Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Digital Switching

Lecture – 10

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Okay so let us move forward from where we left in the last lecture so just to recall the last lecture was about the les's approximation which I did for three stage interconnection network which was clos configuration and the of course we also did a cross point complicity of clos network so those two things we did but lee's approximation is not precise it is error because we are approximating there so then of course I suggested that we can do more precise calculations but this not for time congestion but for the call congestions.

So let us actually do it we are actually considering here again three stage clos network only and we are going to this approaches.

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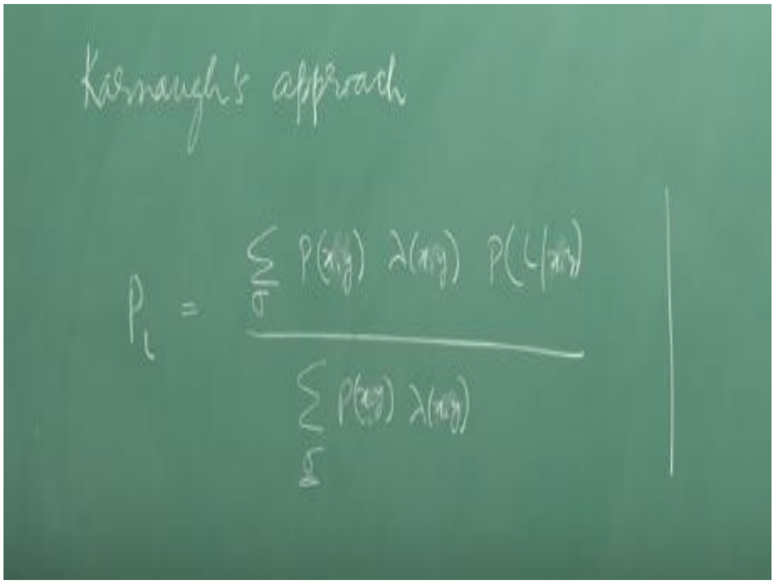

Attributed to Karnaugh's approach and this gives a beautiful result about the call lose or call congestion so in this case the P_L in fact remember P_B when I am actually writing it is always the probability of switch being in blocked state whenever it is P_L is a probability when a call arrives it is going to be lose it a conditional probability okay so P_L will now began as so this we are not computing so P_L we can write as I will tell what this Σ is.

Now σ is something which have now introduced this is known as this represents the state the of the switch now what do you mean by state of a switch, so if I have only one single switch what are the state for this one it can be either ON or OFF their only two possible states okay if all the cross bar of 2/2 each one of them can actually be ON or OFF independently technically there will be 2^4 possible states some of them are not valid states.

Okay so in fact and some of the states will correspond to multicasting scenario so I am not looking into the multicasting scenario here I am actually looking into uni casting case so that many possible states which can happen so normally if the number of cross points are C in general total number of states which will be there will 2^C and of course which input and output are active where also be counted for ram actually assuming that all inputs and all outputs are connected they are actually communicating.

So under that possibility there are 2^C possible states okay so this σ represents this but there is going to be large number of states we need to actually take care of these so we use concept of something non as equivalent states so what do we mean by equivalent state if a switch is in one part of say A and if it is in state B the probability that you are going to be in A or probability you are going to be B in is same.

The blocking which happens is going to be same so basically what you can do is the cross switches consist of a smaller switches if I can move them around I can just do a renumbering I can always go from a state A to state B by doing this transformation we will actually do an example of this while looking at while since non blocking switch and it is prove that time on will be actually doing this thing then these become equivalent states, so I do not need all this possible states to be taken care of when trying to computing this.

I can represent it by much a smaller number of non equivalent states we also go further ahead actually now how this σ can be represented I am coming to that but what does the P(σ) means P(σ) is the probability that your switch will be in state σ what do you may understand by λ σ. λ is the call arrival rate number of calls which are arriving at any point of time okay so this represents that the call arrival rate probability you are in this step average number of calls which are arriving per unit time when you are in a state σ now this is conditional loss probability when you are in a state σ what are the chances that a call will be loss when it arrives is that conditional loss probability for an arrived call.

