Bayesian/MMSE Estimation for MIMO/OFDM Wireless Communications
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Lecture - 35
Example - Orthogonal Frequency Division Multiplexing (OFDM) - LMSSE Estimation of Channel Coefficients and resulting MSE -Part II

Hello welcome to another module. So, we are looking at an example of LMSSE estimation of the channel coefficients HL across sub carrier across each sub carrier l in OFDM wireless system.

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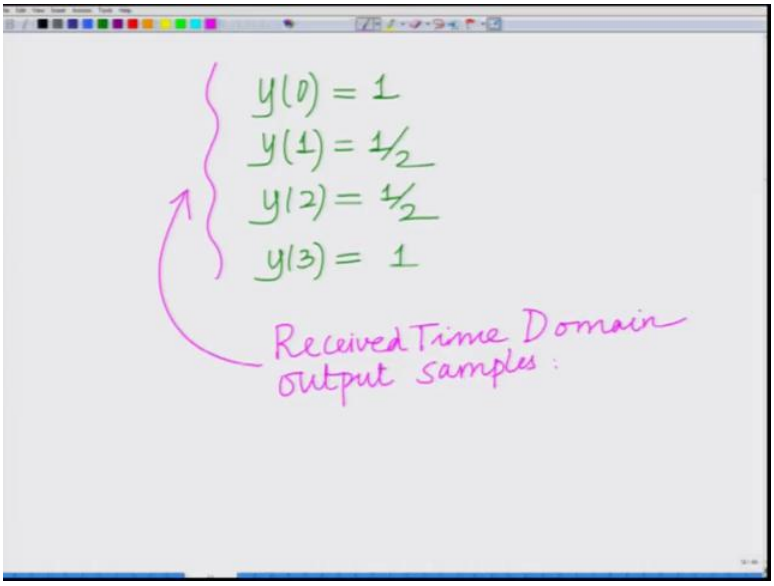

So, let us continue with this example let us consider the received symbols, received time domain samples, received time domain samples to be given as - the domain output sample as y 0 equals 1 y 1 equals half y 2 equals half y 3 equals these are the received time domain, these are the received time domain output samples. Therefore, the corresponding frequency domains symbols (Refer Time: 01:31) across the various sub carriers are given by the FFT of this time domain samples that is what we feel.

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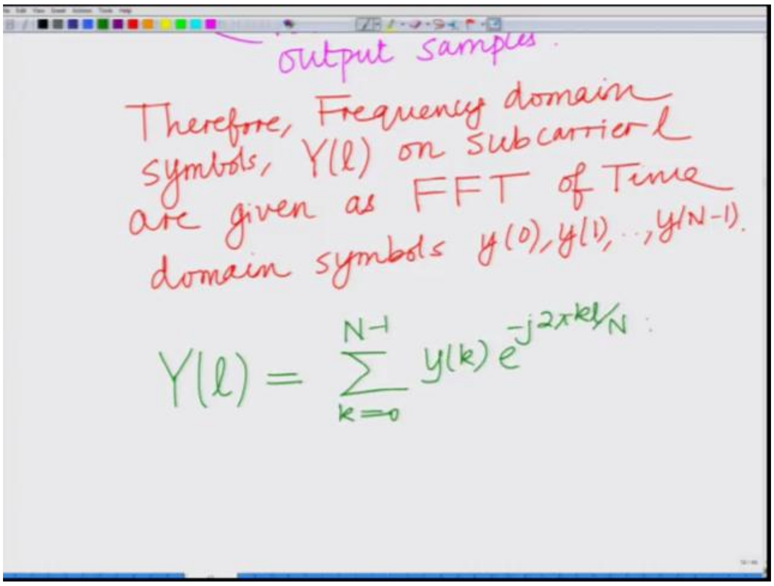

Therefore, the frequency domain symbols the frequency domain symbols y l on sub carrier l these are given as the FFT of the time in the domain symbols y 0 y 1 up to y N minus 1. Therefore, we have y l equals summation k equal to 0, summation k equal to 0 to N minus 1 y k e to power of minus j 2 pi k l over n.

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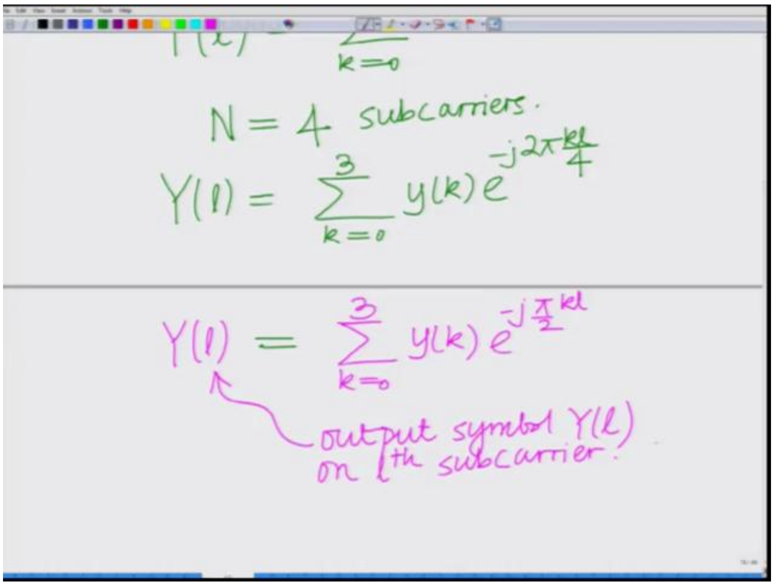

