be the  $l1'$  sorry that is not  $l1'$  that is only this particular component not  $l1'$  sorry and this is only this component  $l1$  by this value so I can now replace this thing by.

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$S_{n-1}$  which comes from there.

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So now let us expand this particular term, so  $L_1$  we had already arrived at so  $L_1$  will be  $S_{n-1} \cdot a^{n-2}$  that is what was the  $L_1$ , okay. so that is  $L_1$ , okay so this is  $L_1$ ,  $L_1$  has to be multiplied by a plus then I had to do  $S_{n-1}$  so this whole thing modulo  $a^{n-1}b$ , okay so I have to multiply by a all throughout so it will become  $S_{n-1} \cdot a^{n-2} \cdot a$ , and this will also be  $a$ , so I can now remove this brace one extra which has been put. So now this  $b$  can be multiplied with everybody, now since I am now doing a modulo operation of  $a^{n-1}b$  so this completes so this term when doing modulo operation will be out actually.

So I can neglected this I can remove this, this will not be of any consequence I need to only look into this particular term. So maximum value of now I need to find out the module let us see what modulo I will get, let us put the maximum values of all essence all destination now less and  $-1$  and let us see I am going to get, okay. So this one is going to maximum  $a-1$  so it will  $a-1$ , so first case value of  $L_1'$  I am trying to estimate, okay so I am only looking into this part so modulo part will look into later on.

So  $a-1 \cdot a^{n-2} \cdot b$  so in fact I can take it out  $a, b$  in fact  $b$  is I can always take out here, this one will be consisting of  $(b-1)a + (a-1)$  that is the term which is they are inside this so I am only considering

that, currently. So let us now compute this what I will get, so this is this one, so this will be now  $a-1$ , this is a series which is starting with  $a$ , so this will be  $a$  into  $a^{\text{th}}$  number of terms will be  $n-1$ , this will be into  $b$   $ba-a+a-1$ , okay. So  $a-1$  cancels with this so you will end up in getting  $a^{\text{th}}b-ab$  so  $-ab$  will be cancel with this  $a$  will cancel with this.

Sorry, this should be  $n-2$  terms this  $1$ , total  $a$  raise power not  $n-1$   $a^{n-2}$  terms so this should be  $a^{n-1}$ , okay let me just do it again, so that for clarity. So when I do this summation  $a-1$  goes as it is,  $a$  will be the first term number of terms here are  $1$  to  $n-2$  so  $a^{n-2}-1$   $/a-1$ .  $b+ab-a+a-1$ , so  $a-1$  cancels with this, this when I multiply will become  $n-1$  this will become  $a$ , so I can actually remove this brace and put a  $b$  here and this goes with this  $b$ , so this cancels with this, you have now ended up with the term  $a^{n-1}b-1$  so in worst case this is the value which you will get and there is a smaller than the operator the modulo operator here  $a^{n-1}b$ .

So when I do modulo operation I should get back the same value which implies that I have  $L_1'$  will be  $S_{n-2} a^{n-2}$  so  $b$  I think I can actually keep later on  $S_1 a \cdot b \cdot d_{n-1} a + S_{n-1}$  so that will be the  $L_1'$ , okay. So this is the value which we will get here at the after the first shuffle exchange, I can actually now use again the switches and find out what is the value of  $L_2$  so I can use this and find out what is the value of  $L_2$  so let us do that,  $L_2$  will be now I will be this switches will be doing routing on the bases of digit  $d_{n-2}$  I have to find out which switch multiplied by  $b$  and then of course  $+d_{n-2}$  so  $L_2$  will now be given by  $L_1'/a$  the floor function into  $b+d_{n-2}$ , okay.

