Bayesian/MMSE Estimation for MIMO/OFDM Wireless Communications
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture – 32
LMMSE Estimation of OFDM Channel Coefficient Across Each Subcarrier – Part I

Hello, welcome to another module. So, in this module we are going to look at IMMSE estimation for an OFDM Wireless Communication System, alright.

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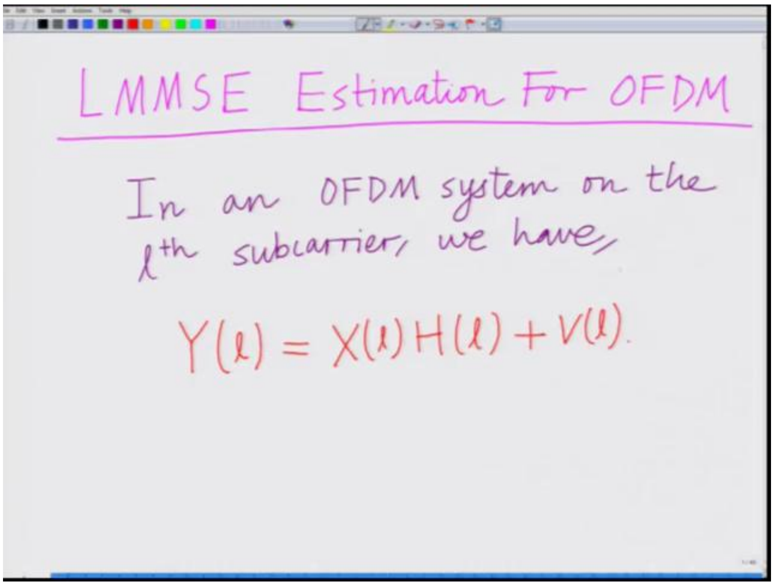

So, we would like to look at the linear minimum means squared error estimation for OFDM. Which we already know is orthogonal frequency division multiplex you know, in OFDM system on the lth subcarrier on the lth subcarrier, we have well what we have? We have y l equals x l into Hl plus Vl well, what is l l denotes your index of the subcarrier l is basically the lth subcarrier that we have talking about y l is the symbol that is received of lth subcarrier Hl is the channel coefficient corresponding to the lth subcarrier x l is the symbol transmitted or loaded on the lth subcarrier and Vl is the noise of the lth subcarrier.

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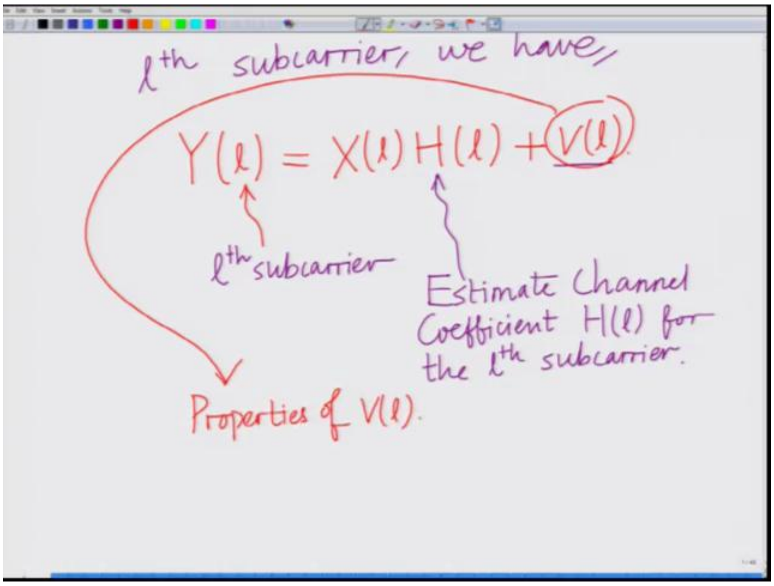

And we have to estimate the channel coefficient on the estimate the channel coefficient Hl for the for the lth subcarrier. So, that is we have to do and towards doing that let us first start by talking about the statistics of the noise that is Vl. So, first let us start with the properties of the noise, let us start deriving the properties just start by deriving the properties of this noise component the noise sample capital Vl on the sub category lets.

