Bayesian/MMSE Estimation for MIMO/OFDM Wireless Communications
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Lecture – 33
IMMSE Estimation of OFDM Channel Coefficient Across Each Subcarrier-Part II

Hello, welcome to another module. We are deriving the IMMSE estimate of the channel coefficient in of dm system that is considering each sub carrier we are deriving the IMMSE estimate of the channel coefficient corresponding to that particular sub carrier.

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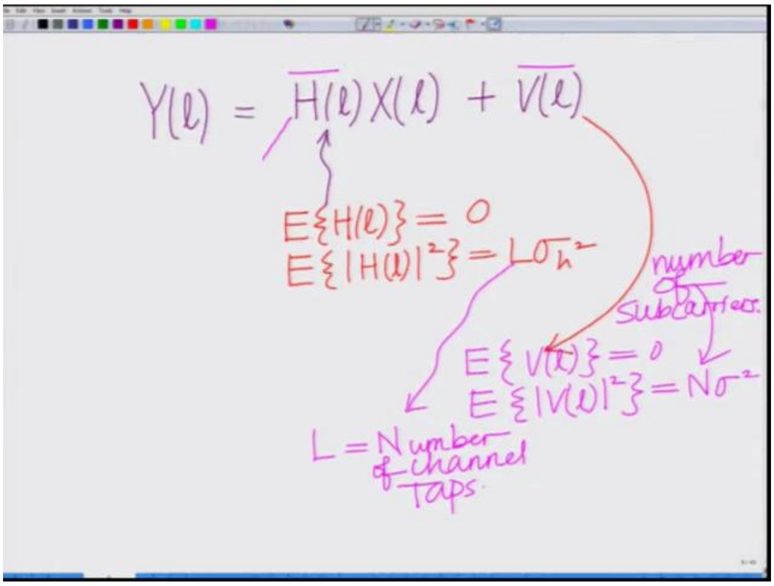

So, across sub carrier l right, we have Yl equals Hl where Hl is the channel coefficient times x l plus Vl right Vl is the noise corresponding to sub carrier l. We have seen that Hl is basically such that expected assuming 0 mean IID taps of variance sigma lth IID channel taps of variance sigma h square each we have seen expected value of Hl is 0. The variance expected value of magnitude Hl square equals l sigma h square. We have also seen assuming IID Gaussian noise samples 0 mean of variance sigma square each expected capital Vl the noise sample on each sub carrier this is equal to 0 expected value of magnitude Vl square, this is equal to what is this equal to this is equal to well n times sigma square, what is l? l equals to number of channel taps and n is equal to number of sub carrier this we have seen. So, far we have seen, alright.

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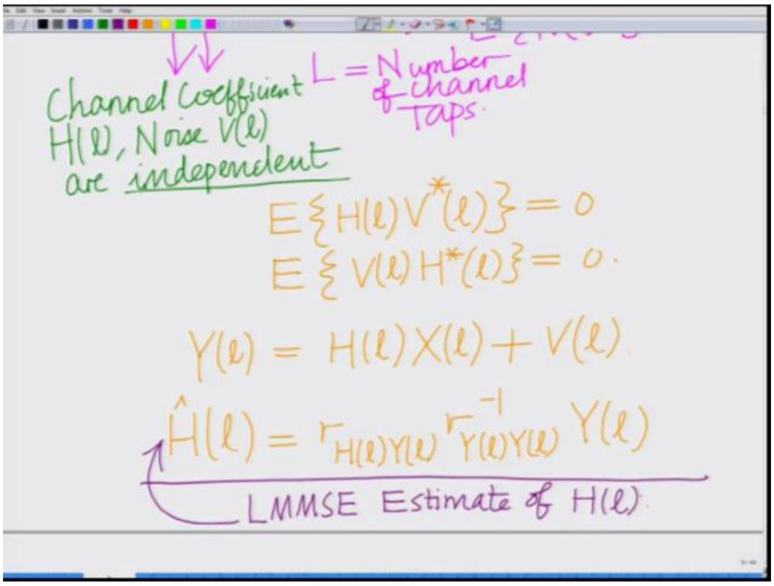

Let us now assume that this channel coefficient Hl and the noise Vl are independents. So, we are assuming going to assume, that similar to what we have assumed before that is correct the channel coefficient h, that these channel coefficient and noise Vl are independent. And we have already justified this several times before we said the channel coefficient arises because of the properties of the ambient scattering environment. the noise arises because of the thermal noise process and the receiver and these 2 phenomena are independent all right. Therefore, the channel coefficient and the noise are can be assume to be independent all right that is a realistic or a realistic and in fact, a very practical assumption. So, we can say that expected value of Hl terms v conjugate l equals zero. In fact, expected value of Vl times h conjugate l is also 0.

