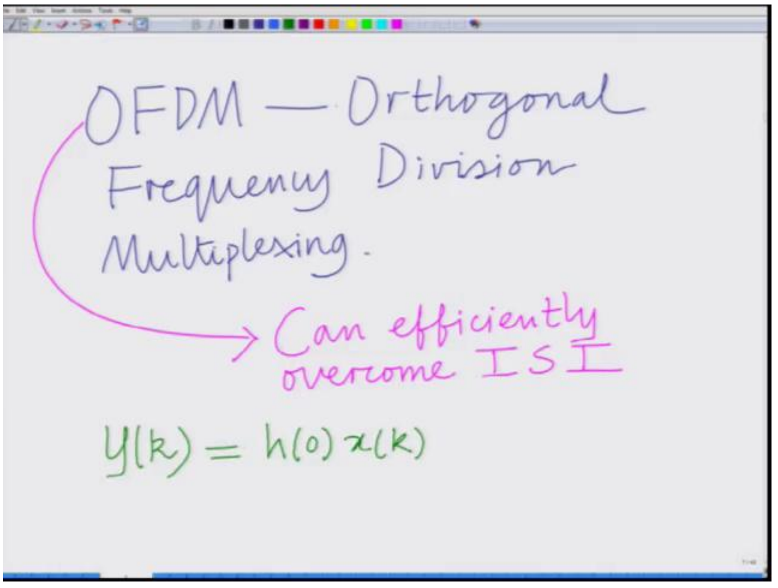
Bayesian/MMSE Estimation for MIMO/OFDM Wireless Communications
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture - 31
Introduction to Orthogonal Frequency Division Multiplexing (OFDM) – FFT at Receiver and Flat-Fading across Each Subcarrier

Hello. Welcome to another module, in this massive open online course. So, currently we are looking at OFDM, or Orthogonal Frequency Division Multiplexing, and we are trying to understand the mechanism of OFDM, and we have also said OFDM, is a modern wireless technology, which can efficiently overcome, inter symbol interference, correct? So, what we are looking at currently is we are looking at estimation in the contest of OFDM, where OFDM stands for Orthogonal Frequency Division Multiplexing.

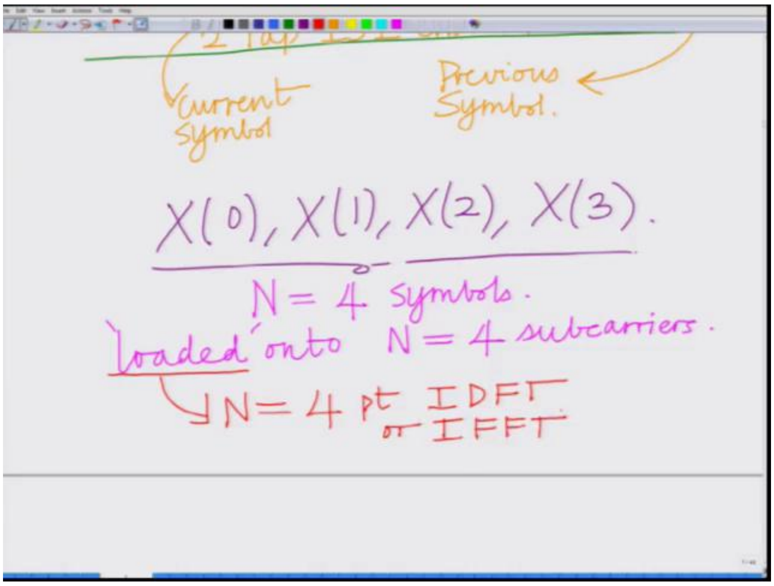
(Refer Slide Time: 00:36)



And we said OFDM can be used to efficiently overcome, can efficiently overcome ISI, or basically, inter symbol interference. For instance, we have our, we have seen this many times before, y k equals h0, x k, plus h1, x k minus 1, plus v k, this is our 2 tap ISI channel. So, this is basically the model for your 2 tap, ISI channel, but the taps are h 0 and h 1, x k, is the current symbol, x k minus 1, is x k minus 1, is the previous symbol.

So, that is a symbol interference, of the previous symbol x k minus 1, on the current symbol.

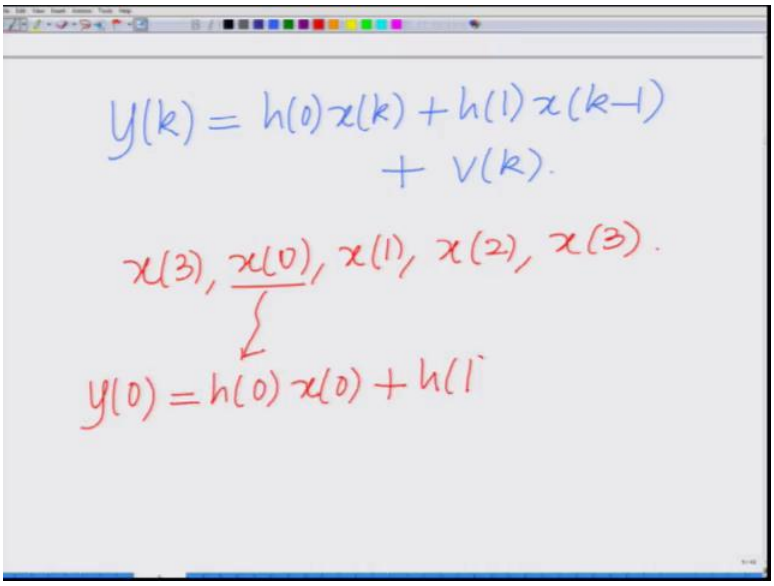
(Refer Slide Time: 02:13)