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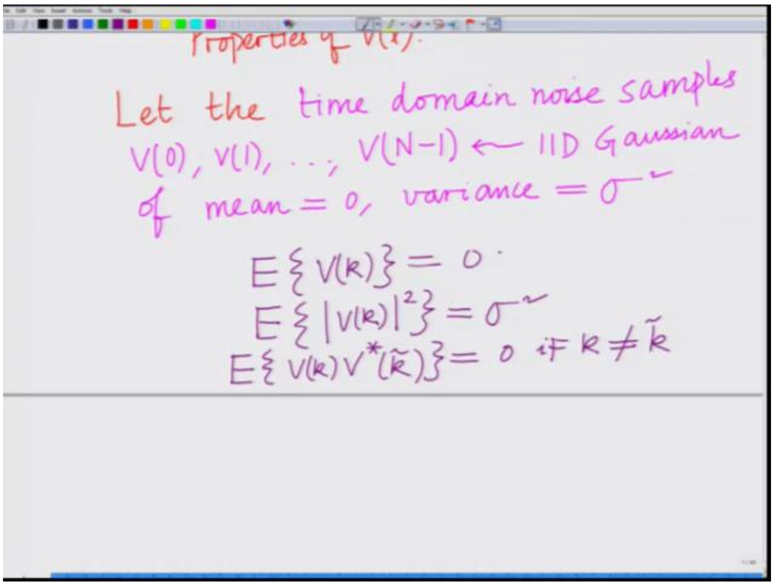

Let assume the time domain noise samples let the time domain noise samples V0, V1 up to remember we have up to Vn minus 1, this are the n time domain noise sample let these be I i d Gaussian of mean equal to 0 and variance equal to sigma square what; that means, at these noise samples a small Vs that is we have expected value of Vk equal to 0 expected value of magnitude Vk square that is the small Vk magnitude Vk square equals sigma square and since they are independent expected value of Vk into v conjugate k tilde equals to 0. If k is not equal to k tilde, So, these we know about time domain samples of small Vk.

Now, what can say about the capital Vl right, which is the noise output on the lth subcarrier remember the capital Vl which are given by the FFT of the noise samples correct.

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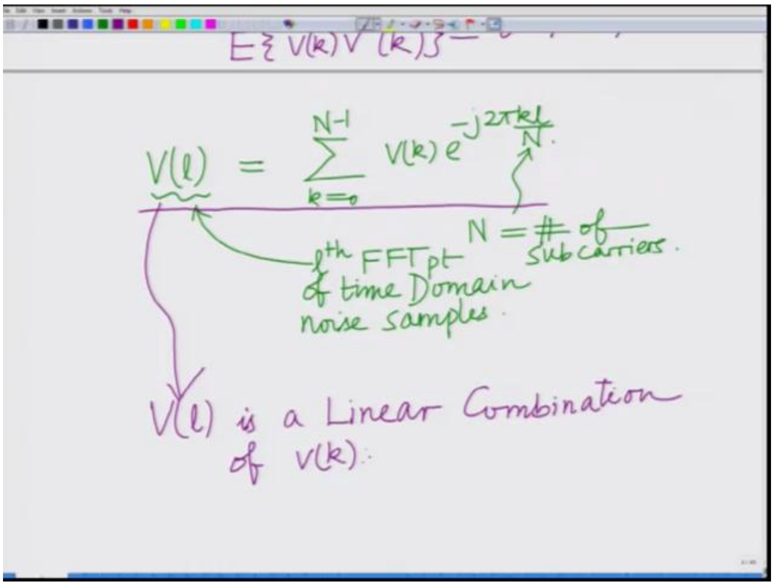

So, each capital Vl you can recall is the lth FFT point of the noise sample is the summation k equal to 0 to n minus 1 Vk e to the power of minus j 2 pi k l over n. Where n equals your number of, So where n equals the number of subcarriers. Now, let us find. So, this Vl is the lth FFT point of the noise samples of your time domain the lth FFT point of the time domain noise samples. Now first observe that Vl is a linear combination that is capital Vl is a linear combination of the small. So, Vl is a the capital Vl is a linear combination of Vk which have the time domain noise sample and this is the small Vk s are Gaussian right linear combination of Gaussian noise sample, Gaussian sample, Gaussian random variable is the Gaussian random variable. Therefore, since the capital Vl is the linear combination of the time domain noise sample of small Vk capital Vl is intern Gaussian noise sample alright this therefore, this implies linear combination of Vk this implies Vl is also Vl is a also a Gaussian random variant alright that is the first 1.