So this will be, so I can take the  $L_1'$  from there and put it so I am dividing by  $a$  so it will become  $S_{n-2} a^{n-3}$  and  $S_1 d_{n-1}$ , so I have already divided by  $a$  all throughout, so this  $a$  is gone so this cancels with this you have no ended up to the term  $a^{n-a}b-1$  so in worst case this is the value which we will get and this is smaller than the operator the module operator here  $a^{n-1}b$  so when I do module operation I should get the same value which implies that I have  $l_1$  prime will be  $s_{n-2} a^{n-2}$  so  $b$  actually I can keep on  $s_1$  of  $a \times b \cdot d_{n-1} a + s_{n-1}$  so that will be  $l_1$  prime okay so this is the value which we will get here at the after the first shuffle action I can actually now use again the switches and find out what is the value of  $l_2$ .



So I can use this and find out what is the value of  $L_2$  so let us do that  $L_2$  will be now I will be switches will be doing routing on the base of digit  $d_{n-2}$  I have to find out the switch multiplied by  $b$  and of course  $+d_{n-2}$  so  $L_2$  will be now given by  $L_1$  prime by a the floor function in to  $b+d_{n-2}$  okay so this will be so I take the  $L_1$  prime from there and put it so I am dividing by  $a$  so it will become  $s_{n-2}a^{n-3}$  and  $s_{n-1}d_{n-1}$  so I have already divided by  $a$ , all throughout so this  $a$  is gone  $s_{n-1} - a$  will be here.

Now all these terms are actually greater than one they will all become integer only this one is the fraction  $s_{n-1}$  can take maximum of  $a-1$  value so when I take floor function this will be going out so it will be  $s_{n-2}$  so since  $b$  has to be multiplied later on so this what you will get  $L_2$  after a shuffling you will get so basically at the output here you have a another shuffle you will get  $L_2$  prime here and you should get  $L_2$  prime let me write it here.

(Refer Slide Time: 50:20)

The image shows a chalkboard with the following handwritten mathematical work:

$$L_2 = \left( L_1 a + \left\lfloor \frac{L_1}{a^{n-2}} \right\rfloor \right) \text{ mod } (a^{n-2} b^2)$$

$$\left\lfloor \frac{L_1}{a^{n-2}} \right\rfloor = \frac{L_1}{a^{n-2}} + \frac{(L_1 a^{n-2} + \dots + 2b^2) + d_{n-2} b + d_{n-2}}{a^{n-2}}$$

$$\frac{L_1}{a^{n-2}} + \frac{(a^{n-2} + 0 + \dots + 1)b^2 + (b+1)b + (b+1)}{a^{n-2}}$$

$L_2 \times a$  so I am just using as  $q \times r$  formula so you have  $a^{n-1} \times b$  number elements here same will be was present here so number of switches which was there  $a^{n-2} \times b$  so number of lines which are going to be here is  $a^{n-2} \times b^2$  okay so that is the number of  $qr$  objects which are present so we will

be actually use that  $qr/a$  so  $n-3$  and this has to be the module operated over  $a^{n-2}b^2$  that is the  $qr$  okay so we can actually do the similar exercise.

Okay so now let us look at this particular factor as usual has we did last time so we will have  $11/a^{n-2}b^2$  with floor function, so I can write down whatever the value of  $l_2$  which we have derived and then divided by this so let us do that so I will have  $s_{n-3}$  sorry  $s_{n-2} a^{-n+3}b^2$  so you just expand this whole term and we will divide by whatever is given here  $a^{n-3}b^2$  so that was for the first one, then we keep on expanding this is  $b^2$  and I have to divided by this is  $b^2$  and I could divide by, so this one actually cancels out, so this one is complete number.

So when I take the four function, this will certainly contribute, I have to worry about only this one, so here again I will do the same thing, I will put as  $i=a-1$  and  $d_i$  maximum value is  $b-1$  and let's compute what I'm going to get. So I will end up in getting a series, so all this  $a-1$  will come out, I will have a series of  $a^{n-4}, a^{n-5}+1, b^2$ , this will be  $b-1$  into  $b$ , this will be also  $b-1$ .

