Bayesian/MMSE Estimation for MIMO/OFDM Wireless Communications
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Lecture – 34
Example - Orthogonal Frequency Division Multiplexing (OFDM) - Transmission of Samples with Cyclic Prefix (CP)-Part I

Hello, welcome to another module in the massive open online course. So, we are looking at estimation in the context of our DM or orthogonal frequency division multiplexing based systems. And we have already seen the theory of basically how what is the system model for an OFDM system and basically out of carry out channel estimation that is estimation of the various channel coefficients of the sub carriers as well as the channel taps in the time domain in an OFDM system.

So, now let us look at a simple example to understand this better. So, what we are going to do in this module or starting in this module is basically look at a step by step implementation of OFDM system as well as channel estimation.

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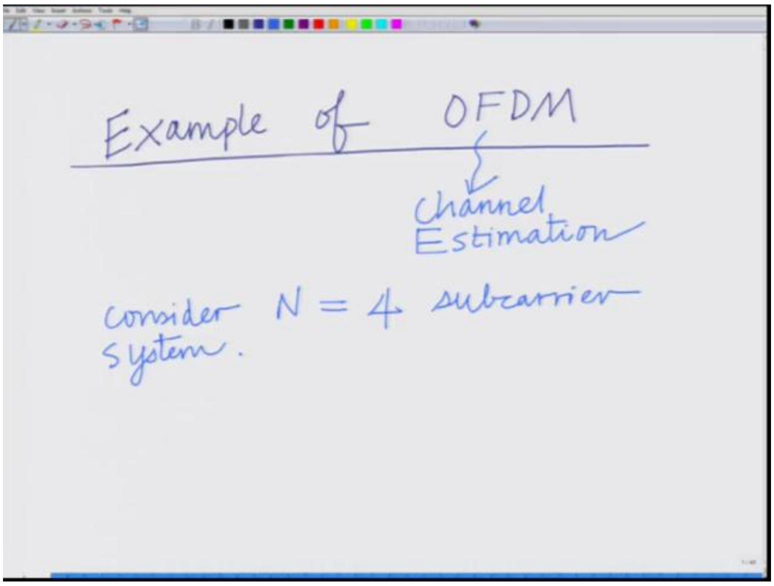

So, let us look at an example - example of OFDM system and in particular what we want to look at is channel estimation in OFDM system. So, as we have been seeing let us consider an n equals to 4 sub carrier system. So, similar to what we have seen in the previous modules consider N equal to 4 and N equal to 4 sub carrier systems and therefore, we have 4 symbols that are loaded on to the sub carrier, 4 pilot symbols. So, let these pilot symbols denoted by capital X be capital X 0 equals 1 plus j, capital X 1 equals 1 minus j, capital X 2 equals 1 plus 2 j, capital X 3 equals 2 minus j.

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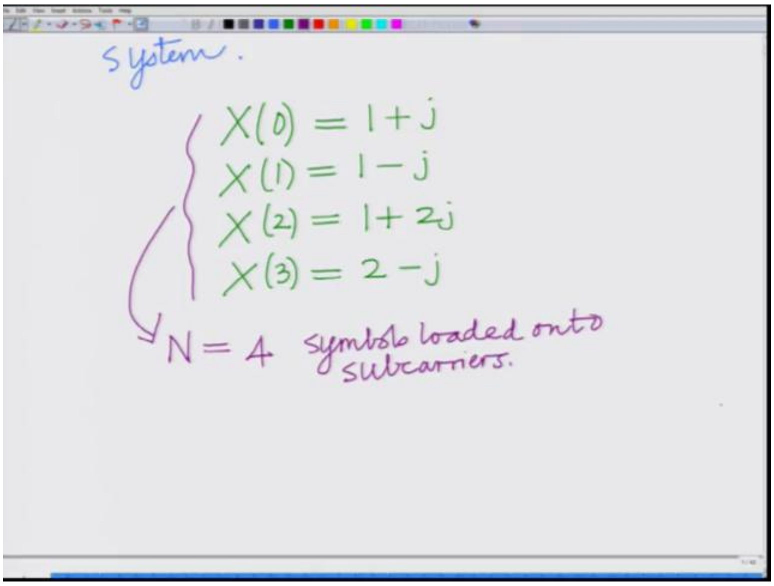

These are the 4 symbols these are the N equal to 4 symbols loaded on to the sub carriers these are the n equal to 4 symbols which are loaded on to the 4 sub carriers correct.