And they said OFDM can be explained as follows. So, OFDM we have, lets consider a typical OFDM system, with 4 sub carriers, that is, we have, that is, we have the symbol X0, capital X0, capital X1, capital X2, capital X3. These are the N symbols. That is these are N equal to, 4 symbols. These are loaded, these are loaded on to, N equal to 4 sub carriers, and the meaning of this is basically, the meaning of this term loaded is basically, we perform N equal to 4 point, N equal to 4 point, IDFT or IIFT, inverse Fast Fourier transform, which is basically the inverse Fast Fourier transform. One can perform IDFT or equivalently IIFT, IIFT is nothing but the same as the IDFT, but it is efficient, it is a Fast algorithm, to perform IDFT that is inverse Fast Fourier transform.

So, now, therefore, the samples from this, what you have, is from this, capital X0, X1, X2, X3, will generate, what are known as the samples, that is, by IFT, that is the samples, x 0, x 1, x 2, x 3, and these are generated by the, these are the N equal to 4 samples, and these are generated by the IDFT as, and all of you must be familiar with IDFT, that is basically we have x of k, equals 1 over N, summation l equal to 0, to N minus 1, capital X of l, that is a symbol noted down to the sub carriers, e to the power of j, 2 pi, k l by N, and now I am going to substitute N equal to 4. So, this gives me basically, 1 over 4, summation l equal to 0, to 3, capital Xl, e raise to j, 2 pi, k l by 4, which is equal to therefore, your x l is basically equal to, 1 over 4, summation l equal to 0 to 3, x l, e raise to j, pi by 2, k l.

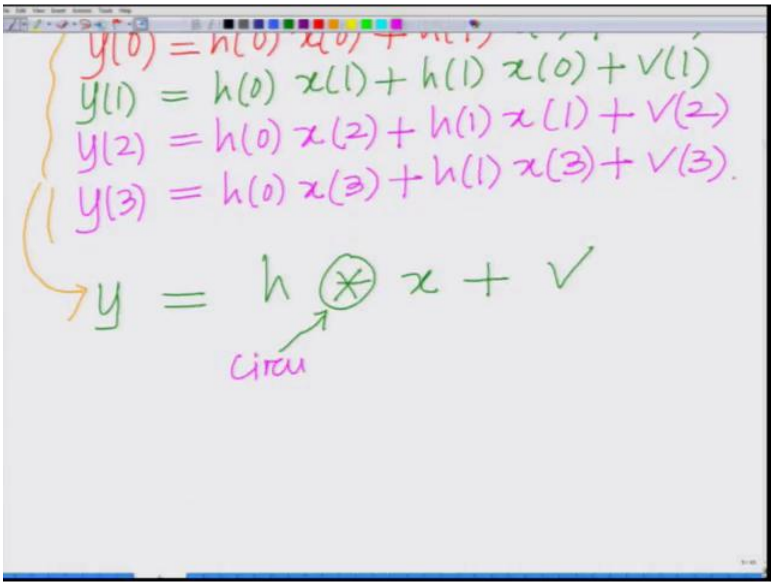
(Refer Slide Time: 05:09)



So, this is basically the samples. So, these are the symbols loaded on to the sub carriers, that is your b p s k, q p s k symbols etcetera, and these are the, these are the samples. And now we said, we are not going to, we want to transmit the samples, but we do not transmit the samples, as it is. We have a small modification before transmission of these samples over the channel, we add the cyclic prefix that is we take a few symbols from the tail of the block, and prefix them at the head of the block, since we have the cycling symbols from the end towards the head, is known as a cyclic. This is a cyclic operation, also since we have prefix in them; it is known as cyclic prefix.

So, in the second step, what we do is you have your samples, x 0, x 1, x 2, x 3, and what I have is now again, I am basically cycling the samples, from the end, towards the beginning. This is known as a cyclic prefix, this is denoted by the term CP, and now this block with CP added, this is transmitted over the, this is transmitted over the channel. Remember our channel is the frequency selective channel, that is y k, equals h 0, x k plus h 1, x k minus 1, plus v k. Now when I transmit the samples, your cyclic (Refer time: 07:47) samples that is x 3, x 0, x 1, x 2, x 3. Now observe the output corresponding to x 0, output corresponding to x 0 is, y 0 equals h 0, times, x 0 plus h 1, times, the previous symbol.