In our particular example we are considering N equal to 4 sub carriers sub carriers correct. So, we have y l equals summation k equal to 0 to N minus 1 that is 3 y k e raise to minus j 2 pi k l by N that is 4 which is equal to which implies that y l is equal to summation k equal to 0 to 3 y k e raise to minus j 2 pi e raise to minus j minus pi k l by 4 which is basically e raise to minus j pi by 2 k l this is the output symbol y N y l on the y l on the l th sub carrier, this is the output symbol y l on the l th sub carrier.

So, now let us compute the various output symbols on the various sub carriers.

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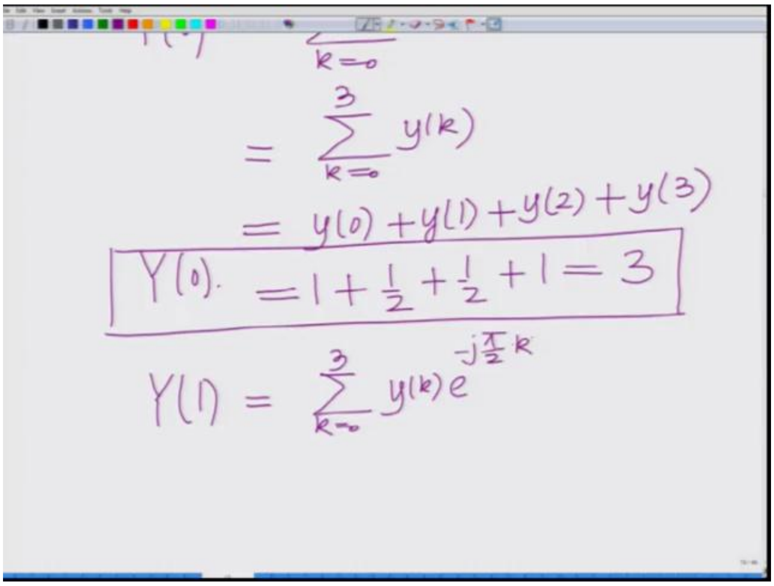

So, we have y 0, we can have y 0 equals summation k equals 0 to 3 y k e raise to the minus j pi by 2 k times l equal to 0. So, e to the power of 0 is 1. So, this is sum simply summation k equal to 0 to 3 y k which is simply y 0 plus y 1 plus y 2 plus y 3 which is equal to well 1 plus half plus half plus 1 equals 3. So, we have capital Y 0 the received symbol on sub carrier 0, this is equal to 3.

Now, similarly y 1 equals summation k equal to 0 to 3 y k e to the power of minus j by 2 pi k into l equals 1.

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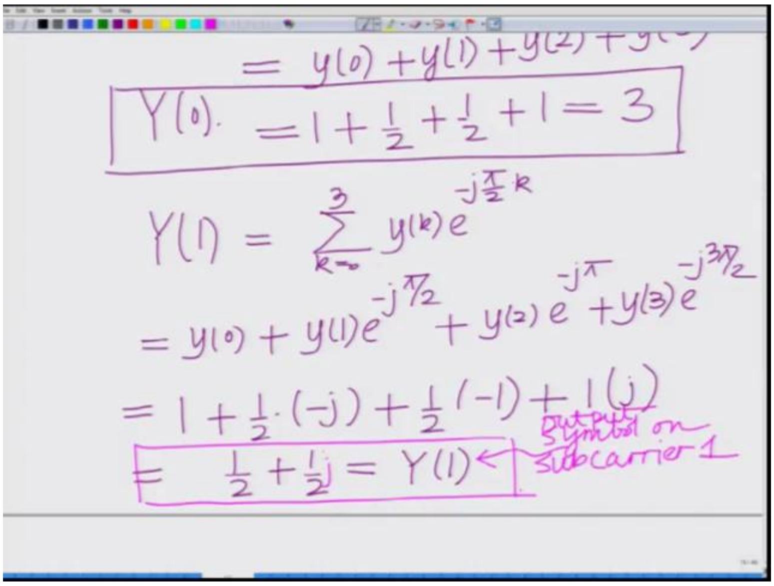

So, this is simply e to the power minus j pi by 2 k which is equal to y 0 plus y 1 e to the power of minus j pi by 2 plus y 2 e to the power minus j pi plus y 3 e to the power of minus j 3 pi by 2 which is equal to y 0 that is 1 plus y 1 half into e to the power minus j pi by 2 minus j plus half into minus 1 plus 1 y 3 that is 1 into j e to the power minus j 3 pi by 2 is j. So, this is half plus half j this is equal to y 1 capital Y 1. And this to be clear symbol on sub carrier or output rather, output symbol received on sub carrier 1. That is y 1 equals half plus the output symbol received on sub carrier 1 that is y 1 capital Y 1 is half plus half j.