So, we have capital X 0 capital X 1 capital X 2 capital X 3 which are the 4 symbols loaded on to the 4 sub carriers. Now we want to generate the samples in the time domain and remember the samples in the time domain I j are generated by the n equal to 4 point IDFT.

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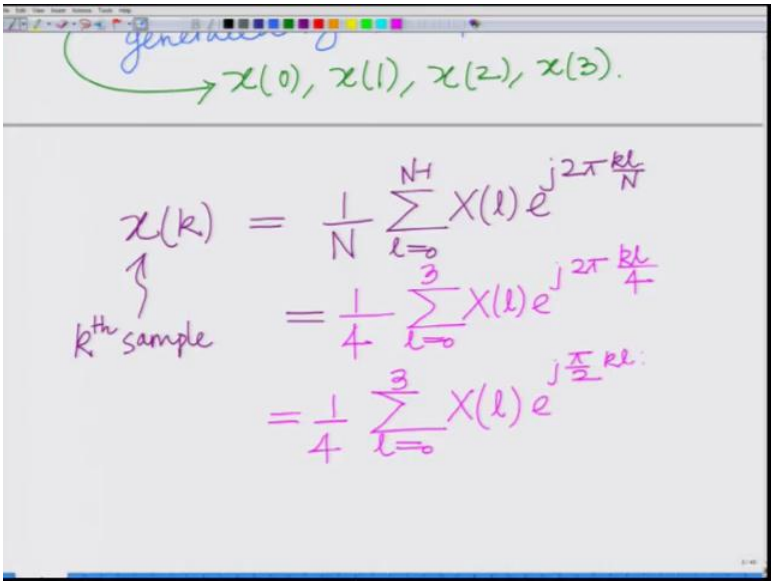

$$x(k) = \frac{1}{N}\sum_{l=0}^{N-1} X(l)\, e^{j2\pi \frac{kl}{N}}$$

$$= \frac{1}{4}\sum_{l=0}^{3} X(l)\, e^{j2\pi \frac{kl}{4}}$$

$$= \frac{1}{4}\sum_{l=0}^{3} X(l)\, e^{j\frac{\pi}{2}kl}$$

So, the samples which are denoted by, the time domain samples let me write it clearly this is - the time domain are generated by n equal to 4 point IDFT that is basically this samples are denoted by X 0, X 1, X 2, X 3 and these are basically your time domain samples which are generated by the 4 point IDFT. And therefore, X k which is the k th sample, what is X k? X k is the kth sample and this is equal to 1 over n, summation l equal to 0 to n minus 1 X l e raise to j 2 pi k l divided by n which is equal to 1 over 4, l is equal to 0 into 3 X l e raise to j 2 pi k l - k l divided by 4 which is in term just simplify this as far as possible this is l equal to 0 to 3 X l e raise to j pi by 2 k l. So, this is basically your X k. So, this is basically your X k this is the sub carrier this is of course, capital X l is the symbol that is loaded on to the l-th sub carrier and the time domain samples are basically given by the IDFT of the symbols loaded on to the sub carriers.