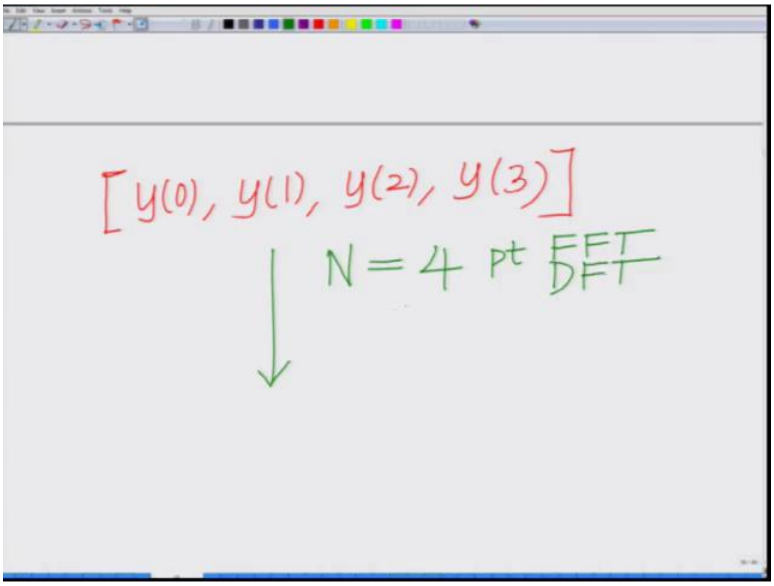
(Refer Slide Time: 08:47)



But the previous symbol to x 0 is x 3, this is the previous symbol because the addition of cyclic prefix, this is your previous symbol to x 0, because of the addition of cyclic prefix therefore, this is x 0, plus, I am sorry, this is x 3, is the previous symbol to x 0, plus v 0, all right? So, what we have done, is basically because you have added the cyclic prefix all right, and x, the sample x 0 as interference from the previous sample, and the previous sample is x 3, therefore, we have x 0, times, x 0 plus h 1 times, x 3 plus v 0. So, that is basically what we have, at the output that is corresponding to y 0.

And similarly I have y 1, now rest can be written as straight forward manner, y 1 equal to h 0, times x 1, plus h 1, times the previous sample, that is x 0, plus, plus v of 1, your y 2, equals h 0, times x of 2, plus h 1 times the previous symbols, sample x 1 plus, v of 2 and y 3, equals h 0, x 3 plus h 1, times the previous sample, that is x 3, plus v of 3, and now these are the samples. Now we said yesterday, that is if you look at y 0, it is, if you look at y 0, y 1, y 2, y 3, that can be that can be basically represented, as the circular convolution, of the channel filter h, with the transmitted samples x, plus the noise. And that is the key operation that is the advantage that this basically, the addition of cyclic prefix is given as. Therefore, we have something which we have seen in an elaborate manner, illustrating it in great detail, is basically the observation that now, because of addition of the cyclic prefix, what I have is y, becomes the circular convolution of h, with x, in the presence of additive noise. So, this is basically nothing, but your, this is your circular convolution.

(Refer Slide Time: 11:09)

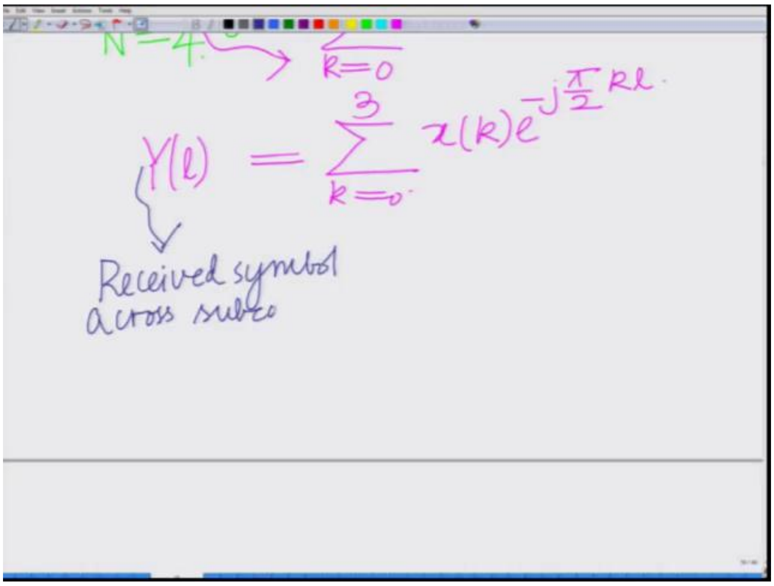


And as a result, now if you take the FFT, we know, in the frequency domain, circular convolution in the time domain, becomes in the frequency domain, it becomes a multiplication, therefore, that is the FFT of h, times the FFT of x, plus the FFT of v, which is basically your (Refer Time: 11:24).

Therefore, you have this interesting property, because circular convolution in a time is basically multiplication in the frequency or a FFT domain. So, once you take the FFT of the samples, at the output, basically the action of the channel in the frequency domain, basically now can be represented as a simple multiplication, therefore, the FFT of the output samples y, is basically FFT equals FFT of channel filter h, times the FFT of the transmitted samples x, plus the FFT of the noise field.