And I submit over σ so over a long range of observation period this numerator will represents a number of calls which will be lost on an average okay because it is being now average over all possible states taking care of the probability of being in that state the arrival rates and the call loss probability now if you come back to denominator this represents the total number of calls which have been attempted so this also includes the calls which were not lost that total calls attempted average over all possible states by using this probability total calls attempted.

So this is the ratio of the failed calls by total calls and this becomes an estimate of call loss probability okay in general now question is how you will represent σ so for doing this I am going to actually take an assumptions so of the efficient known as the we need to take assumptions so one of them is random root hunting now the way it happens is whenever you are going to have switch which will consist of all these states all these switches in three stages typically when a call as to be set up the input and output ports are indentified then how the call will be rooted.

I will be setting up I will be taking up any middle state switch in setting it up I can do it randomly actually so if I take another input ports another input ports where I can take up any one of them randomly so when a call as to be set up the way the links are chosen they will be taken care of they will be done randomly that is what the random root hunting actually means, I will just pick up randomly we will try it out does not go through I will make an attempt again till I get a free call.

If it does not happen certain time I will simply discover it is possible I can always start from the top switch keep on doing it till I find out a free link certain connection everybody does that so most of the time the upper state middle state switches will get occupied lower one will not be so that ordered system or may be when I tried for the first call I use this one next one this one the convert through here and when the next one will come I will start from the next one so that can be happening in cyclic fashion but I am talking random root hunting advantage of this is I can represent the state of the switch by x and y they are independent of how the calls are setup at any point of time when I observe the switch this is actually same it is not changing so time earlier time state it does not matter actually.

So that makes it far simpler for us to do the analysis and I can actually take I need not take all the switches I am only worried about only one of the input ports so one switch here and one switch here we need to take now why we do it and number of links which are occupied here is x number of x which are occupied here is y but remember I have other switches here also so where I look at this combination so input and this input output pair that is independent of the earlier pair for example this A,C which I was talking and BD which I am talking.

So these are independent and there will be in the similar situation so if I compute by estimate for AC this is going to be true even for BD or any other possible pair so I am also taking care of I am also making assumption that it is an uniform kind of routing which is been used so when somebody wants to make a call it is going to make call to anybody else with equally likely probability so it is a uniform loading condition so under that condition the σ now can be very well represented any a pair x, y okay and I need not bother about anything else but accept one pair.

So which is A and C here and this number is x and this number is y and x and y will represent the state, so x can be 1 y can be 1 x can be 0 so all possible pairs and they can take maximum values can be K where 1, 2 the number of switches in the middle stage are K, okay. Under this condition we have to compute. So I can now modify my expression this will become (x, y) and now I think I can compute.

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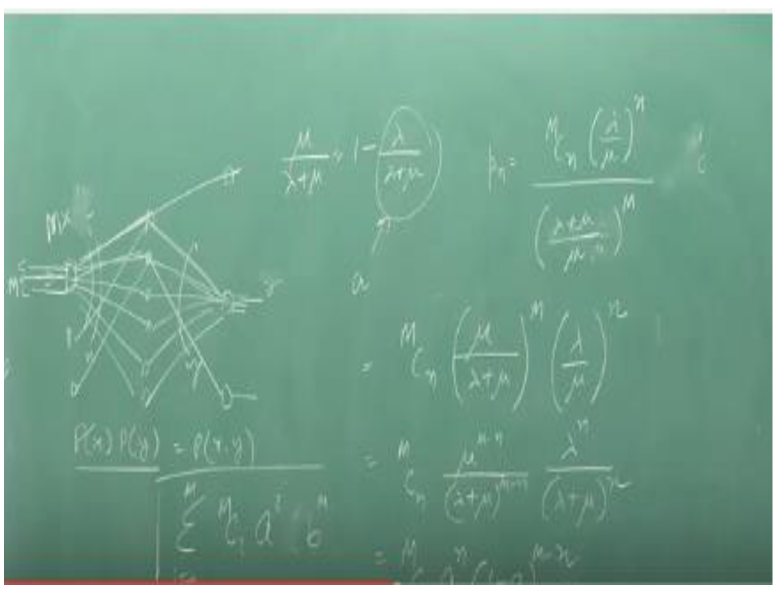

So now if I draw this so this one gives you x this one gives me y, probability of cal, arrival essentially this technically means that a probability of (x, y) that switch will be in a state (x, y) how this will be decided. X and Y are independent Y because when actually I am setting up the calls all the calls which are coming emanating from this switch are not terminating here there might be terminating here also they might be terminating somewhere here also.