Now, coming back to our modal that is Yl equals Hl x l plus Vl, we indeed we can compute the MMSE estimate h hat of l as r Hl, remember this is the standard expression that is the cross covariance.

Where Hl Yl into r the covariance of Yl inverse times Yl this is the standard expression remember, this is the IMMSE estimate of the channel coefficient Hl all right this the r Hl Yl r Hl Yl which is the correlation between Hl and Yl times r Yl Yl inverse, where r Yl y l is basically the covariance of the output Yl of the l sub carrier times Yl where Yl is the output of the lth sub carrier all right this is basically follows from the expression for the standard IMMSE estimate all right the principle of standard IMMSE estimation all right.

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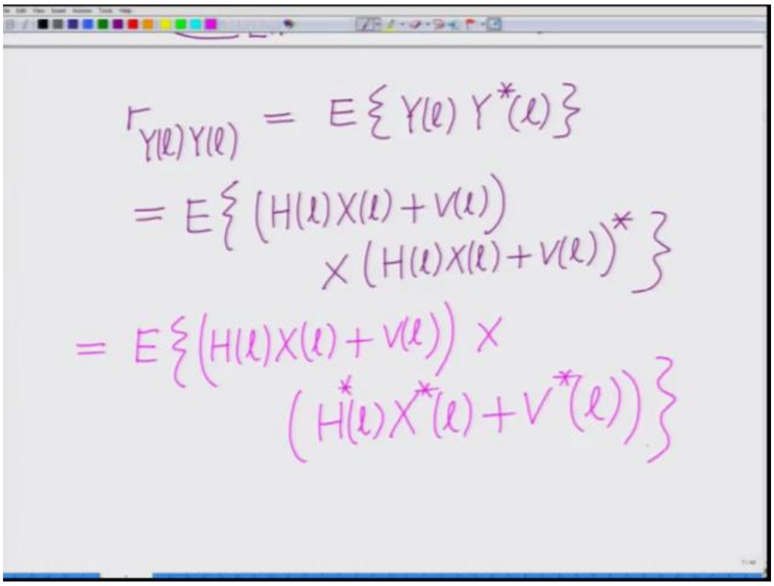

Now, what remains is basically to compute each of these quantities is Yl y l equals expected value of Yl times y conjugate l. This is equal to well substitute Yl expected Yl is basically your Hl x l plus Vl times Hl x l plus Vl conjugate this is equal to the expected value of well Hl. Let us just write a couple more steps in between to be very clear Hl x l plus Vl into h conjugate l x conjugate l plus v conjugate l.

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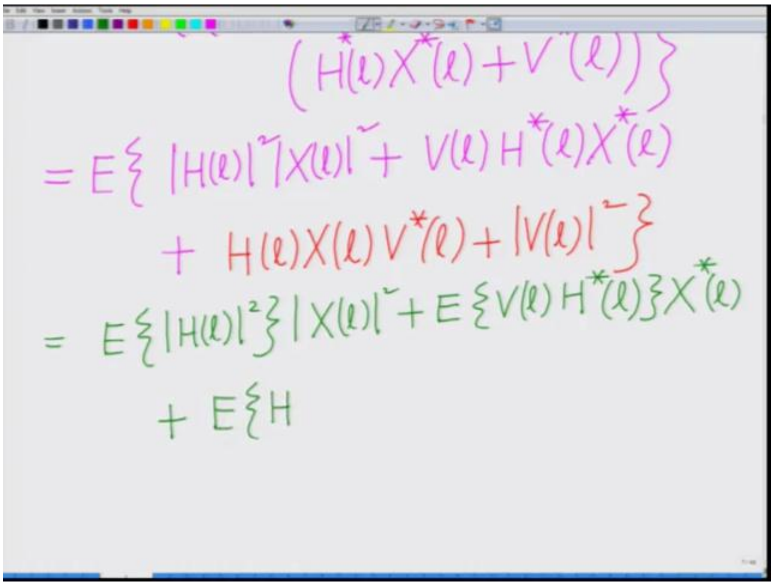

And now I can expand this as expected value of well Hl, we have the first term Hl x l h conjugate l x conjugate l that is magnitude Hl square magnitude xl square plus Vl into h conjugate l h conjugate l into x conjugate l plus well plus Hl x l into v conjugate l plus Vl into v conjugate l that is magnitude v conjugate a magnitude Vl whole square.