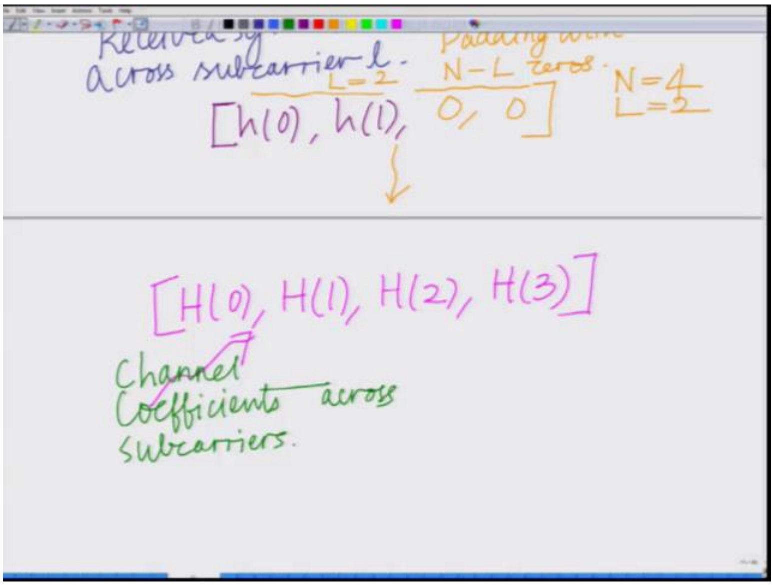
And now, let us see how you get the FFT, how you take the FFT of course, the FFT of y is basically, you have samples, y 0, y 1, y 2, y 3. So, you take the FFT, of this sample so; obviously, we are talking about the N equal to 4 point FFT, where N is the number of subcarriers, so the size of the FFT is always fixed, that is N equal to 4 point, FFT, that is Fast Fourier transform, or basically also the DFT, I do not need to mention this FFT is simply a Fast algorithm, for the DFT that is a discrete time Fourier transform, and that gives you, the symbols across the sub carriers, y 0, y 1 y 2, y 3, where each Y l is basically generated by the FFT, of the small y's, that is basically, summation k equal to 0, and I am writing the expression for the N point FFT, x k, e to the power of minus j, 2 pi, k l, divided by N, and substitute N equal to 4, this is basically your k equal to 0, N minus 1, which is equal to 3, that is x k, e to the power of minus j, 2 pi, k l divided by 4, which is basically equal to.

(Refer Slide Time: 13:08)



So, your Y l equals summation k equal to 0, to 3, x k, e raise to minus, j pi by 2, k l. So, basically this what we are doing here, is what we are substituting N equal to, we are substituting N is equal to 4, then that is what you saying is the capital Ys, the capital Y l, that is the received symbol, across subcarrier l, in the FFT domain is now given by the 4 pointer, N equal to 4 point FFT, or the Fast Fourier transform, DFT Discreet Fourier transform, of the received output samples y 0, y 1, y 2, y 3. So, this is basically, what is this Y l, this is the received symbol in frequency domain across, this is the receive symbol across subcarrier l.

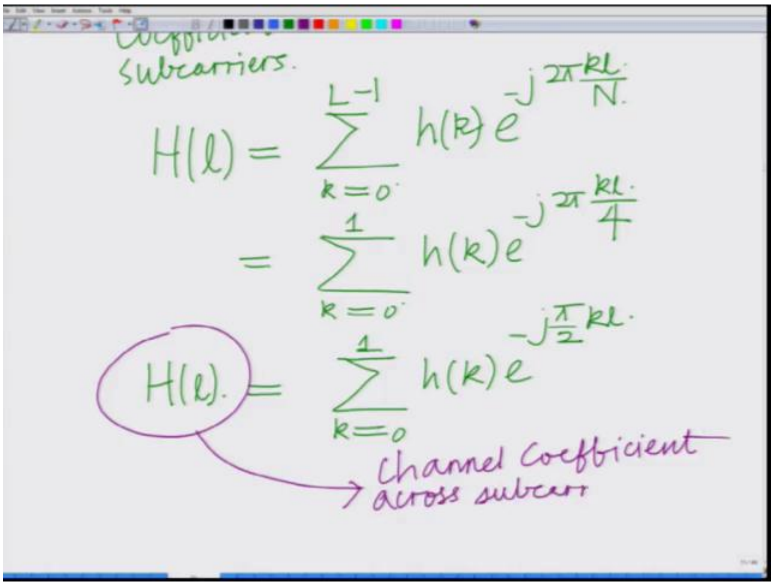
(Refer Slide Time: 14:55)



Similarly, one can do the same thing for, now let us look at, of course, we have to take the FFT of the channel filter. Now look at, the channel filter has, 2 channel taps, h 0, comma h 1, and I have to take the 4 point FFT therefore, I have to; obviously, pad with 0s. So, these are basically l channel taps, where l equal to 2, I have to pad it with N minus l, 0s. So, a N equal to 4, l equal to 2, because basically I have l channel taps, and I have to take N point FFT. So, naturally I have to pad with, N minus l 0. So, this is basically 0 padded FFT.

So, padding with, padding with N minus l 0s, and now you take the FFT of this, to give you capital H 0, capital H 1, capital H 2, capital H 3 and these are the coefficients ,across the subcarriers. These are the coefficients, or you can also say channel coefficients, these are the channel coefficients, across the subcarriers. And how do we generate them, naturally, what we do is, we have h of l, the coefficient across subcarrier l, equals summation, k equal to 0, to l minus 1, k equal to 0, remember we only have channel coefficients l, channel coefficients from k equals to 0 to l minus 1, that is h of l, e raise to minus j, 2 pi, k l divided by N, I am sorry, this is k, which is equal to summation, k equal to 0.