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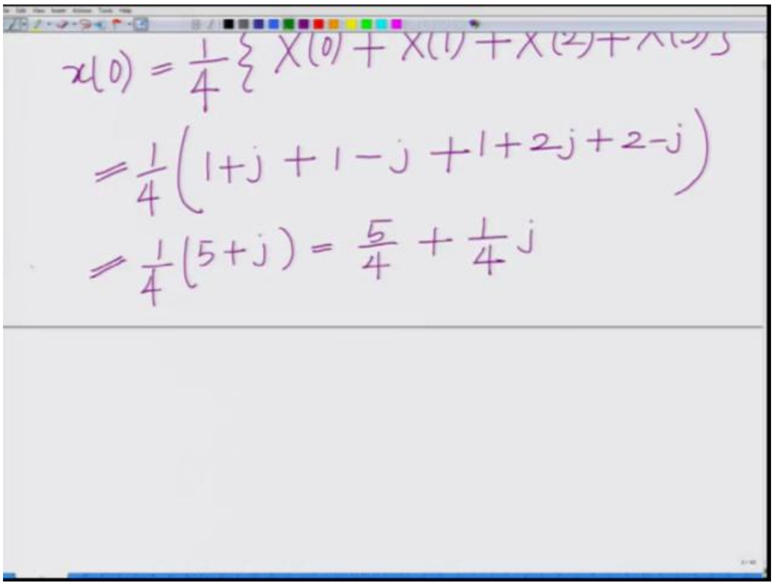

So, we have the symbols loaded on to the sub carriers. Let us now find what the corresponding time domain transmitted samples are. So, X of 0 let us start with example X of 0 that is sample corresponding to time index 0 that is 1 over 4 summation l equal to 0 to 3 X l e raise to j pi by 2 k - for k I have to substitute 0 because I am considering the sample at k equal to 0 because 0 times l which is basically now this quantity is basically 1. So, this is 1 over 4 simply summation l equal to 0 to 3 of X l. Now, therefore, X 0 your X 0 is one over, it is basically 1 over 4 capital X 0 plus capital X 1 plus capital X 2 1plus capital X 3 which is equal to what is this equal to 1 over 4. Now, substitute for capital X 0 which is 1 plus j plus capital X 1 which we said is 1 minus j plus capital X 2 plus 1 plus 2 j plus capital X 3 which is 2 minus j.

Remember we already given the values of capital that is the symbols, these are the symbols that are loaded on to the sub carrier. So, I am simply substituting them over here. So, I have 1 over 4 1 plus j plus 1 minus j plus 1 plus 2 j plus 2 minus j which is equal to 1 plus 4 into 5 plus j which is equal to 5 by 4 plus 1 over 4 times j this is the complex symbol, this is the x small x 0 that is the sample transmitted at time 0.

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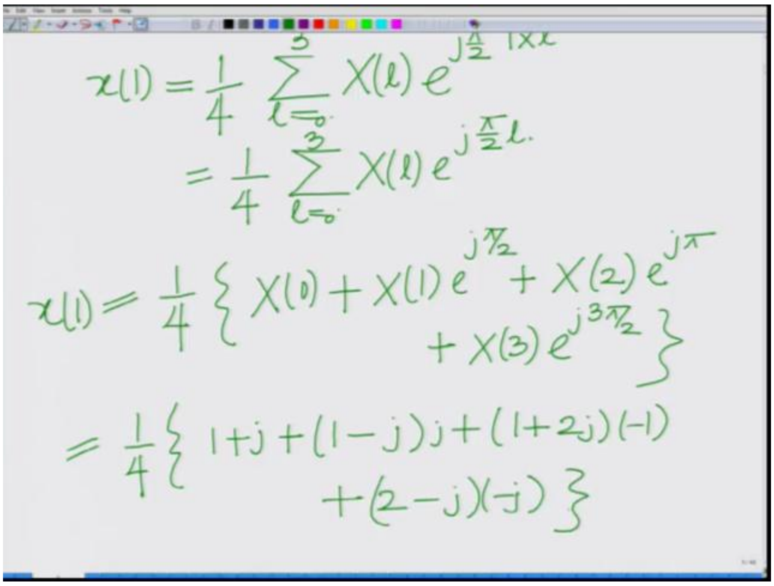