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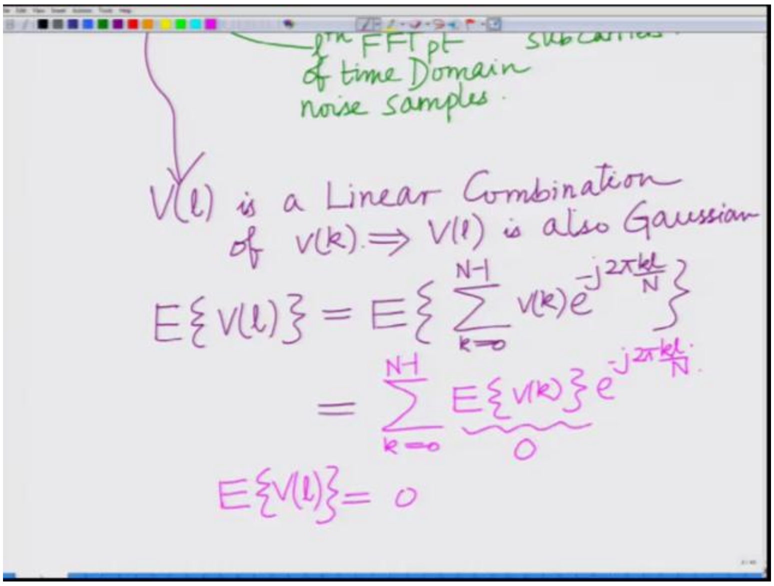

Further if you look at the expected value of Vl that is the nothing, but the expected value of summation k equals to 0 to n minus 1 Vk e to the power of minus j 2 pi k lower n taking the expectation operator inside this is summation k equal to 0 to n minus 1 expected value of Vk e to the power of minus j 2 pi k l divided by n each Vk is 0 mean expected value of Vk is 0 which means expected value of Vl is also 0.

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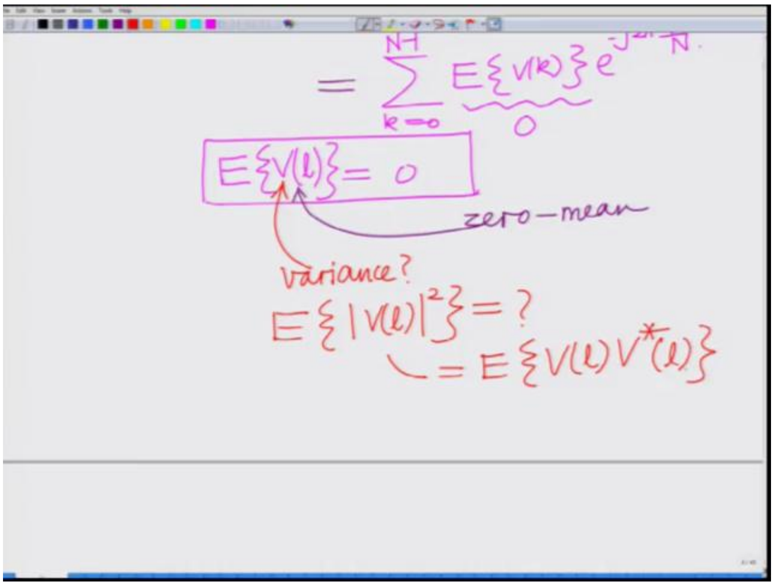

Therefore we can now say that Vl is Gaussian. So, we have deduce 2 proprieties 1, Vl is Gaussian Vl is also a 0 mean random variable, there is a noise output Vl on the l subcarrier is both Gaussian and it is 0 mean now I remains to deduce the variance of each that is, what is expected value of now, we want to ask the question what is the variance of each Vl that is expected value of magnitude of Vl square what is this remember this is also equals to expected value of magnitude of Vl v conjugate of l and I can simplify this as follows expected value of Vl into v conjugate of l.

