Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Digital Switching

Lecture – 15

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So we have done the clos theorem this say that for strictly on blocking switch r2 h2 greater than or equal to m1 +n3-1 and of course if the switch is symmetric m1 should be equal to n3 is equal to n so it will be 2n-1 so that was in the last video now this particular condition is not the condition if I'm going to look into multicast scenario okay so before I go to the multicast scenario let me set up a condition and show that yes it does not happen so we will actually take up an example first to simply whatever I have though clos theorem okay so and then of course we will come over to the multicast case and we will try to simply with multicast case this whole theorem.

Actually breaks this same condition is not god enough and then we will try to see if how we can determine a condition for multicast scenario so I will get a cured bound there are two three more estimates which are there so we will not do that thing in detail but you can look at the papers which are but on the brass site which has been listed in the forum and then actually go through those papers to understand how those conditions are been derived even though such papers so I will give a crude estimate of FA multicast.

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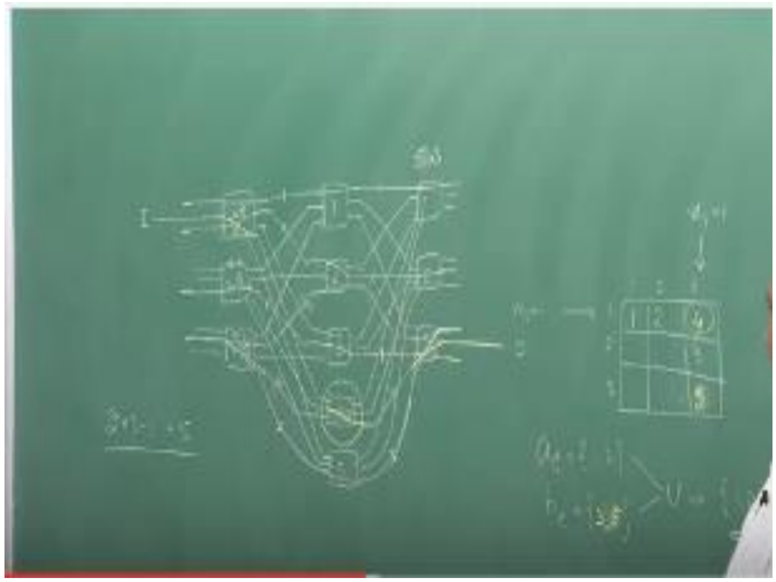

So before that so let up a small switch it is a 3/3 and show you that when I will be able to set up a connection and any path which have been already set up should not be disturbed okay if I am allowed to do any arrangement I can actually do at much lower thing so it will be our next theorem so let us put three switches here so I put everything 3/3 so this also has to be 3/3 now so it is a 3/3 multiple three stage switch so 9/9 basically clos now let us see when the switch when the blocking will happen.

And I will keep on increasing this and ultimately every time I will show you that there is a blocking and when this condition of 2x3 -1 which is 5 when my number of middle stage switches will become equal to 5 I can always set up a path between free incoming and free outgoing port I will just create worst case scenarios so the case here is very simple I want to actually have I here I will also draw corresponding Paull's matrix okay.

So this is the output which I want to connect now can I crate a scenario where the input and output cannot be connected so I have to just create that scenario entire corresponding Paull's matrix so I will have a switch identifier is 1, 2 and 3 here okay I am only worried about the row corresponding to this switch and the row correspond the column corresponds to this one so again these are 1, 2, 3 so the Paull's matrix here will look something like this it will be 3/3 matrix so this is the switch 1, 2 and 3 and this are in the third stage 1, 2 and 3 okay .

So I want to set up a path so if this path as been set up in this fashion first is are I am just going to create a blocking scenario let us see what will happen the second one is being occupied through this okay the third one is can be in this fashion and so the connections the way they are actually happening let me us a colored chalk so this connection is connected already so this lines are occupied okay so from this side these two are occupied the third one is which is free which can used to connect this input to the output.

Okay so only switch which can be uses this one but what I have done is have connected this output through this to this one okay so I have consumed this particular path which was required this was only path through which you could have setup this I to O connection so there is a blocking which has been created okay so now you look these are the1 2 these are 1, 2 there is only 1 which is over lap there is no free middle switch which can be used to connect input and output what is the corresponding Paull's matrix so Paull's matrix from 1 I am connecting to 1 wire 1 so I will 1 here from 1 I am connecting to 2 wire 2.

So I will put 2 here the third is being connected to third via 2, third is being using third to connect via the third one to the second one so three is here, only elements which are available are 1, 2 and 3 which are the middle stage switches, look at the row they contain M1 – M1 in the worst case scenario so which is 3 – 1 which is 2 elements 1 and 2, okay. There are actually 3 elements but 1 and 2 I have been consume so 3^{rd} one was available.