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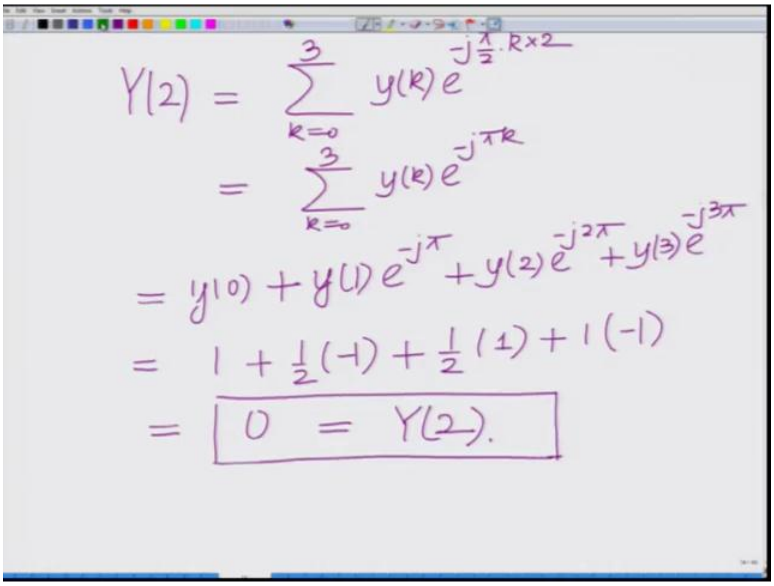

Now, y 2 that is given as summation k equal to 0 to 3 y k e to the power minus j pi by 2 k into 2 which is summation k equal to 0 to 3 y k e to the power of minus j pi k which is equal to y 0 plus y 1 e to the power of minus j pi plus y 2 e to the power of minus j 2 pi plus y 3 e to the power of minus j 3 pi and this is equal to 1 plus half into minus 1 plus half e to the power of minus j 2 pi which is 1 plus 1 into minus 1 and you can see this is 0 therefore, y 2 is equal to 0 that is output symbol on sub carrier 2 this is equal to 0.

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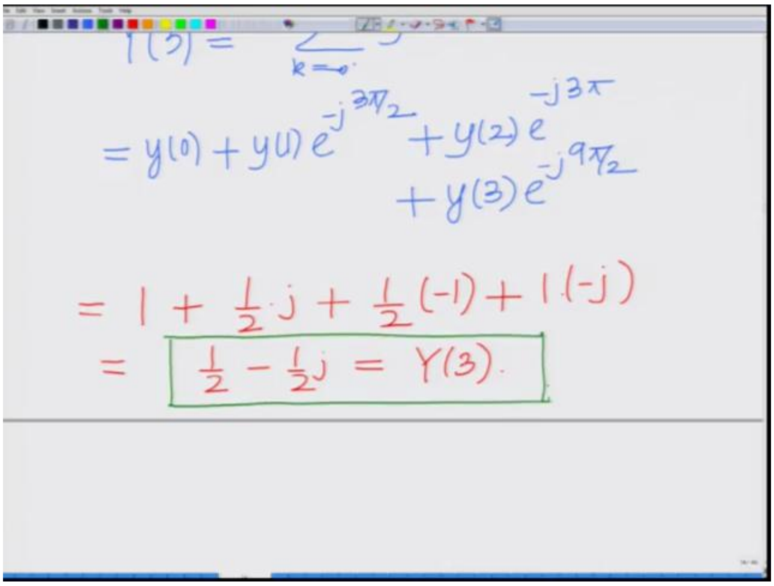

And now let us compute y 3 that is output symbol on sub carrier 3 which is summation k equal to 0 to 3 y k e to the power minus j pi by 2 k into 3 which is y 0 plus y 1 e to the power minus j 3 pi by 2 plus y 2 e to the power of minus j right, e to the power of minus j 3 pi plus y 3 e to the power of minus j 9 pi by 2. This is what we have and this is equal to 1 plus half into j plus half into e to the power of minus j 3 pi right, correct, that is minus 1 plus 1 into e to the power of minus j 9 pi by 2 that is minus j. So, this will be half minus half j that is y 3. So, these are the received symbol of the sub carrier 3.