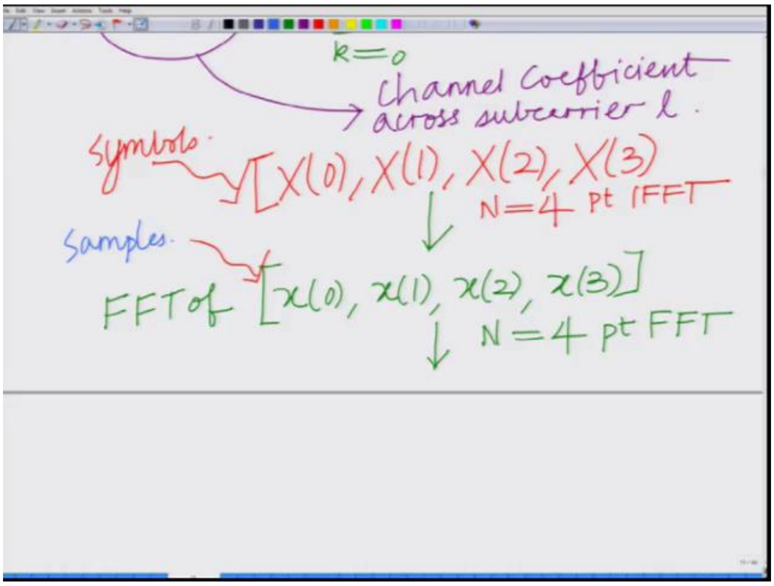
(Refer Slide Time: 16:29)



In this case l equal to 1, so k equal to 0, to basically l 1, and that is h of k, e raise to minus j, 2 pi, k l divided by 4, which is basically equal to, your summation, k equal to 0 to 1, h k, e raise to minus j, pi by 2, k l, this is your h l, as we said this h l is basically the channel partner coefficient, or the effective channel coefficient, across the lth subcarrier.

So, the capital H l, which is the channel coefficient across subcarrier l, in the frequency domain is given by the FFT of the channel taps, in the time domain of course, since there is only capital l, channel at a time, and we have to take N point FFT, right, we have to consider N point FFT therefore, naturally you have to pad it, with N minus l, 0s. And now let us look at the FFT of the samples x, FFT of the samples x. Now we want to take the FFT that is; obviously, N point FFT, N equal to 4 point, N equal to 4 point FFT.

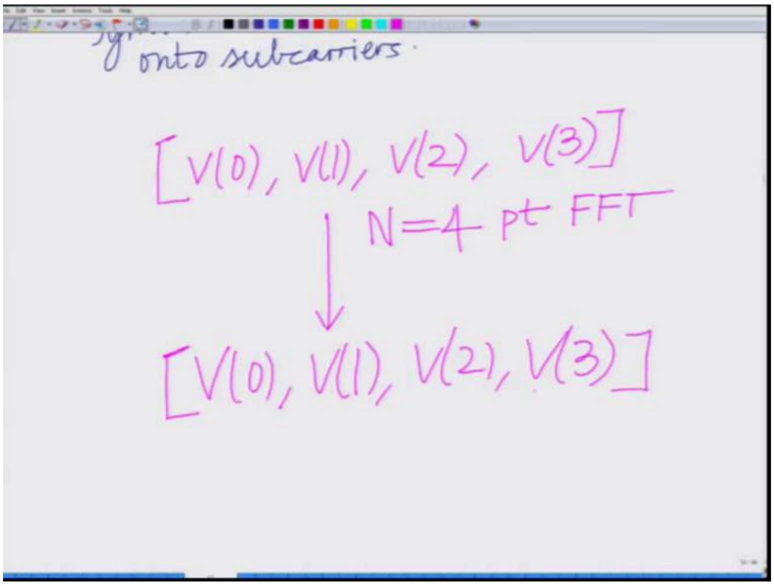
(Refer Slide Time: 18:42)



But observe that, these are basically given by the IFFT of the symbols, remember where we started from, we started with x 0, x 1, x 2, x 3, and we did the N equal to 4 point, IFFT to get the samples. So, these are basically your symbols, and these are basically your samples.

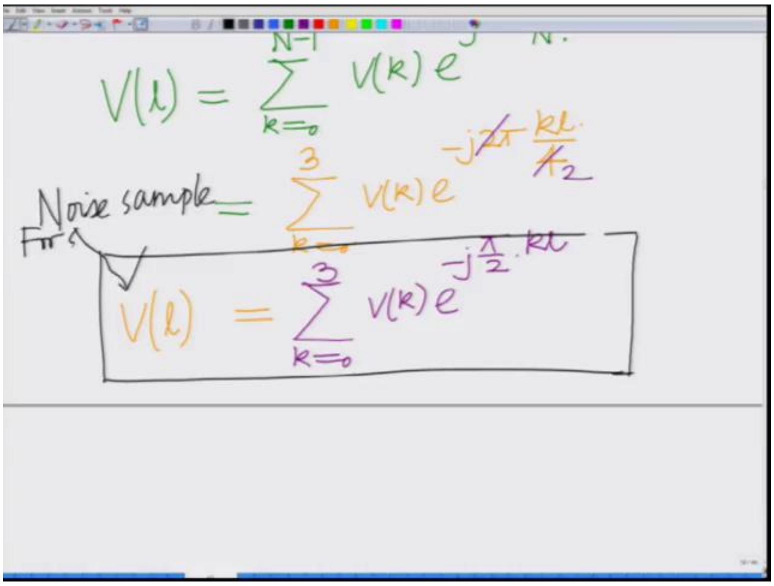
So, remember, the way we started with, in this (Refer Time: 19:22), we considered the symbols, the capital Xs and we performed the IFFT, to get the samples, that is small x's. So, naturally, once you do the FFT of the samples, you get back the symbols, that are loaded to the subcarrier, and that is the other important point to keep in mind. So, once you take the FFT. So, basically we had the symbols, in the frequency domain, which were loaded on to the subcarriers, and you consider the IFFT to get the samples, now once you take the FFT of these samples, you are back in the frequency domain, and you get the symbols, that are loaded all to the various subcarriers. And that is basically the important point to keep in mind. So, this basically, N equal to 4 point FFT. So, this basically, this gives back your x 0, x 1, x 2, and x 3.