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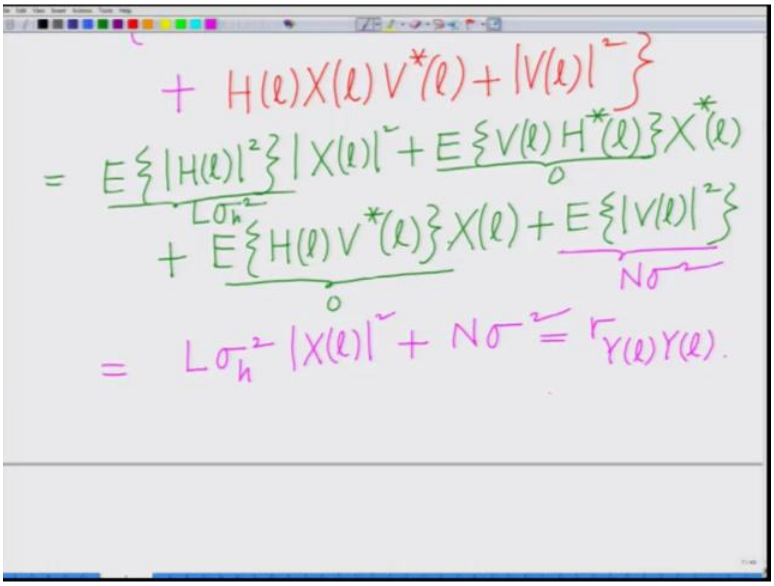

And now I can simplify this as well term by term taking expected value of magnitude Hl square into magnitude xl square plus expected value of Vl h conjugate lx conjugate l plus expected value of Hl into v conjugate l into x l plus expected value of magnitude Vl square.

Now, we know that expected value of Vl Hl conjugate is 0 expected value of Hl Vl conjugate this is 0 because the noise and the channel coefficient are uncorrelated expected value of magnitude Hl square this is l times sigma h square this, we already known already known expected value of Vl square is n times sigma x square therefore, what we have is l times sigma h square times magnitude x l square plus n times sigma square that is your r Yl Yl which is the covariance of which is the covariance right which is the covariance of the channel coefficient, which the is covariance of the channel coefficient y are the observation Yl across the on the lth sub care all right and now we need to.

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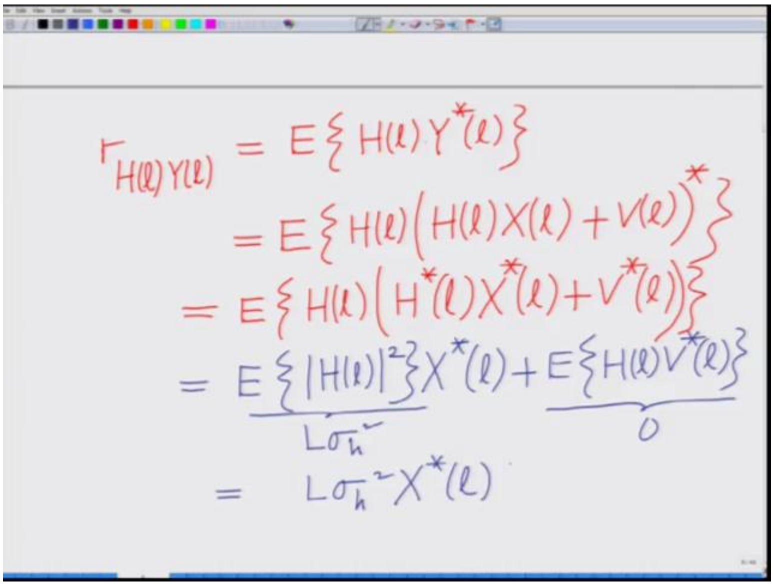