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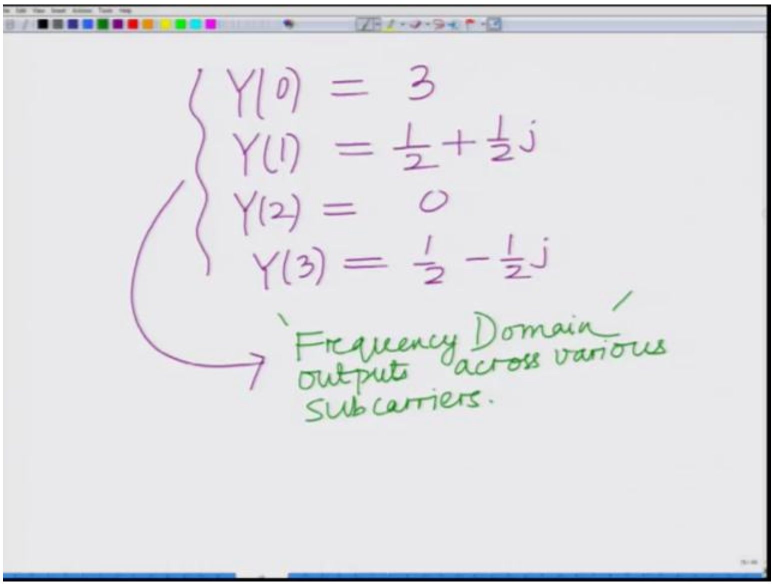

And therefore, now to summarize the frequency domain output that is received across various sub carriers are capital Y 0 equals 3, capital Y 1 equals half plus half j, capital Y 2 equals 0, capital Y 3 equals half minus half j. These are the frequency domain output across the various; these are the frequency domain outputs across the various sub carriers that is we have N equal to 4 sub carriers 0 1 2 3 - capital Y 0, capital Y 1, capital Y 2, capital Y 3, are the outputs symbols in the output symbols across these various sub carriers that is N equal to 4 sub carriers. And now we can compute the LMSSE estimate of the channel coefficients, remember the LMSSE estimate of the channel coefficients that is given as.

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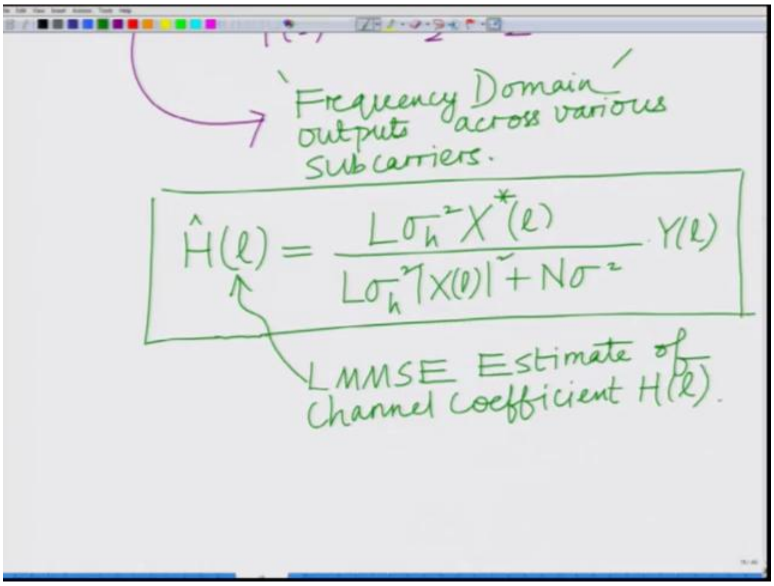

Well we have H hat that is what we have derived in 1 of the previous modules H hat l equals L sigma H square X conjugate l divided by L sigma x H square magnitude X l square plus N sigma square into y l, this is the LMSSE estimate of channel coefficient LMSSE estimate of channel coefficient H l.

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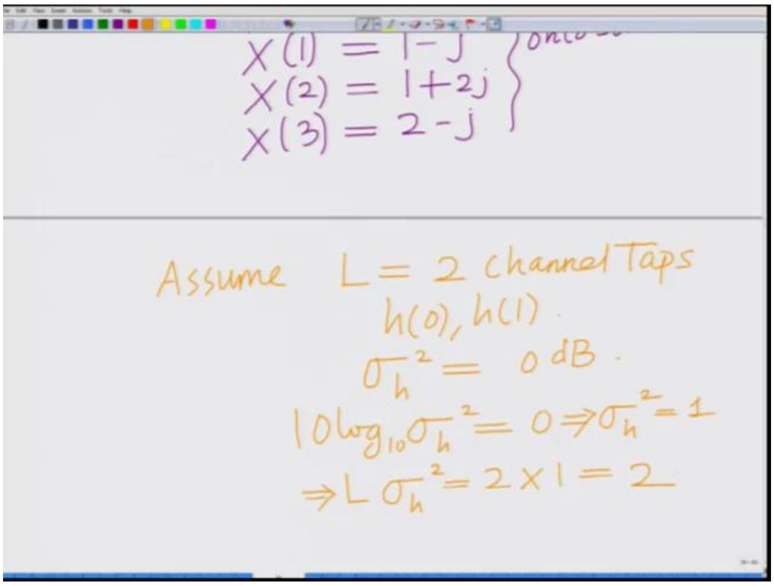