(Refer Slide Time: 20:17)



These are basically your, these are basically the symbols, or the modulated symbols, loaded on to the various subcarriers, and that is what you are getting back, after you perform the FFT, and naturally the only remaining thing is the noise, that is also very simple. You have the small vs which are the noise samples, additive white Gaussian noise samples, in the time domain, you do, you do the N equal to 4 point, let me write it a little more clearly, you do the N equal to 4 point, we do the N equal to 4 point FFT of this, and what you get back is, what you get is the noise, the capital vs, the noise across each subcarrier, the noise samples of across the subcarriers, what are these, these are your noise samples across subcarrier.

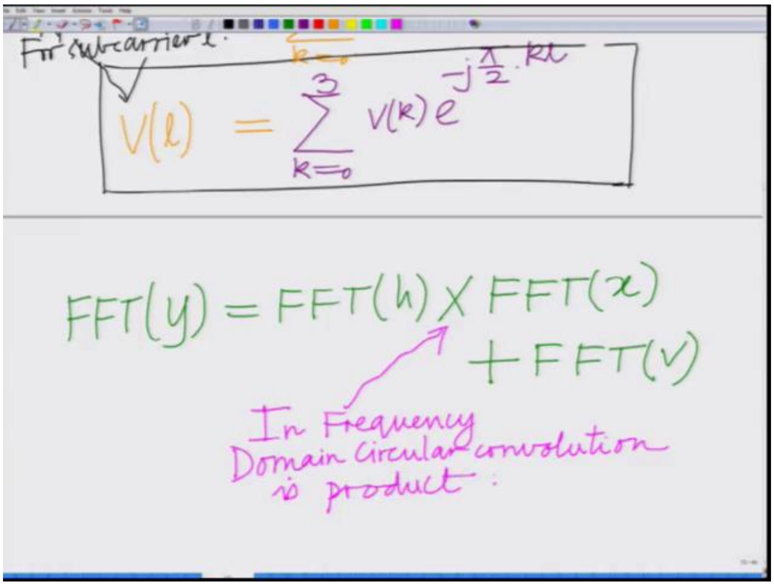
(Refer Slide Time: 21:55)



These are the noise sample across the subcarrier, and of course, we have v of l, which is given by the f of t, that is summation, k equal to 0, to N minus 1, small v k, which is the noise sample, e raise to minus j, 2 pi, k l, divided by N, substitute N equal to 4, and what you have is summation, k equal to 0, to 3, small v k, e raise to minus j, 2 pi, k l, divided by 4 which is equal to. So, v l, your noise, across subcarrier l, that is equal to summation, k equal to 0, to 3, e raise to minus j, 2 pi, or in fact e raised to minus j, pi by 2, because this 2, and you have the factors of 2, so we have pi by 2, k l. This is basically nothing, but your, this is basically your noise sample. This is basically your noise sample, for the subcarrier, for the subcarrier l.

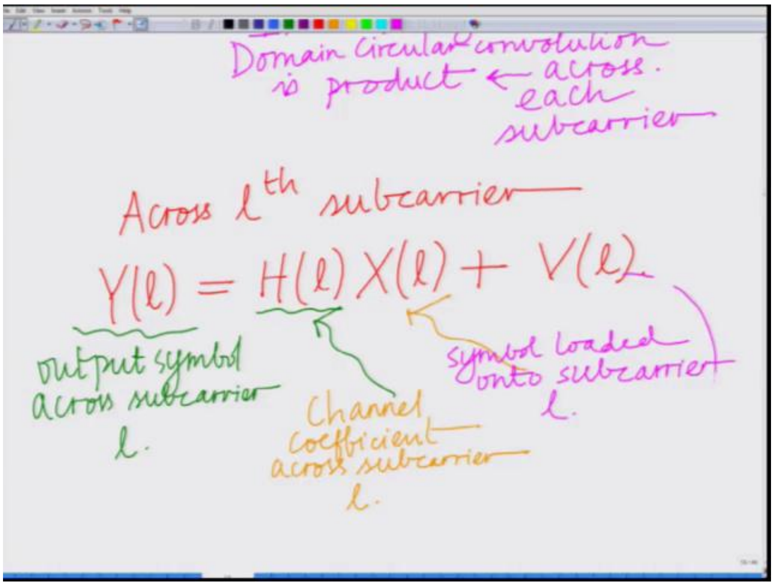
And therefore, now, you have the frequency domain, remember in the frequency domain, we have the FFT of output y, is equal to FFT of the channel h, times the FFT of the samples x, which are basically nothing, but the symbols loaded in the subcarriers, plus, the FFT the noise sample, across each subcarriers and therefore, now rewriting it, I have FFT of y, equals FFT of the channel filter h, product, remember this is important, to remember, in this frequency domain it is simply a product, because circular convolution becomes the product, just to repeat the importance of that, in frequency domain, this product of the corresponding frequency components across each carrier.