Compute r Hl Yl which is the cross correlation between the channel coefficient Hl and observation Yl and that can be also be computed in a straight forward passion as follows r Hl Yl equals expected value of Hl into Yl conjugate equals expected value of Hl times Hl x l plus Vl, conjugate equals expected value of well Hl into h conjugate l x, conjugate l plus v conjugate l which is equal to which is equal to expected value of well magnitude Hl square x conjugate l plus expected value of Hl v conjugate l which is 0 expected value of magnitude l conjugate Hl h magnitude Hl square is l sigma h square. So, this is simply l sigma h square x conjugate l where remember x l is the pilot.

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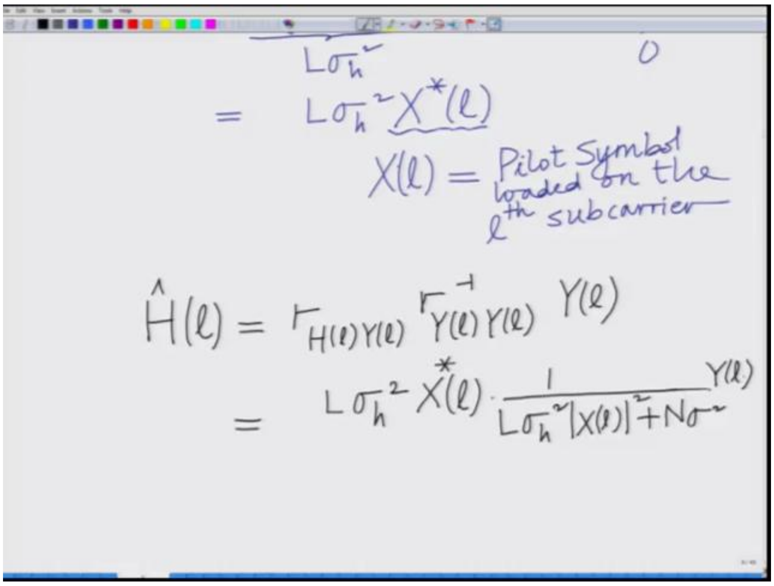

Symbol transmitted or loaded on the lth sub carrier x l equals pilot symbol loaded on the lth sub carrier. This is the pilot symbol remember we transmit pilot symbols specifically for the purpose of channel estimation and this is pilot symbol that is loaded.

On the lth sub carrier therefore, now substituting therefore, we have computed both the quantities r h y and r y y.

Now, substituting an expression for the IMMSE estimate we have the IMMSE estimate that is given as h hat l equals r Hl Yl r Yl Yl inverse into Yl now r Hl Yl is l sigma h square x l conjugate into r Yl Yl inverse is basically into 1, over where r Yl y l is basically your that is that is basically your l sigma h square magnitude x l square plus n sigma square times Yl.

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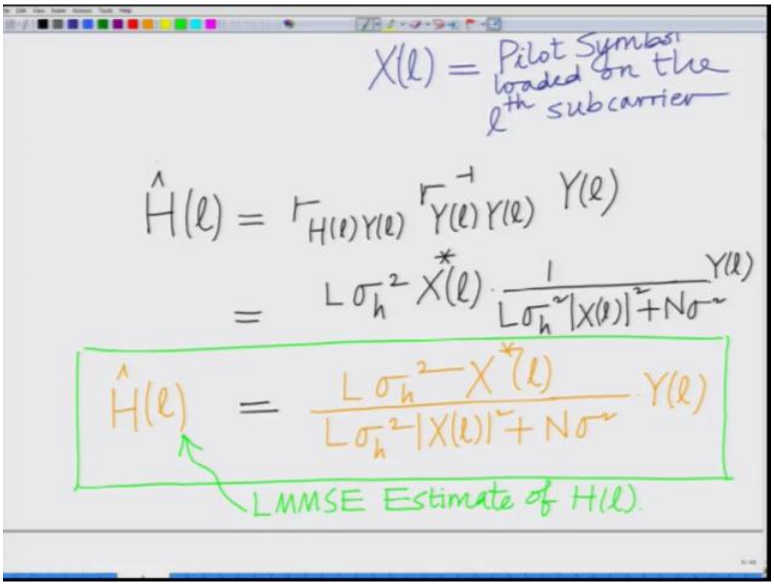