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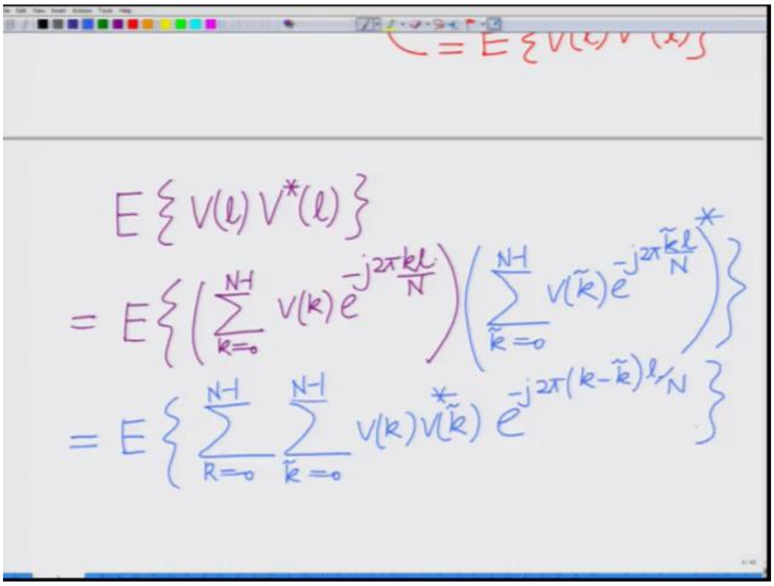

This is equal to well expected value of we know the expression for Vl summation k equal to 0 to n minus one Vk e to the power of minus j 2 pi k l divided by n into its own conjugate. Now I am going to write the conjugate with the different index k is a only a index. So, I going to use here an index k tilde k tilde equal to 0 to n minus 1 Vk tilde e to the power of minus j 2 pi k tilde l divided by n of course, the conjugate.

Now, I am going to simplify this together by multiplying out these 2 the term in these 2 brackets term by term. So, I have k equal to 0 to n minus 1 summation k tilde equal to 0 to n minus one Vk v k tilde conjugate e to the power of minus j 2 pi k minus k tilde l by n.

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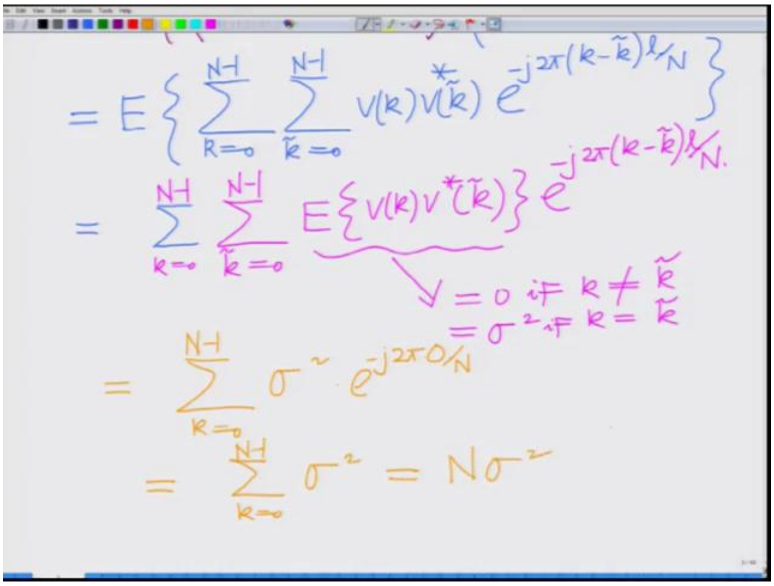

Now, once we take the expectation operator inside I have something interesting I have summation k equal to 0 to n minus 1 k tilde equal to 0 equal to n minus 1 expected value of Vk v conjugate k tilde e to the power of minus j 2 pi k minus k tilde l divided by n and now, we that these 2 noise samples that the time domain noise sample the small v ks are independent which means expected value of that is what we have already seen expected value of Vk times, Vk tilde conjugate equal 0. If k is not equal to k tilde and sigma square if k is equal to k tilde alright.