And then of course divide by  $a^{n-3}b^2$ , so this series, some actually have terms, total number of terms will be  $n-3$ , so if I do the sum of this, this will turn out to be, nothing but  $a^3n-3$  terms  $-1$  divided by  $a-1$ . So I can just replace it here, this will cancel with this, actually remove this one and  $b^2$  gets multiplied to both sides  $-b^2$ , so this also I can now write  $b^2-b$ , so  $b$  cancels with  $b^2$  cancels with this.

I have a term  $a^{n-3}b^2-1$  divided by this, this certainly is less than 1, which implies when I'm going to take this four function, I should only get this term, which is  $s_{n-2}$ . So I can actually use this  $n-2$  and put it here and let's see what's going to happen, and I can replace this by  $s_{n-2}$ , so I need to explain this one  $f_4|l_2$  prime, so I will put  $l_2$  from the there into  $l_a$ , so this will be  $s_{n-2}, a^{n-2}$ , so remember it was  $s_{n-2}, a^{n-3}$ , so I'm multiplying by  $a$ .

(Refer Slide Time: 56:48)

The image shows a chalkboard with handwritten mathematical work. At the top, there is an expression:  $L_2 = (L_2 a + \frac{L_2}{a^{n-2}}) \text{ mod } (a^{n-2} b^2)$ . Below this, a large fraction is shown:  $\frac{L_2}{a^{n-2}} = \left[ \frac{L_2 a^{n-2}}{a^{n-2}} + \frac{(L_2 a^{n-2} b^2 + \dots + L_2 b^{2n-2}) + d_{n-1} b + d_{n-2}}{a^{n-2} b^2} \right]$ . A large arrow points from the first term of this fraction down to a circled expression:  $\frac{L_2 a^{n-2}}{a^{n-2}} = L_2$ . To the right of this, another circled expression is shown:  $\frac{L_2 a^{n-2} b^2 + \dots + L_2 b^{2n-2}}{a^{n-2} b^2} = \frac{L_2 (1 + \dots + b^{2n-2})}{a^{n-2} b^2}$ . An arrow points from this circled expression to an exclamation mark (!).

And there is a  $b$  term, come at the end, this whole thing will be  $b^{2+dn-1}$ , so there is a  $b$  multiplied by  $a$ , because of this  $a$ , this also multiplied into  $a$ , this  $sn$  of  $-2$  has come because of, this term, and then I have to the module of  $a^{n-2}b^2$ . So once I do it, so one term which will come out as  $sn-2$ ,  $a^{n-2}b^2$  plus the remaining stuff, so you can see here  $a^{n-2}b^2$  also comes here, so when I take modular operation, this anyway has to go out.

(Refer Slide Time: 59:06)

The image shows a chalkboard with handwritten mathematical work. The top line is  $L_1 = (L_2 a + L_2) \text{ mod } (a^{n-2} b^2)$ . The next line is  $-(L_2 a^{n-2} + L_2 a^{n-3} + \dots + L_2 a) b^2 + d_{n-1} b a + d_{n-2} a$ . The third line is  $L_2 = (L_2 a^{n-2} + \dots + L_2 a) b^2 + d_{n-1} b a + d_{n-2} a + L_{n-2}$ . The bottom line is  $L_3 = \left\lfloor \frac{L_2}{a} \right\rfloor b + d_{n-1} = (L_2 a^{n-3} + \dots + L_2) b^2 + d_{n-1} b + d_{n-2} b + d_{n-3}$ .

Because this will become an integer, so I need not even bother about it, I have to just check whether what's the value of this,  $x^n$  value of this, so you can actually now do the same thing, for all  $s_i$ 's put  $x^n$  value is  $a-1$  for all  $d_i$  put value as  $b-1$  and solve and  $c$  what you're going to get. You will have  $a-1$  coming out of here, obviously this one will become,  $b-1$ ,  $b-1$  and  $a-1$ . So once you solve this series consists of  $n-1$  to  $n-3$ ,  $n-3$  terms this can be now return as, the first term "A" total number term  $(n-3-1)/(a-1)$ , so I can replace  $(a-1)$  will be cancelled with this  $b^2$  will come here, so I can write down this thing as  $a^n - 2b^2 - ab^2$ , this term will give me  $(ab^2 - ab)$  I can replace this. This one become  $(ab-a)$  "a" cancels with this, this cancels with this.