So this x becomes independent of this y this outputs might get connected to the difference switches via this route. So y need not have any overlap with x so they can be independently varying in this case, okay. So the probability of x and probability of y can be written as the product of probability being in x and probability of being in y, that comes because of the independence.

Any two independent random variable they are join probability is nothing but the product of the probability of each one of those independently and that is what I have written, okay. So that is one important thing we have to estimate what this p(x) and p(y) also. Now next look at this P(l, x, y) how this will be estimated? So you have a switch here and you have 1, 2…kn and of course these arte A links and these are B links.

And there x calls which are progressing here there y calls which are progressing here, I have to find out the probability that call vector get lost if I want to set up a connection between these free incoming and out coming ports so this is a free incoming port this is a free outgoing port what are the chances that when you randomly select a pair you would not be able to find out a path, okay. So that is comes from a Jacobeans approximation this is also known as Jacobeans approximation. So let us see how we actually figure it out, now if my x + y this is the x number of links which are busy y number of links which are busy if x + y is less than K if this condition is true then what is going to happen?

You will always be able to find out a switch in worst case x are occupied y are occupied and they are not connected to the same middle stage switches in that case you can always find out something for which A link and B link both are free and I can use that to set up a path between from input to output, so blocking will not happen if this is true blocking will one can only happen when it is x + y > = k.

But when this x and y are actually being the links which are occupied A type and B type are happening independently we have to find out the chances that I may find out a path from input to output, okay. So let us see how this will be done, so let me take an example so I have three links which are occupied three connections and in total we have five so these three are occupied, three are occupied of from this side out of these five.

So these 3 can be like this so this one possible combination. Now you see I cannot set up the path set up a connection between a free incoming port and free outgoing port in this example. But as I said we are using random route hunting and this happens with the same probability as this there is a path available I can set up the connection, there is possibility that this will happen with the same probability.

So essentially if I freeze this that many possible ways this now can these three connections can actually be distributed across these five, in some of the ways you will find out you will not be able to set up the path in some of the combinations you can set up the path, so probability a call will be list or you cannot set up a connection, essentially now can be written as total number of patterns.

In which the blocking will happen divided by total number of possible patterns which can exist, okay. So this we can compute for this combination, now instead of this if this would have been this scenario and I would have allowed all possible patterns here I would have got the same ratio it does not matter is a same case. So I need not bother about x so I will just fix it x and this is y the remaining balance will be k – x.

And out of this y in how many ways this y actually can be arranged among these case middle stage switches this will be k come pictorial why, that is a total number of possible which you can arrange and of course now you have x occupied versions and k – y unoccupied once, so k – 1 y which are unoccupied once in how many ways they can be arranged across this occupied A links. That is the time when the blocking will happen, okay.

So for example when you have this possibility these are blocked there is another possibility this way this blocked if you do this is also blocked, so I am not trying to arrange the free once across this busy stuff so in how many possible ways that can happen, there was another number of blocked possibilities. So if I take this ratio for which will be xc_{k-y} this should give me the probability that your call will be lost given (x, y).

This is what is known as Jacobeans approximation. So I can solve this and we will get x factorial by k-y factorial, x-k + y factorial so in fact I can write this as x + y – k factorial and then of course k factorial, y! (k – y)! So this one will cancel and that is what with the probability x! y! x + y–k ! / k! now you can actually see x + y has to be greater than k than only this make some sense. Otherwise blocking does not happen, okay. So this will be the expression which we can use it for PL given (x, y).

Now to find out that probability that x calls will be here how to estimate this so I will you can actually go to the some of the earlier lectures we are have done m/n composite switch where I have done the state probabilities for a m/n composite switch now here important thing is that you will might have here certain number of inputs I call them m and this is actually k here so k = n so I can use same m/n composite switch result to find out P(x) so let see how that will come so instead of this m/n this is now m/k switch okay.