Similarly, let us compute small x 1 which is the sample transmitted at time 1. So, again it goes without saying small x 1 corresponds to k equal to 1 that is the sample corresponding to your time instant 1 l equal to 0 to 3, X l e power j pi by 2 instead of k I substitute 1, 1 times l which is basically 1 over 4 summation l equal to 0 to 3 X l e power j pi by 2 l which is now if I substitute the symbols loaded on to the sub carriers I have X 1 equals 1 over 4 times X 0 plus X 1 into e power j pi by 2 plus X 2 e power j pi plus X 3 e power j 3 pi by 2, which is equal to basically your 1 over 4 times 1 plus j plus 1 minus j into j plus 1 plus 2 j into minus 1 plus 2 minus j into minus j and what is this? This is equal to your 1 over 4 plus 1 plus j plus j plus 1 minus 1 minus 2 j minus 2 j minus 1.

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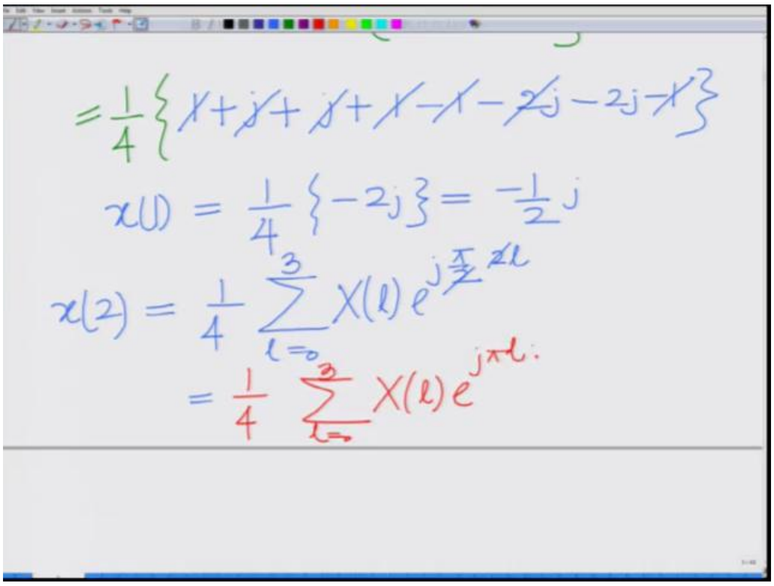

And this equal to basically, let us write it down this is X of 1 which is equal to 1 over 4 you can see there is basically 1 minus 1 which goes 1 minus 1 which goes 1 minus 1 which goes, left with this j j minus 2 j. So, this is basically minus 2 j divided by 4 equals minus half times j. Again you can check all these calculations please do them yourself and try to check these calculations. So, you have calculated the sample X 1 small X 1 corresponding to 3 equal to 1 as minus half j.

Now, let us compute small X 2, small X 2 equals well 1 over 4 l equals 0 to 3 capital X l e power j pi by 2 substitute k equal to 2 times of l twos goes away and what we have is 1 over 4 l equal to 0 to 3 capital X l e power j pi l that is small x 2.

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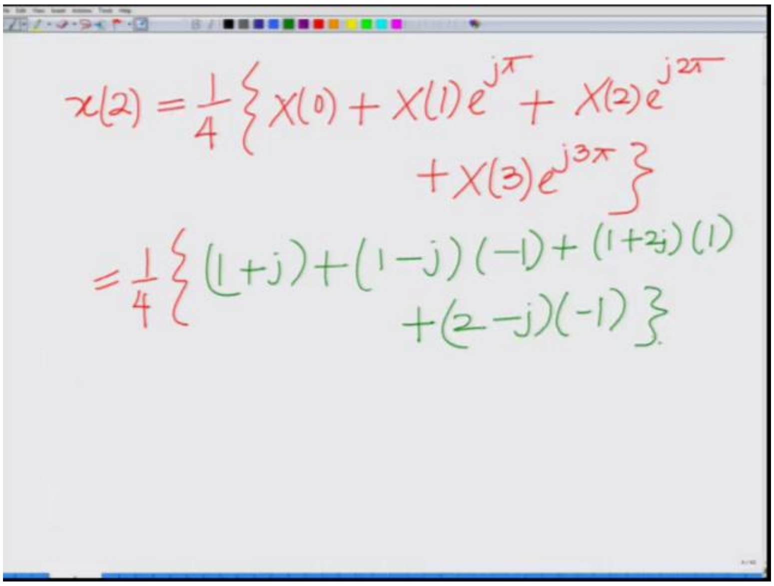