So, this is equal to equal to 0, if k is not equal to k tilde equal to sigma square if only if k is equal to k tilde which means only terms which survive in the summation are when k is equal to k tilde and therefore, this is for each k only one term corresponding to when k tilde equal to k will survive. So, this will be summation k and when k equal to k tilde this sigma square. So, this is summation k equal to 0 to n minus one sigma square e to the power of minus j 2 pi k minus k tilde is 0 divided by n which is basically 1 e to the power of 0 is 1. So, this is summation k equal to 0 to n minus 1 sigma square which is equal to sigma square.

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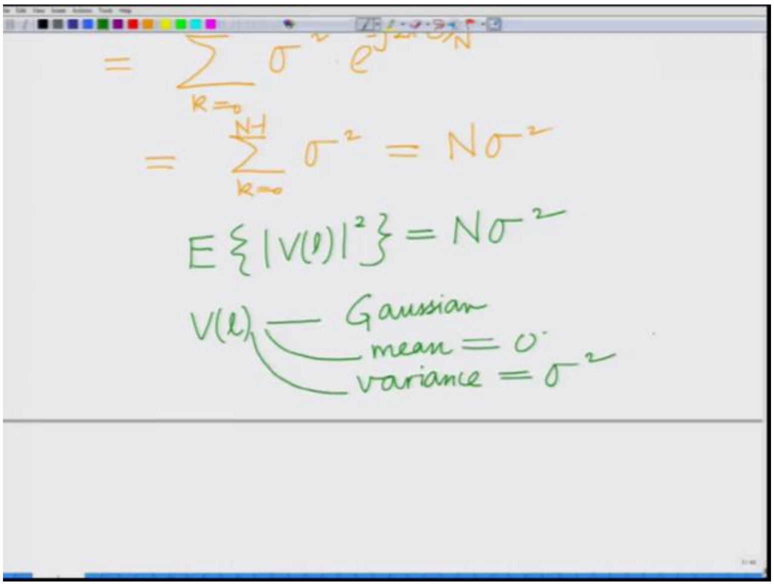

Therefore what we have is if you look at this result we have expected value of magnitude Vl square equals n sigma square. So, Vl is Gaussian that is written by the symbol n its normal Gaussian is; in fact, complex Gaussian. So, let simply write Vl is Gaussian mean equals 0 variance is equal to sigma square its mean equal to 0 and variance is equal to sigma square correct right, v l. So, we have characterized the noise the properties of this noise Vl then Vl is remember this is the noise coefficient across the remember, we need this properties to in order to find the IMMSE estimate because IMMSE estimate depends on the depends on knowing remember we need to know the variance alright the mean and the variance of the noise.

Now, let us go back and let us look characterize the prior variance let us characterize the properties of the channel coefficient Hl corresponding to subcarrier l. So, we would like to characterize the properties of the channel coefficient Hl.

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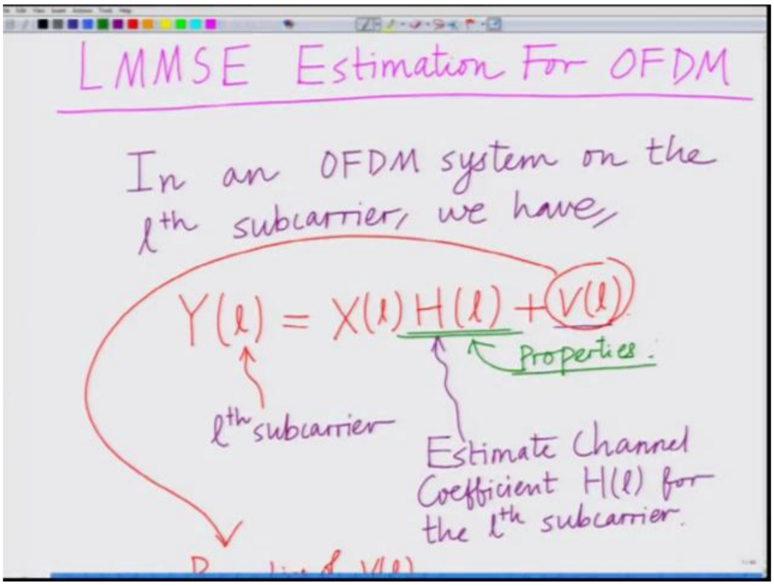

So, we would like to characterize the properties of the channel coefficient Hl now remember Hl?