Okay let because my notes I have actually n so let me keep it n and we will solve from here at some point I will change the variables I will do the transformation for m/n probability that this switch will be in state n where n means outgoing n number of links are occupied and remember in this case n is greater than m okay greater than equal to m that is the case which we are looking at so I can write down t(n) probability of being in the state and outgoings are occupied if you recall from there this will be M_{CN}.

γ/μ λ is arrival rate μ is the departure rate or one over μ is the average call duration okay and this will be raise power n I can write any variable let me put i because k have been we have been using there important thing it was n M_{CN} sorry this will be i now one thing which we need to note that n is going to be larger when m so this has to be change it m actually okay and once I change it m because that is into maximum number which you can get you cannot actually have a state board than m.

So this is becomes a complete binomial so binomial expression is MCI a^i $b^{m-i}\sum$ I I goes from 0 to m this will be always return as a+ b^m so this is the binomial expression we can use this to solve this expression for P_n so I can actually at here one raise power m- i so this becomes a and this becomes b so a+ b so 1+ λ/ μ^m okay I can actually further solve it I can write this as $\lambda+\mu/\mu^m$ and I can take it up so this will M_{CN} let me write it down at the bottom.

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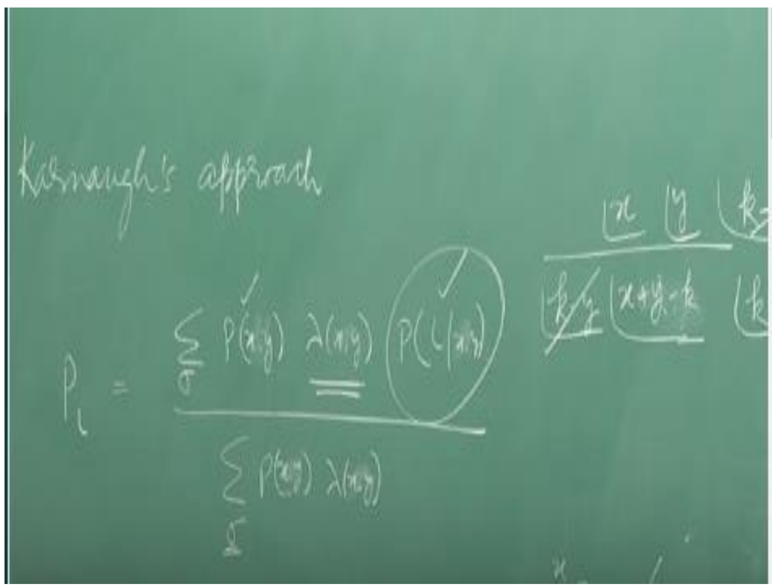

So we can write it as MCN so I just split it into m and n parts the bottom one $\mu^m - \mu^n$ comes here λn comes here now μ/ λ+ μ there is nothing but one – λ/λ+μ and I call this as e I will give the interpretation of it what does it mean it technically means loading of the telephone line or line which is coming here at the n input okay so if I do this so this becomes 1- a and this becomes a M_{CN} a^n that is what it will be the probability of x P_x now whatever is true from this side this is a symmetric system.

Then same should be true for p_y also okay because a technically when I am talking about p_y that is because of the calls arriving it all other input ports which are terminated at the outgoing line which I am currently observing and that should be same as this is the input line n all outgoing lines to which this call is being rotate p_x is because of that so both are symmetric systems so p_y as be equal to p_x and they both are independent so I can write down p_y also as M_{CN} $a^n 1$ – a m- n so p_x and p_y.

We have found out we have found out Pl given x and y which is this using karnaugh's approximation so we have got this now we have to find out this and we have to ultimately put it into the expression for getting my answer so in this case.