In the column, since N3 = M1 so its N3 is 3 so you again 2 elements have been occupied but this time it is 2 and 3, I need to put something here. So do I have a free element I have only three elements 1, 2 and 3, so union of this which is you're A_e in this case 1 and 2, B_e in this case is 2 and 3. So union of these two will give me 1, 2 and 3 I had do not have any free element to put so connection cannot be set up, is a blocking switch.

Now if I increase the number by 1 in this example I can make it 4, now interestingly the switch is not no more going to be 3/3 it will be ¾, ¾, ¾ this will become all 4/3 switches, okay. Intermediate switches will still be 3/3 so this will also be 3/3 but there will be four elements now. So I have to now connect from here in this fashion, now can I still create a blocking system here or not?

So let us see what I can do is, I have 1 and 2 as been occupied here so if this by chance is not using this particular path but it may decide that I am going to set up a path in this fashion from here is all the way here and then in this fashion, so what I will have is, I will have this has occupied link this is occupied here. So the changes in the Paull's matrix now I can have 4 elements in the Paull's matrix, okay.

So I will have 1 and 2 here and this will contain instead of 2 this will become 4, so I have 1, 2 and 3,4 so union now consist of 1, 2, 3, 4. Oh1 in this scenario again I cannot set up a path between the two, okay. Reason being again my number of elements are not satisfying the Clos condition, I can increase one more now because that is the limit which comes from the Clos theorem so if I can put another one which is a 5^{th} element.

So each one of this switch is now will become 3/5 this will also become 5/3 this is still remain 3/3 but 5 switches, so I can now connect in this fashion. So now since these two and these two are do not have any overlap even if this is actually start try in any way to occupy. So if I actually instead of here I want to connect this one through this last one then I will free up this one actually then I will end up in connecting this.

So there will be always one free switch which is available which can be used for connecting input to output I will be able to make a connection if I have 5 what does that mean? I have 1, 2 of course the last fifth is being used so it is the 5 so it will become 3 and 5 which have been occupied UN consists of 1, 2, 3, 5 there is a element 4 which is available which can be put here that is what I have done.

And total number of pattern this is now satisfying the condition given by Clos theorem, okay. And this actually gives us example this is a proof that yes Clos thrown thus whole true, so you can actually try different example sin C and you should be able to create the blocking system if my number is not matching this, the moment your R2 is less than this you can always create a scenario whereby switch is in the blocking state.

So I can take up one more example this time I am doing x-temporal here itself I will take it dimensions which are different and prove that I can create a blocking system. In fact if you are clear about the theorem intuitively you should be knowing that how this has to be done, okay.

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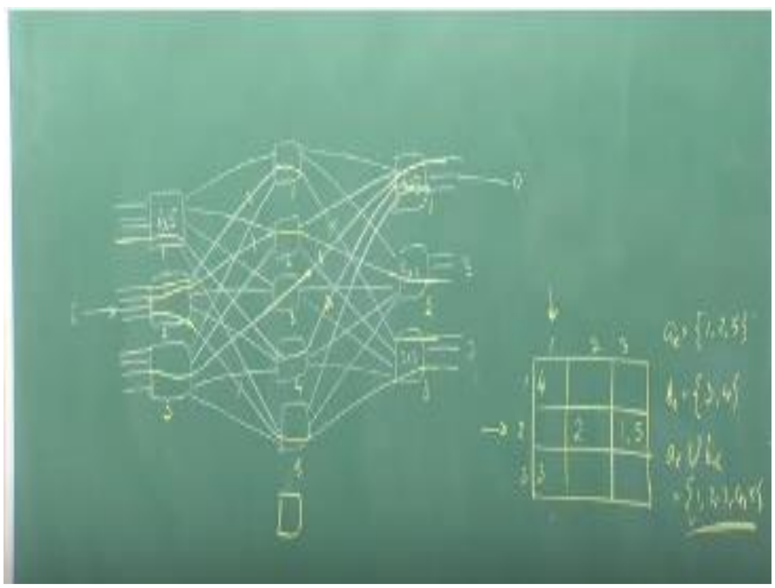

So let us take a small example, so this time I am taking 4/4 I have taken a symmetric system here at their this time I am taking a symmetric system, okay. So I am going to use 4/4 3 switches, okay. And this time I am using here on this side not 4/4 not 4 inputs but say 3 inputs, okay. So I need some middle stage switches so as per clos theorem I require 4+3 which is 7 minus 1 atleast 6 which is will be required. So I can put 5 of them so I do not have 6 and I will be able to I should be able to create a blocking scenario with 5, okay.