And therefore, now remember we already know that we have the symbols loaded on the various sub carriers these are given as x 0 equals 1 plus j x 1 equals 1 minus j x 2 equals 1 plus 2 j and x 3 equals 2 minus j. These are the symbols loaded, these are the symbols loaded on the sub carriers therefore, and we have H hat 0 equals. Now further we will need the prior variance remember we will need also the prior variance sigma H square the number of channel taps and the noise variance sigma square. So, let us assume l equal to 2 channel taps H 0 comma H 1 which sigma H square equals well sigma H square equals that is d B variance sigma H square equals 0 d B which implies 10 log 10 sigma H square equal to 0 which implies sigma H square equals 10 power power 0 equals 1 therefore, l sigma H square, since we have l equal to 2 channel taps 2 into 1 that is equal to 2.

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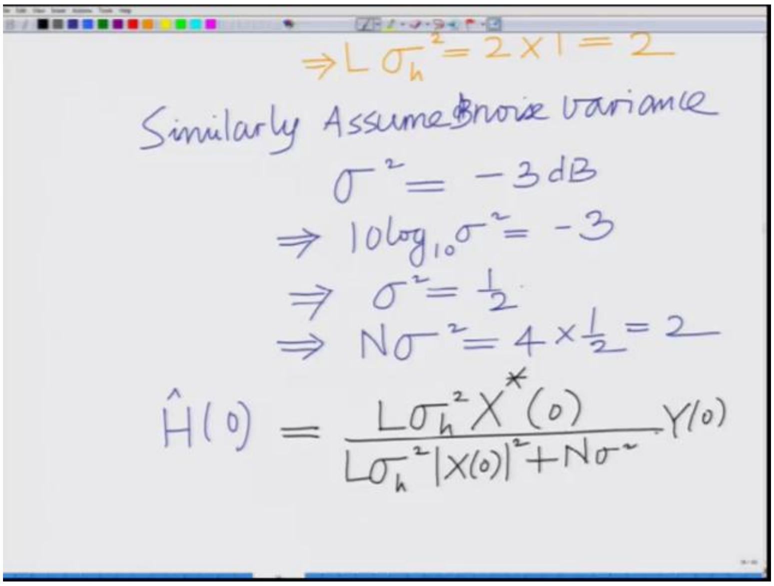

Similarly, assume the noise samples time domain. Similarly assume noise variance for rather d B noise variance sigma square equals minus 3 d B implies ten log to the base 10 sigma square equals minus 3 implies sigma square equals half 3 d B is 2 minus 3 d B is half implies N sigma square where N is the number of sub carriers remember is 4 times half that is 2. Now we can derive H hat of 0 estimate of channel coefficient across tap 0 this is l sigma H square x conjugate 0 divided by l sigma H square magnitude x 0 square plus N sigma square into y of 0.

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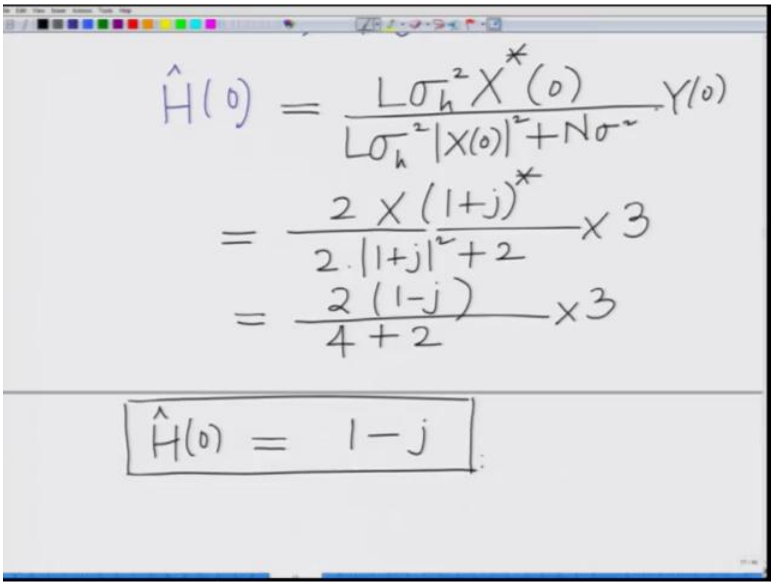

This can be divided as l sigma H square is 2 times x conjugate that is 1 plus j conjugate divided l sigma H square divided 2 times magnitude of x 0 square that is 1 plus j magnitude whole square plus 2 into y 0 which is basically you have derived y 0 above that is 3.

So, this is twice 1 minus j divided by twice into 1 plus j magnitude square that is twice into 1 plus j magnitude square into 2, so 4 plus 2 into 3. And now we can see there is 6 there is this basically you have 3 into 2 is 6 in the numerator 6 in the denominator so this basically simply your 1 minus j H hat of 0 equals 1 minus j this is the LMSSE estimate of channel tap H hat of 0 H hat of 0 is the LMSSE estimate of the channel tap H 0 or the channel coefficient H 0 across sub carrier 0.