You have a term this  $-1$  which is smaller then this when you do modular operation you should get the same thing so the  $l_2!$  Will now be exactly whatever is left over here, okay, except this particular term which will go out because it turns out to be  $(a^n - 2b^2)$  it becomes integer multiple of that.

So  $l_2!$  Will be from there onwards, so I will have  $(sn-3a) (n-4)$ . And from there  $l_2!$  Which is here I can actually again have the set of switches and then get what's the output of that so I am

interested what  $L_3$  is!  $L_3$  actually, so  $L_3$  will be now  $L_2!$   $/a_{b+bn-3}$  so this will turn out to be you divide this whole thing by “a”, and then take the floor function.

So when you divide everything by “a” you will get, this become  $(n-2)$  this become a there “a” will cancel here only thing which not integer is  $(n-2)/a$  because  $(sn-2)$  can take maximum value of  $(a-1)$ , which implies when I take the floor function this will be going out rest everything divide by “a” will present here. So I can actually do that  $(S_{n-3}) (a^{n-4}) + \dots$  so on  $(s_1 * b)$  it has to be  $b^3$  now plus these “a” going out now, I will have  $d_{n-1}b^2 + d_{n-1} - 2b + d_{n-3}$ .

So now I can actually use i have proven this particular expression for 1 that what is the value for 1 what will happen for 2 what will happen for 3, now I can use induction to prove it for if it is true for “I”, it is going to true for  $(i+1)$  and henceforth i can do it what is going to happen after the  $n^{\text{th}}$  stage.

(Refer Slide Time: 01:04:31)

The image shows a green chalkboard with handwritten mathematical expressions. The top expression is  $L_i = (d_{n-i} a^{n-i} + \dots + d_1) b^i + d_{n-1} b^{i-1} + d_{n-2} b^{i-2} + \dots + d_{n-i}$ . Below it is  $L_i' = (d_{n-i+1} a^{n-i+1} + \dots + (2a^2 + 2a) b^i + (d_{n-1} b^{i-1} + \dots + d_{n-i}) a + d_{n-i}$ . The bottom expression is  $L_n = \underline{d_{n-1} b^{n-1} + d_{n-2} b^{n-2} + \dots + d_0}$ . There is a small note  $(a^{n-i} b^i)$  on the left side.

So it  $L_i = (n-i a^{n-i-1} + \dots + s_1)$  you can actually read see from here  $(n-i, n-3-1)$  from there itself you can derive it last term will be  $s_1$ . Since it is 3 it has to be  $b^i +$  you will be having  $(d_{n-1}b^{i-1} + d_{n-2}b^{i-2} + \dots + d_{n-i})$  .okay. So once you know this you can always find out what is going to be  $L_1$ ! Using the same procedure and using same kind of conditions in the maximum values in the “s and d’s” so this  $L_i$ ! Should be  $(S_{n-i-1} \dots a^{n-i-1} \dots d_{n-1})a + s_{n-i}$ , and of course once we know this we can find out that what will happen after the  $n^{\text{th}}$  stage what will be  $L_n$  ,so this can be actually proven using same procedure .

So once you have  $L_n$  will now be given as all these terms will vanish and you will end up in that so you will have  $d_{n-1}b^{n-1}$  ,okay,when  $(i=n+d_{n-2}b^{n-2}+d_{n-3}b^{n-3}+\dots+d_0)$ .Now you can clearly see this number is nothing but the output address ,you started with this you connected from any input port any source port ,you keep on doing the same procedure basically using at every switch one of the digits here will be used for outing.

So first stage this was used second it was used last one this was used you ended up in going to the same output so this shuffle net is indeed a delta B network ,okay, so that’s a proof which we have.

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