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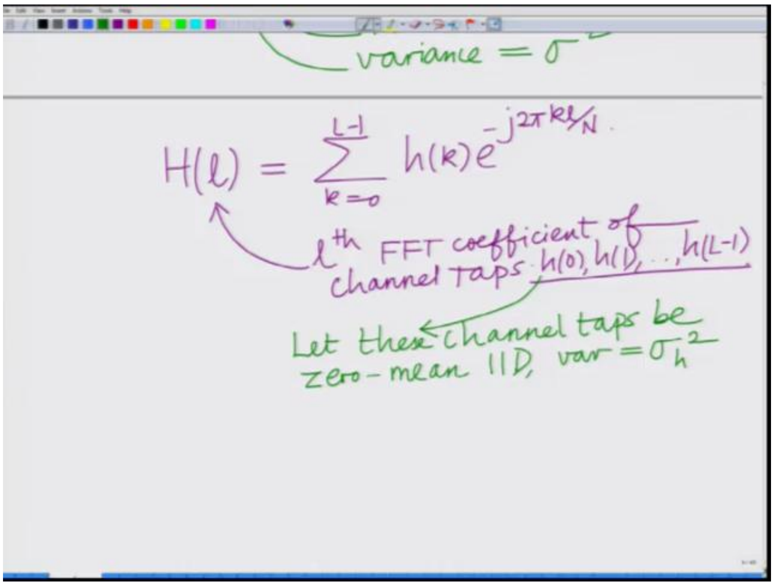

Now, let now remember Hl again similar to the noise Hl is the lth FFT coefficient of the channel tabs h k there is summation k equal to 0 to l minus 1 h k e rays to minus j 2 pi k l this is the lth FFT coefficient of channels taps h 0 h 1 up to Hl minus 1 now what we are going to assume is.

We are going to assume let these channel taps let these channel taps h 0, let these channel taps be 0 mean I i d variance is equal to sigma h square that is we are assuming the channel taps where 0 h 1 up to Hl minus 1 to be independent identically distributed with mean 0 and variance sigma h square as we know because, we considering the IMMSE estimate these need not necessarily be Gaussian in nature right. If we are considering the IMM if they are additionally Gaussian then this becomes the MMSE estimate alright. So, for a general IMMSE estimation scenario these need not be Gaussian.

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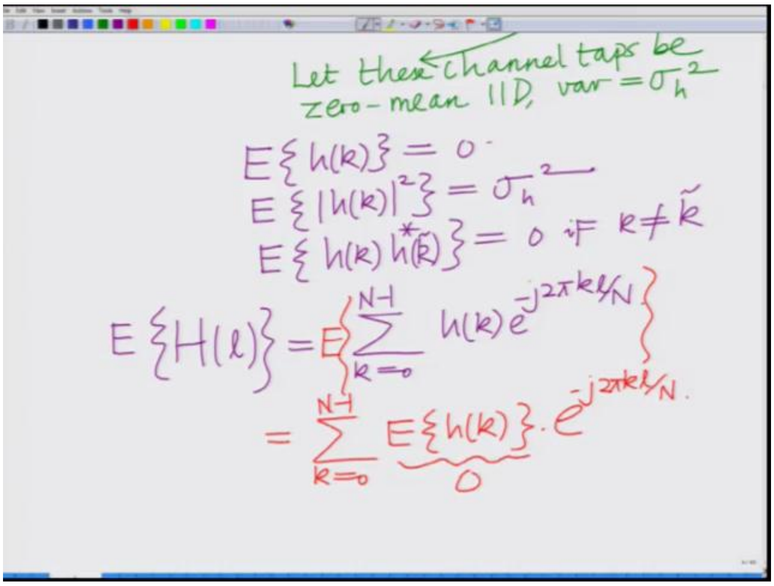

So, we are assuming expected that is the channel taps the small h s remember these are the channel taps expected h k equal to 0 expected magnitude h k square equals sigma h square expected magnitude h k into h lets call this k tilde conjugate equals to 0 because they are independent if k not equal to k tide; obviously, if k equal to k tide then, this becomes sigma h square alright.