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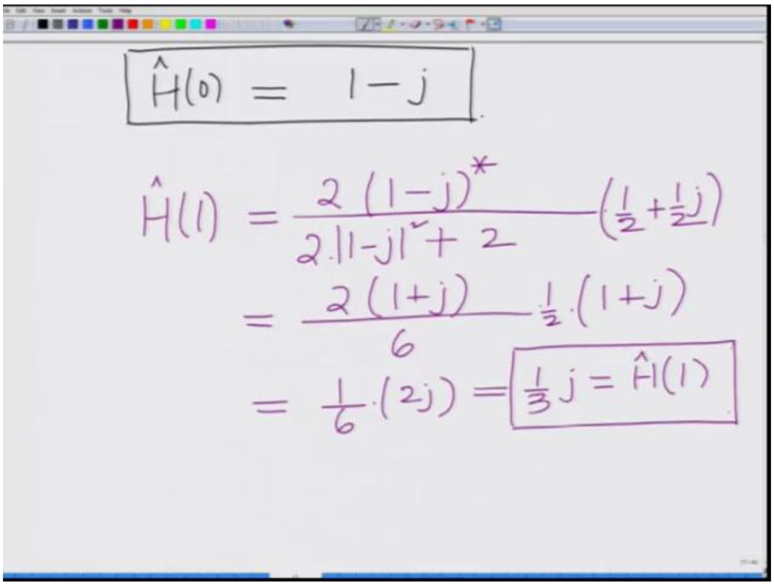

Similarly, we can derive the estimate LMSSE estimates for the rest of the sub carriers. So, let us quickly derive them we have hat of 1 this is equal to well twice x 1 conjugate 1 minus j conjugate divided by twice magnitude x 1 square plus N sigma square that is 2 times y of y of 1 that is basically your half plus half j. So, this is twice 1 plus j divided by 2 into 2 4 plus 2 6 into half into 1 plus j. So, this is going to be 1 by 6 into 1 plus j square 1 plus j square is basically 2 j that is basically gives 1 by 3 j and this is your H hat, this is equal to your H hat 1 estimate of channel coefficient across sub carrier 1.

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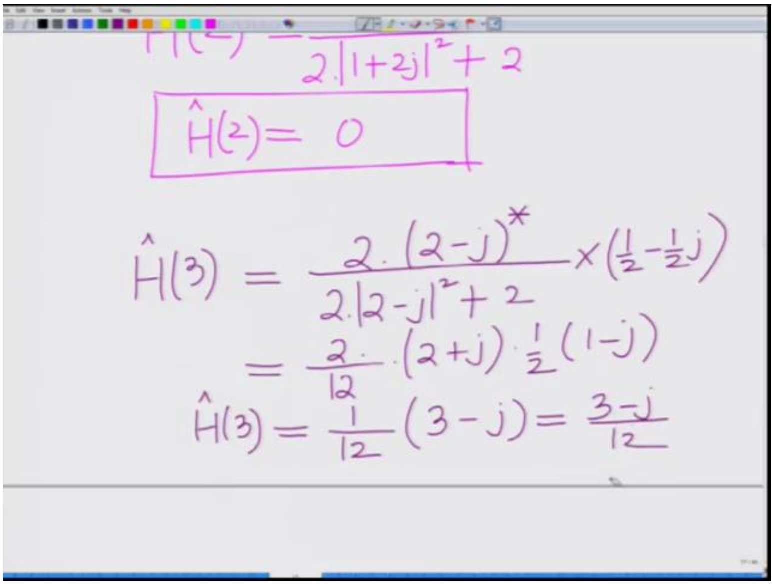

Let us quickly do for H hat 2 and H hat 3. So, H hat 2 equals well that is equal to twice 1 plus 2 j that is x 2 conjugate divided by twice l H l sigma H square plus 2 j magnitude square plus N sigma square what is N sigma square that is twice into y of 2, but y of 2 is 0. So, therefore, this is simply 0 you do not need to compute this elaborately and the remaining 1 is H hat of 3 H hat of 3 is l sigma x square 2 times 2 minus j that is x 3 conjugate divided by twice magnitude 2 minus j whole square plus 2 into y of 3 that is half minus half j that is 2 divided by magnitude 2 minus j square is 5. So, 2 into 5 plus 10 2 that is 12 times 2 plus j into half into 1 minus j this is equal to 1 over 12 times 2, times 2 plus 1 that is 3 minus j plus j minus 2 j minus j.