Now, I can write this estimate as your h hat l equals well l sigma h square divided by l sigma h square plus l sigma h square into l sigma h square x conjugate l divided by l sigma h square magnitude xl square plus n sigma square into Yl that is the output on the lth sub carrier and if you look at this, this is the expression for the once again IMMSE estimate of Hl this is the IMMSE estimate of Hl and once again you can see in this expression for the IMMSE estimate. So, we have derived the expression for the IMMSE estimate you can see once again for the expression for the IMMSE estimate that if at high SNR again similar to at high SNR.

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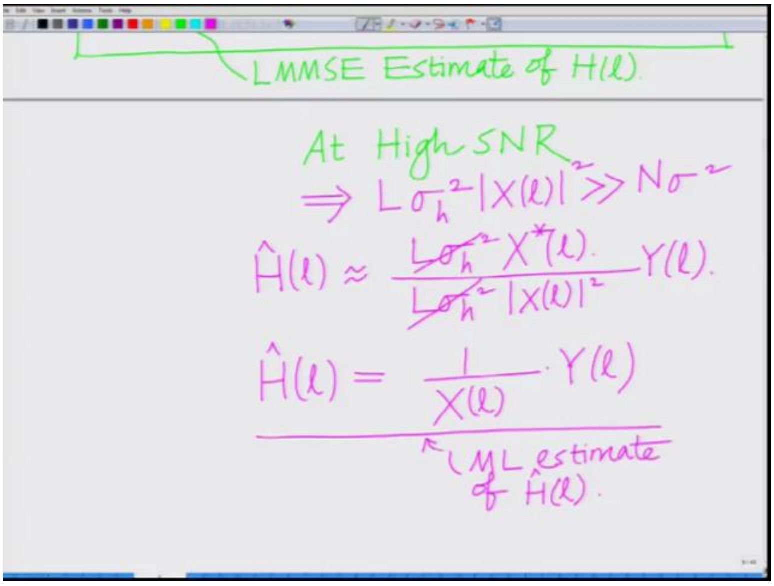

What do you mean by S SNR implies if l sigma h square magnitude x l square is much greater than n sigma square then h hat l becomes approximately your, I can ignore n sigma square in comparison to l sigma h square magnitude x l square. So, this becomes l sigma h square well x conjugate l divided by l sigma h square magnitude x l square.

Into y l, but x magnitude x l square which is nothing, but x l into x conjugate l therefore, this becomes well l x sigma h square l sigma h square cancel. So, this becomes 1 over x l into Yl and this is simply the ml estimate that is this is simply the expression for the maximum like load ml estimate that is once again at high SNR that is when, your l sigma h square magnitude xl square is much, much greater than noise power n square on the sub carrier l. Then what happens it again reduces to the ml estimate the maximum likelihood estimate for the channel coefficient Hl which is simply Yl that is the output of sub carrier l divided by xl which is the capital xl which is the symbol loaded on the sub carrier l.

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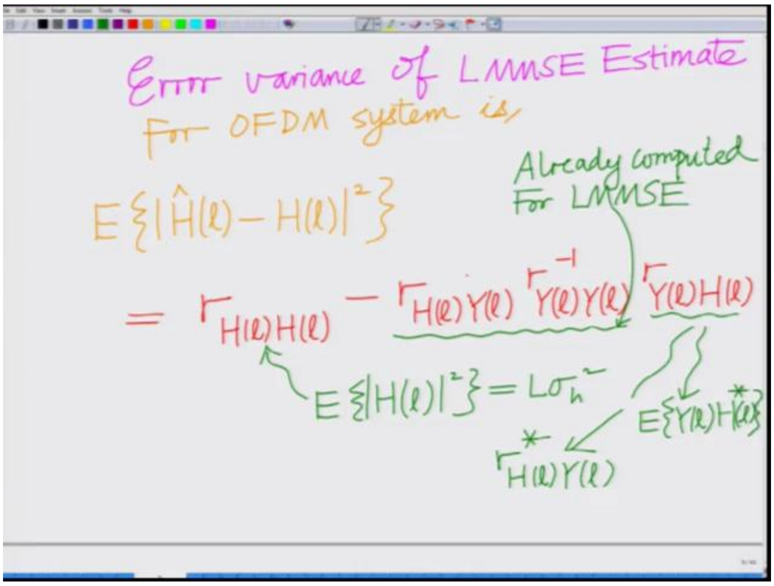