Now, again we have Hl equals summation k equal to 0 to n minus 1 h k e raise to minus j 2 pi k l by n. If I take the expected value on the left I also get the expected value on the right which is equal to, now taking the expected value inside expectation operator inside k is equal to 0 to n minus 1 expected value of h k e raise to minus j 2 pi k l by n expected value of each h k equals 0 because we assuming the these 2 0 mean random variables.

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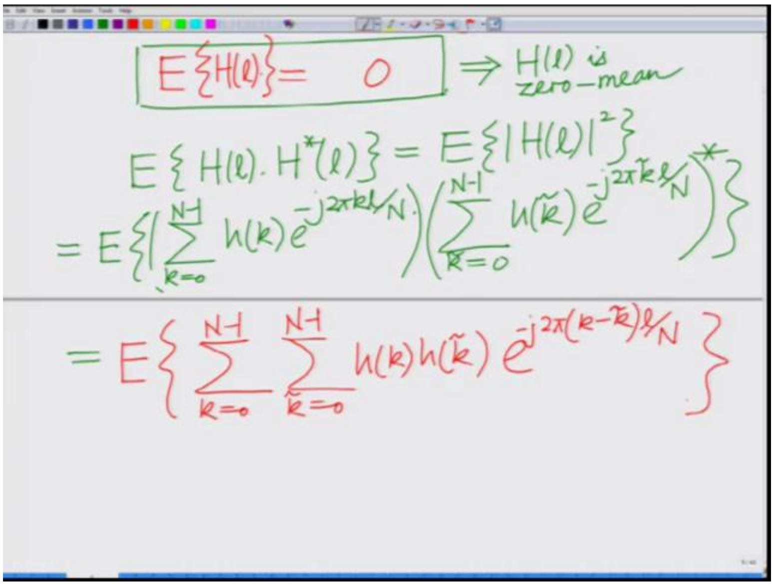

Therefore expected value of Hl where Hl is the lth channel coefficient on the lth subcarrier expected value of each Hl is also 0. So, which means this each Hl is also implies Hl that coefficient on the lth subcarrier channel coefficient on the lth subcarrier is also 0 mean.

Now, we have to the variance that is what is the expected value of Hl h conjugate l which is the expected value of magnitude Hl square expected value of Hl into h conjugate l which is expected value of magnitude Hl square what is this? This is equals to the expected value of well let us again write at use the same trick or technique that we used earlier we can write this as expected value of summation k equals to 0 to n minus 1 h k e raise to minus j 2 pi k l divided by n into same thing, but changing the index k tilde equals to 0 to n minus one h k tilde e raise to minus j 2 pi k tilde l divided by n conjugate which is equal to well.

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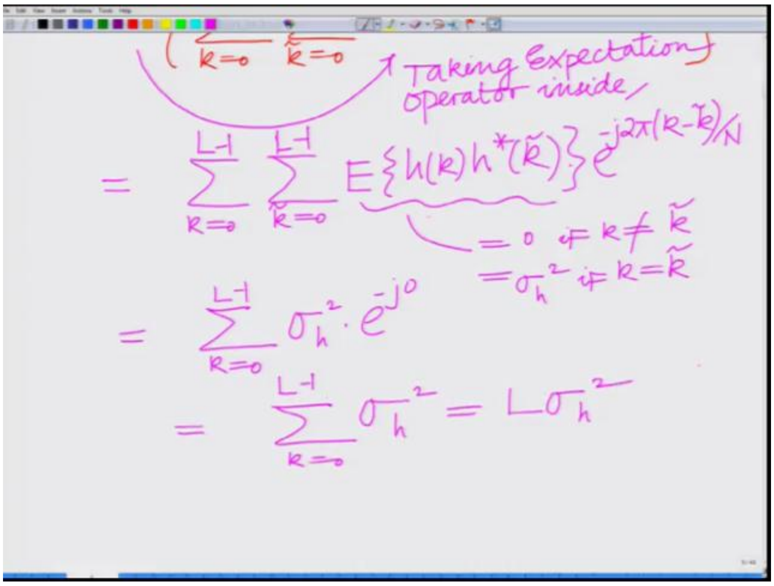