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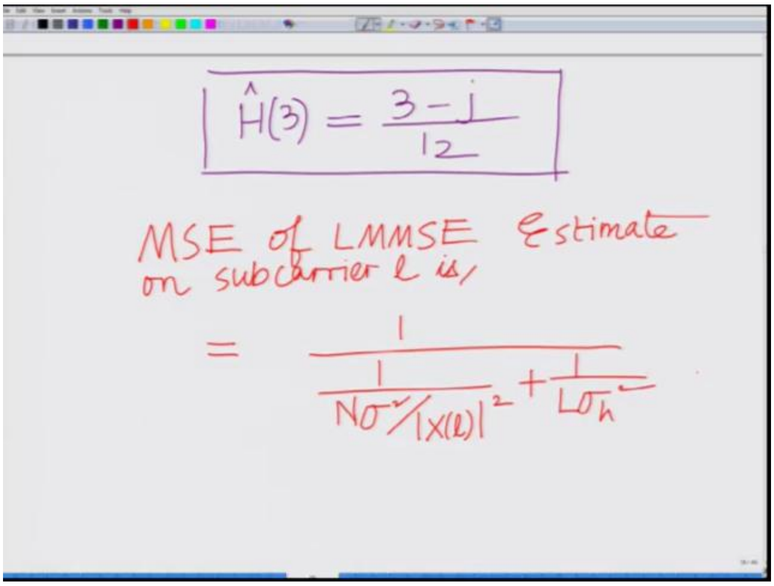

So, this is 3 minus j divided by 12, this is the estimate H hat of 3, this is the estimate H hat of 3. So, the estimate H hat of 3 across sub carrier 3 is 3 minus j divided by 12. There by we can compute the LMSSE estimates of the channel coefficients across the various sub carriers.

We can also compute the corresponding mean square errors of the LMSSE estimate and we also derived the expression to compute the mean square of the LMSSE estimate, remember. The MSE of the LMSSE estimate on sub carrier l, MSE of the LMSSE estimate MSE of LMSSE estimate on sub carrier l that is equal to 1 over N sigma square divided by magnitude x l square plus 1 over l sigma H square. So, let us compute this

MSE - MSE for sub carrier l equal to 1 that is well what is that? That, definition of that is H hat 1, I am sorry for l equal to 0.

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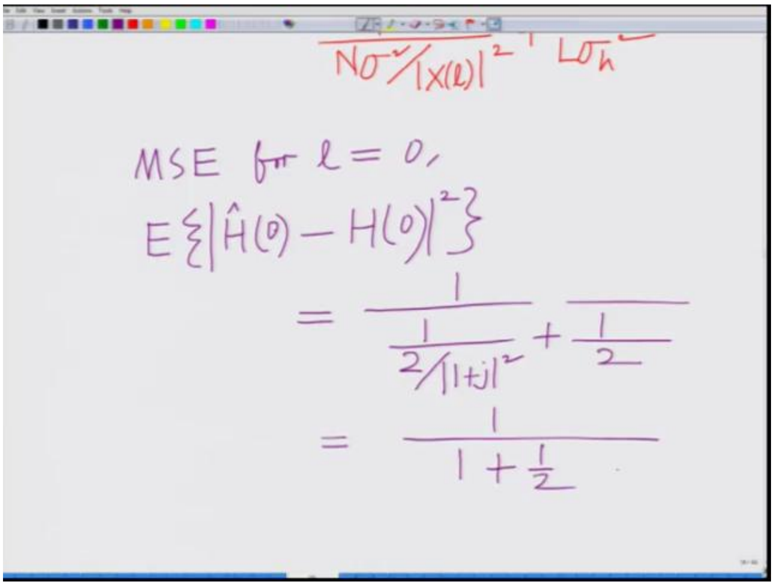

You have to start with sub carrier 0 H hat 0 minus H 0 magnitude square that is equal to 1 divided by N sigma square while we know sigma square that is 2 well 1 divided by 1 divided by 2 divided by magnitude x 0 square, magnitude x 0 square is magnitude of, that is magnitude of 1 plus j whole square plus 1 divided by H square that is 2. So, this is 1, 2 divided by magnitude 1 plus j square is 2. So, this is 2 divided by 2 that is 1 plus half so that is 3 by 2 this is equal to 2 by 3.

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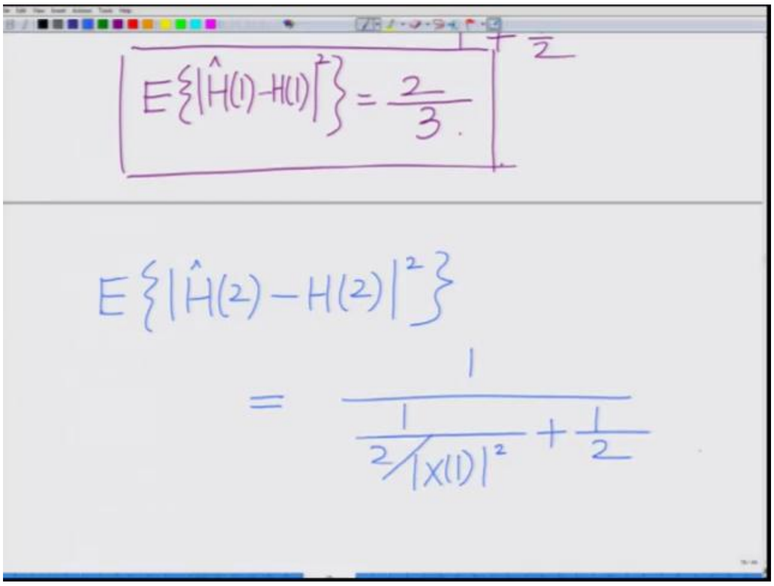

So, this is what this is? Expected value of magnitude H hat 1 minus H 1 whole square the MSE associated with the LMSSE estimate corresponding to sub carrier 1. Similarly, expected value of magnitude H hat 2 minus H 2 whole square this is given as 1 divided by 1 2 divided by magnitude x 1 square plus 1 divided by 2 - this is again 1 divided by 1 plus half which is equal to 2 by 3.