Now we can again similarly find out what is the error what is the variance of the error corresponding to the estimate of channel coefficient l. So, let us now complete this problem find that finding the variance of that error the error variance the error variance of IMMSE estimate for the of dm system is well we have expected value of magnitude h hat l minus Hl whole square this is equal to r Hl h l minus r Hl Yl into r Yl y l inverse into r Yl well Hl.

Now if you can observe this this quantity is what we have exactly computed above that is r Hl Yl into r Yl y l inverse this is the quantity that, we computed for the already computed for IMMSE estimate this is the quantity that we have already computed. Now r Hl h l this is nothing, but expected value of your magnitude Hl square and this is equal to l sigma h square and this quantity r Yl Yl expected value of Yl into Hl conjugate this is nothing, but r Hl Yl conjugate. We have already computed r and this is equal to basically r Yl that is basically your r Yl Hl is basically r. if you look at this quantity r Yl Hl this is expected value of Yl into h conjugate l which is basically r Hl conjugate into r Hl Yl conjugate and we have computed all these quantities before that is r Hl Yl and each of these quantities we have computed before.

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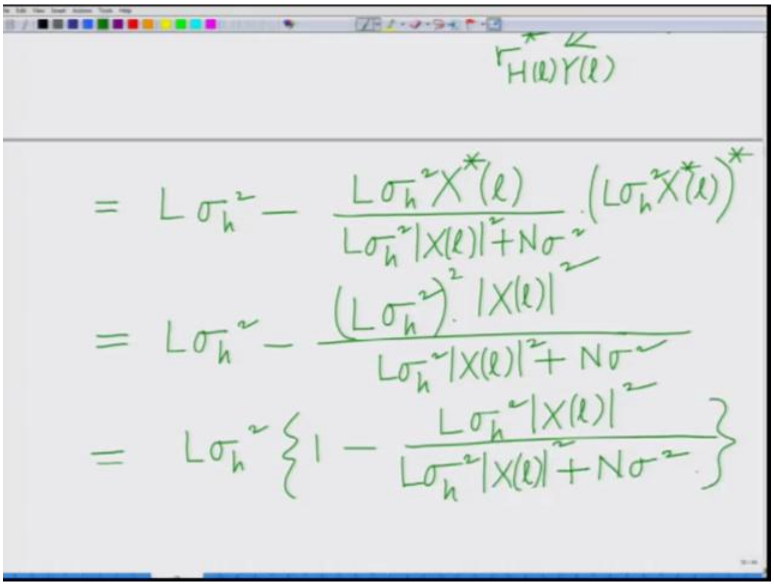

So, all it remains basically remains it remains to substitute these quantities which I am going to do now. So, r Hl Hl that is basically your let me write this again r Hl Hl that is equal to l sigma h square minus r Hl Yl into r Yl Yl inverse that, we already seen above l sigma h square x conjugate l divided by l sigma h square magnitude xl square plus n sigma h square times r Yl Hl which is r Hl Yl conjugate that is l sigma h square x conjugate l conjugate.

And therefore, that gives us l sigma h square minus l sigma h square magnitude xl square plus n sigma square times l sigma h square whole square into magnitude xl square which is equal to l sigma h square into one minus l sigma h square magnitude xl square divided by l sigma h square magnitude xl square plus n sigma square and this is equal to well, if you look at this this is simply your l sigma h square into l sigma h square magnitude xl square minus l sigma square magnitude this cancels.