Now, substituting of course, the capital X l is which are the symbols loaded on to the sub carriers I have 1 over 4 X 0 plus X 1 e power j pi plus X 2 e power j 2 pi plus X 3 e power j 3 pi which is equal to 1 over 4. Again hoping that I am not making mistakes, this is 1 plus j plus 1 minus j times minus 1 e power j pi is minus 1 plus 1 plus 2 j into e power j 2 pi is 1 plus 2 minus j into minus 1 and what is this? This is therefore, equal to 1 over 4 1 plus j minus 1 plus j plus 1 plus 2 j minus 2 plus j.

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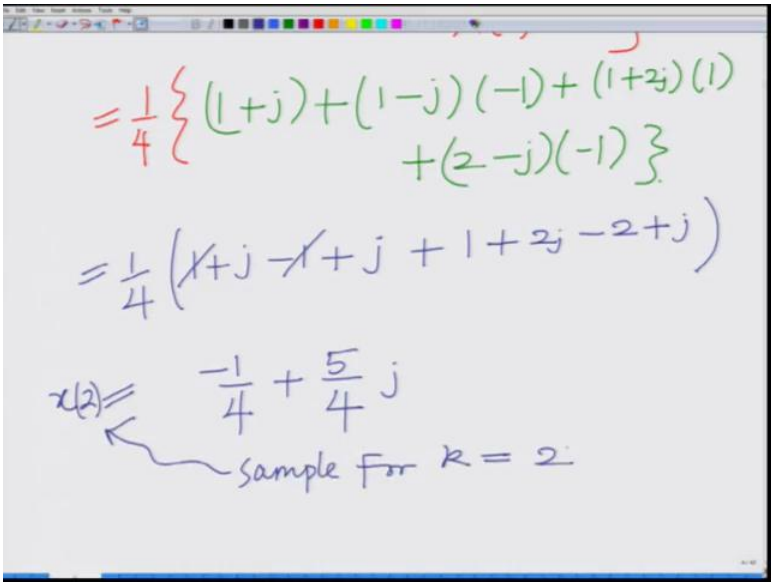

And this is basically if I correct again they have 1 minus 1 that is 0. So, basically I have 1 what is this this is basically equal to well I have over here I have 1 minus 2, so this is minus 1 over 4 and I have basically I have 2 j plus 2 j plus j that is 5 j. So, plus 5 by 4 divided by j and this is in fact, your X 2 that is sample corresponding to k equal to sample for k equal to 2, this is the sample corresponding k equal to 2 and similarly I think we can do the sample small X 3 corresponding to equal to 3. Just to be little bit more explicit to illustrate how to compute these things, although I am sure that all of you can do the simple 4 point IDFT, just to explicitly illustrate the process, this is X of 3 equals 1 over 4 again summation l equal to 0 to 3 X l e power j pi by 2 3 times l which is 1 over 4 summation l equal to 0 to 3 X l e power j well 3 pi by 2 times l.