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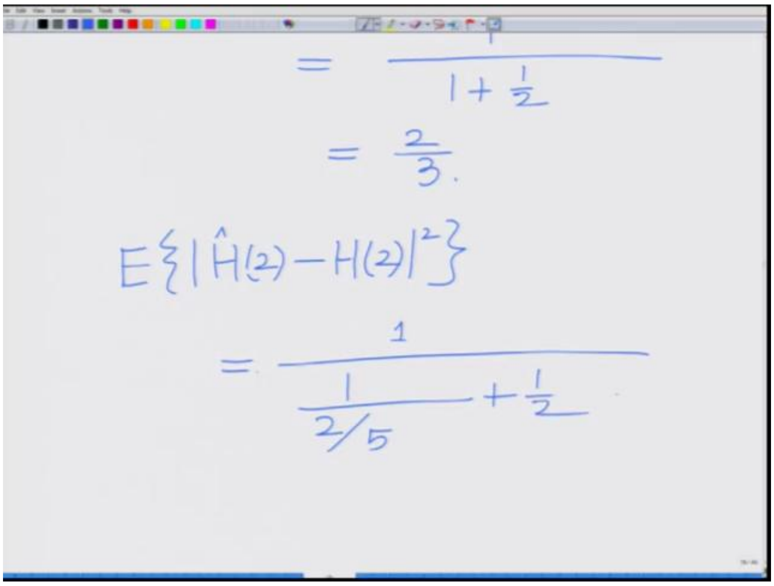

Now expected value of magnitude H hat 2 minus H 2 whole square. Now this is equal to well this is equal to 1 divided by 1 divided by 2 divided by magnitude, well x 1 square magnitude x 1 square is magnitude 1 plus 2 j square that is 5 plus half that is 1 divided by basically 5 by 2 plus half which is 1 divided by 3.

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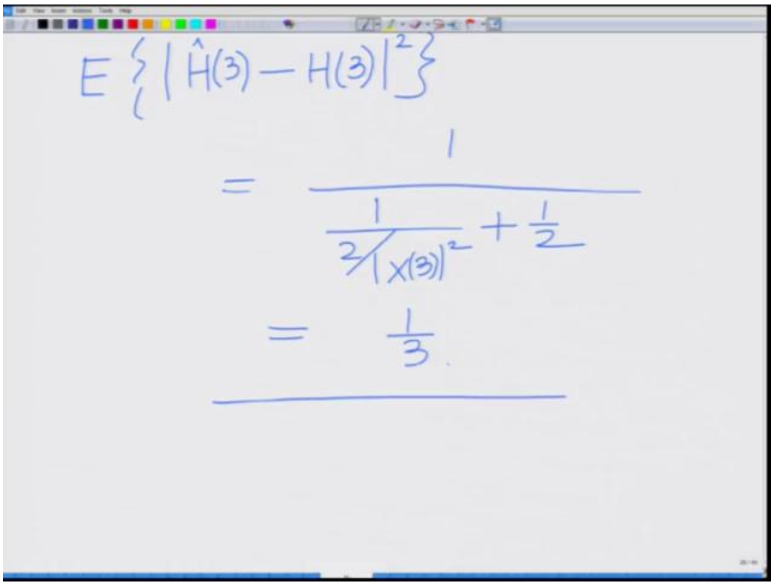

So, this is this MSE is this mean square error and finally, expected value of H hat 3 again we find that equals 1 divided by 1 divided by 2 divided by magnitude x 3 square plus half again you can check this will again be 1 by 3.

So, basically what this example tells us what we have done in this example is we have done an elaborate example, we will considered the transmission or the symbols loaded onto N equal to we have considered N equal to 4 sub carrier OFDM system with l equal to 2 channel times. Considered the symbols loaded onto the sub carriers derived what are the samples where transmitted in the time domain, consider the output samples received in the time domain, derived what are the corresponding output symbols across the various sub carriers in the frequency domain. From that using the properties of prior variance of the channel of the coefficient of the channel taps and the noise variance we have computed the LMSSE estimates of the channel coefficients capital H S across the sub carriers. We also derived the expressions for the mean square error in the LMSSE estimate of each channel coefficient.

So, that completes this compressive example on the MSSE estimation for an OFDM orthogonal frequency division multiplexing wireless communication system.

Thank you.