So each one of them will be connecting here so the witch sizes 4/5 and this is 5/3 okay so okay let me do it actually completely so all metals switch are 3/3 all of these are 3/3 in this fashion so if I want to connect for example say this one which is the input and this one why I can arbitrary anything this is the output I have to just prove that there is a blocking condition here so is not a strictly opn blocking switch that is good on a fs proof.

So if I want to do that I will take these three and they can be connected to anything anywhere actually so this is occupied in fact I need not bother because I should only look at these links which are occupied this one is occupied it is okay, and for the output links which are immediately from here so next one I can connect where this and connect and when in coming anything so this link gets occupied and their total five links remember so third one I can take this so this link get occupied.

Okay the only these two are free so there only two links which are available here so I need to just prove that the two links which are connected to this only these are the possible links which can actually be use to setup the connection of these are somehow occupied than it is not possible so I can set up the connection between already existing path so this gets occupied to some path here I can use other one to connect where this to this one now you can see these two are occupied and these three are occupied there is no free intermediate switch by which.

A connection can actually be made this a blocking system and how the Paul's matrix will look like so in fact you can actually do it for any arbitrary system in this case it is a 3/3 Paul's matrix you will have I call it number one two and three I call it one two and three okay these switches are one two three four and five and they do not satisfy the Clos theorem so you can actually look at I am looking at this two versus one so two is already occupied to as actually connected where one and this is for the third switch.

Two is connected where as is to for the second switch two is connected wire of five to this stage three switch and look from this one side in the column the two elements which are missing here see you cannot have more than three in this case m1 – 1 so from here it will be again n3 – 1 which is two here so this one is getting connected wire three to three and the next one it is connecting wire four to one so set e in this case is one 2 and 5 be will three and four union of these two one two three four and five no free element available you cannot set up a path from input to out so that is again a blocking system the movement I had one more six and this is the first case scenario remember these two are connecting to.

These switches these three are connected for different and their some total is equal to total number middle switches I created one extra which is free now this can always be use to set up the path and when I put it b = 6 this satisfies the Clos condition so it will be 4+3 m1+n3 – 1 4+3 is 7 – 1 which is 6, 6 number of switches it will enable the path to be set up but remember in real life all we do no0t actually create is it known blocking switches we always create blocking switches.

Control blocking so the amount of hardware you say want the switches hardware and with limited blocking you reduce the cost of switch so now let us back to the multicasting scenario after this example so in multi casting what is going to happen so let us understand how the multicasting happens in a three state system.

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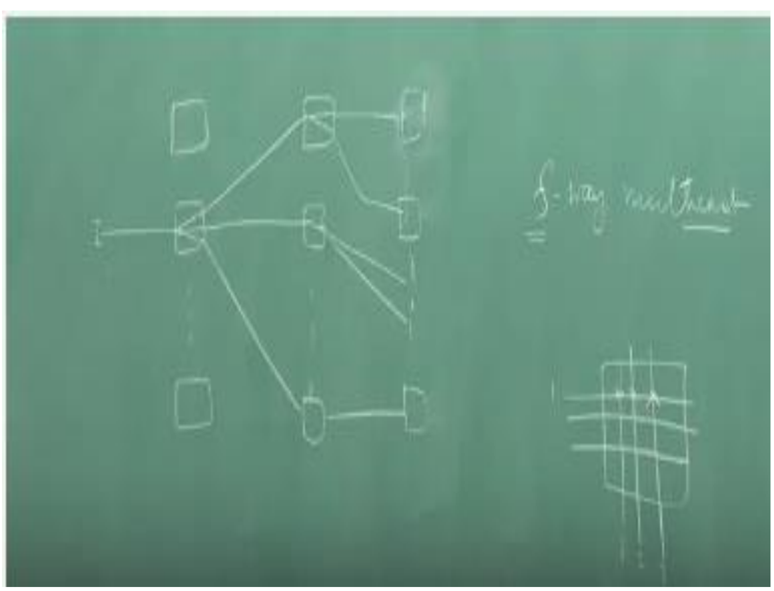

So now let us look at the equivalent of a Clos theorem for a multicast system so in this case first fall we have to understand when a multicasting happens how it happens in a three state system so when we build up a switch so normally a switch will have the first stage the second stage and third stage that is a Clos configuration so how the multicasting will happen in this case so multicasting basically means there is one input and I would like to connected to multiple outputs so where we have only one transmitter.

The multiple receivers now if you look at here from one line I will be able to get the signal from the input here and I can because is the broadcast is a cross bat system when a cross bar when I want to do a multicast from one to say call ports I just snap all the cross points together so the duplication are multiplication in a multicast system happens within the cross bar so one input can be fed can be feeding the outputs and so this the strictly non blocking system so I should not worry about the multicasting if the multiple inputs are here okay.