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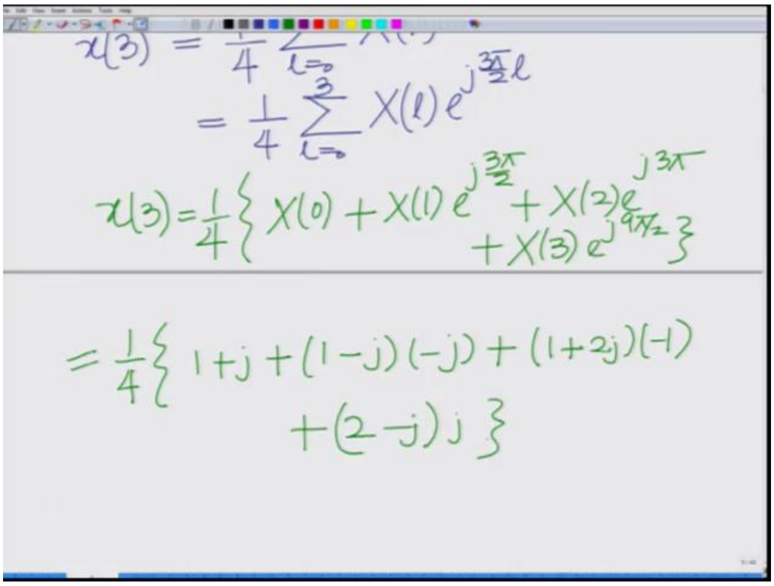

And now again when I substitute the symbols of the sub carriers that is X 3 equals 1 over 4 what do we have X 0 plus X 1 into e power j 3 pi by do plus X 2 into 2 power j well l equal to 0. So, this is 3 pi plus X 3 into e power j 9 pi by 2 because l equal to 3 this is equal to 1 over 4 X 0 is 1 plus j plus 1 minus j times minus j that is e power j 3 pi by 2 plus well 1 plus 2 j into minus 1 plus 2 minus j into e power j 9 pi by 2 that is j and when we simplify this what do we have when you simplify this. So, that is 1 over 4 and once again you can check this 1 over 4 1 plus j minus j minus 1 minus 1 minus 2 j plus 2 j plus 1. and what do we have over here?

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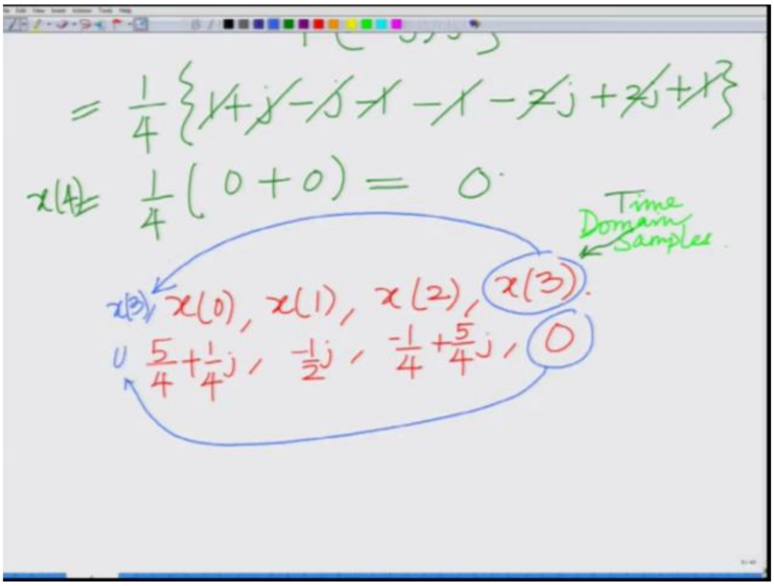

In fact, you have one by 4 times 0 plus 0 you can see this is j minus j 1 minus 1 minus 1 plus 1 2 j minus 2 j. So, this goes. So, what we have 0 and this is basically your X of 4. So, what you have is basically let me summarize what you have you have the transmitted samples in the time domain which I have given as X 0, X 1, X 2, X 3 and therefore, these samples now let me write down the values that we have computed 5 by 4 plus 1 by 4 j comma minus half j comma minus 1 by 4 plus 5 by 4 j comma 0 this is X 3. So, what I have done is (Refer Time: 16:18) in each time domain sample I have written down, these are your time domain samples. These are basically here - time when it each time domain sample I have written the corresponding the value of the sample. And now what we do as we said right. So, we generated the samples small x 0 small x 1 small x 2 small x 3 as the IDFT of the symbols capital X 0 capital X 1 capital X 2 capital X 3 which is the loaded of the sub carriers.