(Refer Slide Time: 23:57)



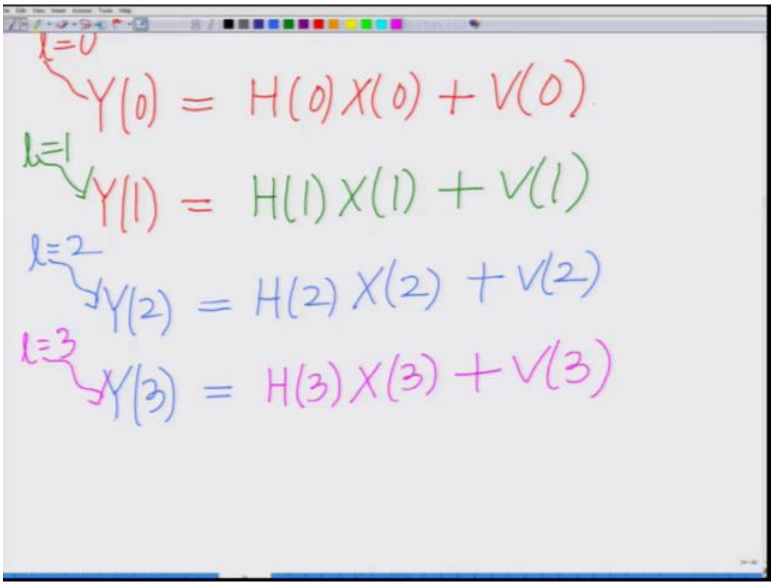
Basically product across each subcarrier, that is the important aspect, because each subcarrier represents a frequency component, and therefore, across each subcarrier, what do we have, across for instance, across l th sub carrier, and we have, remember N equal to 4 subcarriers, across l subcarrier, we have y l, equals h l, times x l, plus v l.

(Refer Slide Time: 24:34)



And this is the most important result, where y l, output symbol, symbol across subcarrier l, h l, is basically your, what is h l? It is the channel coefficient, across sub carrier x l is the symbol loaded on to sub carrier l, and v s, v l, naturally v l not to forget v l, v l is the noise across sub carrier l. So, v l is basically your noise, v l is basically the noise across subcarrier l.

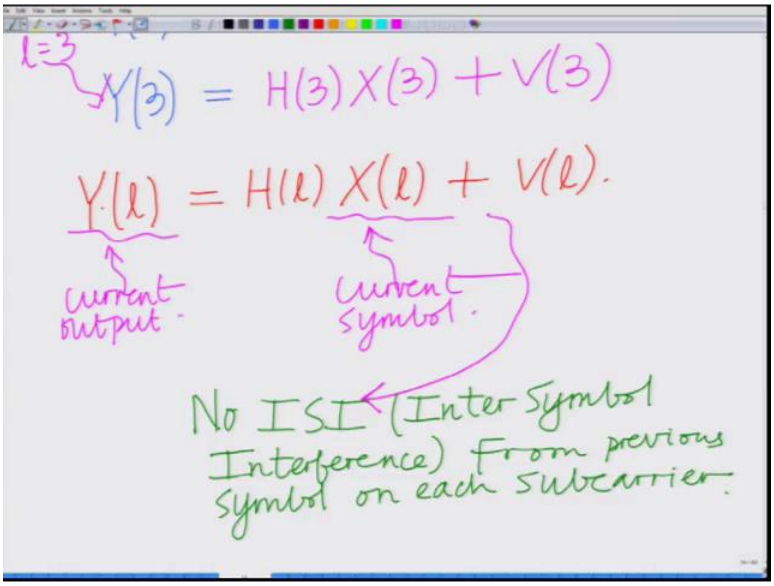
(Refer Slide Time: 26:00)



And therefore, we said, the number of subcarriers, is basically the same as, basically equal to capital N, where N equal to 4, all right? So, number of subcarriers, is N equal to 4, which means we have l equal to 0, 1, 2, 3, corresponding to the l equal to 4 subcarrier, and therefore, for each subcarrier, what we have, is y equal to y 0, equals h 0, times x 0, plus v 0, this is corresponding to subcarrier l equal to 0, y 1, equals h 1, times x 1, plus v 1. This corresponds to subcarrier l equal to 1, then we have, naturally again, the same thing, y 2 across sub carrier 2, y 2 equals h 2, x 2, plus v 2, across subcarrier l equal to 2, and just to finish this, I have y 3 equals h 3, x 3, plus v 3, across subcarrier l equal to 3.