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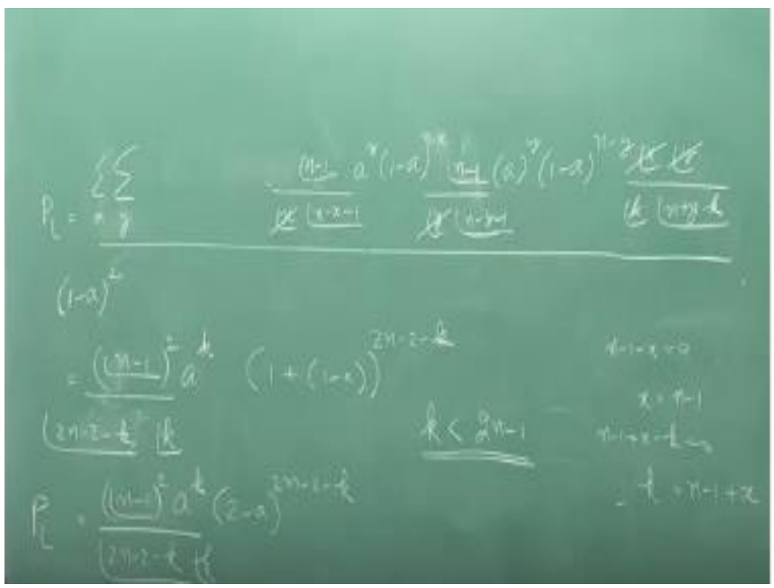

Now x λx is we talking about that is the call arrival probability so if I talk about the switch here x links are busy their total number of m inputs so what is going to happen is how many input links are free here it should be m – x so the call arrival rate per line is λ in fact I can call it λx so that will be the arrival probability similarly on the other side m outputs are there one of them is free which I am trying to connect.

Links which are basis y so how many are the free links available here is m – y so this is the arrival probability here so I can write this λx λy this λx is this, this λy so I can write λx λy so this can be written as δ m – y I am also got my third term so I have to just actually now use all these three to compute my expression so let us write down the expression and solve it so P(L) so I have to write down summation since α = x, y I am going to put I have to do summation over x and y.

This actually means I need to do summation of with one with x and one with y okay so forward timing let me put it this feel at some point I will just into double summation oh maybe now itself I can do it, it does not matter so sorry this is I am using γ so it will be γ now be very carefully what I am actually doing is I am doing now transformation of variables the middle state which is here rk n number here the number of inputs are n number of inputs are n we have to be careful because I have use this fall n.

Slightly earlier when I am doing transformation and n is now going to mean something else okay so it will be n – x so this is just to be consistent with my notes so I do not end up in making some mistake n – y while solving these will also understand certain things which we can observe in the process. So this was nc something so it will become ncx and then of course you have x! y!/k!x+y this coming from Jacobian approximation denominator we consist of except PL rest everything will be there and you can also observe very careless about this λ this is ultimately it concerns out this is simply proportionality constant does not matter actually here.

Yeah I made a mistake here okay so now it is a correct expression. Now look at A, A is written as λ/ λ+μ what does it mean. So on the trunk line or on telephone line λ is arrival rate and remember I had actually mentioned in one of my previous lectures that λ is because of poison distribution. So if I look at on time scale the events which are happening or the calls which are arriving λ calls per second which are arriving.

So λ represented 1/λ represents the inter arrival time will be exponential distributed with mean value of 1/λ. So the gap between two calls is 1/λ, 1/μ simply is also in the same fashion I had that 1/ μ is the call duration when the call starts is going to be occupied for some time before again the line goes to free and the new call came coming that call duration that also exponential distributed with average 1/ μ

So or we can say the calls are been processed very fast the μ number of calls per second or per unit time will be processed okay. So because exponential distribution and Poisson distributions are interrelated. So I can actually modify this and write do the inversion I can actually put this as $1/\mu/1/\lambda+1/\mu$ okay, so I can do this. So once I do this and this is nothing but as good as A you can actually proved this one, so this term has to be same.

So this is on a timing scale when a call arrives this is the call duration, so average duration for this is $1/\mu$ and when the next call will arrive that gap is $1/\lambda$ and then, the next call will be arriving. So what I am saying is the busy duration divided by busy plus free duration on the line that is what is A. So A becomes technically a trunk utilization and its value will be between 0 and 1 okay, this value will be between 0 and 1.

So it is busy period divided by this period between two calls busy + free periods so that is what the A is, so it is line loading actually. So let us now start solving it. So we can see it the γ and γ will cancel, so that is very good let us see now what we have to do next I can explain the Cx so that I can solve these n-y and n-x thing. So let me do that, I can write as n!/x!n-x! I can do same thing for this and same thing can here also.