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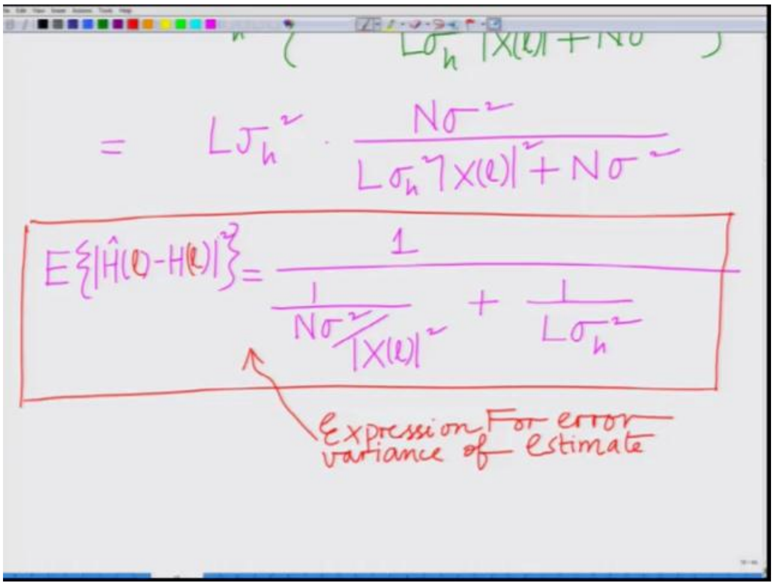

So, what is left is n sigma square divided by l sigma h square magnitude xl square plus n sigma square. Now bringing l sigma h square and n sigma times n sigma square to the denominator what we have is this is equal to 1 over 1 divided by well n sigma square divided by magnitude xl square plus 1 over l sigma h square and this is basically your equals your expected value of magnitude hk h hat k minus hk whole square. So, what we get is this is equal to this quantity this is the expression for the error variance expression for error variance this is the expression, expression for error variance of estimates sorry this has to be h hat of l that is the estimate of the lth sub carrier yeah magnitude h hat l minus Hl square still vary expression for the error variance of estimate h hat l corresponding to corresponding to sub carrier l.

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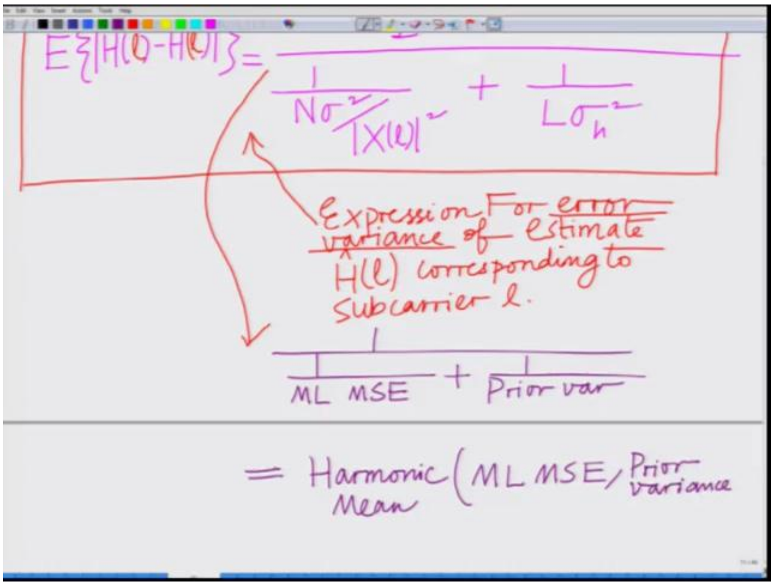

So, this is the expression for the error variance the variance of the error of estimate h hat l corresponding to the corresponding to sub carrier l and once again you can see, one over l sigma l sigma h square is basically the prior variance that is l sigma h square is basically the prior variance of Hl right. We have already seen that n sigma square is basically n sigma square divided by magnitude xl square is the variance of what is the mean square error of the maximum likelihood estimate.

Therefore, again you can see it is the harmonic mean of the mean square error of the maximum likelihood estimate and the prior variance all right and this is something that we have seen several times before and once again well this is true of all IMMSE estimation in general. So, this is basically your again to reconfirm that one by ml MSE plus one by prior variance that is this is basically equal to your equal to the harmonic mean of MSE of m l coma where m l denotes the maximum likelihood estimate that is it is the harmonic mean of the mean square error of the maximum likelihood estimate and the prior variance of the channel coefficient.

So, basically this completes IMMSE estimate of IMMSE estimation of the channel coefficient Hl corresponding to sub carrier l and also the derivation of the variance or derivation the mean squared error rather of the estimate of the IMMSE estimate all right we will continue in this subsequent module.

Thank you very much.