So, basically now what we have done is we have written down explicitly the relation corresponding to each subcarrier that is y l equals h l, times x l, plus v. Now if we look at this system, what you can observe is that, for each subcarrier the output y l, is simply the channel coefficient, times, the input symbol excel. There is no inter symbol, there is no inter symbol interference, from the previous symbol on each sub carrier, therefore, now if you can look at this model, incredible thing about this, about this model, y l equals h l, x l, plus v l. This is only current symbol, that is current symbol loaded on to subcarrier l, current symbol, and this is your current output across subcarrier l.

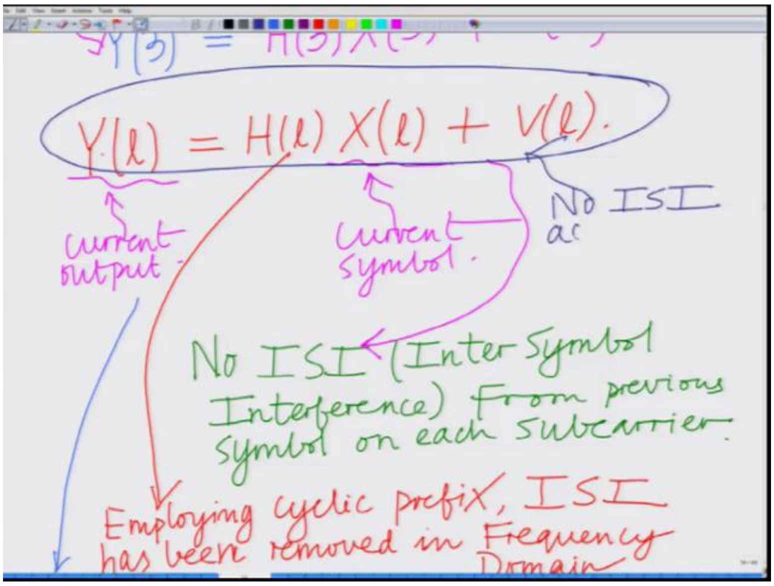
(Refer Slide Time: 28:39)



And therefore, what we have, therefore, there is no inter symbol interference from the previous subcarrier, no inter, no ISI, or basically your inter, no inter symbol, interference from previous symbol, on each subcarrier. Remember in the time domain, you still have the inter sample interference, the time domain, the inter sample interference is there, but the intelligently, working the frequency domain, by adding the cyclic prefix, and converting it back into the frequency domain at the receiver, you are eliminated ,or one has eliminated, the inter symbol interference in the frequency domain, across each subcarrier, and this is the important aspects of OFDM, that is it eliminates the inter symbol interference, in the frequency domain.

And what is efficient about this? This is efficient because it based on IFFT and FFT. So, IFFT at the transmitter, FFT at the receiver, and since IFFT and FFT, that is the inverse Fast Fourier transform, and Fast Fourier transform, that can be performed in a very fast fashion, that these are the efficient algorithm, as a result, the entire OFDM architecture, the entire OFDM system transmission scheme, is very efficient, since it does not employs any matrix inversion, anywhere, it is simply based on IFFT and FFT, which are Fast operations.

(Refer Slide Time: 30:23)



So, basically, to summarise, employing the cyclic prefix, it has been converted into a, employing cyclic prefix, ISI, has been removed, has been removed in the, ISI has been removed in the frequency domain. Further, this is a very efficient, OFDM is very efficient, to highlight that point, that further OFDM is very efficient, OFDM it employs IFFT slash, FFT, that is, IIFT at the transmitter, and FFT at the receiver, which are, and both of these algorithms are very fast and efficient, which are very fast and efficient algorithms.

And to note, that there is no matrix inversion, though inversion which is a very which is adds to the complexity, there is no matrix, there is no matrix inversion, in OFDM and so basically, that is the. So, basically if we look at this equation, this equation here, summarises OFDM which is basically, it is no ISI across, each subcarrier, each subcarrier is ISI free. There is no ISI across each subcarrier, each subcarrier is ISI free, and basically it is a efficient algorithm, since it based only on IFFT, and FFT, which can be done in a very fast and efficient manner.

So, basically what we have done, in this module, is basically, we have completed description of the OFDM system model, which is based on, loading the symbols on to the subcarriers, that is basically performing the FFT, IFFT operation transmitted followed by the addition of the cyclic prefix, that results in a circular convolution at the output of the channel, when you take the FFT of the output, basically what you get, is basically in the time domain, in a frequency domain, is basically can be represented across each subcarrier, as the FFT of the channel, times, the FFT product, times, the FFT samples, which is basically nothing, but the symbols, plus, the FFT of the noise.

Therefore, across each subcarrier you have y, l equals the channel coefficient, h l times the symbol, x l, loaded on to the subcarrier, plus, the noise v l, and therefore, the transmission across each subcarrier is inter symbol interference free, and this is done very efficiently done by use of the IFFT and FFT algorithms, which makes OFDM overall very efficient scheme for transmission, in modern wireless communication systems. In fact, as we talked about, in the previous module, several 4G standard basically, such as (Refer Time: 34:26) and modern wi fi standards, such as a 2 dot 11 n, a 2 dot 11 a c r, all based on OFDM.

So, we will stop here, in this module, and in the next module, subsequent modules, we will look at the estimation aspects of OFDM, that is, how do you estimate these channel coefficients, in an OFDM system. So, thank you, we will conclude this module, here.

Thank you very much.