So I can take this n-x and n-x! and I can cancel it out this will become n-x-1, I can do the same thing with this, same I can do on this side. But this is not full combinatorial, so let me take n out and make it n-1, same I can take here n^2 so this n^2 can come out of the summation and we can cancelled out. So this is what I am going to have so let us solve it for that n^2 I can remove from here and what I need to have here is a^x so this is the combinatorial which I am using if I want to make it a full combinatorial sum full binomial sum it has to be n-1-x.

So it has to be n-1-x, it has to be n-1-y which actually implies I have to take $1-a^2$ out which I can take out of the summation it independent of x and y. Now these two are this summations now can be taken and I can put one summation here and this become summation over y and this is over, this is over y, this is over x. So this summation now can be written as $a+1-a^{n-1}$ same I can do for this and I can write $a+1-a^{n-1}$.

So I can remove this one this turns have to be nothing but unity this is also turns out to be unity so this is gone and these what we are left with okay. Now solving it further out of these now writing it $1/1-a^2$ this k! is independent of x and y, so this can come out I can write it here rest everything I will kept inside $n-1!^2$ can also come out okay. So but rest everything has to be kept inside.

So I will have a^{x+y} yeah this x! and y! we cancel it from here so these will not be here these have gone out and that is what we are left with okay. So we will now actually split into two separate parts, so what I will do is I will take $X+y^{-k}$ which actually means k s ^{k} has to come somewhere here I will put 1-a ,n-1 –y here so I need to take somewhere here 1-a, n-1-x this is because $1-a^2$ has to come up so these -1 have setting in so I have to remove this okay, we will also add a factorial here so that we can solve it later on n-1+x-k so ill put the same thing in denominator here and I will keep this n-x so I will take this part now you can see there in no the x is something which is the in this summation i have to solve this thing first with respect to y okay.

This is the complete binomial you can actually add these two a and 1-a this terms out to be n-1-y will cancel with this n-1 +x –k okay x+ y-k is here n-1-y is here so this forms a binomial actually but what's the range of y in this case so that's a one of the interesting question so range will be when x+ y –k because this is something which has to be certainly observed otherwise you know it's a binomial, I can simply solve it by adding a+1-a s something okay.

So when the x +y –k 0 that's one possibility okay secondly this can go from 0 to the largest value which is this one ,so second possibility is that x+ y-k will become equal to n-1 + x-k, in this case that k will cancelled x will cancel y will be equal to n-1 is the range here so y will be ,so y will range now n-1 coming from here n y = k-x y ,so infect by you should note here if y is not greater then k-x switch cannot be in blocking states which can only be in blocking states it is greater then, k-x remember with this is we did while estimating the Jacobian approximation .

So this is the range in which y will be valid so this range has to come here in the summation and this forms a complete binomial so complete binomial means this plus this a+1-as power whole term so I can represent. So I can actually re write it, a+1-a,n-1+x-k so this is one 1s power anything going to be 1 so this is a material so this is go away .only thing is remains its summation with respective to s and I can rewrite is this in this case I can actually sum these two things up and make a add a factorial here multiplied by that I should now remove this one.

This will be two end x will cancelled 2n-2-k that's what it will be , so I have to divide it also so but this is independent of x I can write it here now here x will be actually also going taking up a range so x can take ,now a^k going to come out which is a second term here 1-a this so 1-a this so 1something is the second term so that second term is 1^{2n-2-k} this actually it will be n-1+x-k okay.

So these forms complete binomial so what's a range of x here so when n-1-x goes to 0 so x will be equal to n-1, that's one possibility secondly when this term will go to 0 which is n-1 + x –k so k=n-1 +x so that's equivalent to the range in which is going to operate .

So this also become a full binomial I can replace it by 1+1-a 2n-2-k,so which is in case be the expression for t (coral loss) and remember this you can only give you a value if you are going to have k<2n-1,okay if k> that then it will make since otherwise it won't and of course one important thing this is the call loss probability there has been a another approach by which a computation can be done for the switch being in the blocked state okay .

So I have not done that here and if you do that I will end up in a expression which will be PB which will turn out to be a^k and an interesting expression it will be $n!^{2}$ 2-a 2n-k/k!2n-k and you can actually clear the relationship is still holds the relationship which we had PBn-1 ,this relationship which we have derived earlier also holds to here okay. So that what is the method of estimating the blocking probability using Carlos approach?

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