Now, the next step is of course, adding the cyclic prefix that is the I take one sample, I mean typically it is not one sample, but for this example one sample is enough when you take the sample from the tail of this block of samples and again repeat it or prefix it at this is known as the cyclic prefix for instance here I am going to take this X 3 repeat this X 3 over here or in other words take this 0, and repeat this 0 over here.

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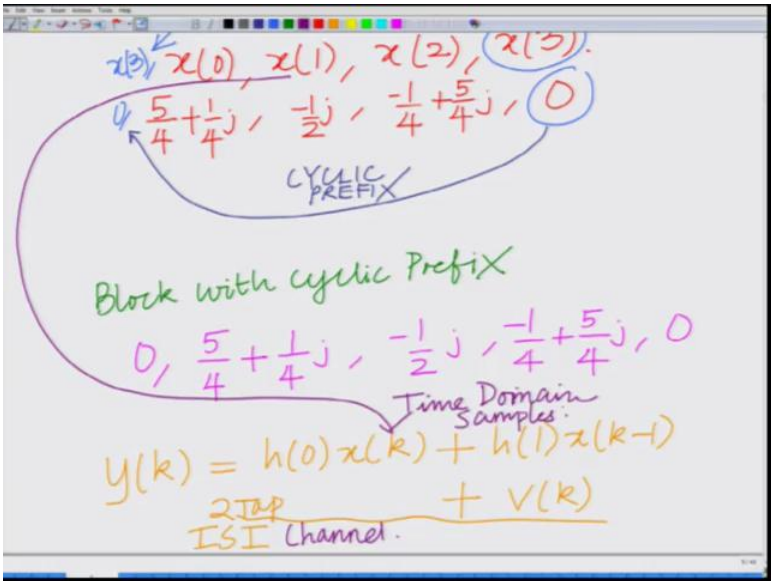

So, this is basically what we said and I think you should be familiar with this by now, this is basically what we said is the cyclic prefix. The block of transmitted samples over the channel with cyclic prefix, the block, and this is important to remember what it is this is block with the cyclic prefix - the block with cyclic prefix is basically your 0.

Let me just write it down very carefully this is 0 5 by 4 plus 1 by 4 j minus half j minus 1 by 4 plus 5 by 4 j comma 0 this is the block. So, now, what we are doing is we added the cyclic prefix to the time domain samples transmit them over the channel. And remember the channel that we are considering in particular that is the channel that we are considered in the example that we have seen for this illustration of this OFDM is the simple 2 tap channel which are the taps h 0 and h 1. So, the channel that we are considering let me remind you that is y of k this is a 2 tap h 0 X k plus h 1 X k minus 1 plus v k. Remember this is a 2 tap channel, 2 tap.

In fact, ISI channel inter symbol interference channel, this is your 2 tap ISI channel and now once you transmit these are the samples remember these X k these samples are nothing but what me let me just illustrated one more time, this X k's are basically this nothing but these are the time domain samples that you have got after the - this is what you have got after the IDFT of the symbols loaded on to the sub carriers.