So for me even if their outputs multiple outputs which have to be connected to same input this is as good as I should only bother about that how I will be able to send the signal here so for me I will define something called a multicast the way that the FA multi cast we call it so this actually means the number of switches in worst case to whom in the output stage to whom I need to copy my input so that’s the FA multicast whit this number is F so I am giving a crude estimate of how many middle estimate switches required for implementing this of course this can actually where find further.

So in this case there is one possibility that i actually can take a signal here and split it here itself in to multiple copies send it here then I can do a split again and send it to the multiple places in this fashion in general this is what is actually going to happen okay so that’s the case which we need to essentially now use false matrix to figure out an estimate of how the FM multi cast is going to work okay.

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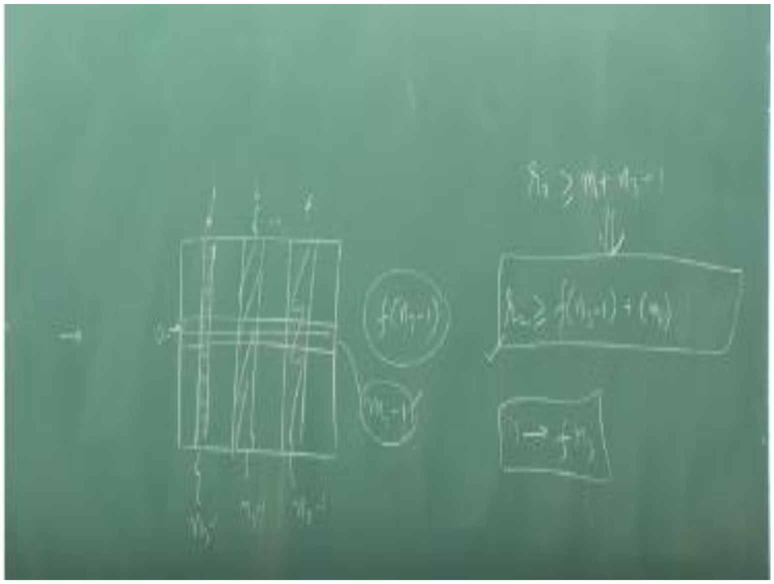

In this case if I draw the false matrix this is the a switch from where the input is multi casted now this can be multi casted to F different elements okay so these where various outputs in worst case the multi cause actually F separate output third state switches are output stage switches so 1 2 and F .

So what I need is i need I am not actually going to divide by signal here I don't want to do that I can certainly do that but i am not considering the case so if I permit that probably i can have a much more efficient multi casting so my bound actually improve in that case but I am not considering so this is not permitted you have to just find out once switch pass the signal; onto that and that will then connect to all F switches to whom you want to communicate.

You have to find out when all these lines actually will be free you have to find out that middle state switch that is our strategy as of now but in reality this actually estimate can be improved if part of these are permitted by this switch and part of can be permitted by other switch and we can actually better bound okay so this is possible but I am actually doing it in a very simplified manner so there f such possible output switches now how many elements which can exists here?

Remember the two inputs cannot combine into one single output so all the elements have to be distinct here.

So number of elements which can be there is n3-1 ,and if you take the first case scenario in that case number of element which are here all these elements don't have any overlap so once if you create the union of these and find out that cardinality that will be nothing but some of cardinality of each one of these okay so basically if you take intersection of one of these F rows F columns it turns out to be null .

So that actually means number of elements in all these F columns will be f*n3-1 ,okay and then of course a is also communicating so there is only one input port which is free so there will be remaining elements which can be here is m1-1 ,so if this m1-1 element and f*n3-1 element also don't have any overlap in that case the total sum of the middle stage elements which have been occupied are f*m3-1 +m1-1 ,but remember I am talking about, FA multi cast okay , so these many elements already occupied so I need one element there which can be put in all these F column or this cells ,it has to be repeated same element.

So in that case if I have one element extra I can just put it here and then my r2≥ this and i will be able to set up a FA multicast in this scenario okay it is independent of any other condition or any other multi cast which has already been set up okay I will always will be able to setup a connection this cases this is the first case scenario which I have put in fact they can be multi cast which have been setup by other inputs of this A but those will be bone already with a different element m1-1 distinct element .

Elements which off course might have been repeated but if we actually take only distinct elements the duplicate ones are been counted only once there will be m1-1 already be there present in this case okay so this is what is going to be my bound for a multicast implementation in a clos network, so the earlier condition we had m1+n3-1 ,so this condition gets converted into this one if you want to actually have a FA multi cast so this will become f*n3-1 +m1where f is number of output third state switches to whom the multi cast has to be done i am not counting the number of ports here okay .

So if all the ports are being occupied and multi casting is being done so from one input technically I can get into f*n3 number of ports that multi cast is technically feasible so 1 to f*m3 port that kind of map is possible okay so this have parameter essentially governs how many middle state switches will be required.

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