Now, multiplying term by term summation k equal to 0 to n minus 1 k tilde equals to 0 to n minus 1 h k h k conjugate e to the power of minus j 2 pi k minus k tilde l by n taking the expectation operator inside alright taking the expectation operator this gives summation k equal to 0 summation k tilde equals to 0 expected value of. Of course, there has to be conjugated expected value of h k h conjugate k tilde e to the power of minus j 2 pi k minus k tilde by n and now, you can see this sorry this can only be an one point FFT sorry this has to be because, there are only l channel taps this can only be till l minus 1 this can only be till l minus 1 or the summations are only till l minus 1 because we have only l channel taps.

So, this is still l minus 1 l minus 1 l minus 1 and again this is also till l minus 1 this also till l minus 1 and you realize that this is equal to 0. If k is not equal to k tilde equal to sigma h square if k is equal to k tilde hence the only term, which survive are the once is k is equal to k tilde again similar to the previous derivation. Therefore, this will again be summation k equal to 0 to l minus 1 sigma h square again e to the power of minus j 0 equals summation k equal 0 to l minus 1 sigma h square which is l times sigma h square.

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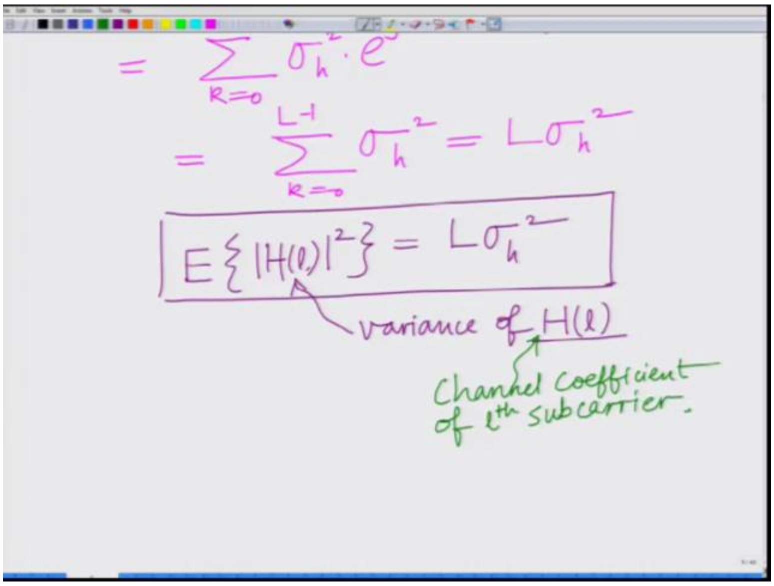

Therefore, what we have now is basically we have expected value of magnitude Hl square equals l sigma h square. So, we have the variance of the channel coefficient on the lth subcarrier assuming IID channel tabs small h 0, small h 1, up to small h n minus small Hl minus 1 that is assuming one IID channel tabs small h 0, small h 1 up to small Hl minus 1. So, this is the variance, variance of the channel coefficient Hl h l is the channel coefficient of the lth channel coefficient of the lth subcarrier, alright.

So, what we have done today is we have consider the OFDM system model accuracy is subcarriers assumed IID time domain noise sample 0 mean variance sigma square, derive the properties of the noise Vl at the output of the FFT and we have shown that the noise Vl on the each subcarrier is Gaussian 0 mean, variance n times sigma square and we also assumed IID, one IID channel tabs and demonstrated channel that the channel coefficient Hl is also 0 mean and has a variance l sigma h square alright we are going to use these properties in the subsequent module to compute the IMMSE estimate of the channel coefficient Hl on subcarrier l. So, we will stop here.

Thank you.