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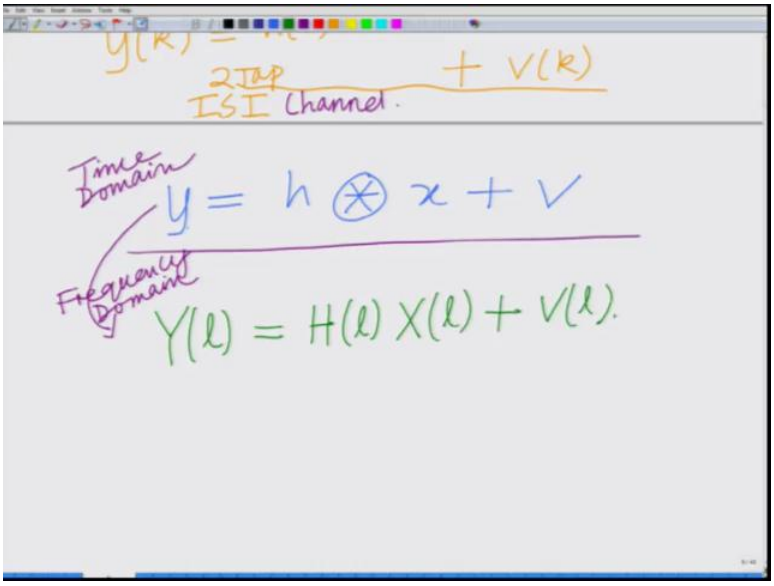

And therefore now what we are saying is action of the channel is one of the circular convolutions because of the cyclic prefix we have y equals h circularly convolved with X plus the noise which means now you take the FFT at the receiver. So, basically this is time domain - on the top is the time domain, on the bottom is the frequency domain and in the frequency domain across each sub carrier remember we have that is the fundamental equation of OFDM by l equals h l into X l plus v l. This is across sub carrier l, across sub carrier, l across sub carrier l.

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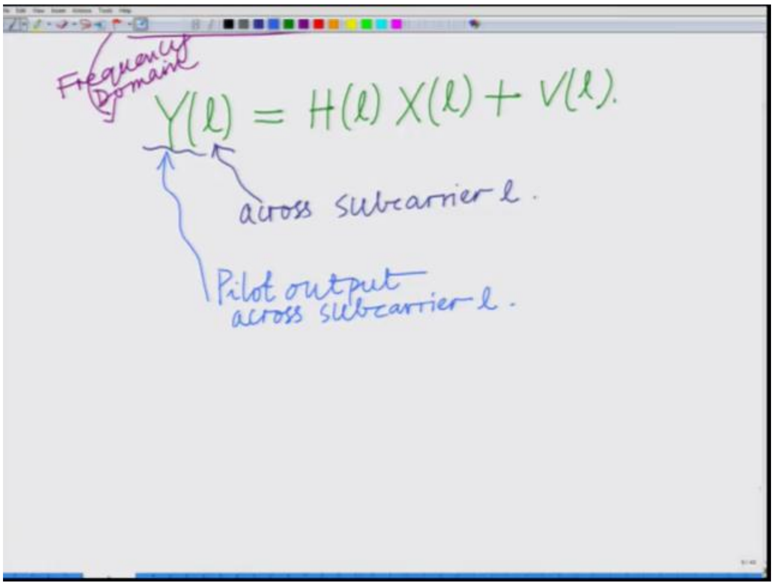

What are these y l's these y l's are the symbol pilot output received across the pilot outputs that are received across the sub carrier, across the sub carrier l. So, we have capital y l which is the pilot output received across sub carrier l.

So, what we have done so far is we have considered a simple n equal to 4 sub carrier OFDM systems. We have loaded the symbols X capital X 0 capital X 1 capital X 2 capital X 3 to basically of the sub carriers that are basically perform the IIFT or the IDFT to generate the time domain samples added the cyclic prefix and we looked at what happens at the receiver once you take the FFT.

Now, in the next module or in the sub sequent module we will look at what is the operations, what are the step by step operations at to be performed at the receiver in the simple OFDM example, starting with basically computing the pilot outputs received a across each sub carrier and subsequently estimating the channel taps or the channel coefficients in the frequency domain as well as the channel coefficients of the channel taps in the time domain. So, let us stop this module here and we will continue this in the sub sequent module.

Thank you.